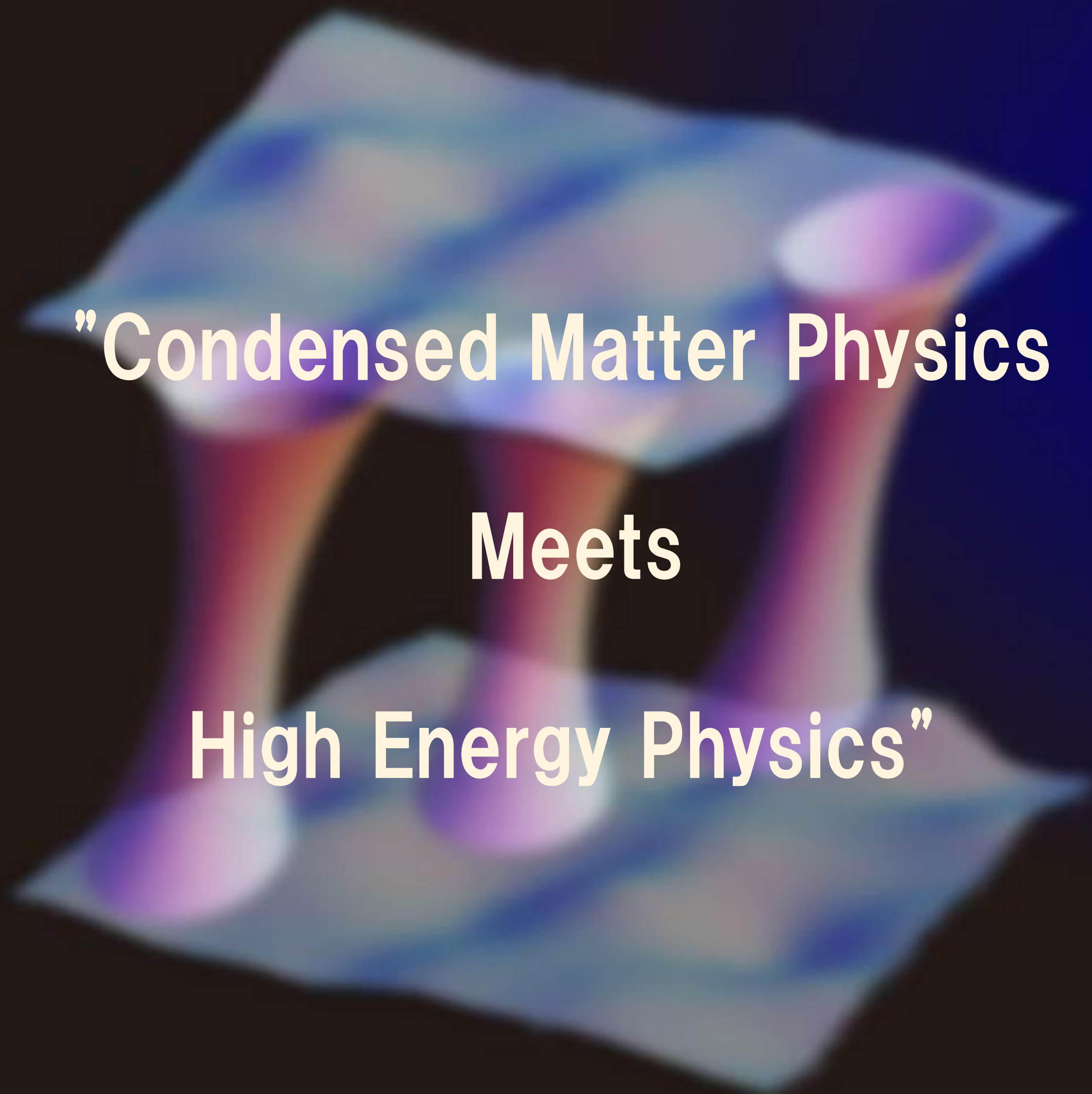
IPMU Focus Week Feb 2010



"Condensed Matter Physics

Meets

High Energy Physics"

Focus Week
"Condensed Matter Physics Meets High Energy Physics"
IPMU, 8 Feb 2010

How can condensed-matter problems interact with field theoretic ideas — an overview

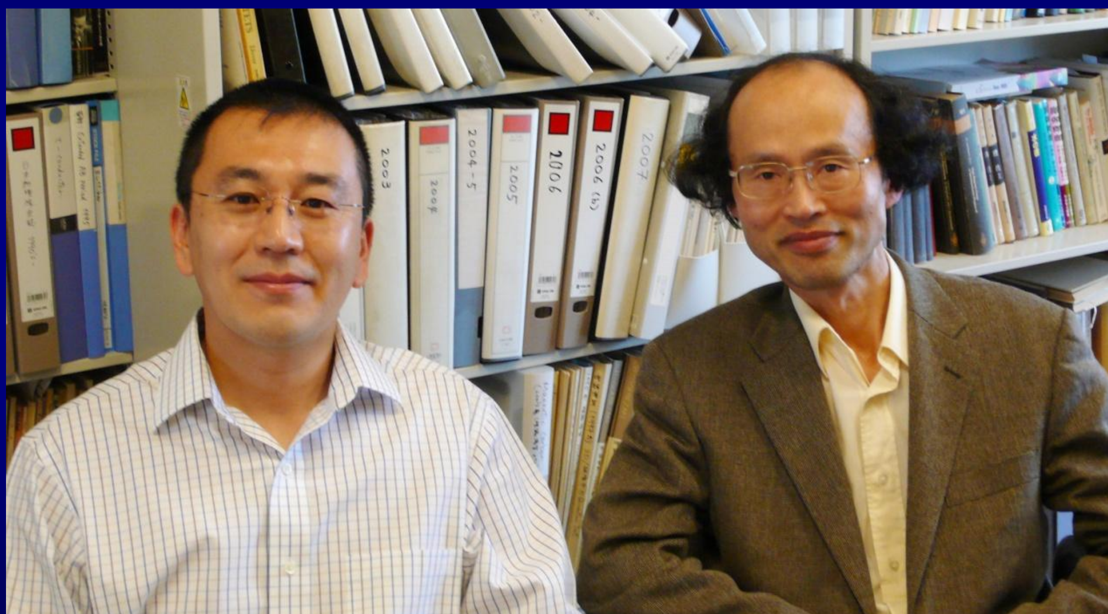
Hideo Aoki
Dept Physics, Univ Tokyo

My talk today ---

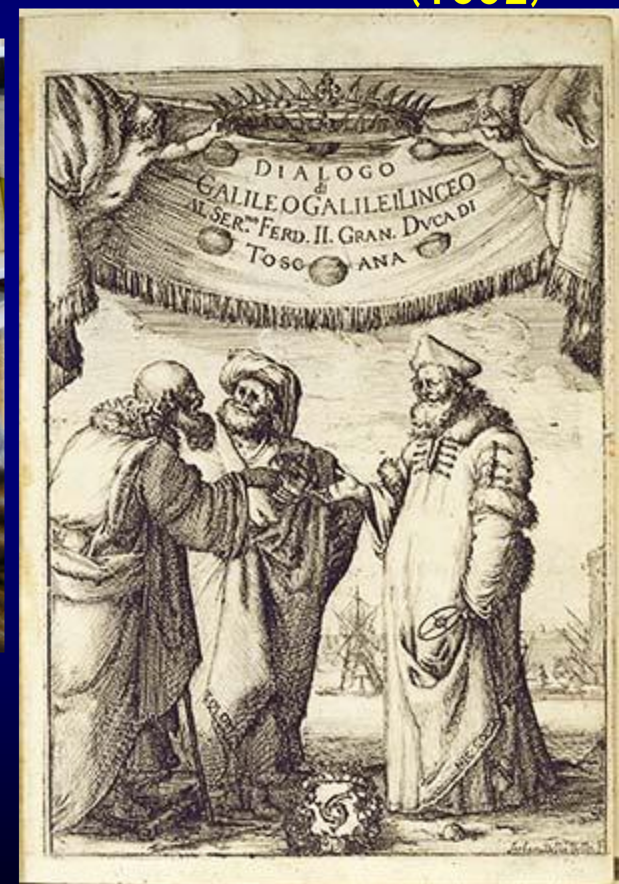
- raise and classify
- give (personal) views on problems in CMP

Hideo Aoki & Hiroshi Ooguri:
CMP and HEP
— a dialogue on diversity and unity in the physical world

(1632)



(2007)



Panel discussion (Wed morning)

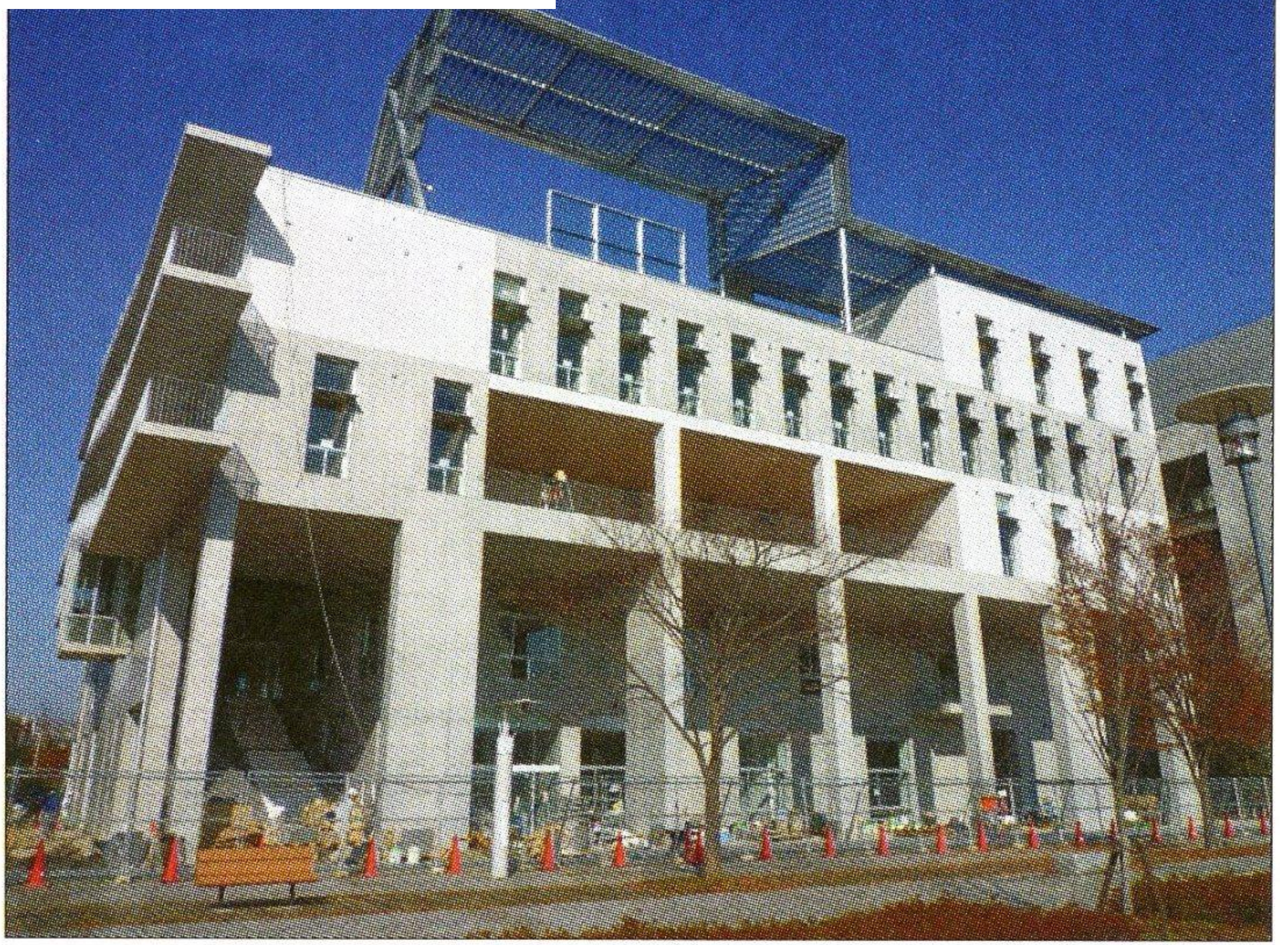
David Hilbert 1900
Mathematische Probleme
(@ Int Cong Mathematicians, Paris)

Weyl 1951

CMP meets HEP 2010

physicsworld
physicsworld.com
Volume 23 No 1 January 2010

Jan 2010, News & Analysis



Centre of the universe

the Physics and Mathematics of the Universe's new building is expected to open this month.

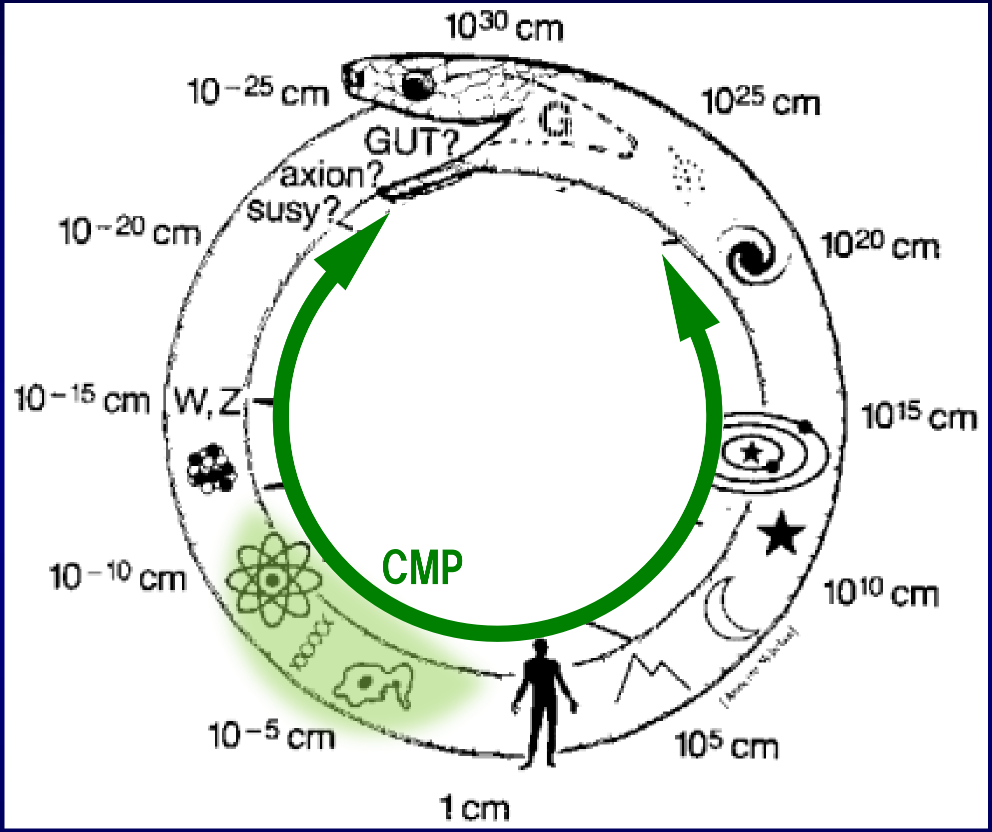
CMP -- *many world*

i.e., diverse effective theories
on orders of mag smaller E scales

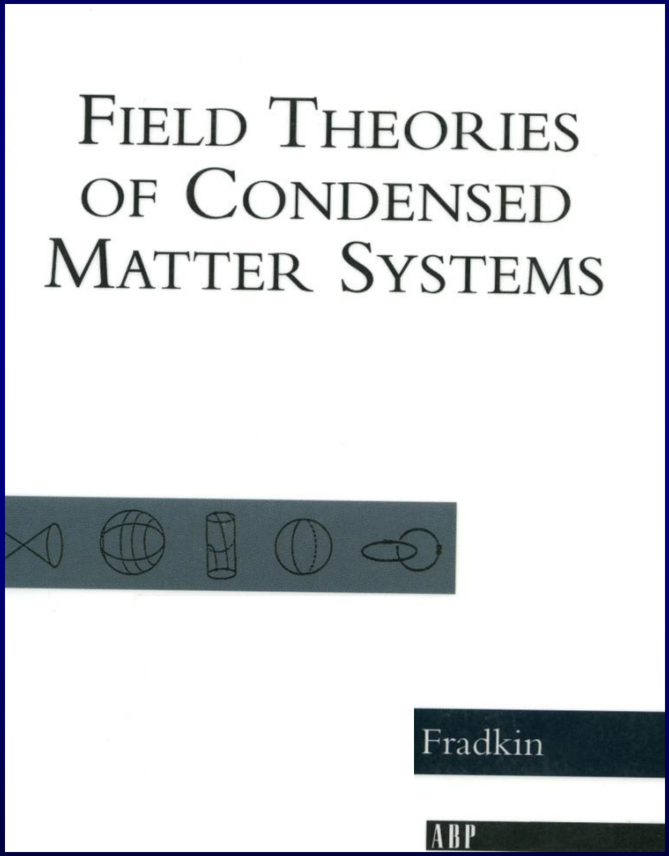
destined to be related
due to ∞ deg of freedom
-- interaction fruitful & intriguing

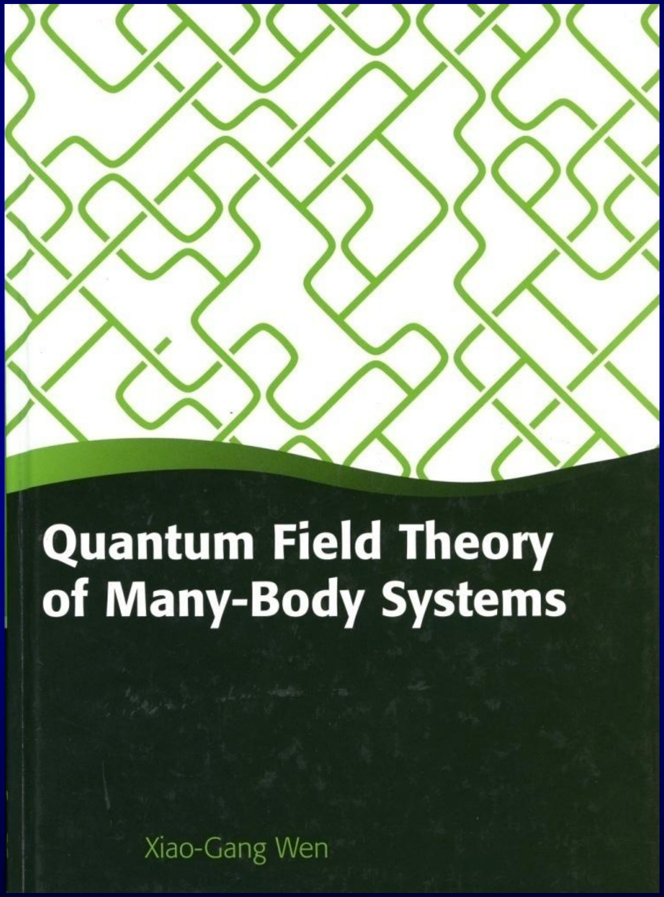
HEP -- *quantum field theoretic world*

Many worlds understandable --- look at powers of ten



Many field-theoretic approaches, textbooks for CMP





・・・

Various phenomena

even within gauge symmetry breaking

SC

Interacting electrons

Magnetism

QHE

Symmetries

* gauge
  - superconductivity / superfluidity (U(1))
  - flux phase
  - FQHE (Chern-Simons)

* chiral
  - some phases of graphene?
* T (ferromagnets, T-broken SC, …)

* P (ferroelectrics, noncentrosymm SC, …)

SC

Exotic SC:

- T-rev broken (p+ip, ···)
- FFLO (nonzero total momentum)
- noncentrosymmetric
  (spin singlet mixes with triplet; Fujimoto)
- multi-band (as in the iron-based)
- ferromagnetic SC
  (two broken gauge symms coexist)

QHE

* Integer QHE

↓

Fractional QHE

Chern-Simons gauge field in (2+1)D

← anyons, nonabelions, etc (Read)

* More generally, topological phases / insulators

i.e., spin-off into a wider family of Hall effects

- anomalous Hall effect (AHE)
- spin Hall effect (SHE)
- quantum spin Hall effect (QSHE)

(Zhang)

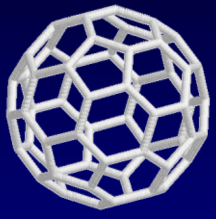
Topological phases
(as opposed to broken symmetries)

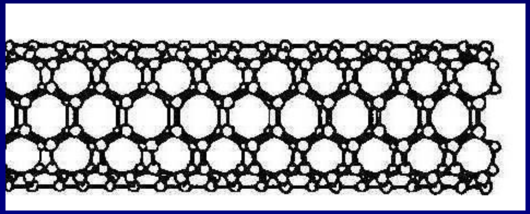
* topological phases
  -- no broken symmetries, no classical order p's;
    (Berry's phase relevant)

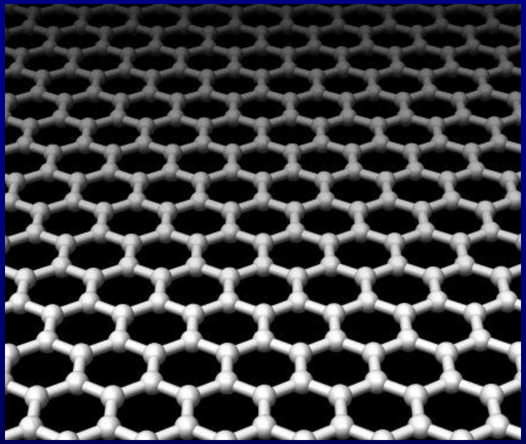
Spatial dimensionality

--- realisable in CMP as

* 0D: quantum dots
* 1D: quantum wires, nanotubes
  → Tomonaga-Luttinger
* 2D:
  - ✓ electron gas at interfaces
    → QHE,
  - ✓ layered crystal structures
    → high-Tc cuprates
  - ✓ atomically single layer
    → graphene → massless Dirac
* 3D:



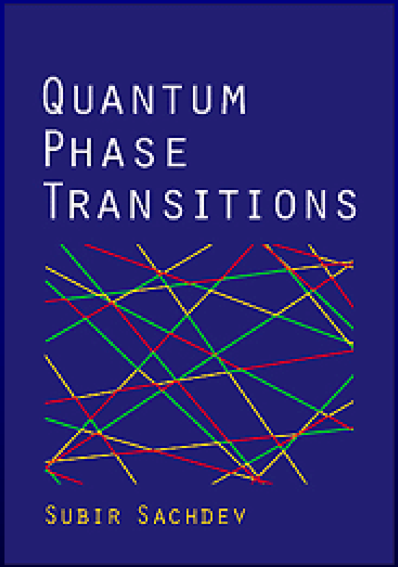


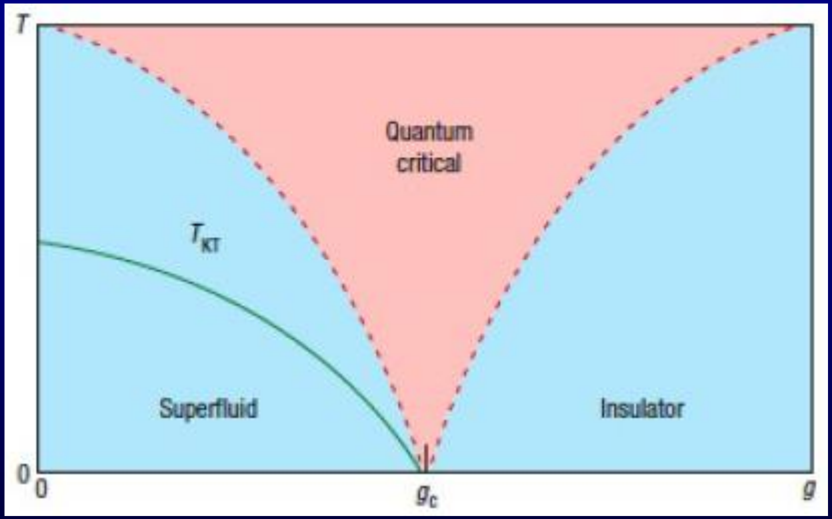


Quantum criticality / strong coupling

* Scale-invariance at quantum critical points

* AdS/CFT



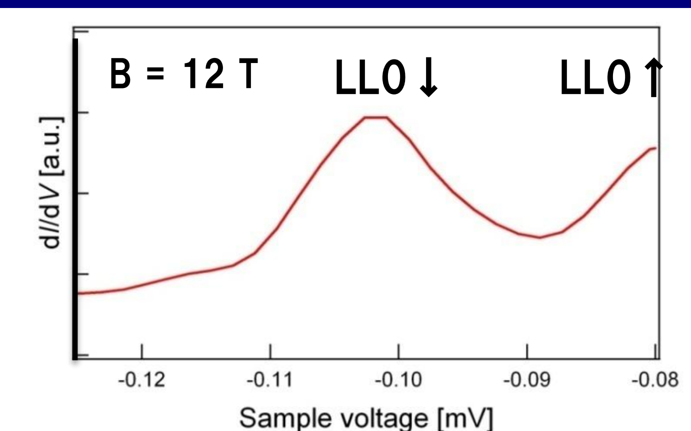


Hartnoll, arXiv09

Real-space imaging experiments on QHE

Centre of a Landau level
← critical point for localisation
(with scale-inv fractal states)
(Aoki, 1980's) ;

STM: Hashimoto et al, 2008



courtesy K. Hashimoto

Boundary states

* Bulk states dictate how boundary states appear (their existence "topologically protected" for topological bulk states)

  - a prime example: QHE edge states

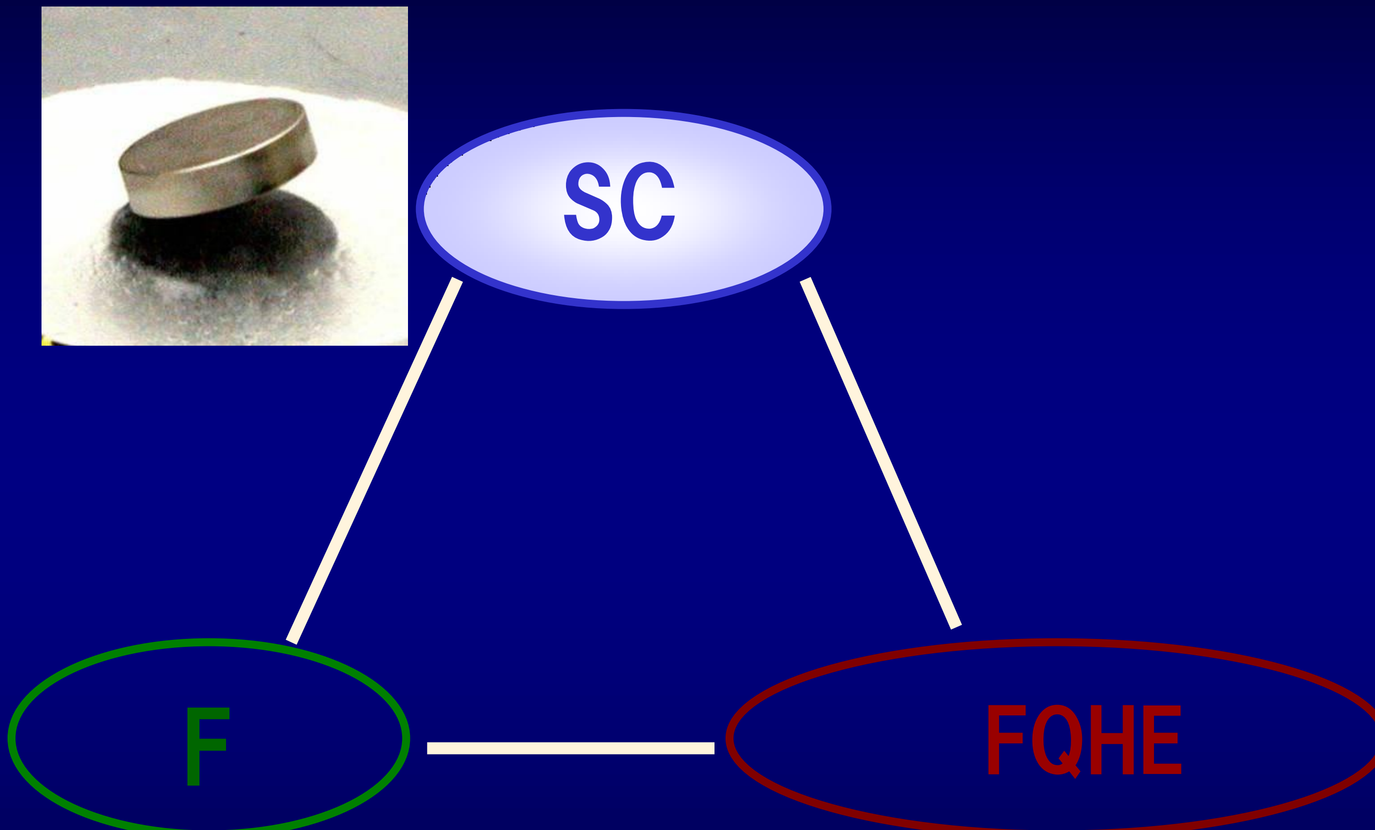
Non-equilibrium

* non-equilibrium phases / phase diagram

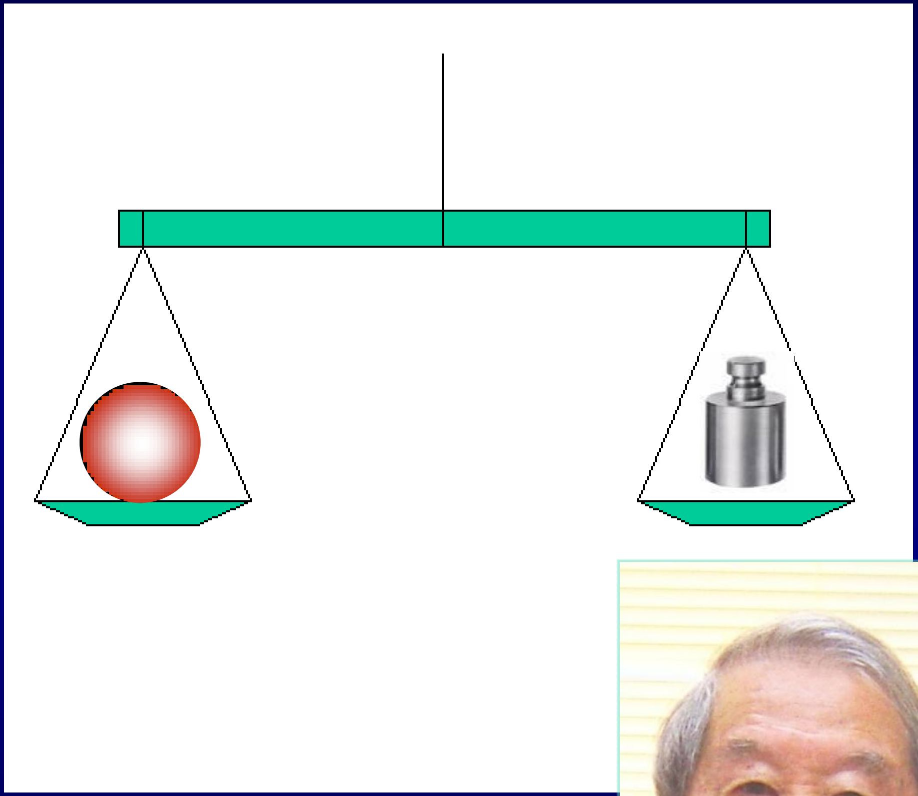
e.g., dielectric breakdown of a Mott insulator
← Schwinger's QED vacuum decay
+ Landau-Zener (Oka)

* non-equilibrium collective modes

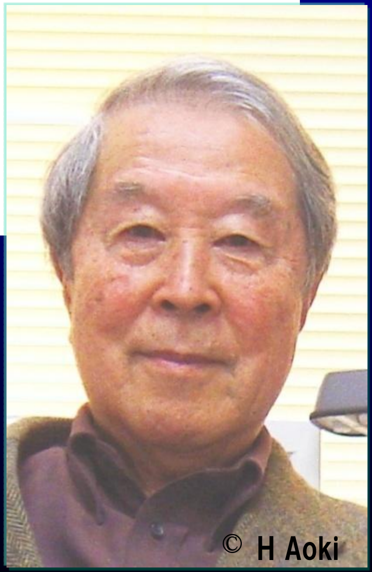
* non-equilibrium in strong ac fields

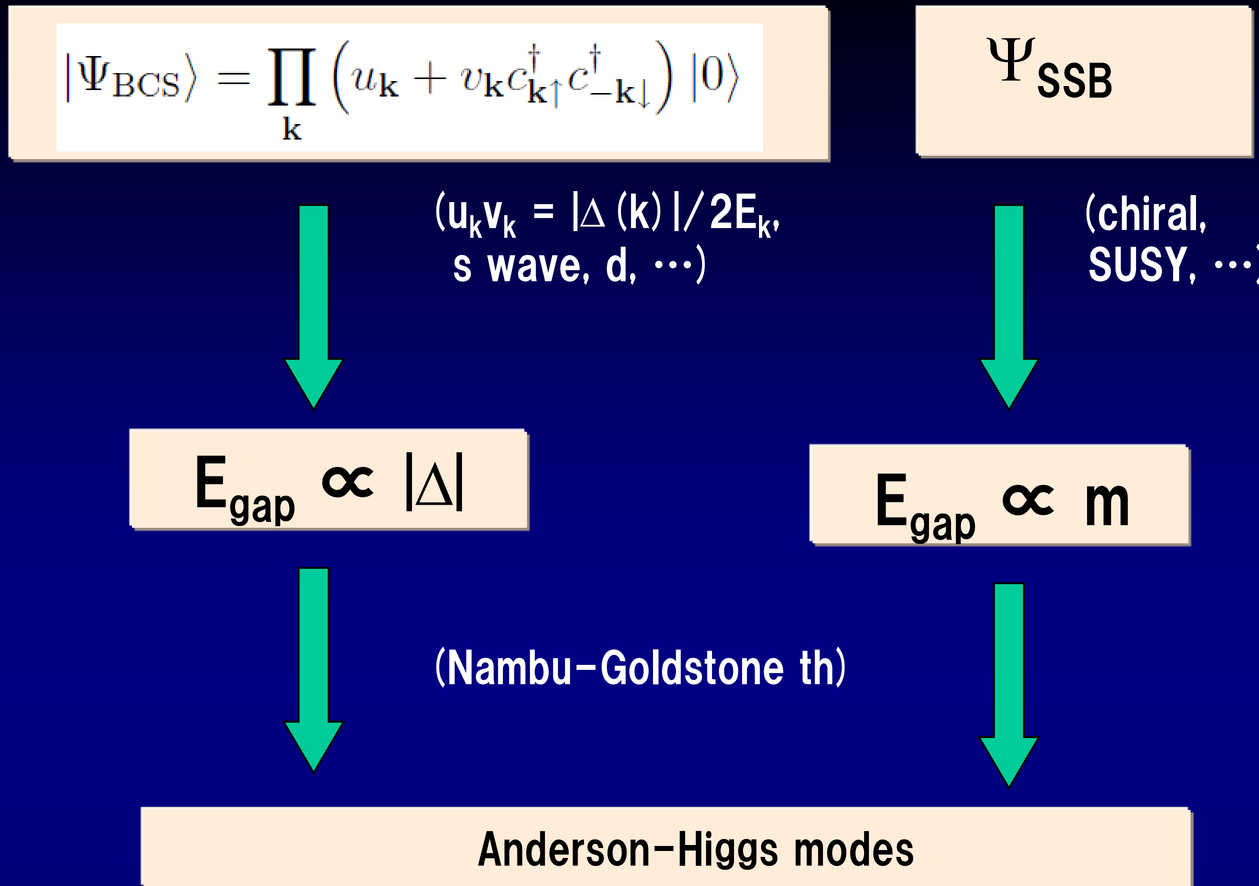

SC
F
FQHE

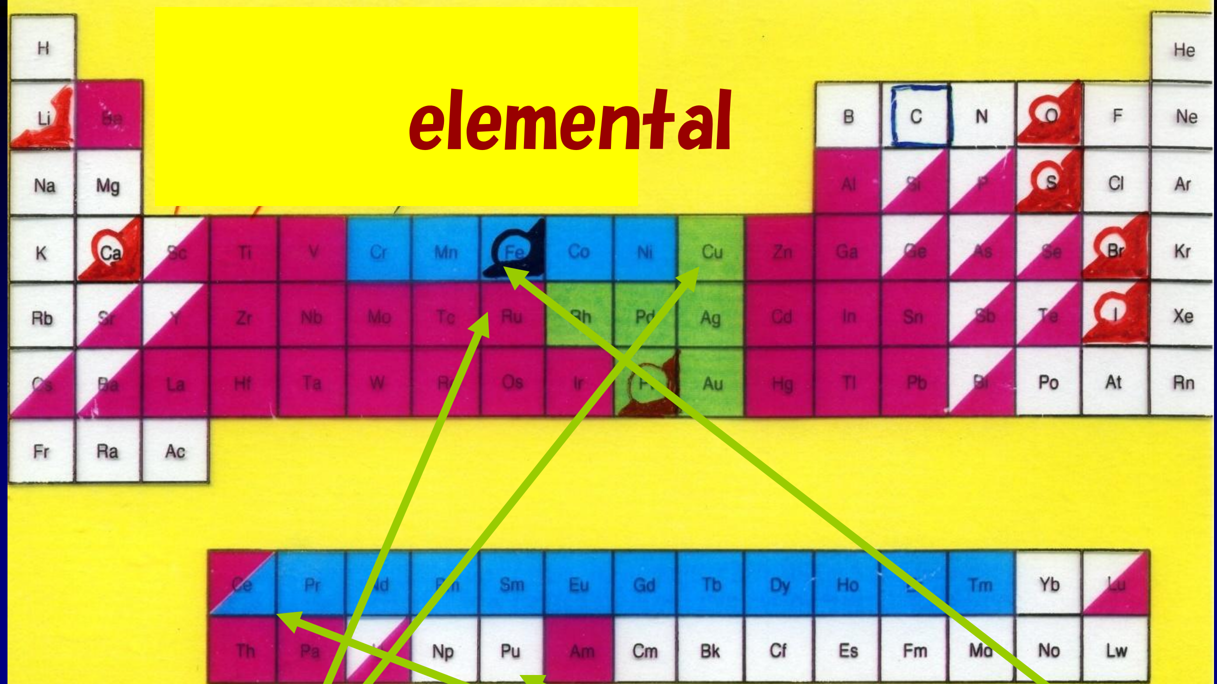
CMP



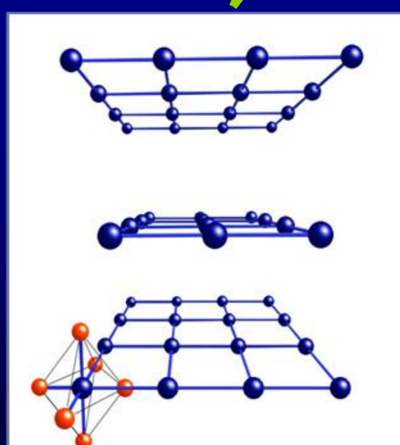
HEP



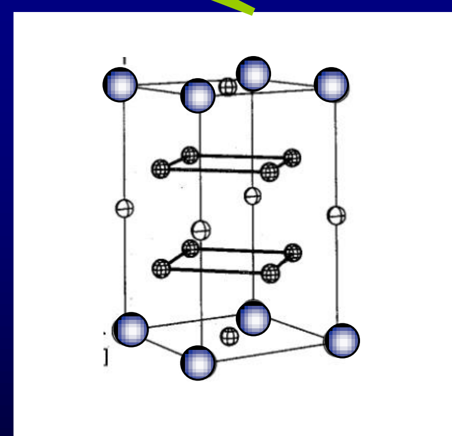

$|\Psi_{\mathrm{BCS}}\rangle = \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} \right) |0\rangle$
Ψ_{SSB}
$(u_k v_k = |\Delta(k)|/2E_k$,
s wave, d, ⋯)
(chiral,
SUSY, ⋯)
$E_{gap} \propto |\Delta|$
$E_{gap} \propto m$
(Nambu–Goldstone th)
Anderson–Higgs modes



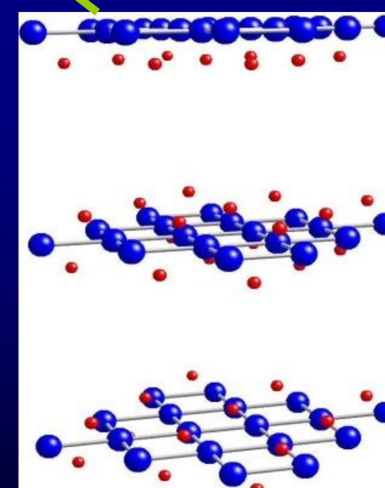
compounds



Cu, Ru compounds

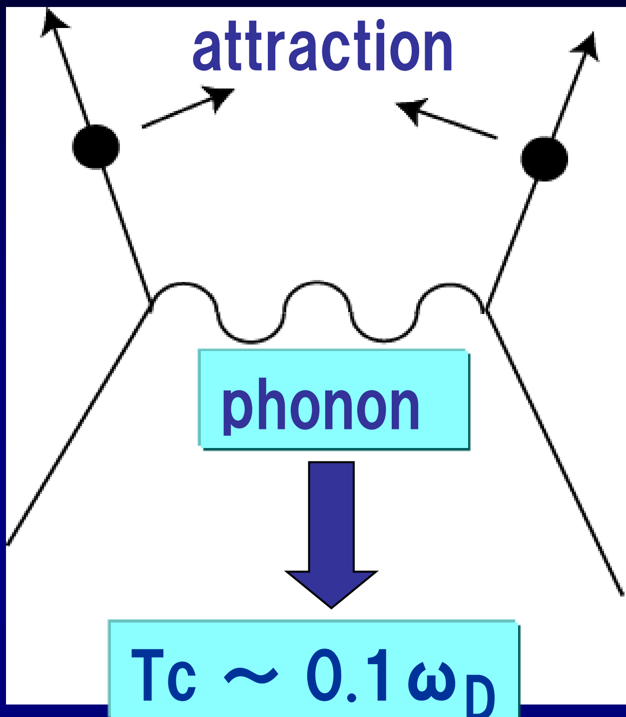


Ce, Pu compounds

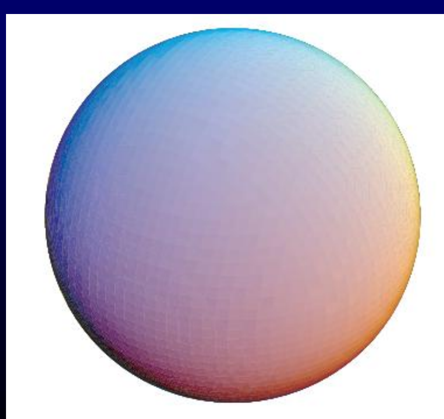


Fe compound

* phonon mechanism

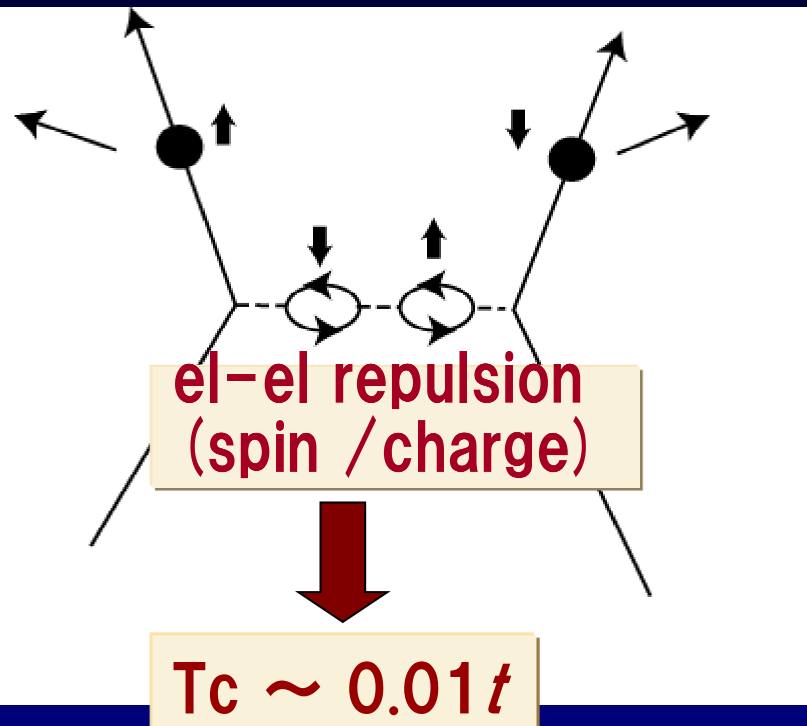


10K ← 100K

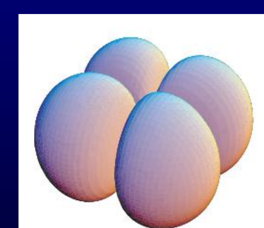


isotropic pairing

* electron mechanism



100K ← 10000K



anisotropic pairing

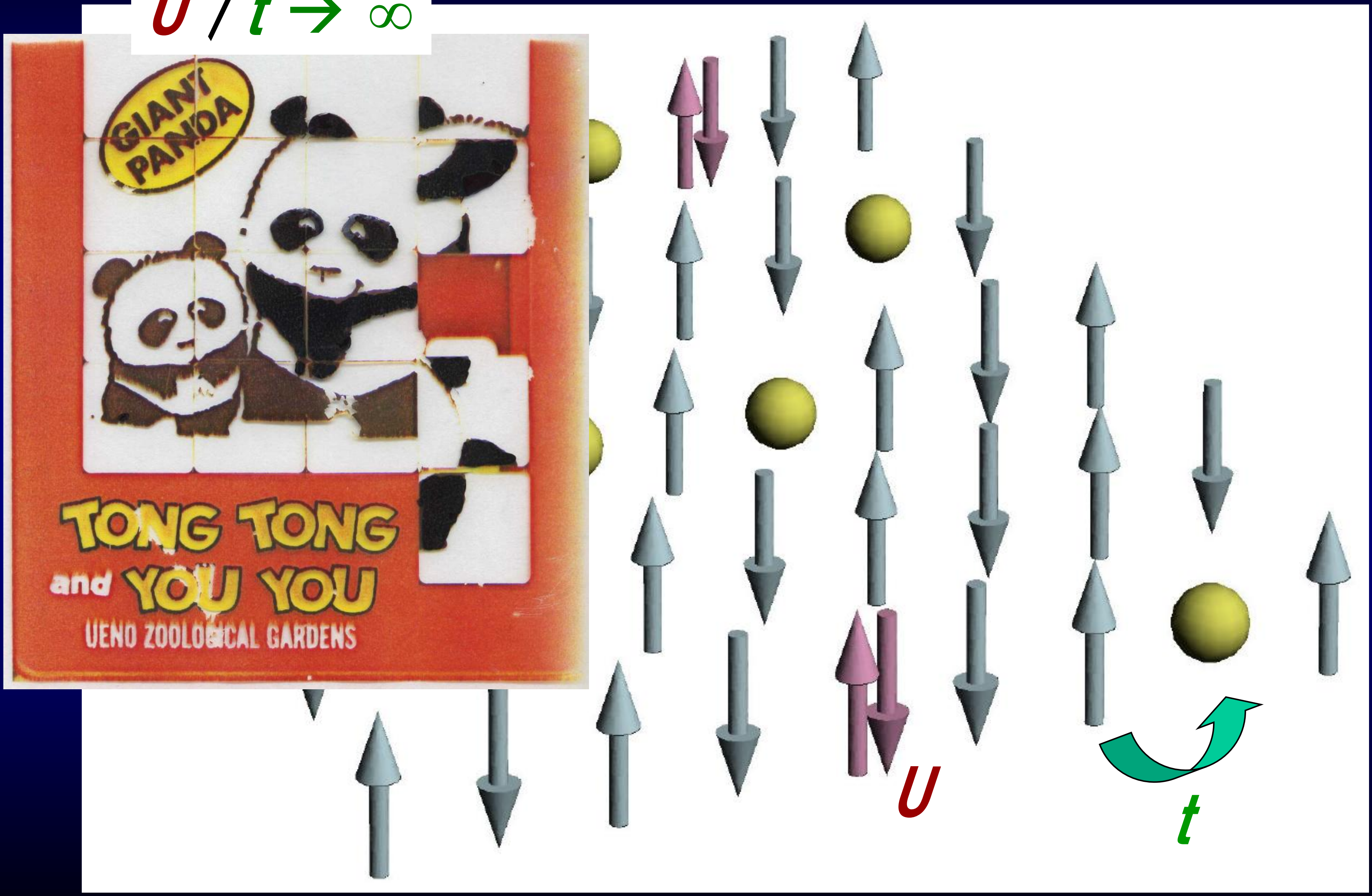
Non-phonon mechanism SC / SF

1. Heavy fermion
2. Superfluid ^3He
3. High-Tc cuprates
4. SC in the Coulomb gas

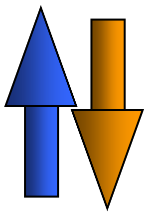
Kohn & Luttinger 1965; Chubukov 1993:
Repulsively interacting fermions
→ Attractive pairing channel exists for $T \to 0$
(weak-coupling, dilute case)

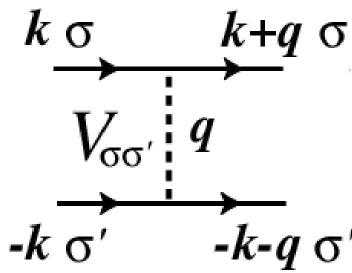
Hubbard model (a generic model)

$U / t \rightarrow \infty$



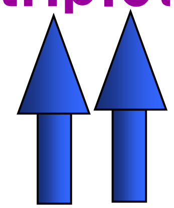
Spin- and charge-fluctuation mediated pairing

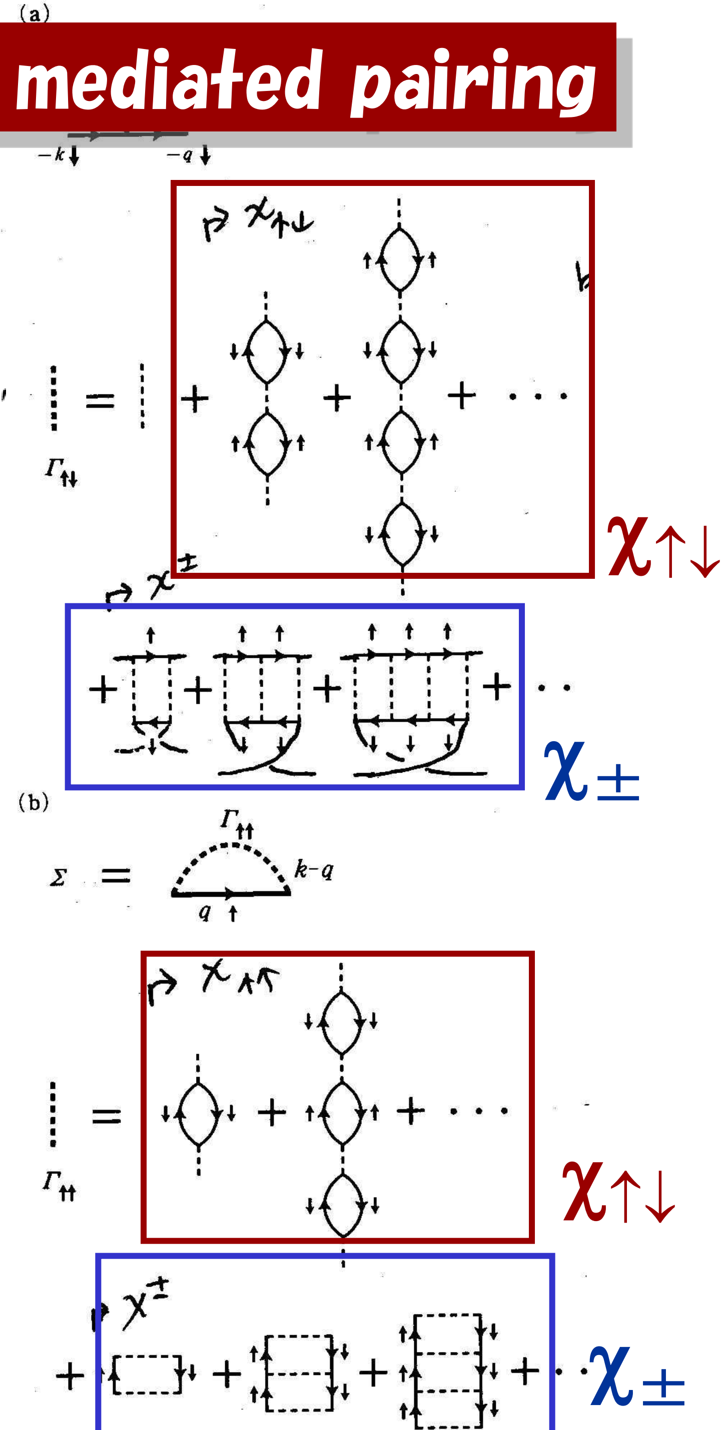




V_{singlet} : $\left[-\frac{3}{2}V_{\text{spin}}(k-k')+\frac{1}{2}V_{\text{charge}}(k-k')\right]$

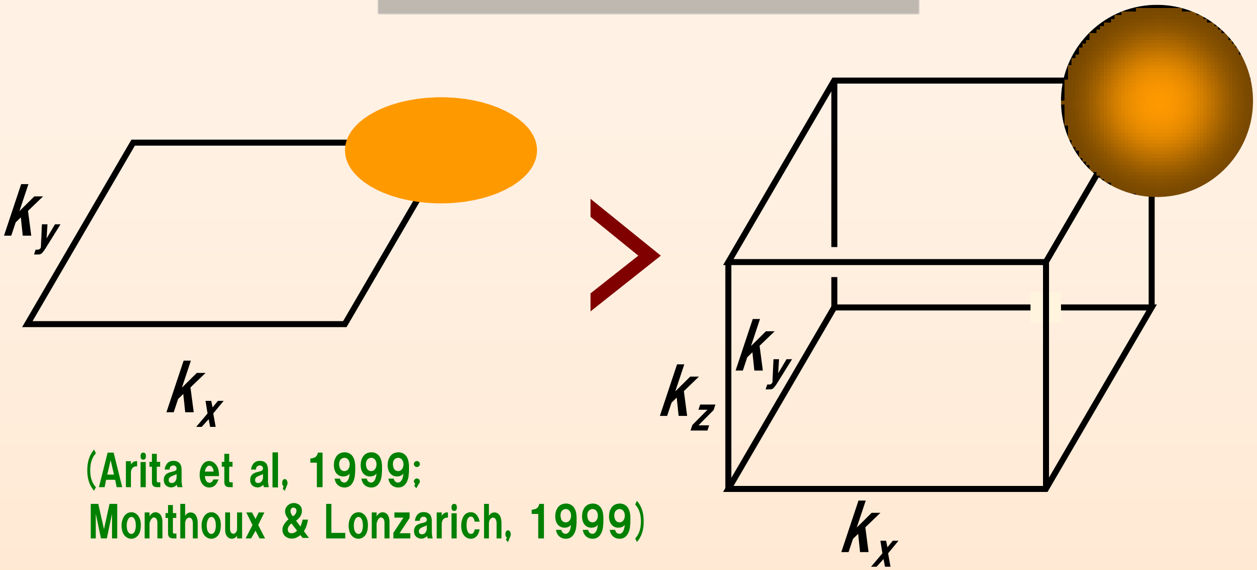
V_{triplet} : $\left[+\frac{1}{2}V_{\text{spin}}(k-k')+\frac{1}{2}V_{\text{charge}}(k-k')\right]$



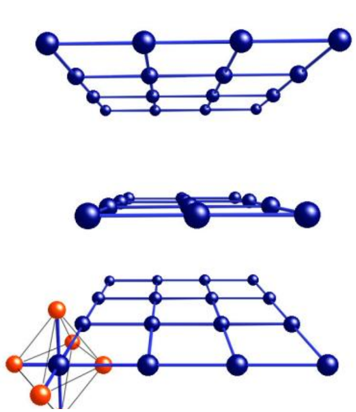


2D or 3D ?

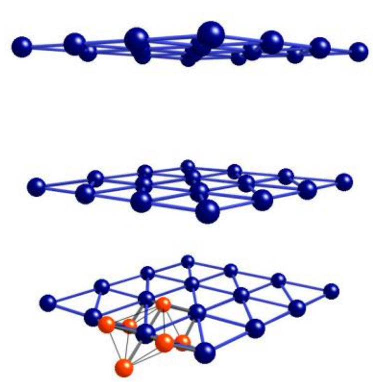
spin-fluctuation mediated pairing interaction



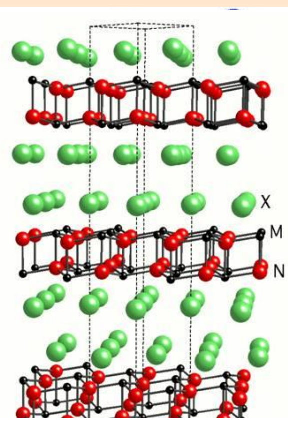
(Arita et al, 1999;
Monthoux & Lonzarich, 1999)



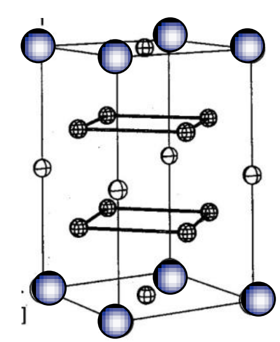
Cu, Ru compounds



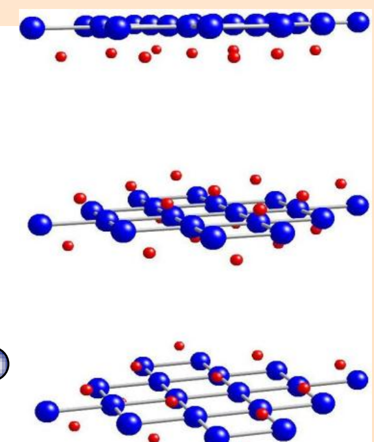
Co compound



Hf compound



Ce compound



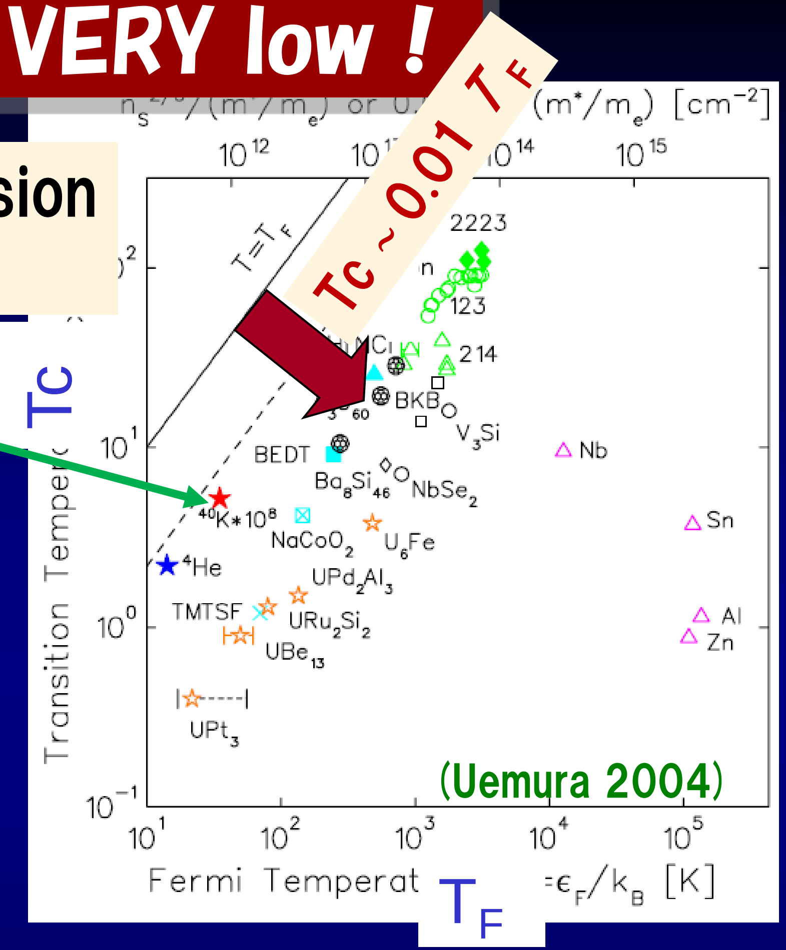
Fe compound

$T_c \sim T_F/100$ is VERY low !

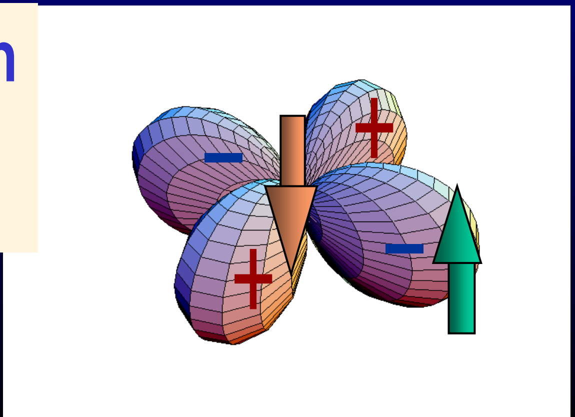
(1) Pairing int'action from el-el repulsion
= weak

Cf. Laser-cooled Fermi gas (2004)
← $T_c \sim 0.1\ T_F$
← attractive int'action
↑ Feshbach resonance

(2) Self-energy correction
→ quasi-particles short-lived

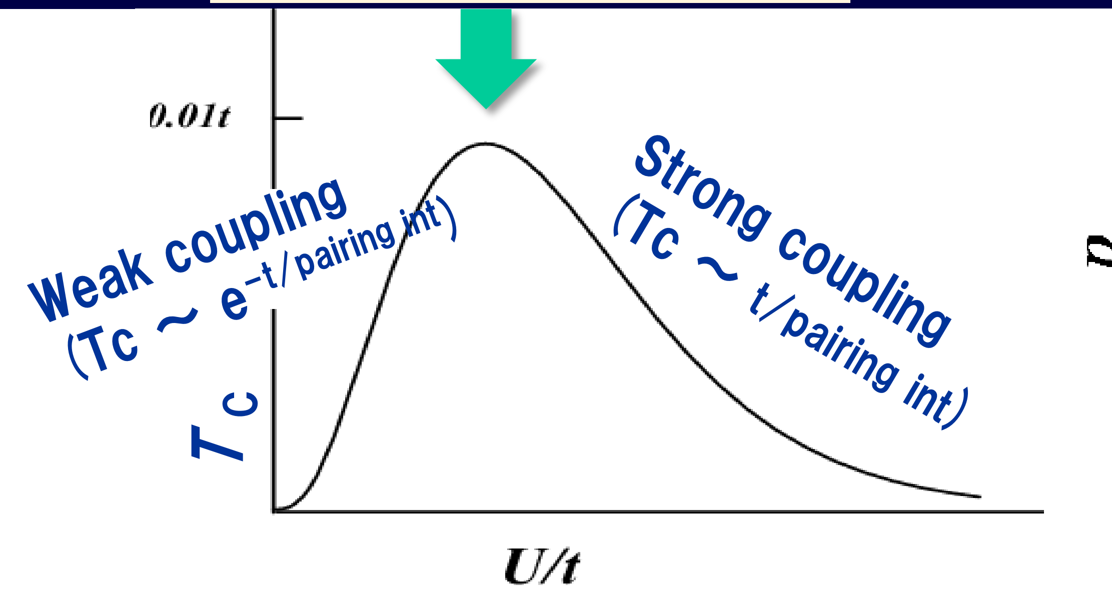


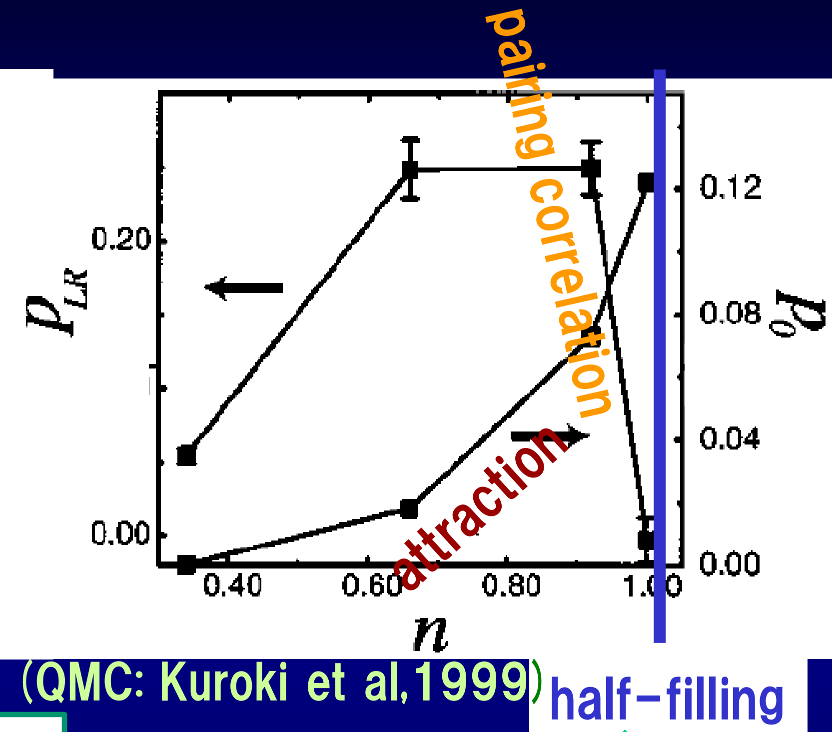
(3) Pairing from el-el repulsion
= anisotropic
(i.e., nodes in Δ_{BCS})



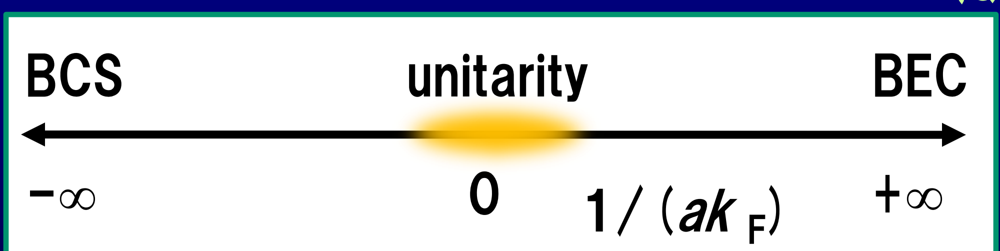
Tc in SC from repulsion

(if duality, self-dual pt?)





(QMC: Kuroki et al,1999)



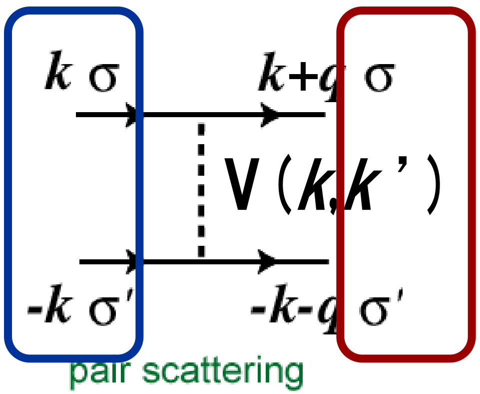
The closer to SC, the worse negative-sign problem in QMC !

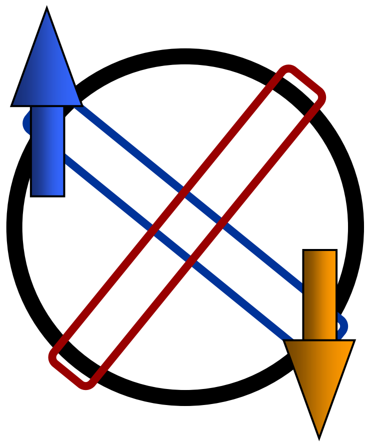
--- ***Murphy's law*** in many-body physics,

not an accident but strong Q fluctuations inherent ?

SC from repulsion: nothing strange

Attraction → isotropic SC

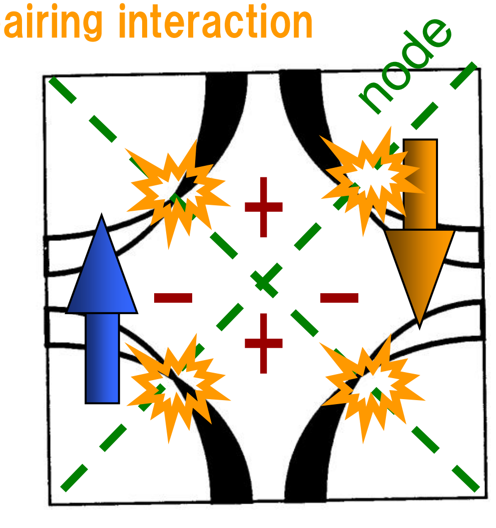




$$\Delta(\boldsymbol{k}) = -\sum_{\boldsymbol{k}'} V(\boldsymbol{k}, \boldsymbol{k}') \frac{\Delta(\boldsymbol{k}')}{2E(\boldsymbol{k}')} \tanh\left(\frac{1}{2}\beta E(\boldsymbol{k}')\right)$$

Repulsion → anisotropic SC

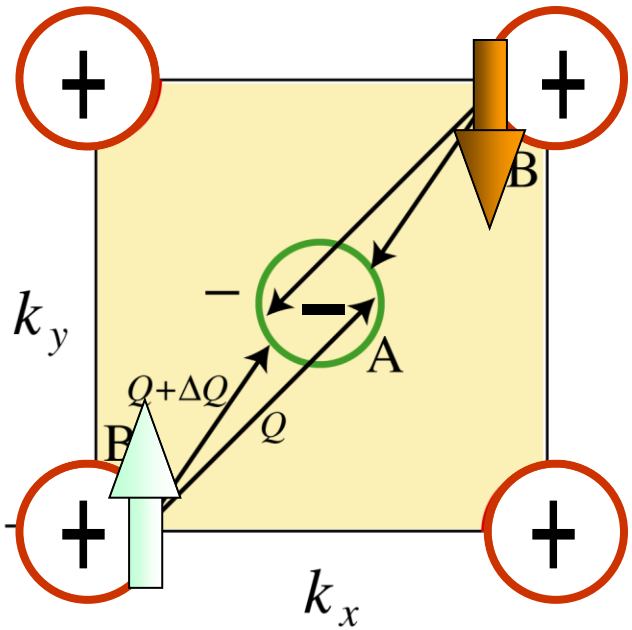
spin-fluctuation mediated pairing interaction



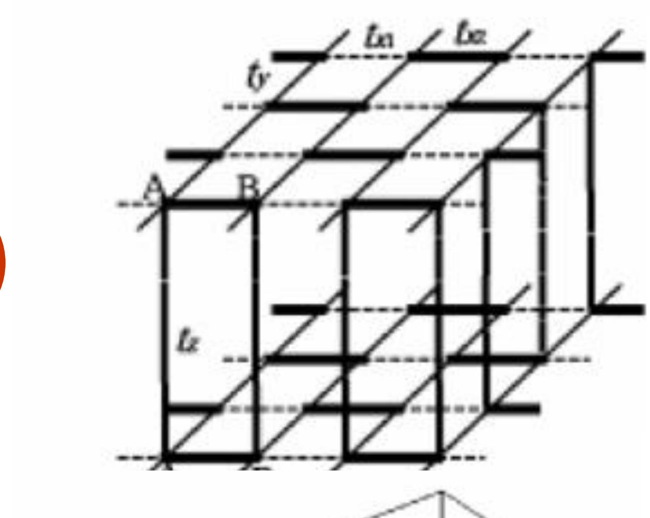
attraction if Δ changes sign

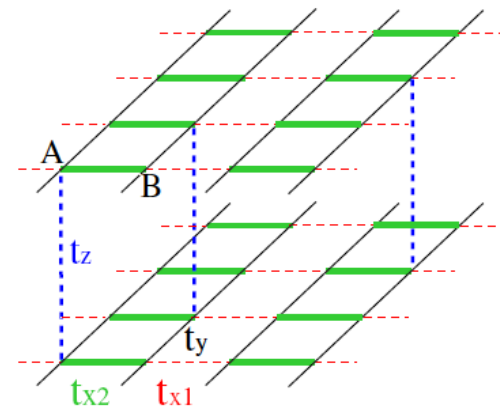
eg, d wave pairing in cuprates

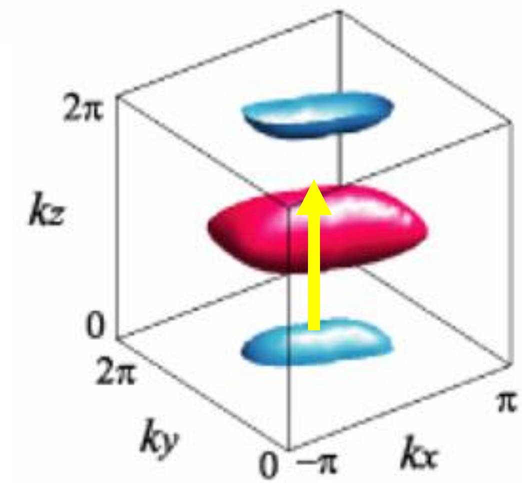
Disconnected Fermi surfaces

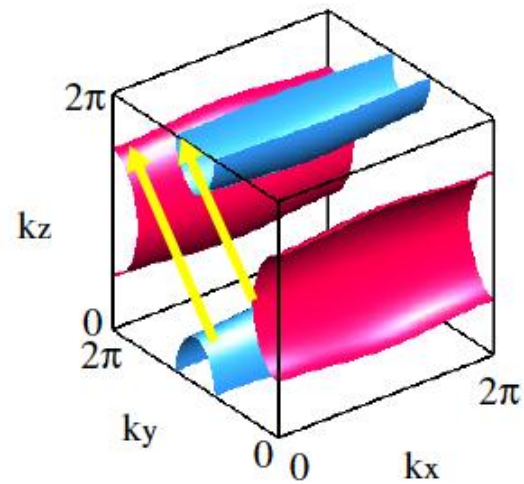


(Kuroki & Arita, 2001)





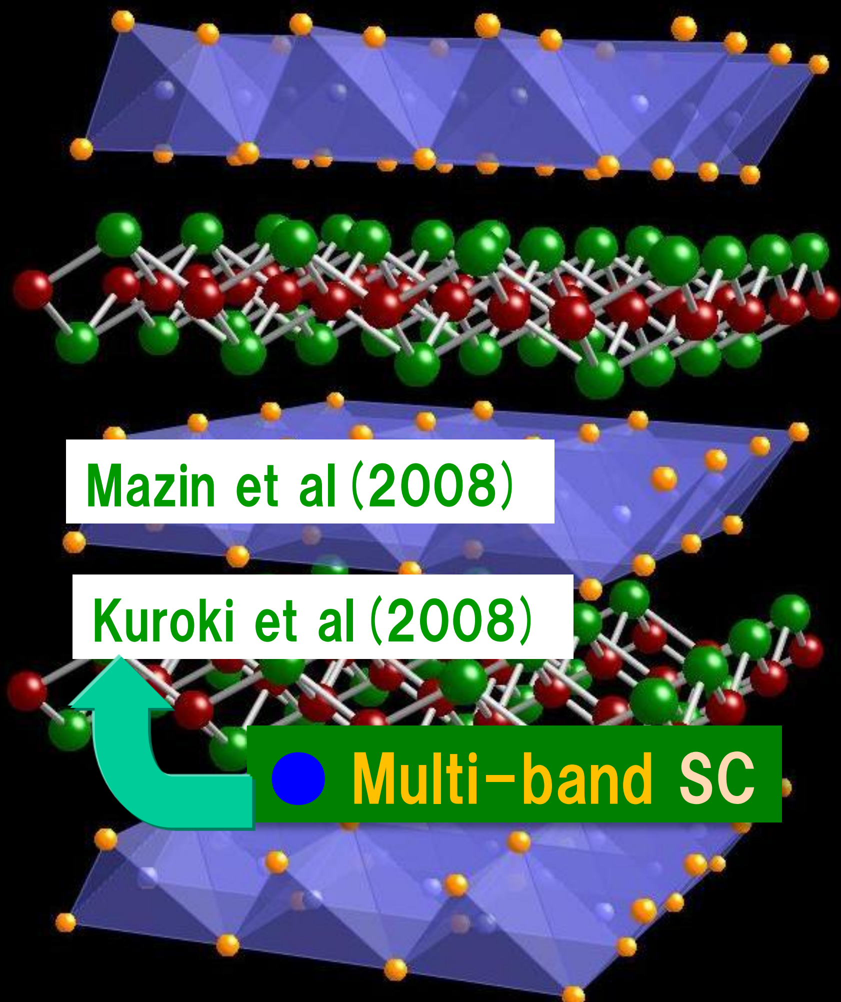




(Onari et al, 2003)

(Onari et al, 2003)

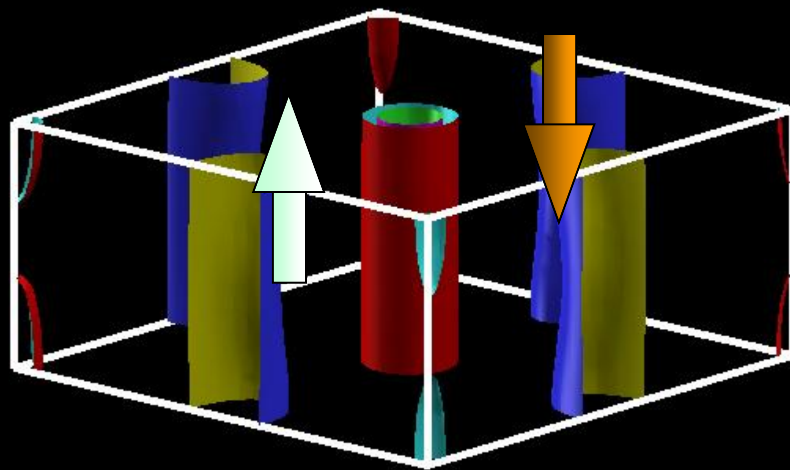
Fe-compound



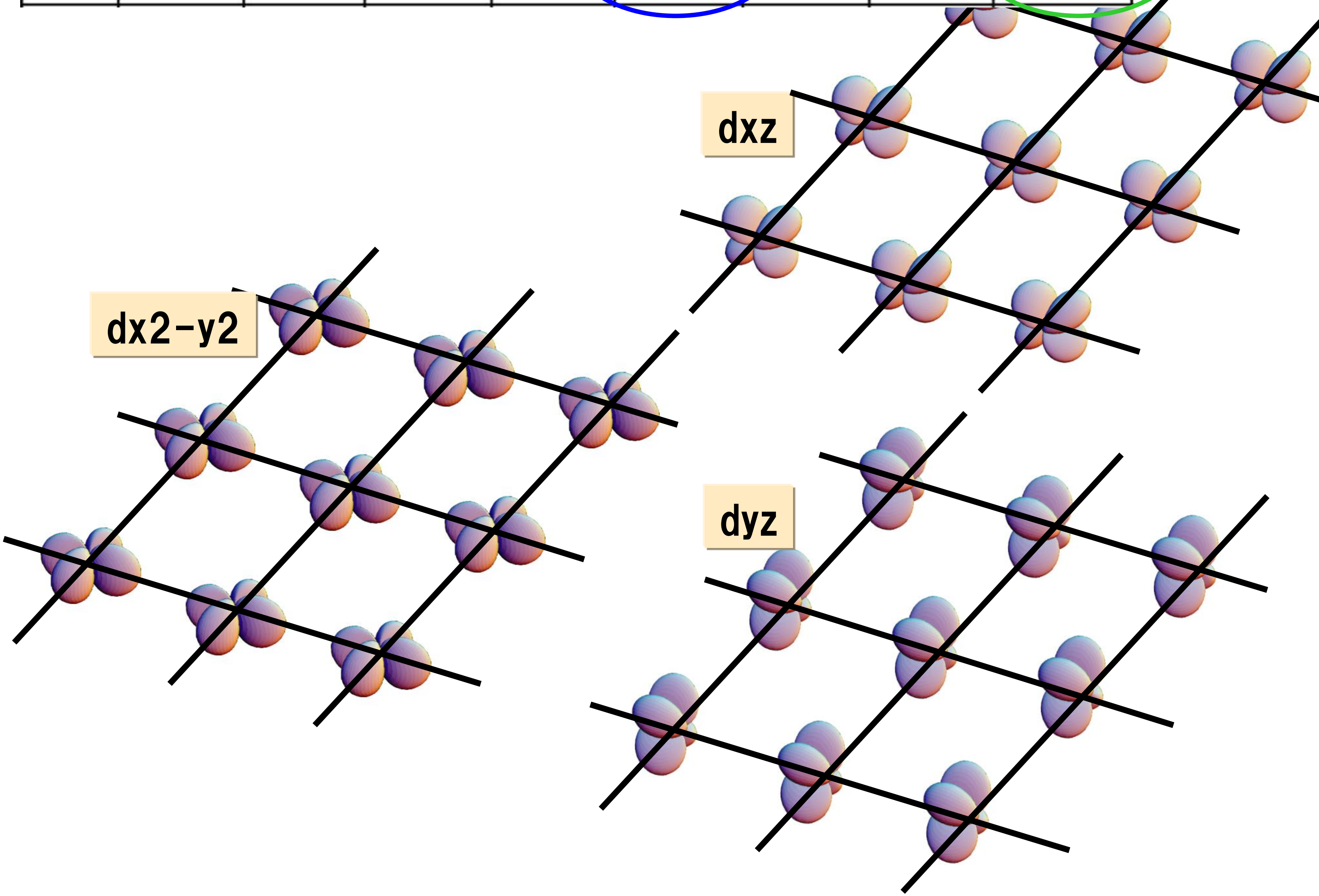
Mazin et al (2008)

Kuroki et al (2008)

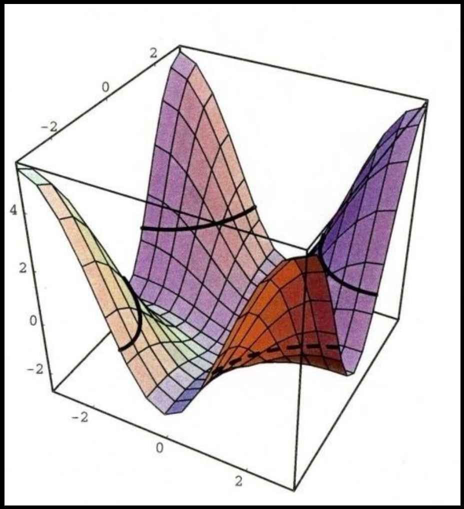
● Multi-band SC



4	Ti $3d^2$	V $3d^3$			Fe $3d^6$	Co $3d^7$	Ni $3d^8$	Cu $3d^{10}$



Cuprates: single band



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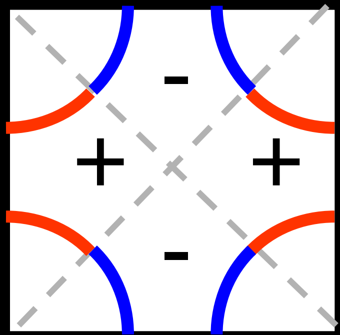
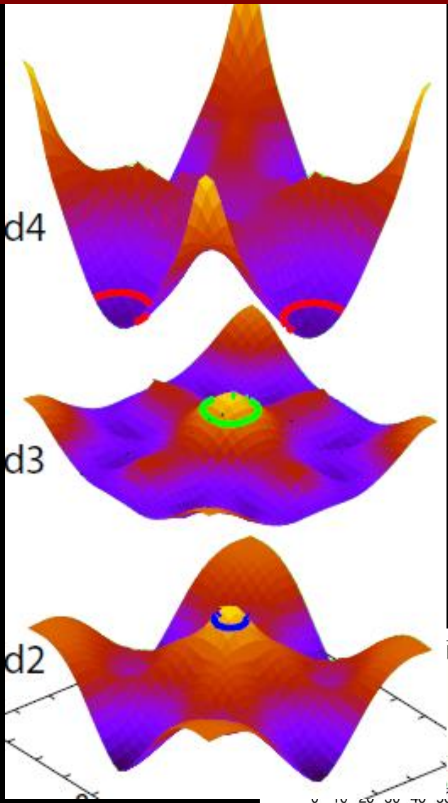
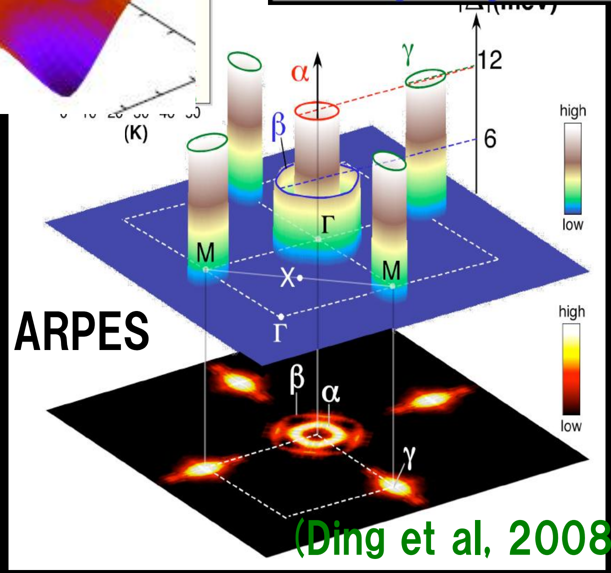


Iron-based: multi-band





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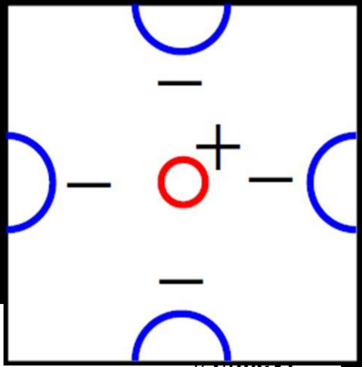
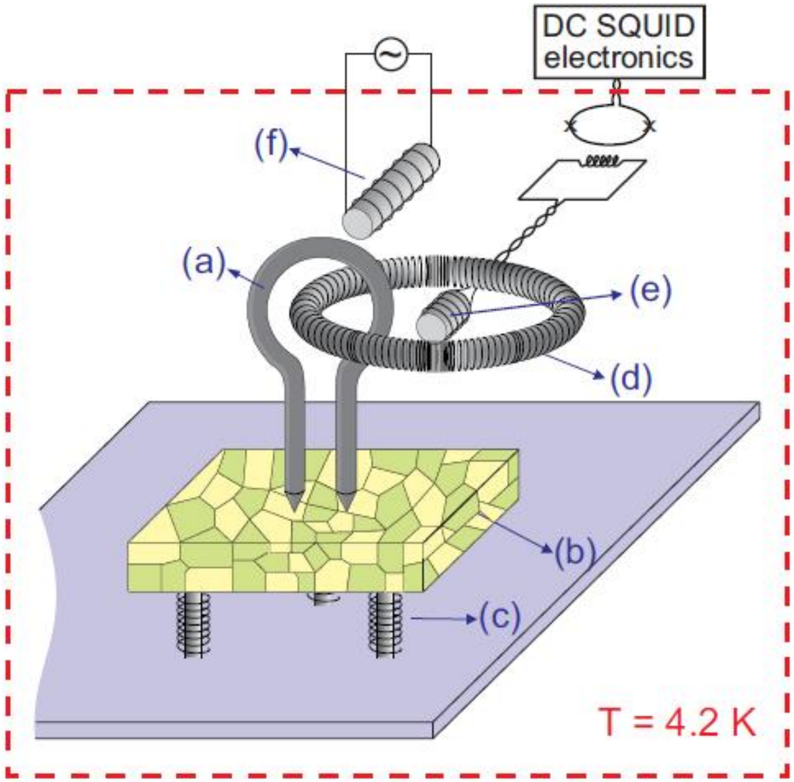
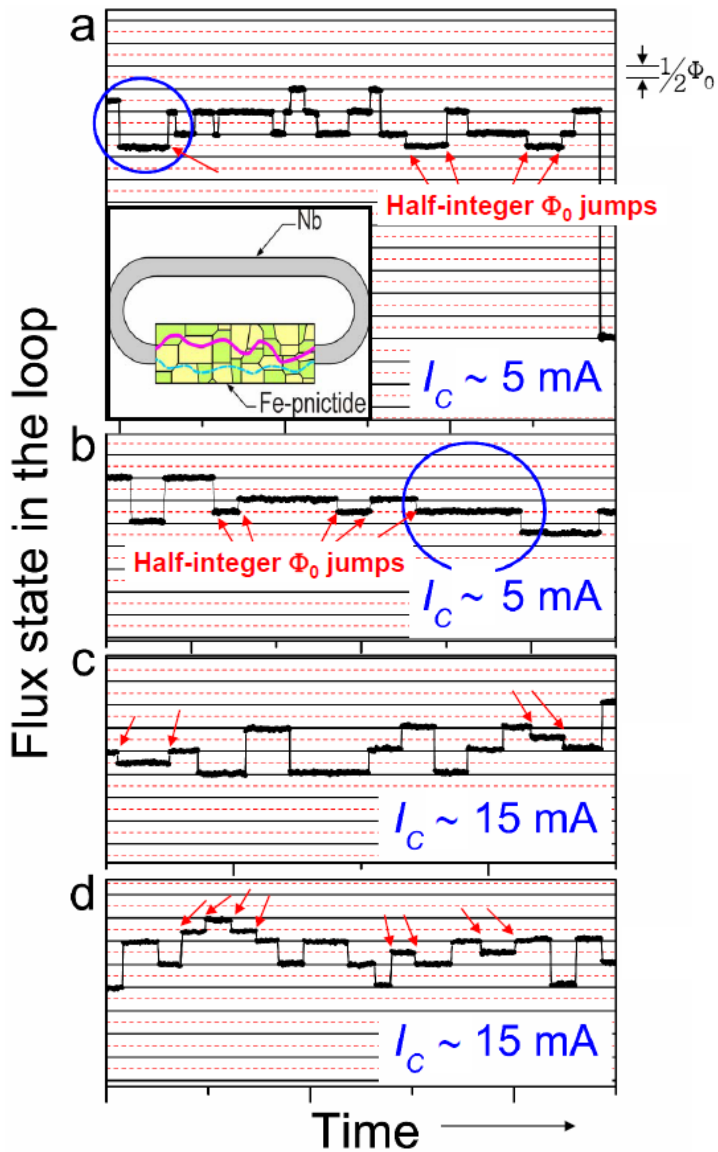


First phase-sensitive measurement

Chen, Tsuei et al, 2010





Leggett's collective mode in multiband SC

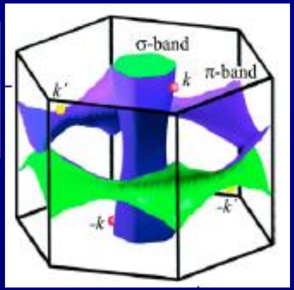
(Leggett, 1966)

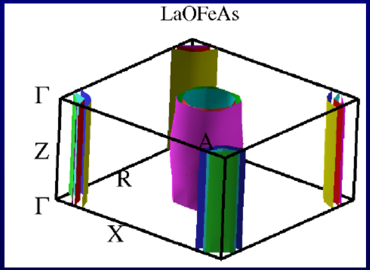
Collective modes in one-band SC

Phase modes (Bololiubov 1959, Anderson 1958)
= massless Goldstone mode for neutral SC
→ massive for real (charged) SC (Anderson-Higgs mechanism 1963)
as observed in $NbSe_2$

Collective modes in multi-band SC

Out-of-phase (countersuperflow) mode (massive, Leggett 1966)
as observed in MgB_2 (Blumberg et al 2007)

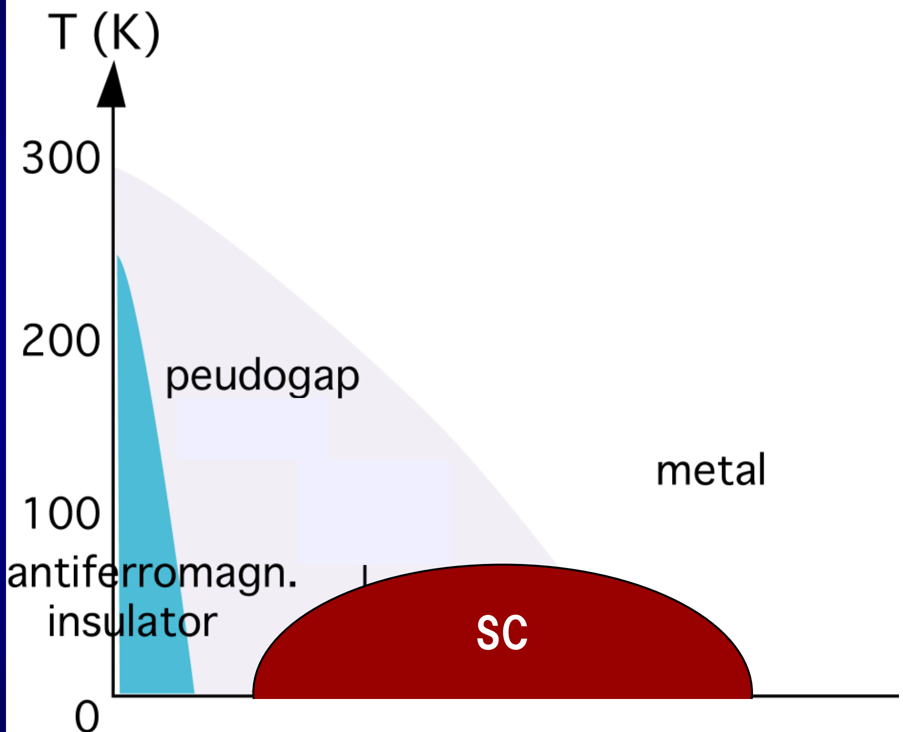




Josephson currents in multi-band SC, esp for the iron-based
(JAEA-Tokyo collaboration under way)

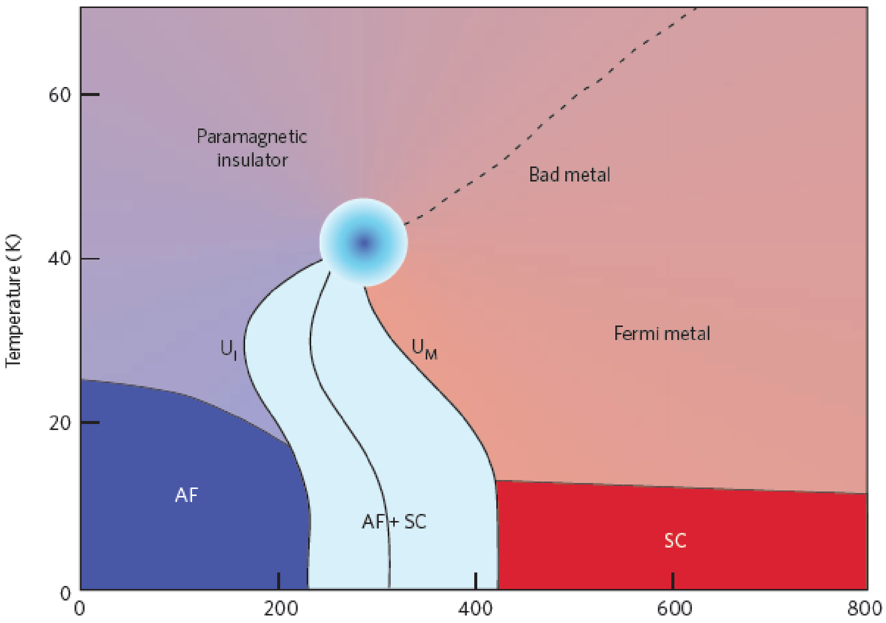
Phase diagram

High-Tc



hole concentration

Organics (BEDT-TTF)

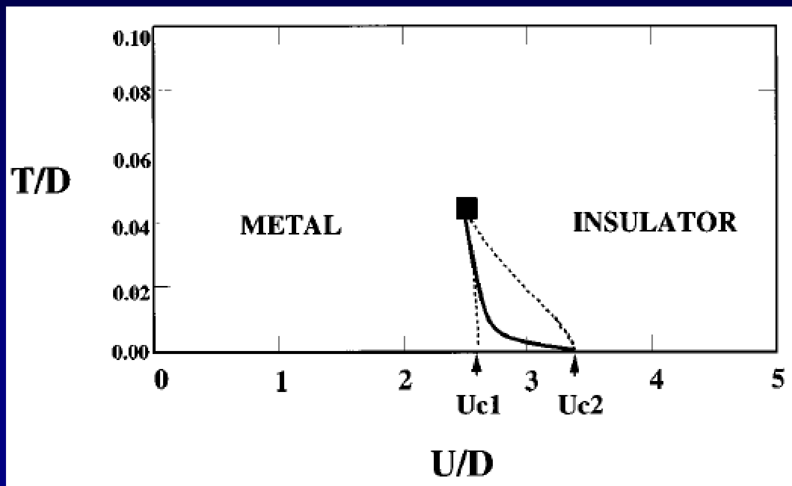


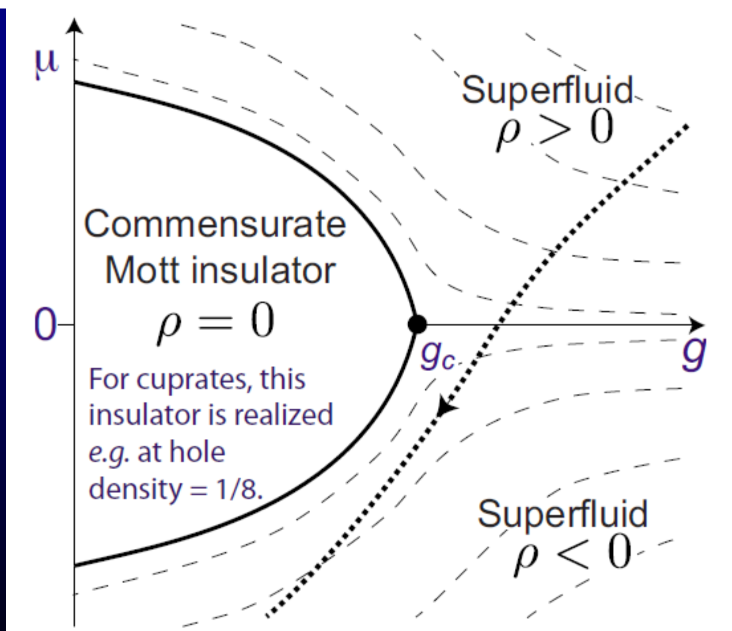
pressure

(Jerome, 2009)

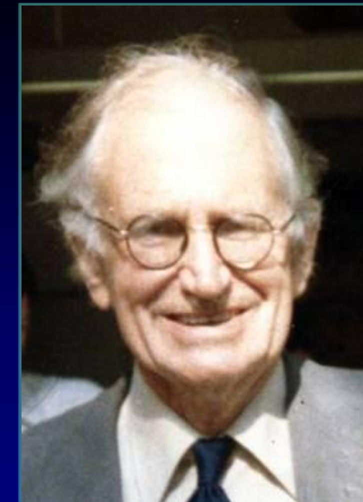
Mott transition

DMFT (exact in ∞D)
(Georges et al, 1996)



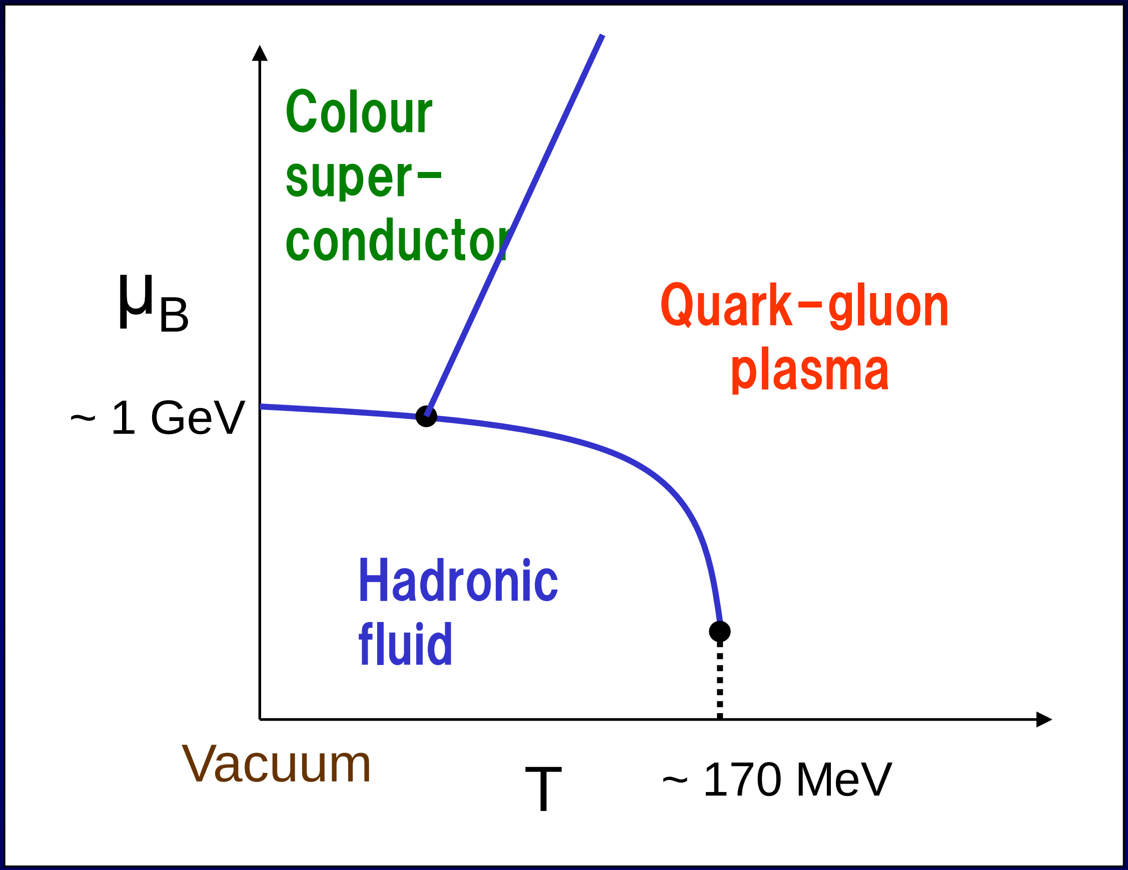


(Hartnoll et al, 2007)



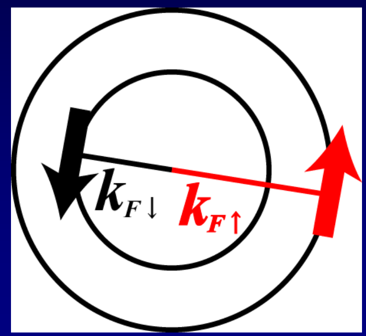
(© H Aoki, 1979)

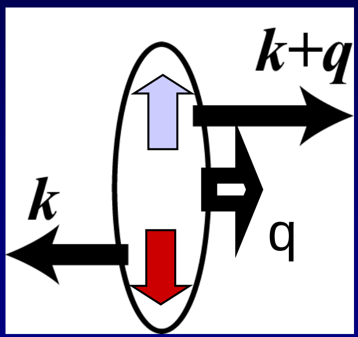
Colour SC in hadron physics



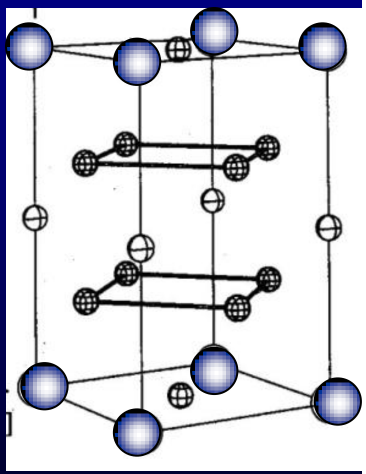
FFLO: Cooper pair with a nonzero momentum

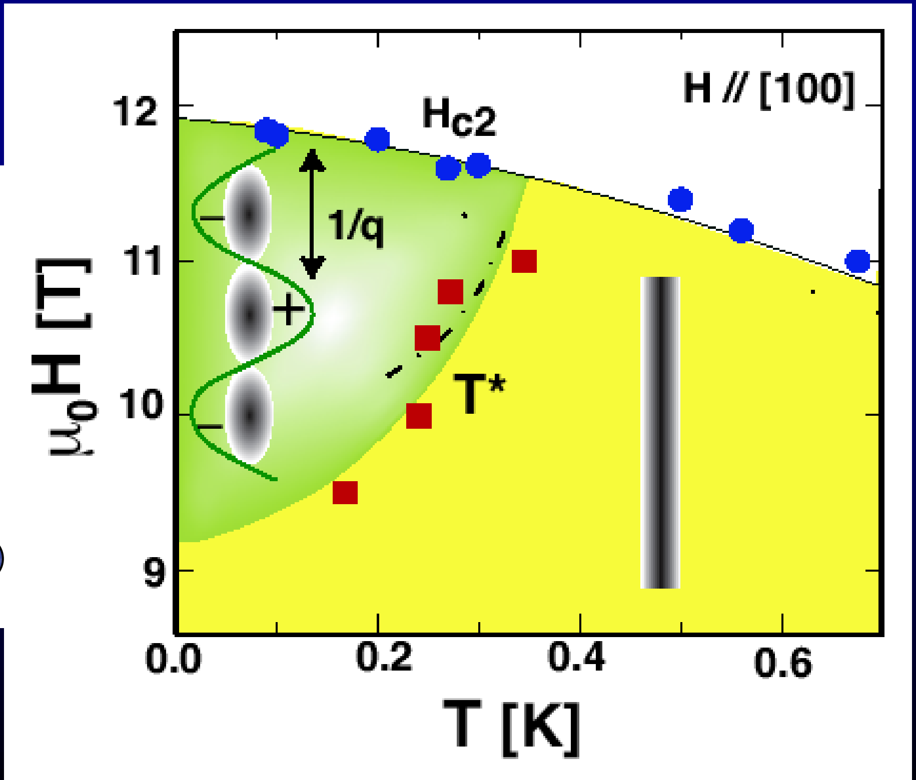
Fulde-Ferrell-Larkin-Ovchinikov





$CeCoIn_5$



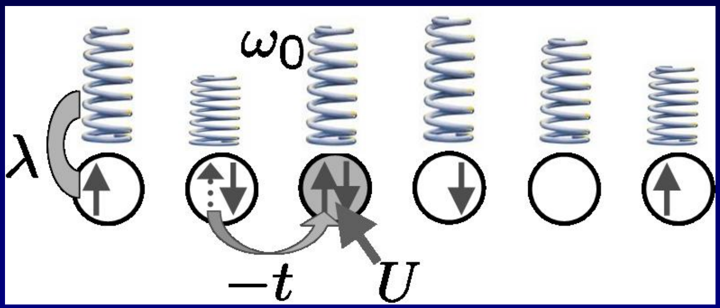


Neutron star ?
(Casalbuoni et al, 2004)

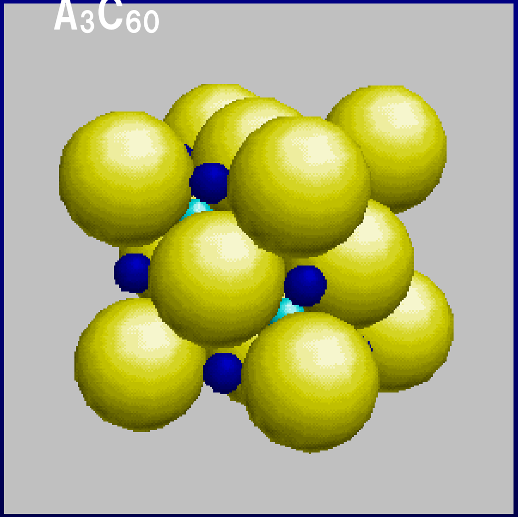
Coexisting el-el and el-ph interactions

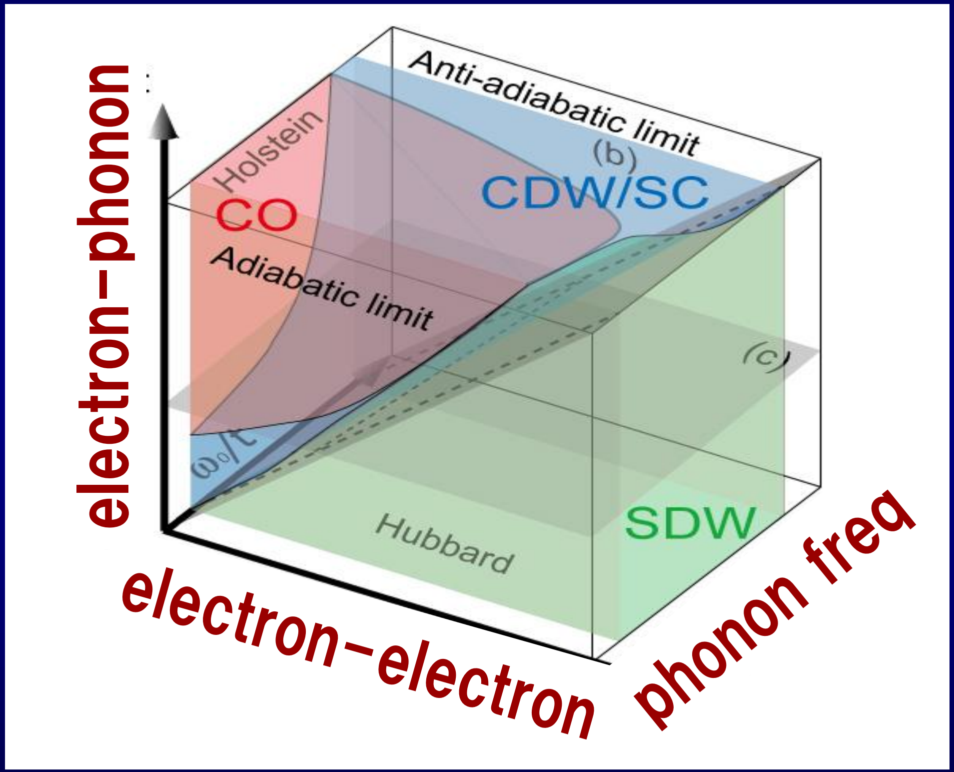
(Tezuka et al, 2007)

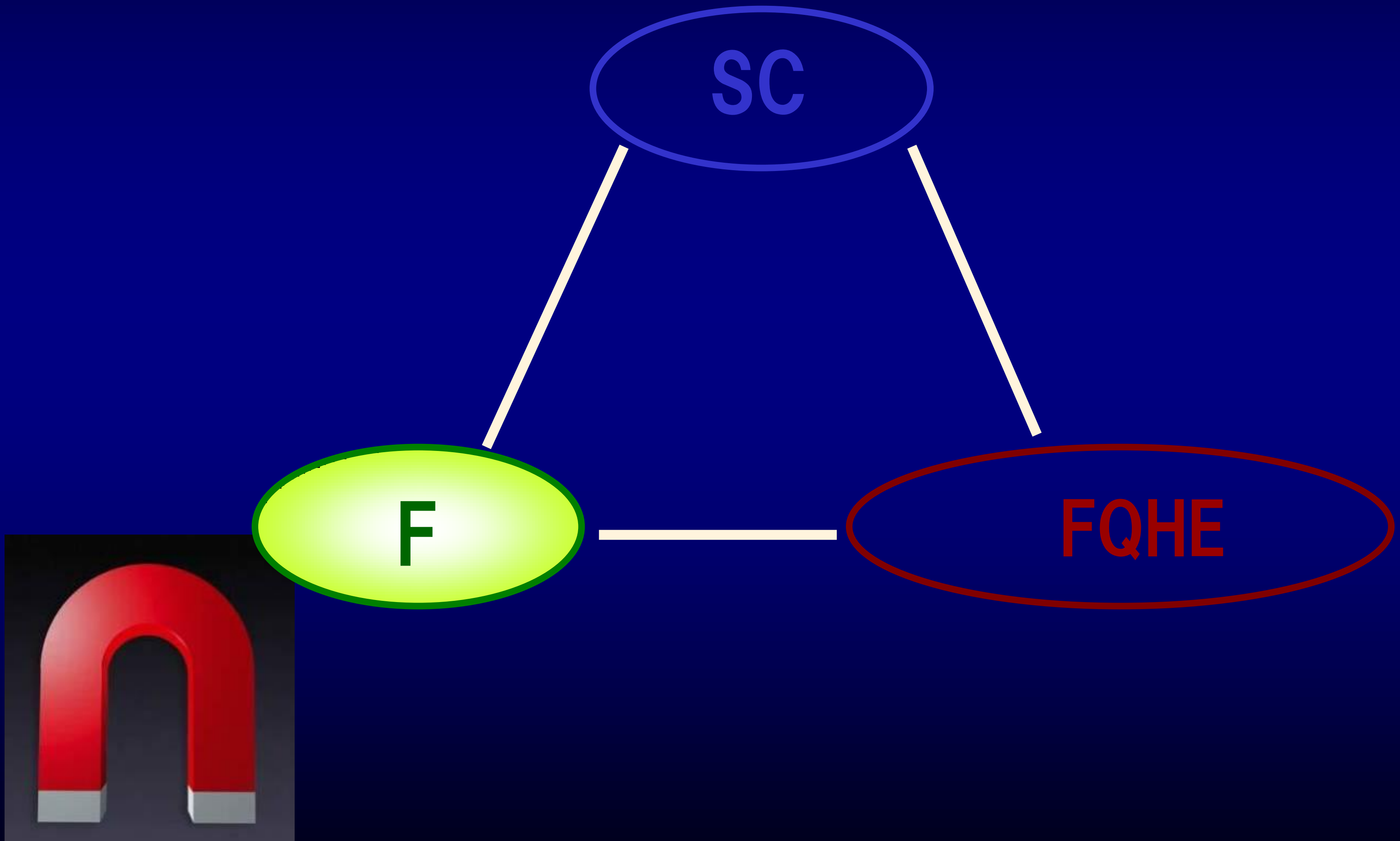
1D Hubbard-Holstein model ← DMRG



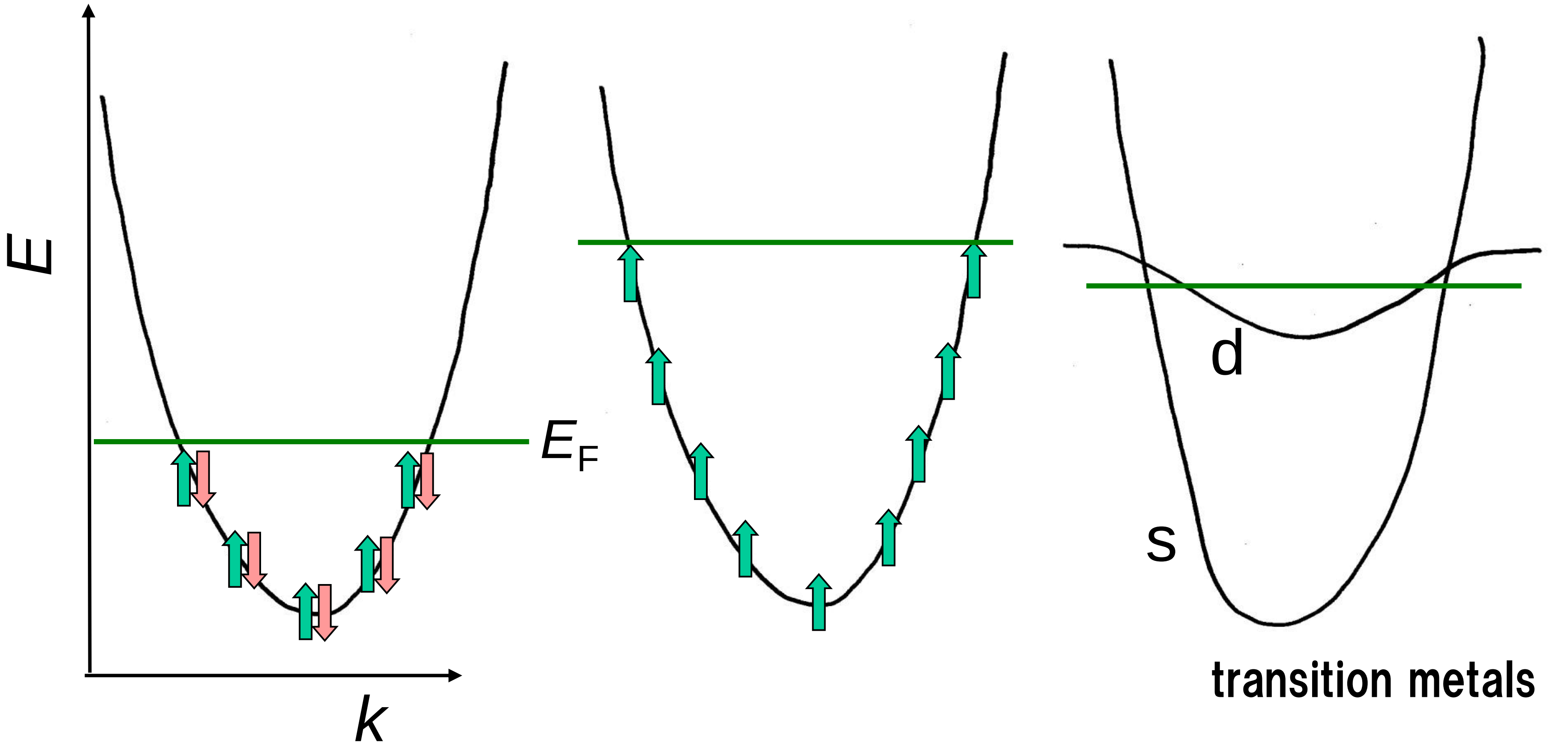
A$_3$C$_{60}$



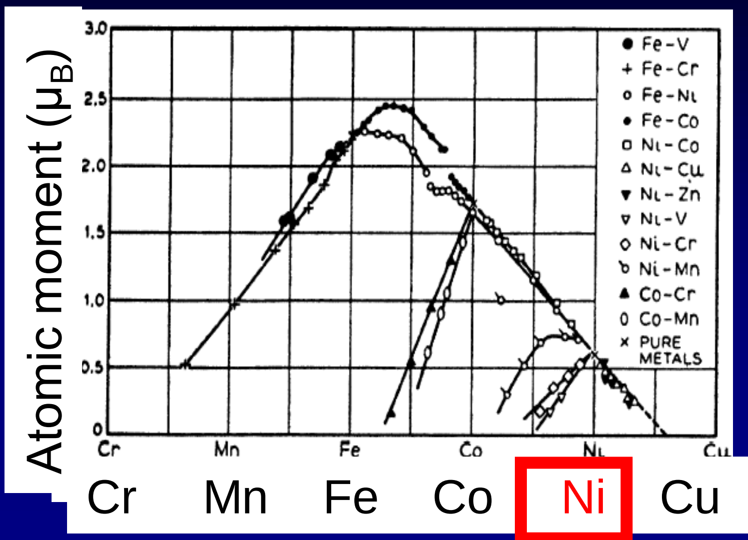


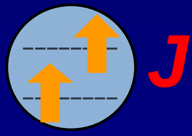

SC
F
FQHE

Ferromagnetism ← very difficult to realise

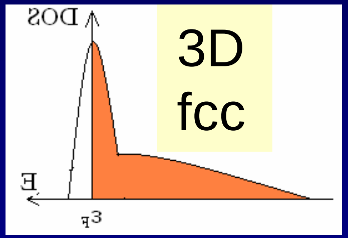


Itinerant ferromagnetism in transition metals





Multi-band → Hund's coupling



U + Lattice structure

* Auxiliary-field method for multi fermion species

(Multi-orbital DMFT: Sakai et al, 2007)

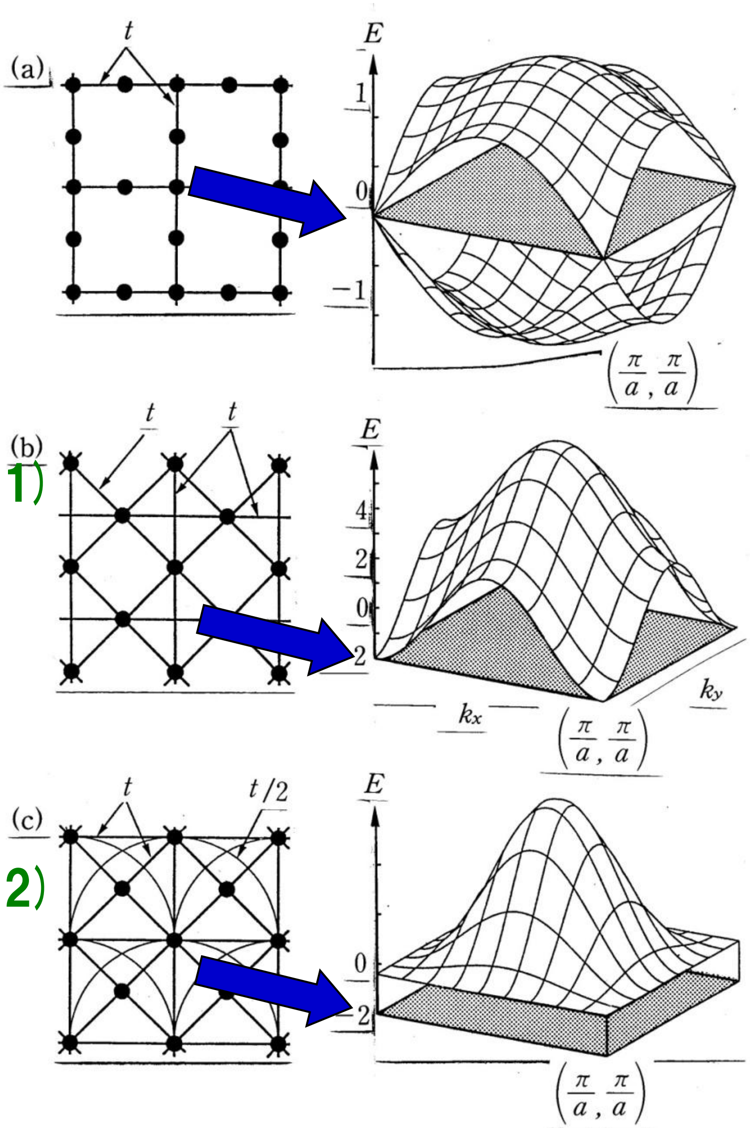
Flat-band ferromagnetism

Ferromagnetism rigorously guaranteed for $< U < \infty$

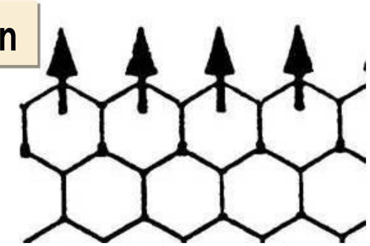
(Lieb, 1989)

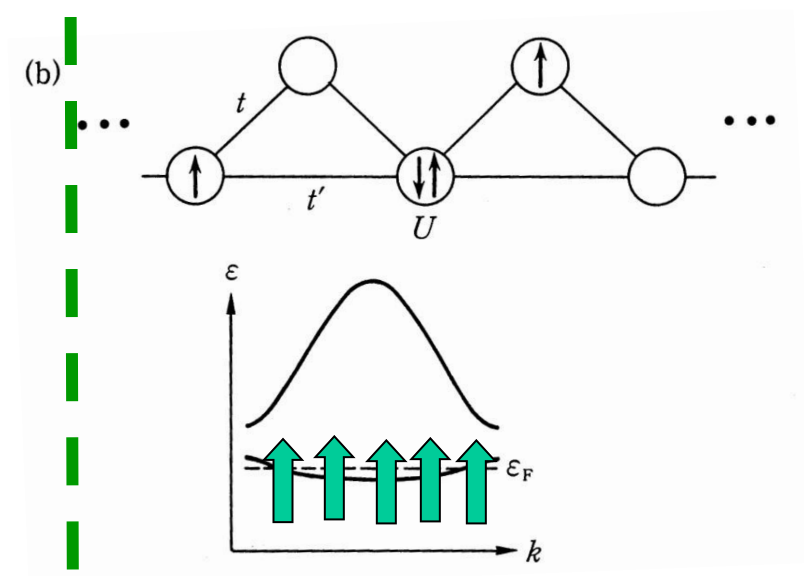
(Mielke, 1991)

(Tasaki, 1992)



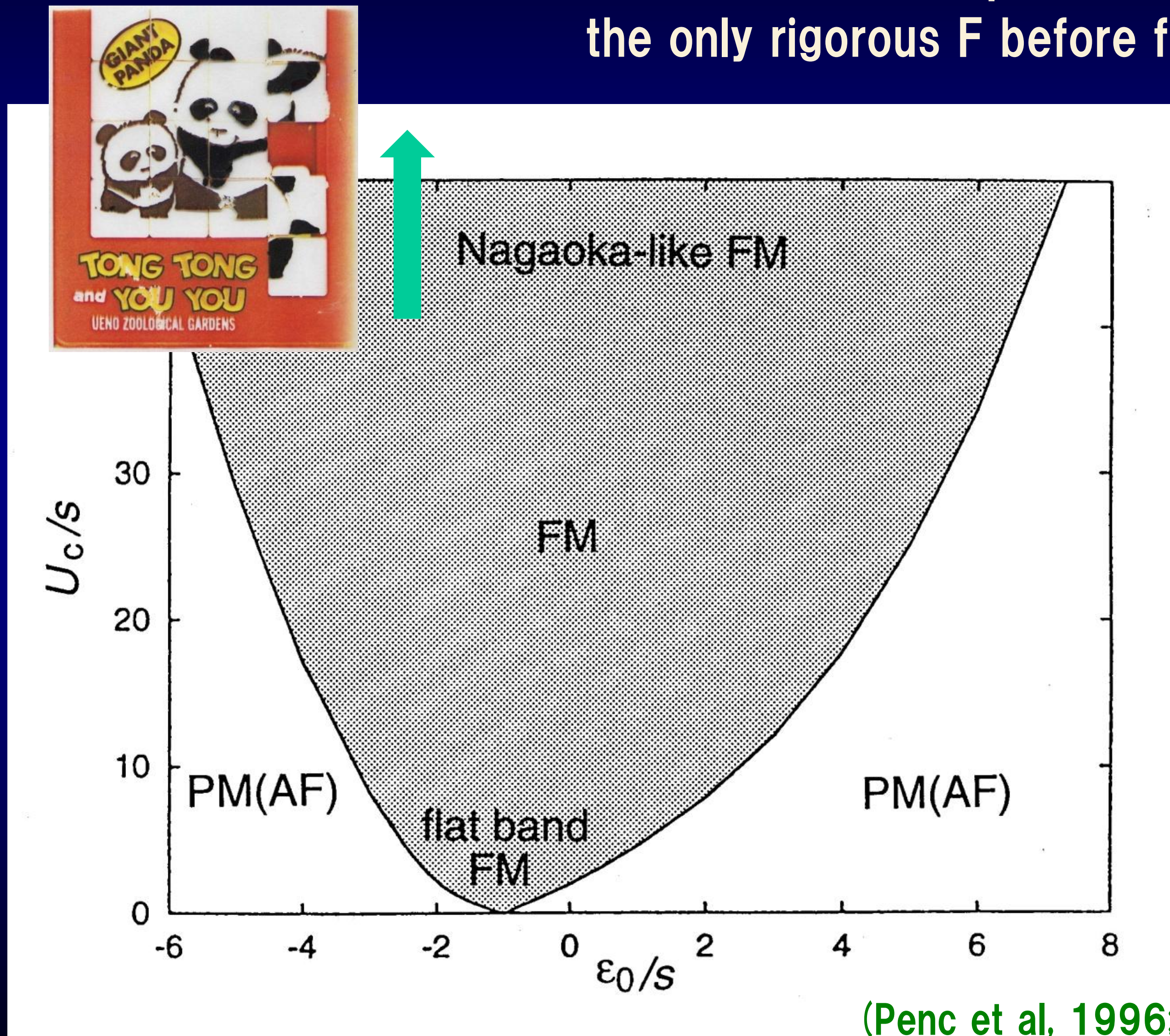
1D version



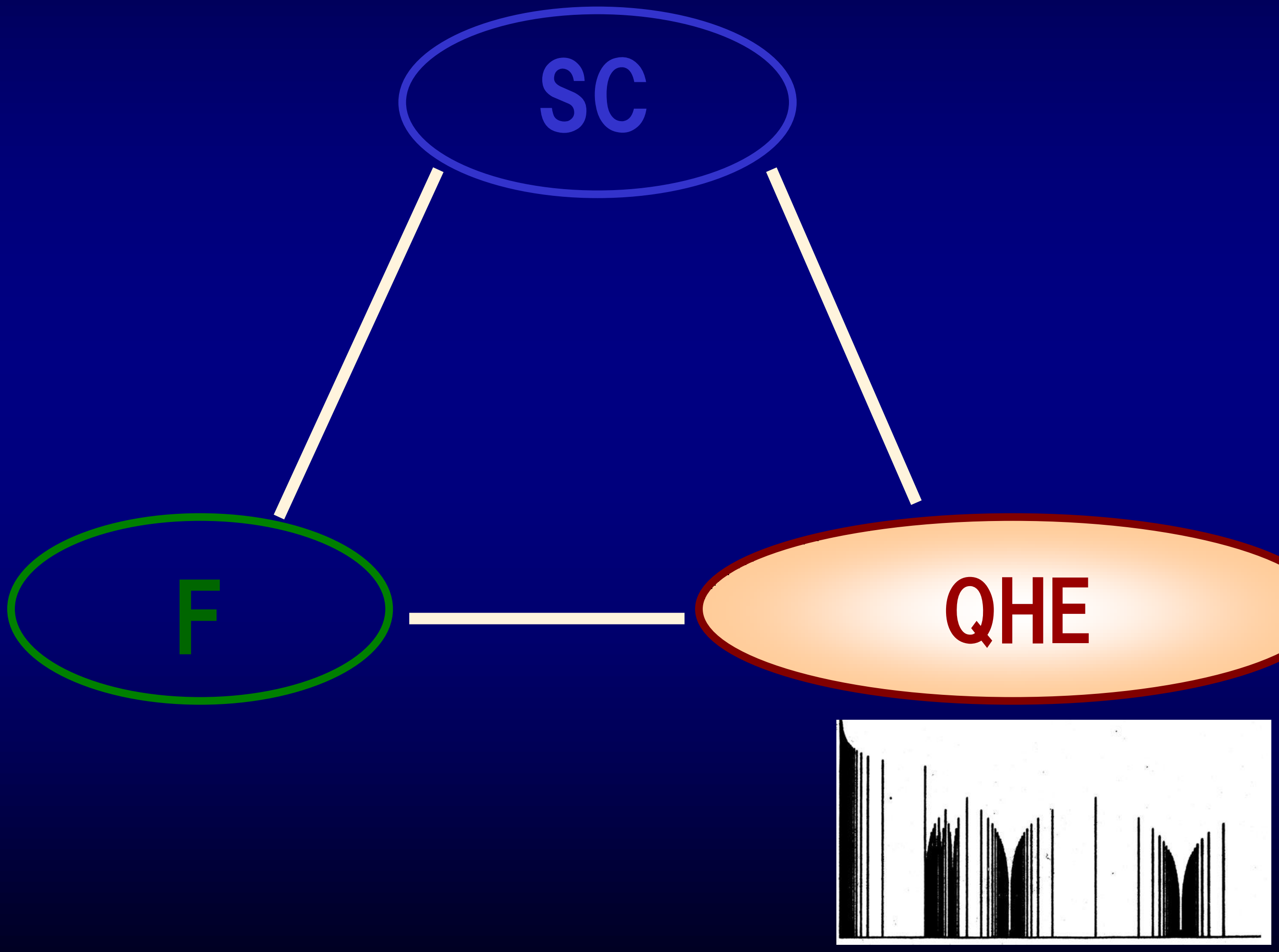


Flat-band F ←→ Nagaoka's F

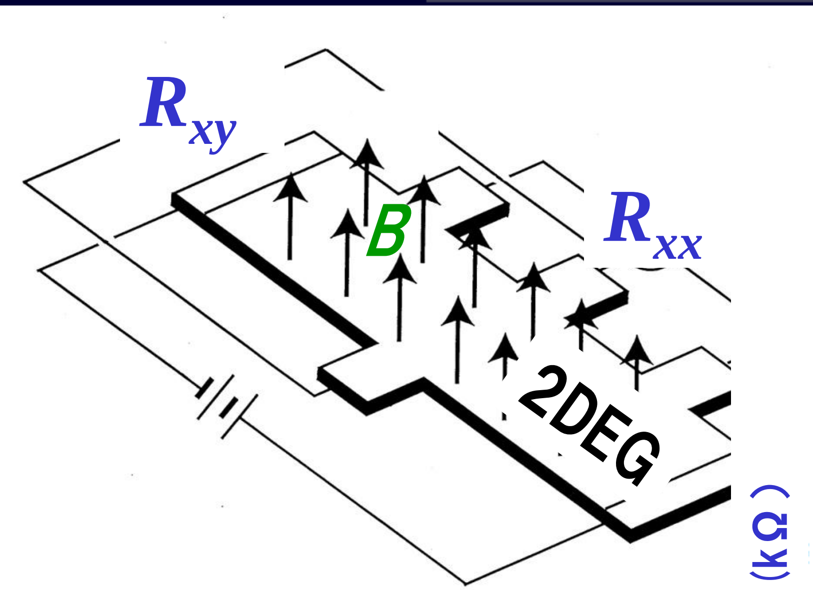
↑
the only rigorous F before flat-band F

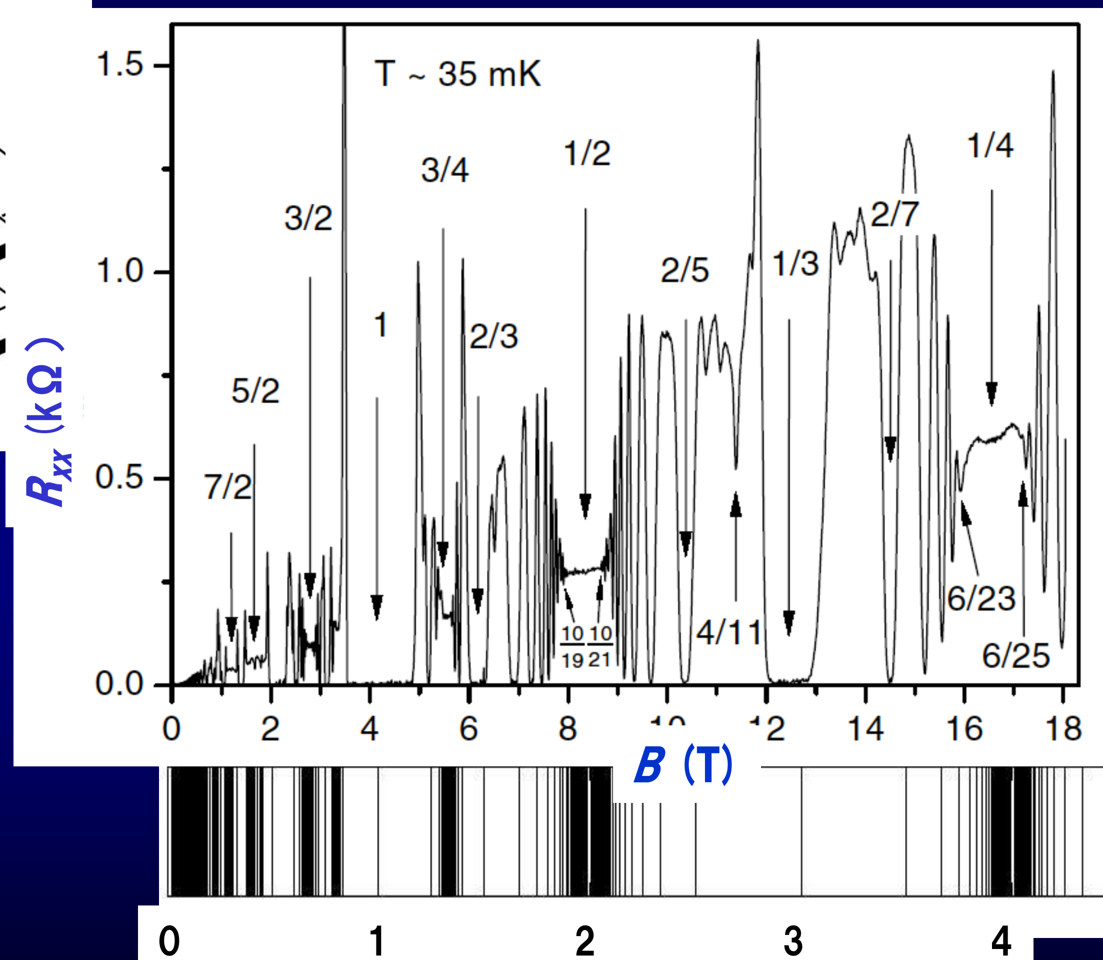


(Penc et al, 1996;
see also Vollhardt 2007)


SC
F
QHE

Fractional quantum Hall effect





$1/\nu$ ($\propto B$)

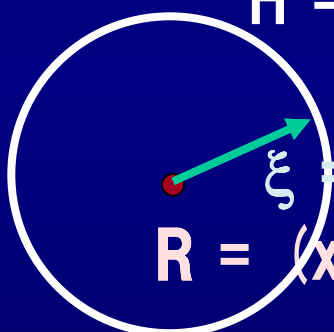
FQH liquid ← Coulomb repulsion + quantum zero point

Liquid He ← $[x, p] = i\hbar$

FQHE ← $[x, y] = i\hbar$

Non-commutative space

$$H = (1/2m)\,\pi^2, \quad \pi = \mathbf{p} + e\mathbf{A}$$



$$\xi = -(1/eB)\ \mathbf{e}_z \times \Pi$$

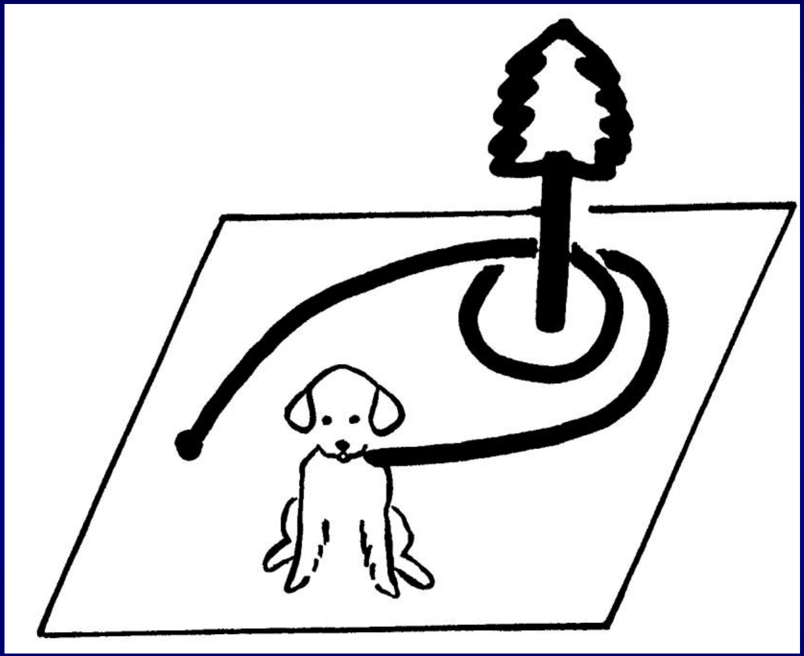
$$R = (x, y), \quad [x, y] = i\,l^2$$

$l = (h/eB)^{1/2}$: magnetic length ($\cong$ 80A for B=10T)

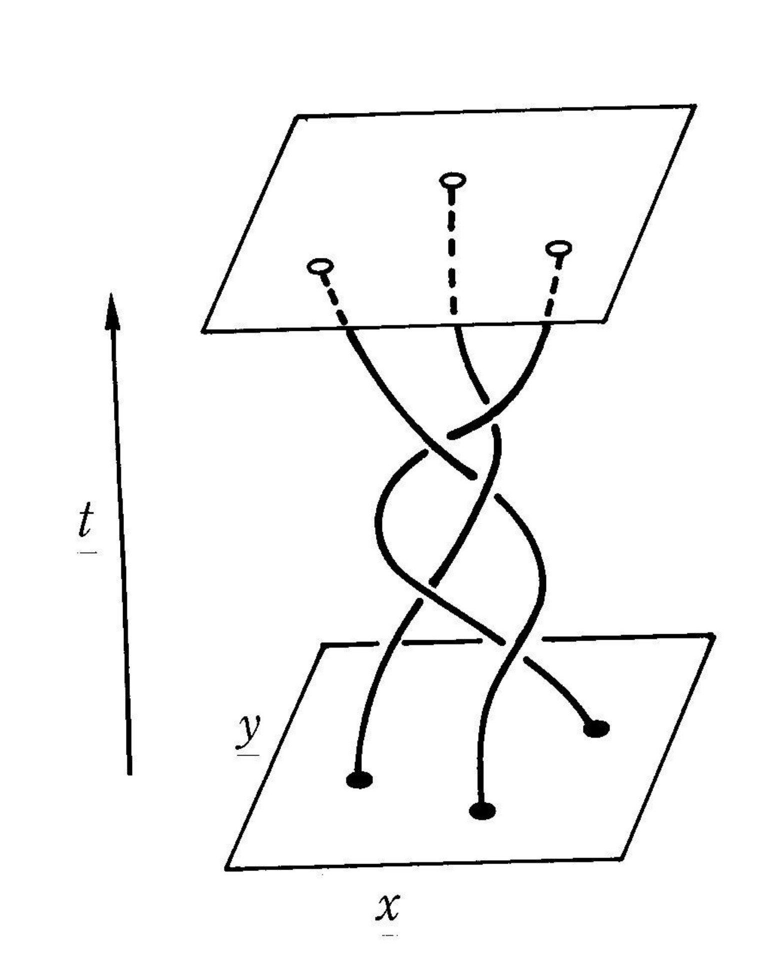
FQHE = repulsively interacting fermions with uncertain (x, y)

Speciality about 2+1D ?

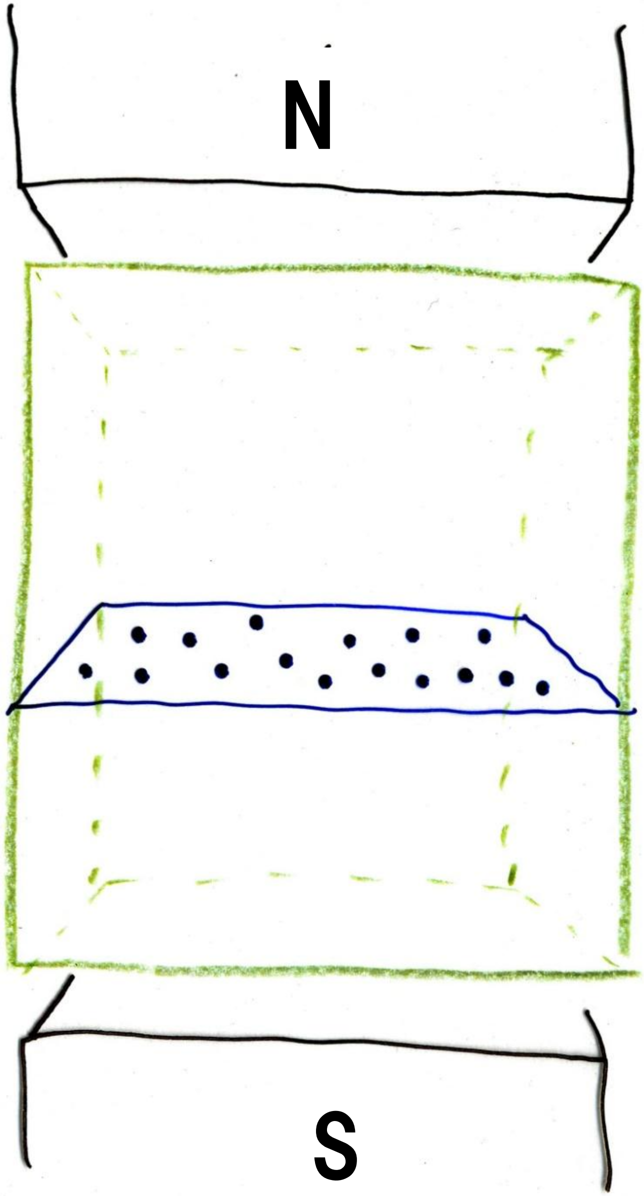
braid group rather than permutation group

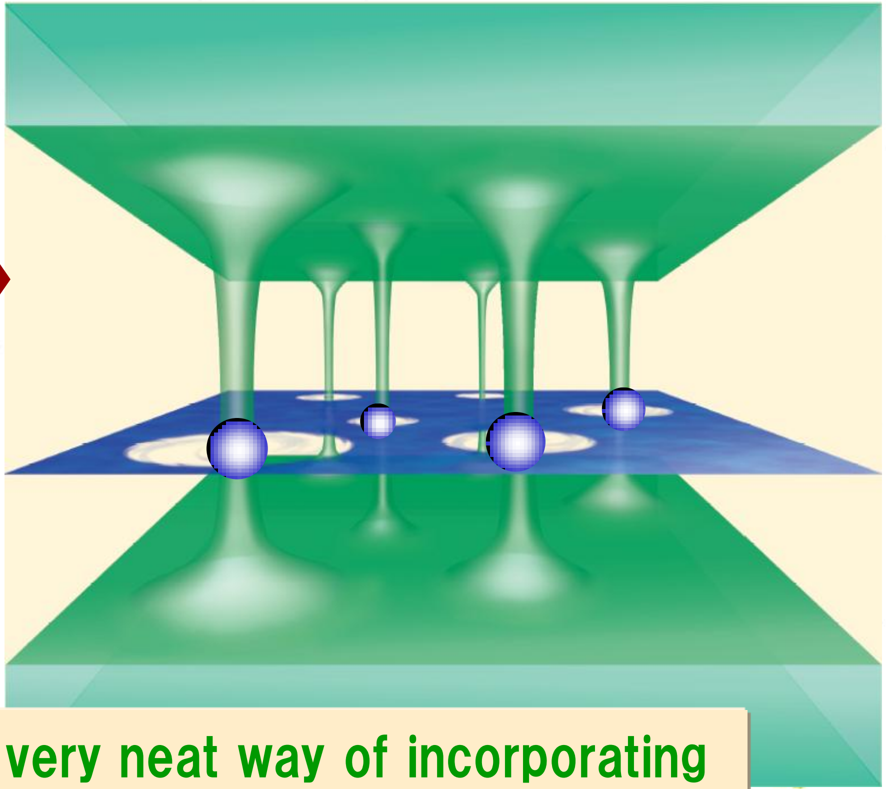


© H Aoki



Composite fermion picture

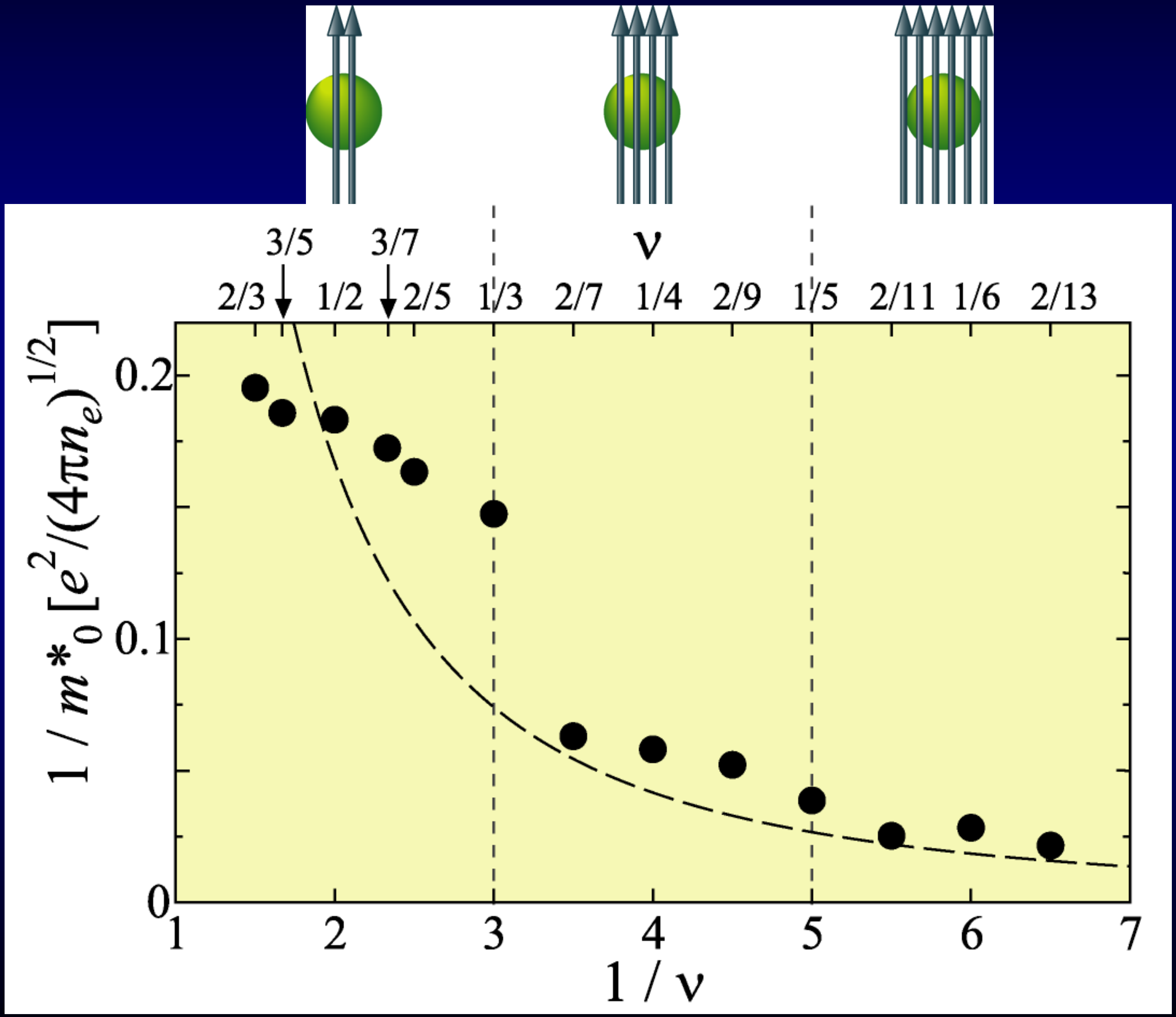




A very neat way of incorporating (short-range) part of interaction

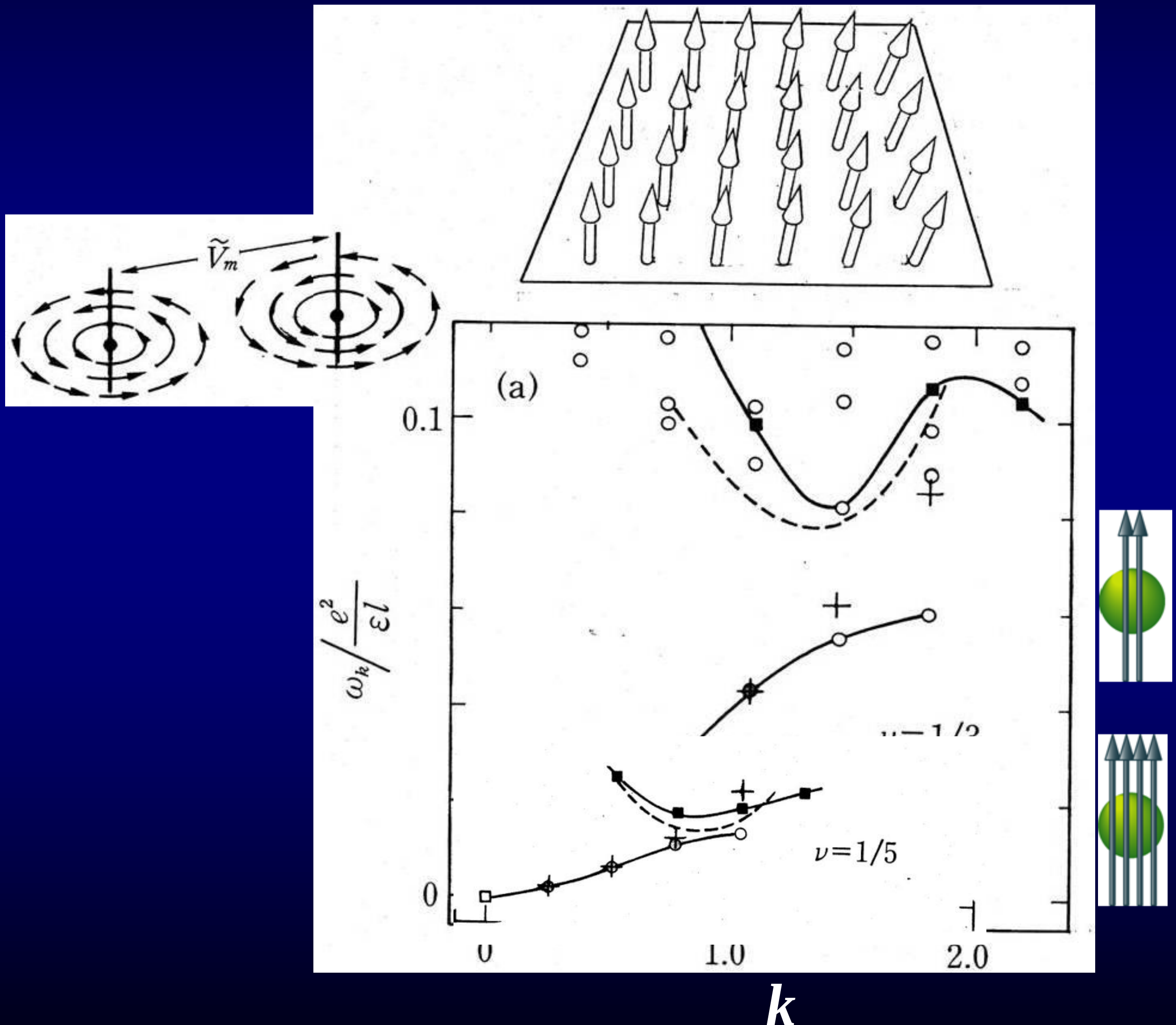
Effective mass of the composite fermion

(Onoda et al, 2001)

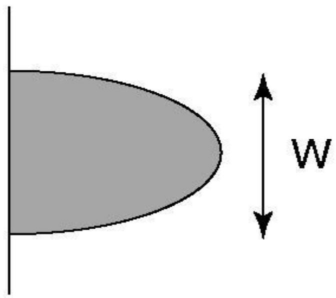


Composite-fermion picuture for the spin wave

(Nakajima & Aoki 1994)



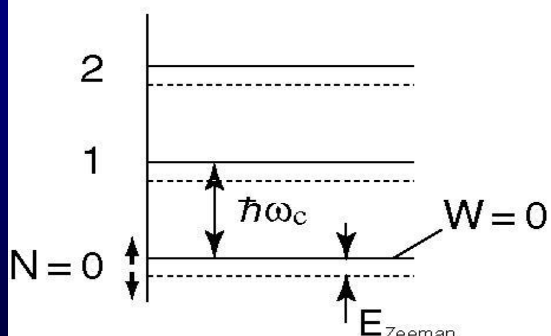
Correlated electron systems



$$\frac{U}{W} = \text{large}$$

$\rightarrow$ strong correlation

FQHE systems



Infinitely strongly correlated

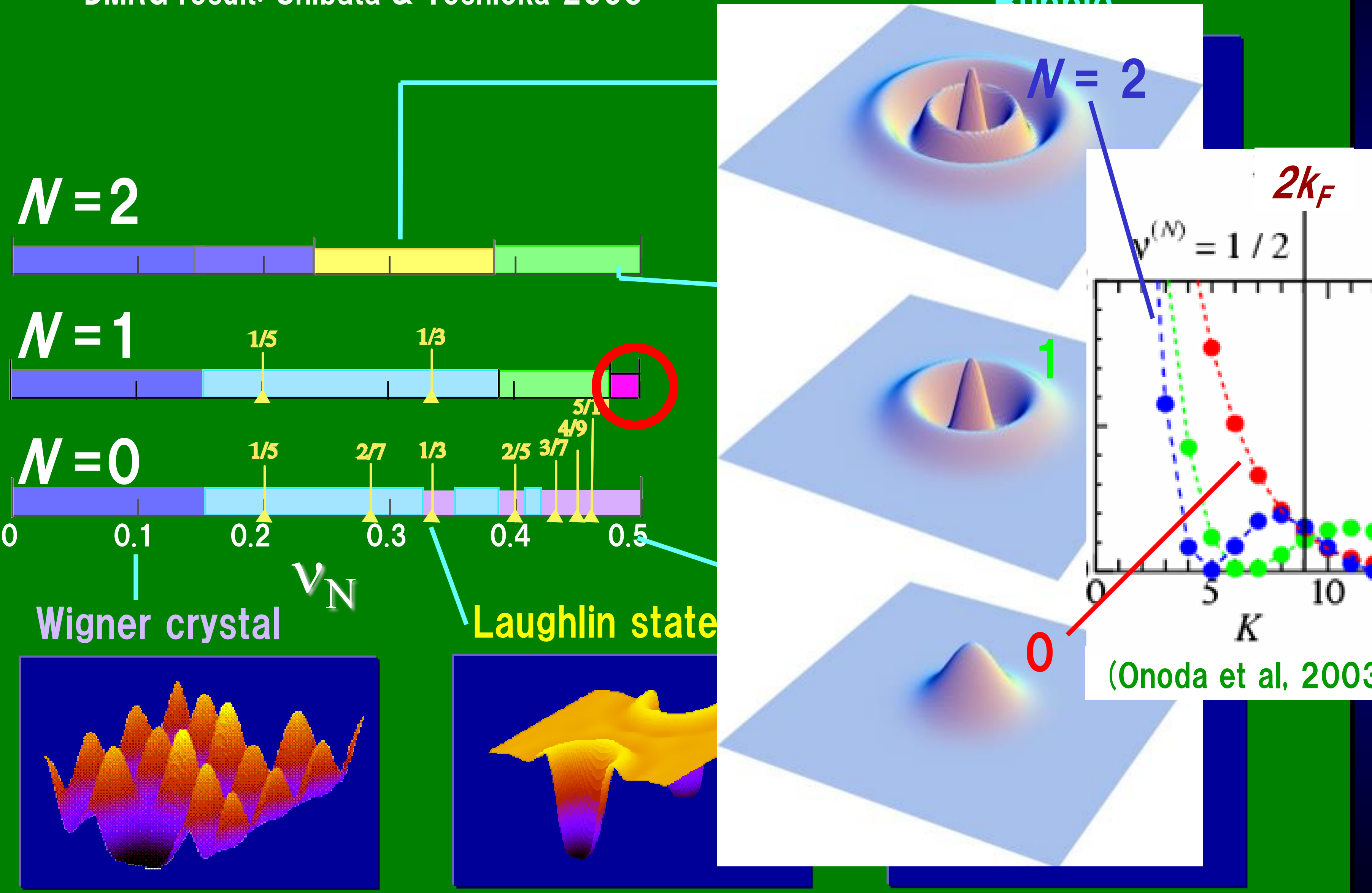
$$\frac{U}{W} = \infty$$

→ many competing phases

$$\frac{U}{\hbar\omega_c}, \quad \frac{U}{E_{Zeeman}}$$: controllable

Various phases in the quantum Hall system

DMRG result: Shibata & Yoshioka 2003



BCS paired state at $\nu = 5/2$

Triplet p-ip (Pfaffian state)

CF CF

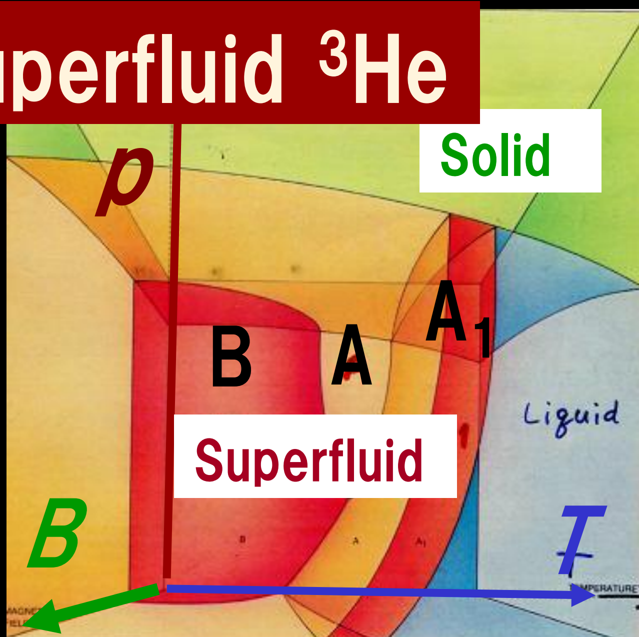
Trial wf: Moore-Read, Greiter-Wen-Wilczek 1991

Numerical: Morf 1998, Rezayi-Haldane 2000; Onoda-Mizusaki-Aoki 2003

Experiment: Willett-West-Pfeiffer 1998, 2002

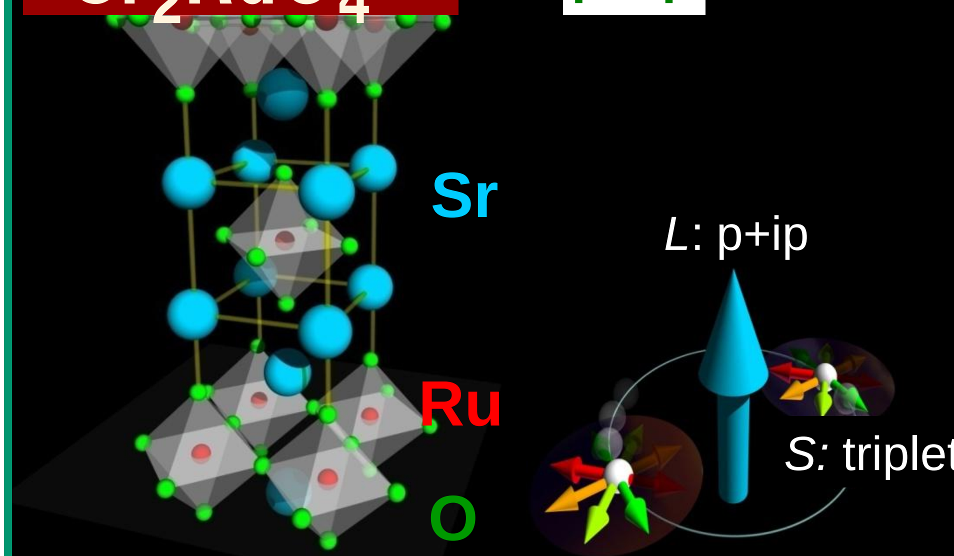
~ ~

superfluid ^3He



Sr_2RuO_4

p+ip



Integer QHE
--- Hall conductivity as a topological invariant

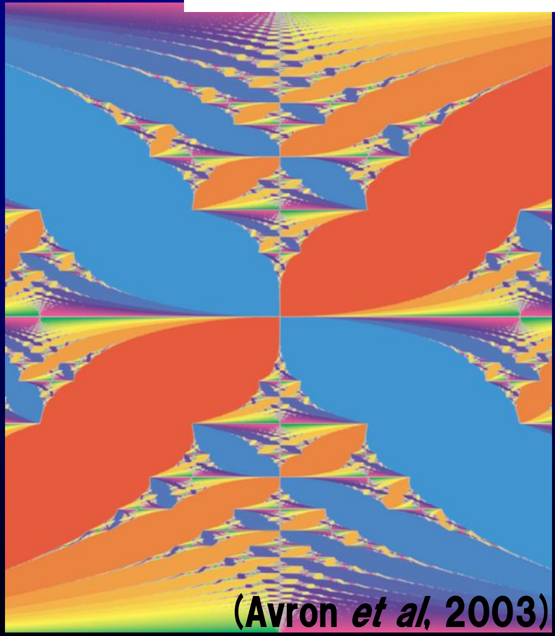
Linear response

$$\sigma_{xy} = -i\frac{1}{L^2}\sum_{\alpha,\beta} f(\varepsilon_\alpha)\frac{\langle\alpha|J_x|\beta\rangle\langle\beta|J_y|\alpha\rangle - \langle\alpha|J_y|\beta\rangle\langle\beta|J_x|\alpha\rangle}{(\varepsilon_\alpha - \varepsilon_\beta)^2}$$

clean, periodic systems

Berry's "curvature"

$$= -i\frac{e^2}{L^2}\sum_{n,\vec{k}} f(\varepsilon_{n\vec{k}})\left[\vec{\nabla}_{\vec{k}} \times \langle n\vec{k}|\vec{\nabla}_{\vec{k}}|n\vec{k}\rangle\right]_z$$



(Avron *et al*, 2003)

"Gauss-Bonnet"

$= (e^2/h)\ \Sigma_n^{\text{band}}\ (\text{Chern}\ \#)_n$

Hall conductivity = a topological number

(Thouless, Kohmoto, Nightingale & den Nijs, 1982)

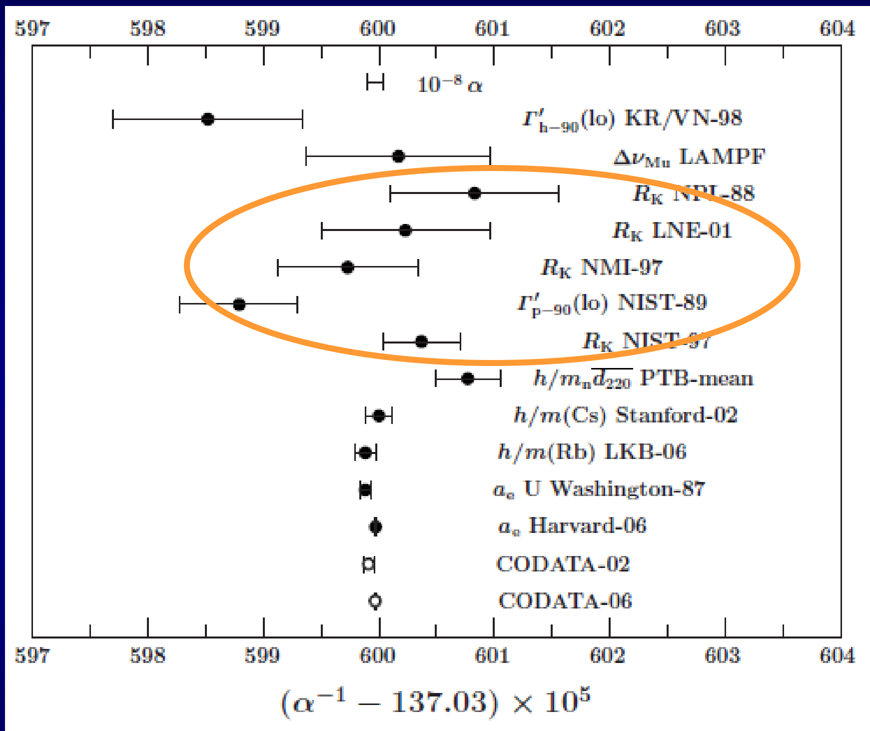
disordered systems

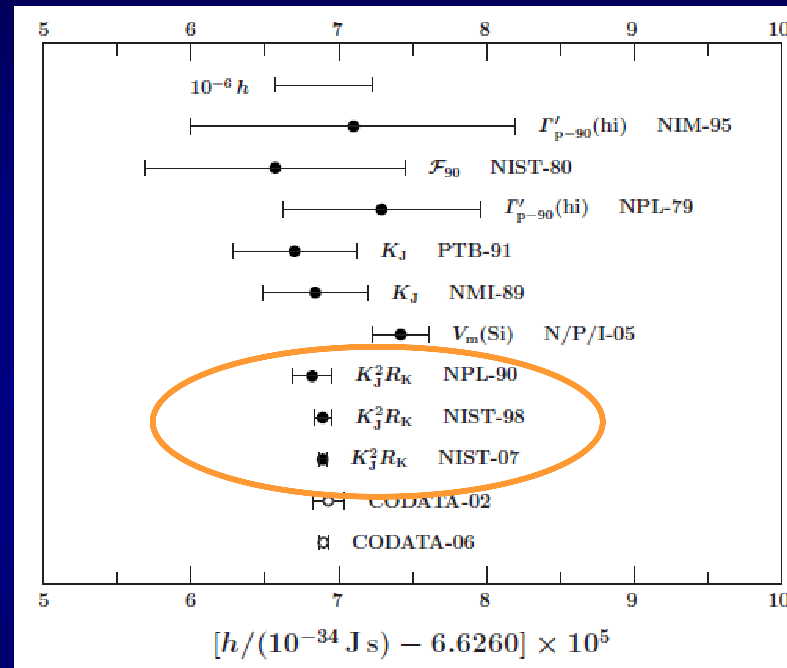
(Aoki & Ando)

distribution of topological # in disordered systems
(Aoki & Ando, 1986;
Huo & Bhatt, 1992;
Yang & Bhatt, 1999)

QHE (von Klitzing const) → fine-structure const

Fine-structure const, $\alpha = e^2/hc$ + Josephson const $K_J = 2e/h$ → Planck's const, h

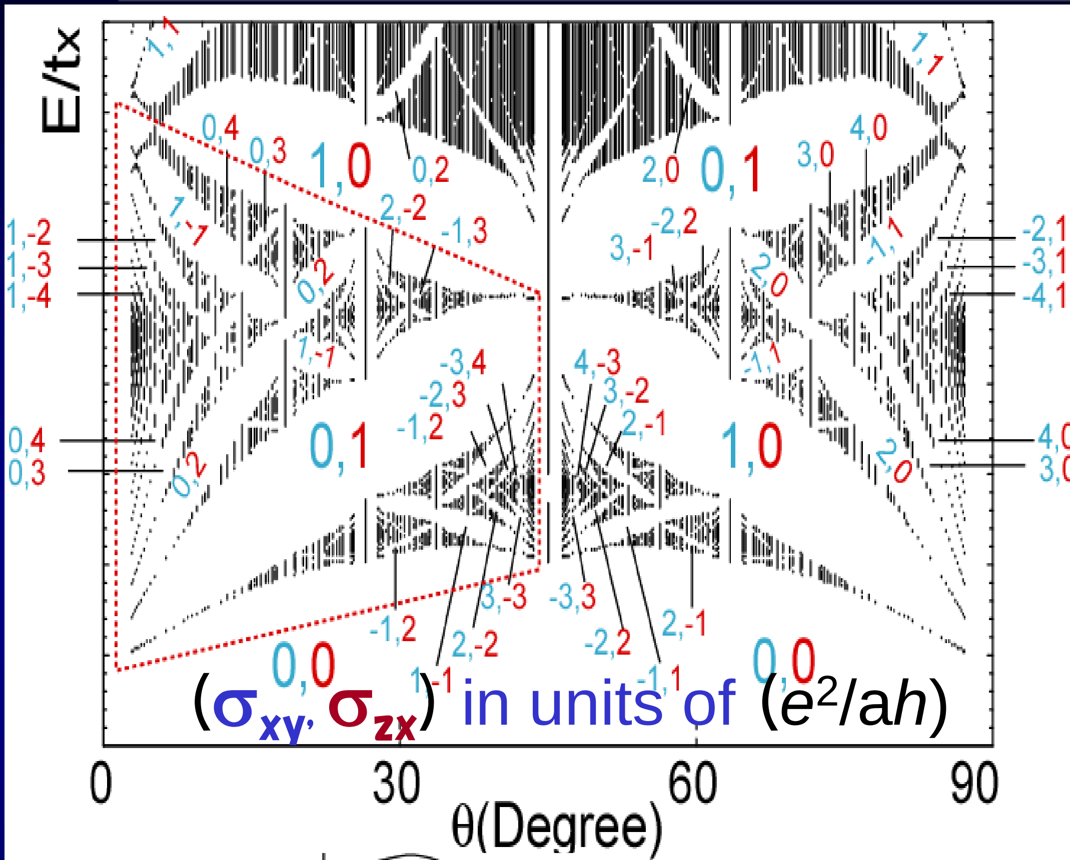


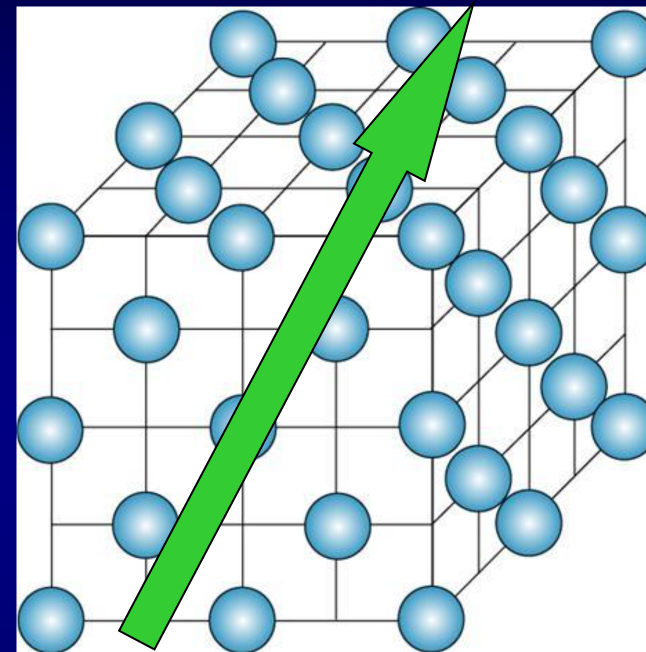


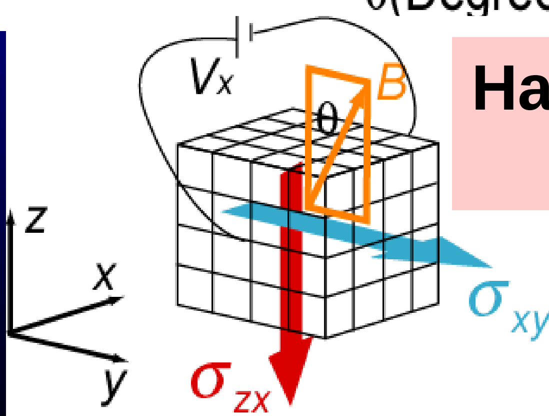
(Mohr et al, RMP 2008)

Integer QHE in 3D periodic systems

(Koshino et al, 2001; 2003)







Hall current in 3D

$$(\mathbf{j} = -\hat{\boldsymbol{\sigma}} \times \mathbf{E})$$

QHE

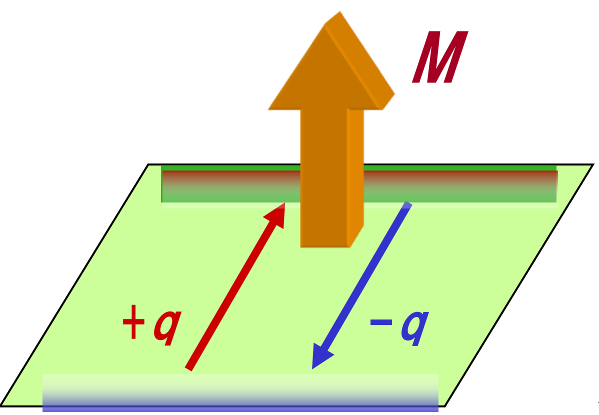
Berry's phase in Ψ

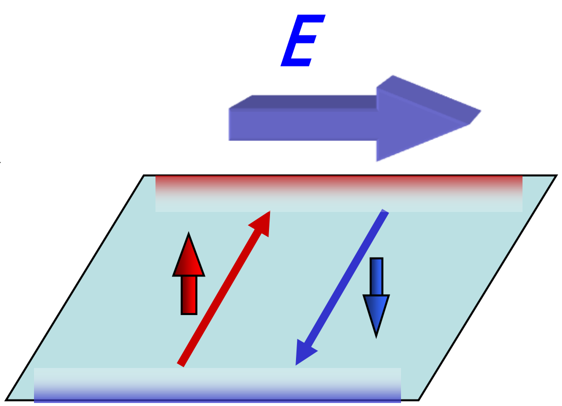
spin

polarisation

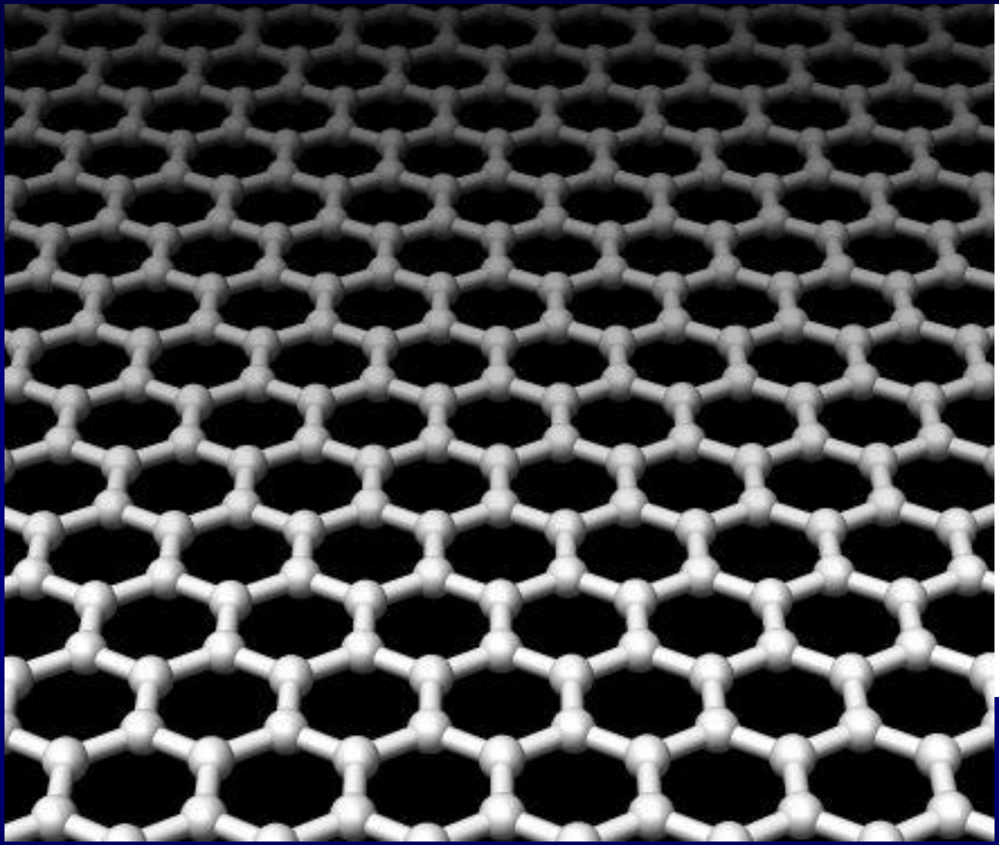
(Resta, 1992, '98;
King-Smith & Vanderbilt, 1993)

AHE **SHE**

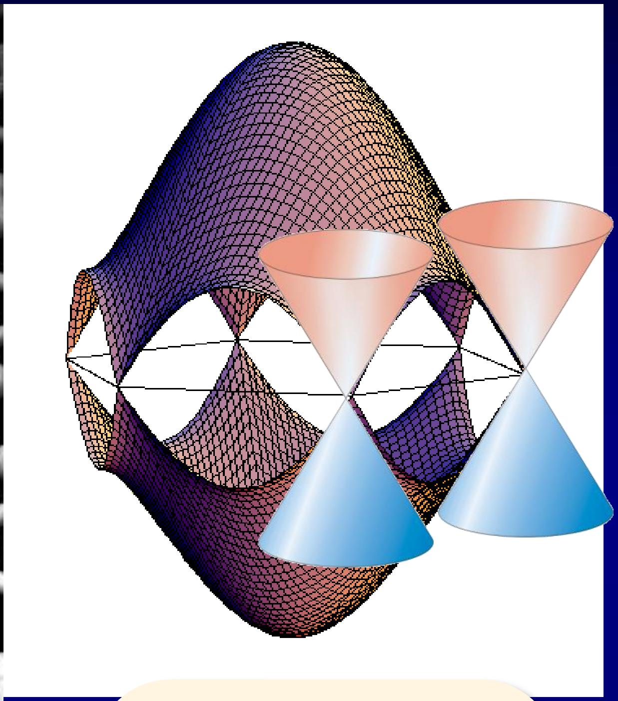




Graphene – massless Dirac

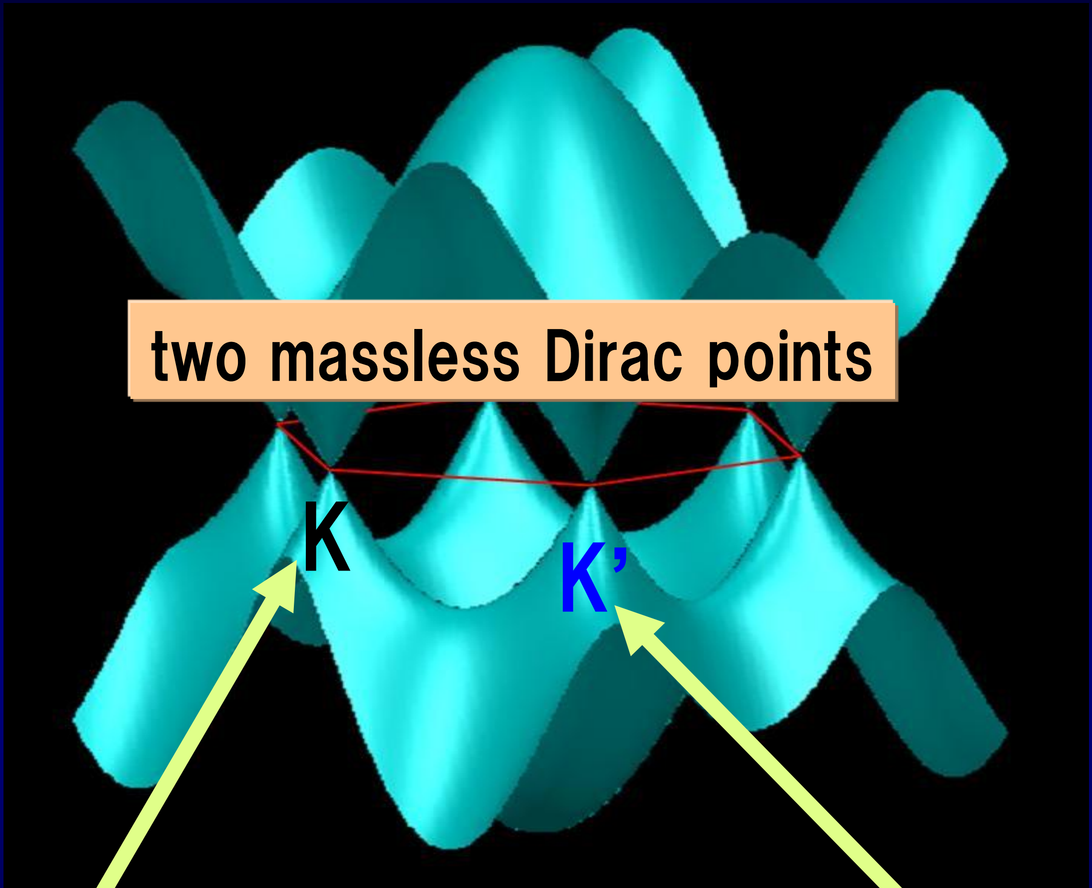


(Geim)



Weyl neutrino

Graphene – Dirac eqn



two massless Dirac points

Effective-mass formalism

$$H_K = v_F(\sigma_x p_x + \sigma_y p_y) = v_F \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

$$H_{K'} = v_F(-\sigma_x p_x + \sigma_y p_y) = v_F \begin{pmatrix} 0 & p_x + ip_y \\ p_x - ip_y & 0 \end{pmatrix}$$

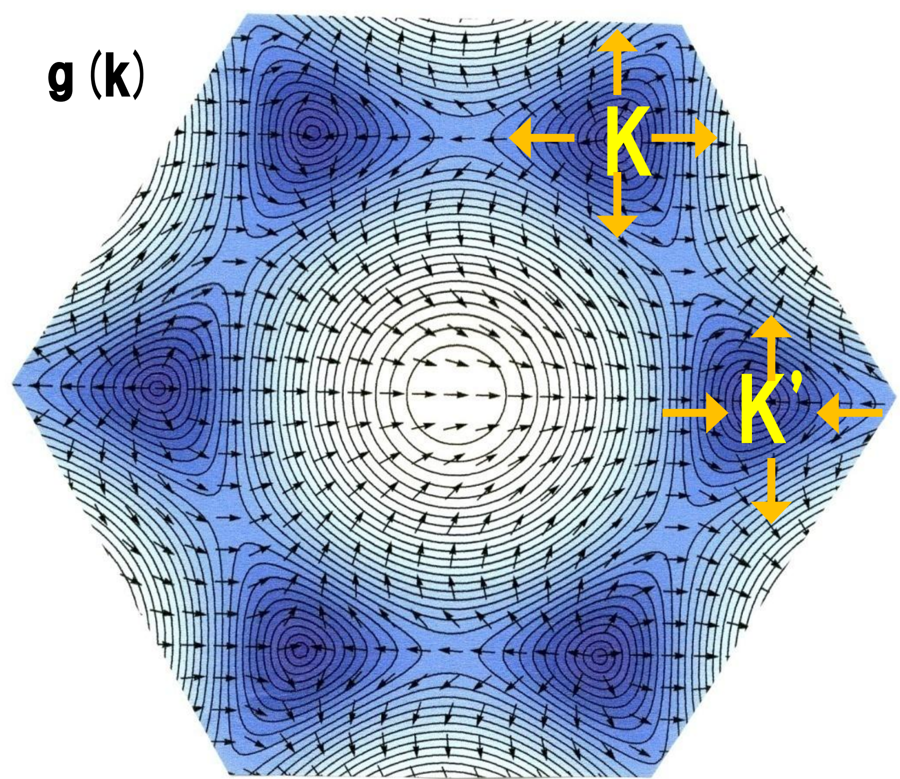
Chirality in graphene

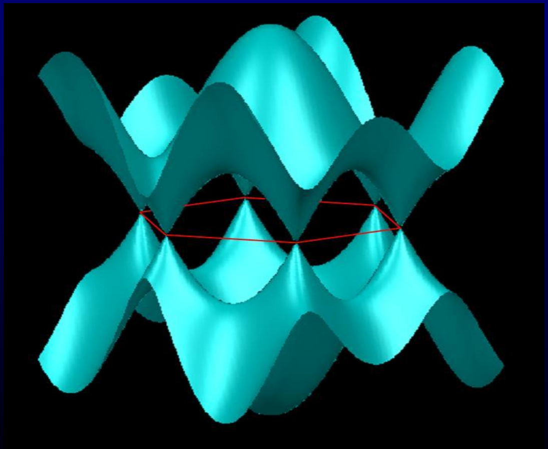
$\mathcal{H} = \sum c_k^{+} g_\mu(\mathbf{k}) \gamma_\mu c_k$

→ eigenvalues: $\pm m|g(\mathbf{k})|$, m: integer
→ degeneracy at $g(\mathbf{k}) = 0$.

(3+1)	(2+1)
γ_0	σ_1
γ_1	σ_2
γ_2	
γ_3	
γ_4	
γ_5	σ_3

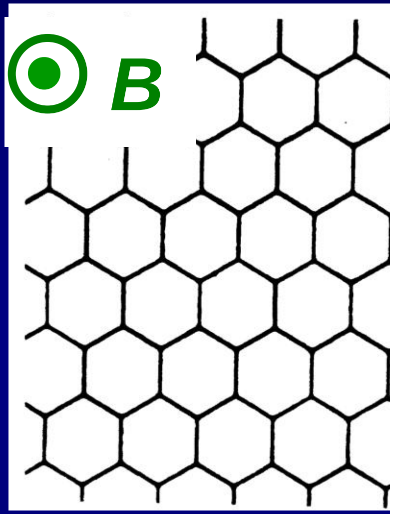
Honeycomb lattice

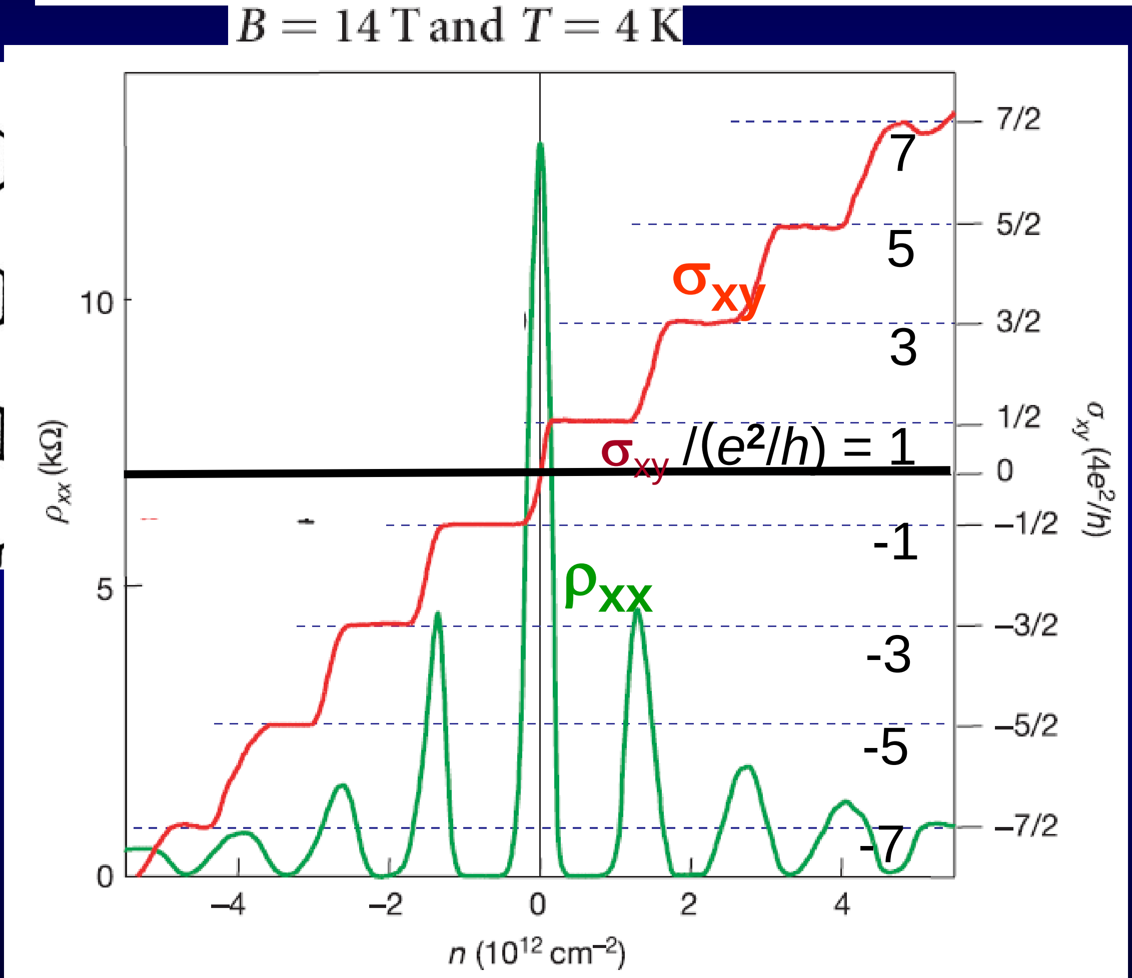




QHE in honeycomb lattice

(Novoselov *et al*; Zhang *et al*, Nature 2005)





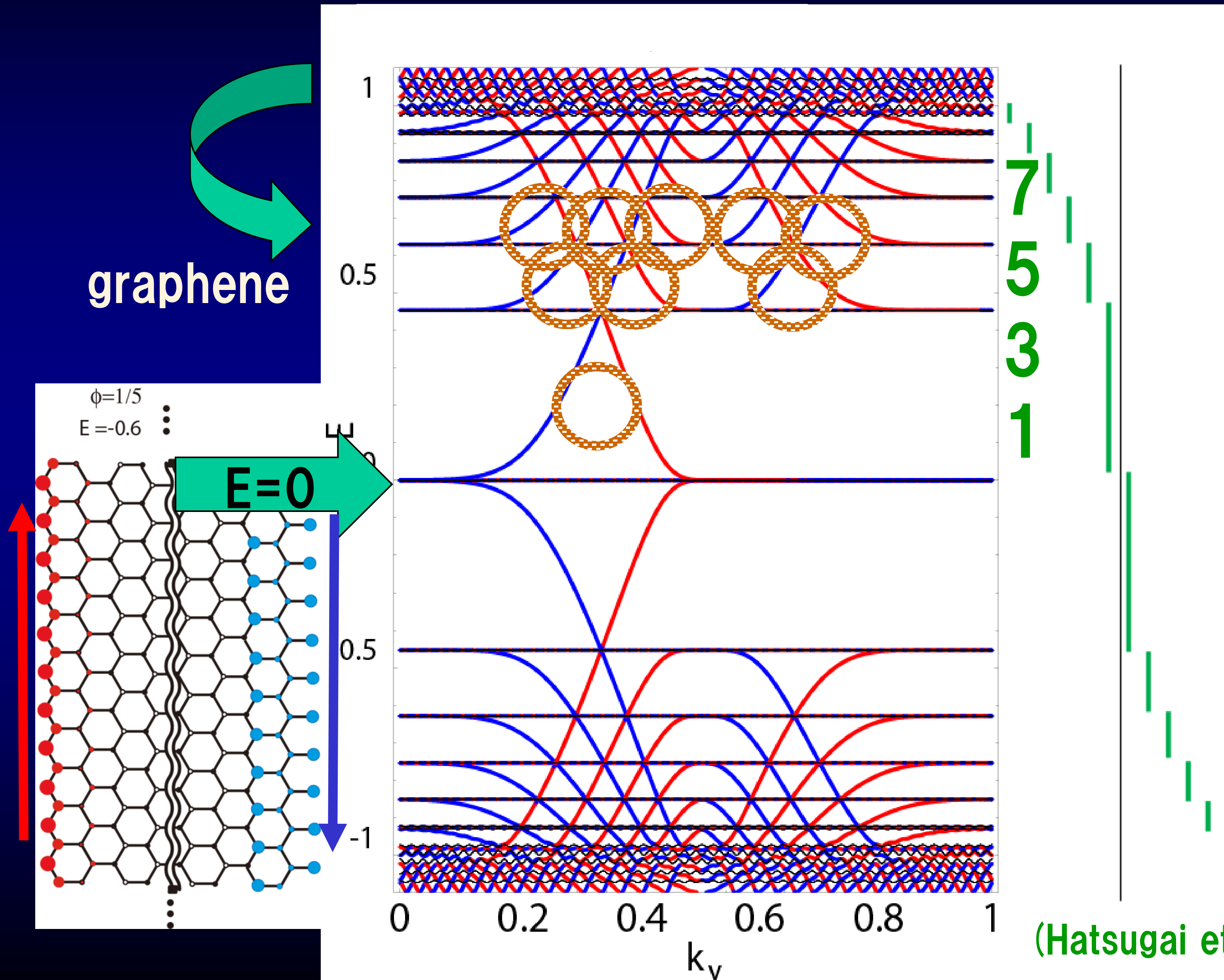
$\sigma_{xy}^{bulk} = \sigma_{xy}^{edge}$

Ordinary QHE:

$\sigma_{xy}^{bulk} = \sigma_{xy}^{edge}$ Hatsugai, 1993

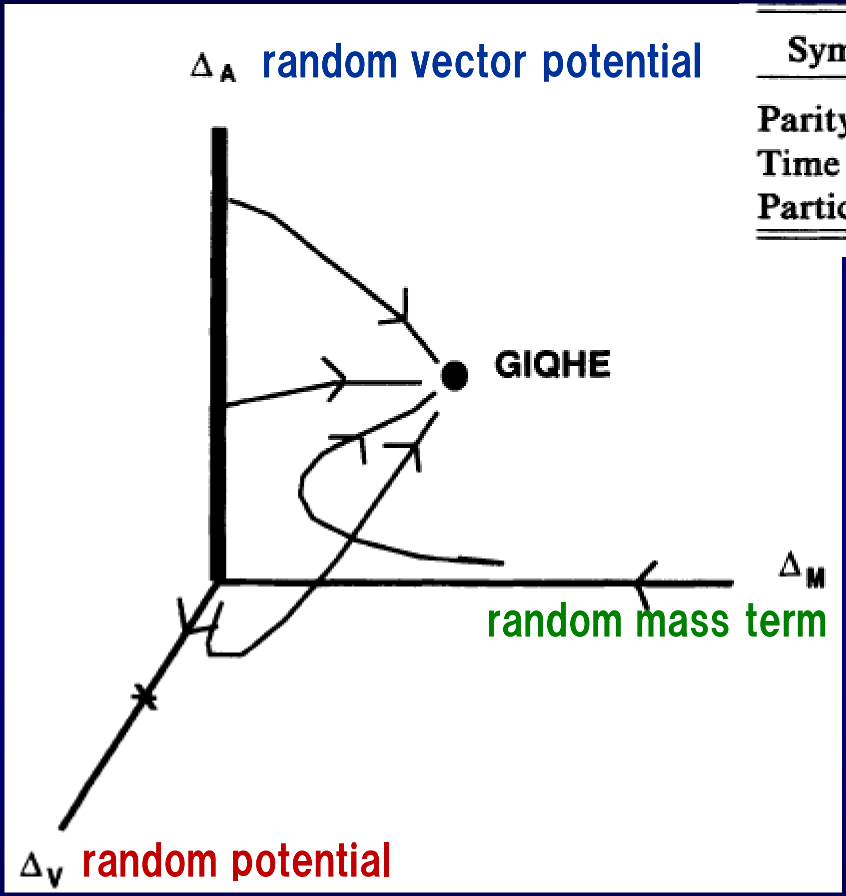
with Laughlin's argument

$\sigma_{xy}^{bulk} = \sigma_{xy}^{edge}$ *in honeycomb*



(Hatsugai et al, 2006)

Dirac particles with randomness



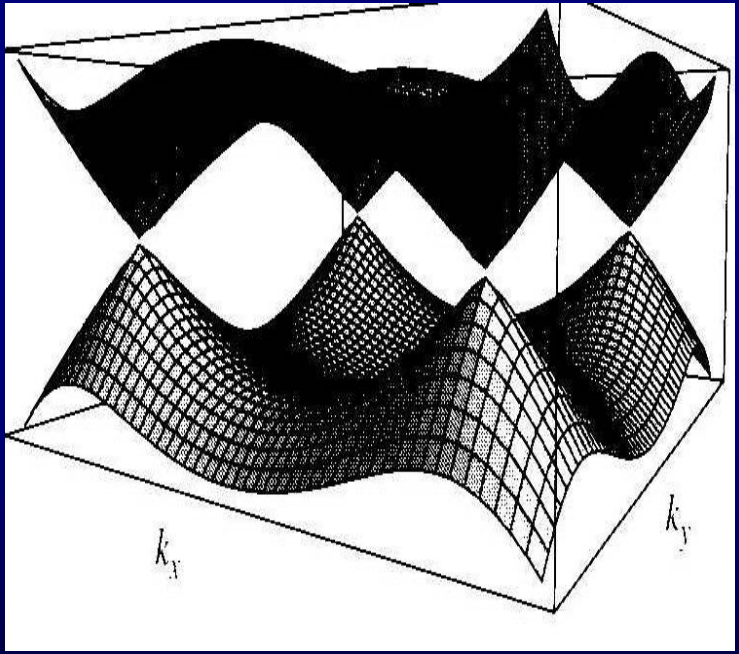
Symmetry	$\sigma\cdot p$	$M\sigma_z$	$M(x,y)\sigma_z$	$V(x,y)$	$\sigma\cdot A$
Parity	Yes	No	No	No	No
Time reversal	Yes	No	No	Yes	No
Particle hole	Yes	No	No	No	Yes

See also the universality classes due to Evers & Mirlin, RMP (2008)

(Ludwig et al, 1994)

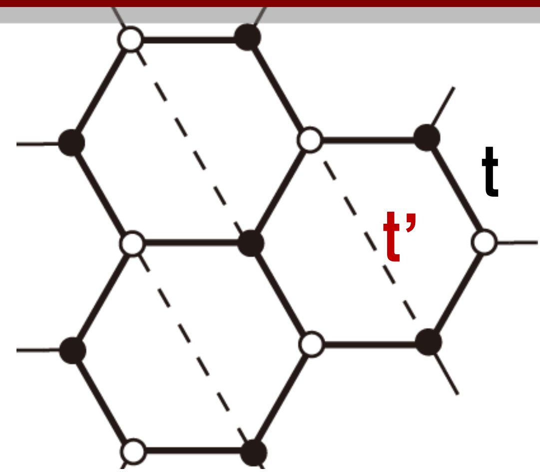
Dirac cones in other / higher-D systems ?

d-wave superconductors

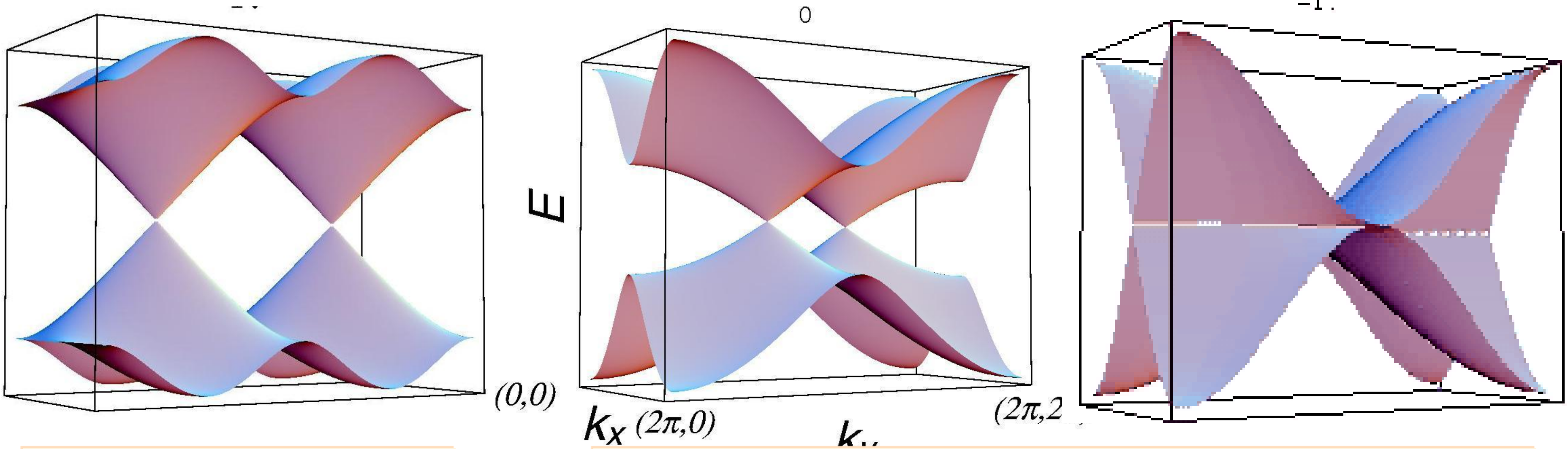


Dispersion of the quasi-particle in the Bogoliubov Hamiltonian
→ edge states

"Massless Dirac" sequence



t'=−1: π−flux lattice ↔ t'=0: honeycomb ↔ t'=+1: square



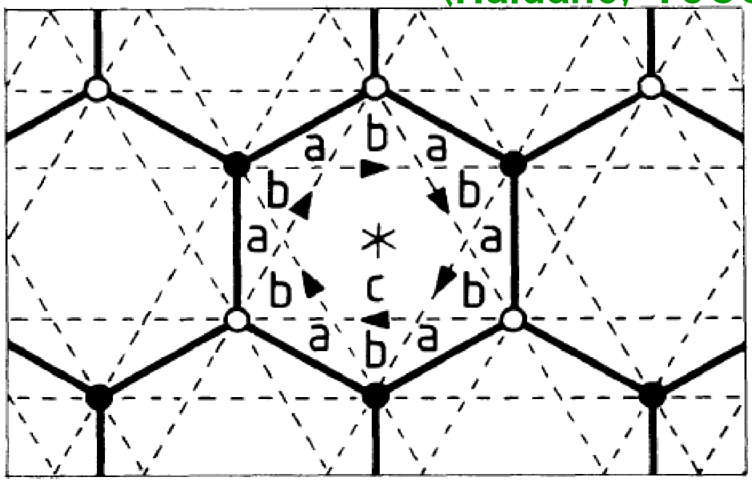
* Flux phase
  cf Kogut−Susskind

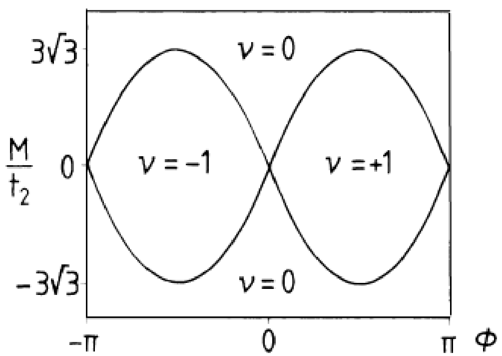
* Dirac cones seem to always appear in pairs
  -- Nielsen−Ninomiya

Can we go around Nielsen-Ninomiya ?

QHE without Landau levels

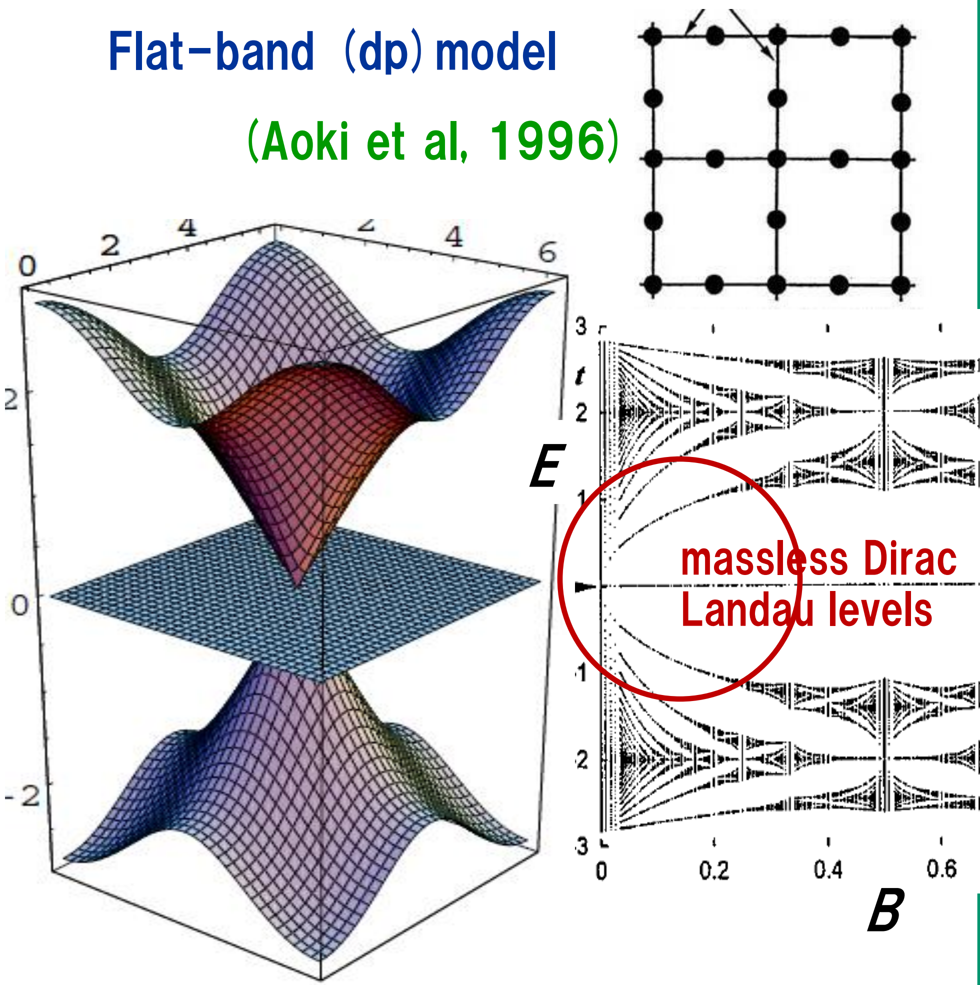
(Haldane, 1988)





Flat-band (dp) model

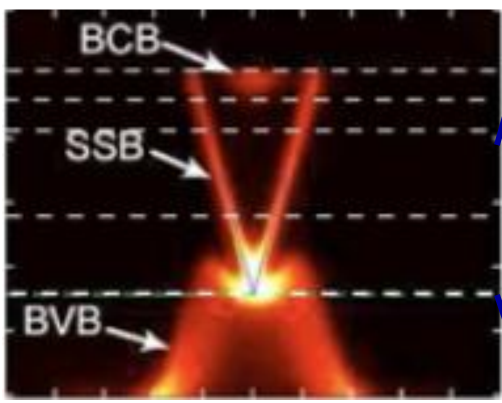
(Aoki et al, 1996)

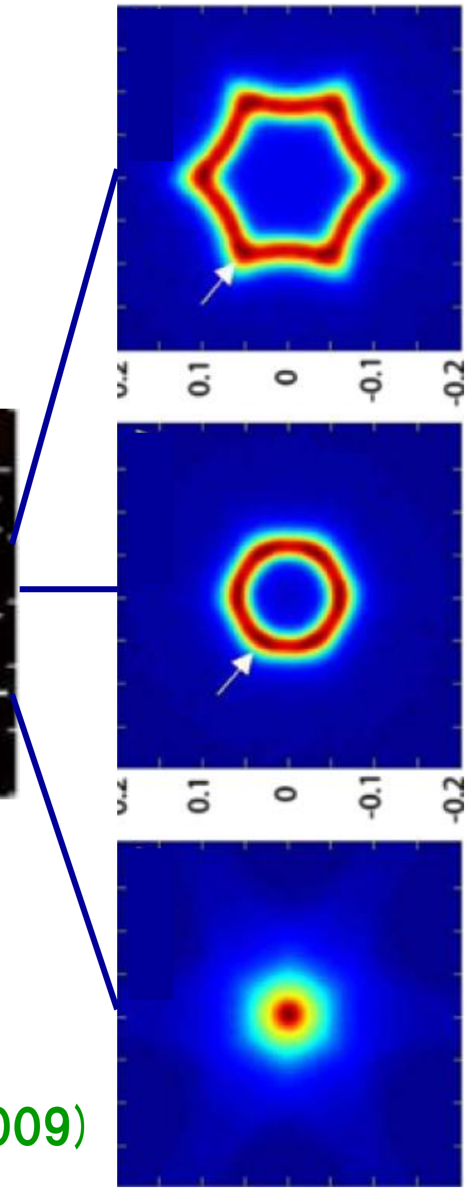


"Dirac cones" in 3D crystals

Bi_2Te_3

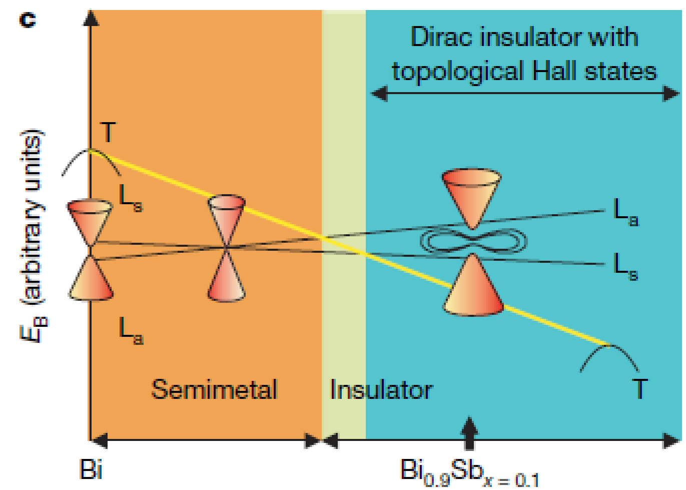
(Liang et al 2008; Chen et al 2009)





(Alpichshev et al 2009)

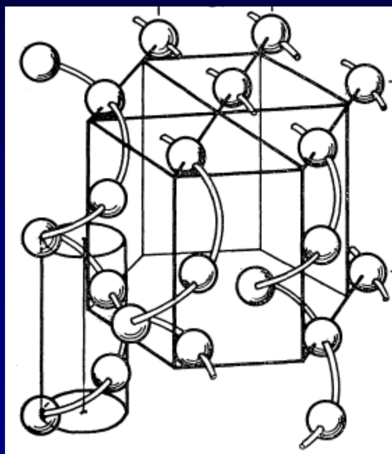
$Bi_{1-x}Sb_x$



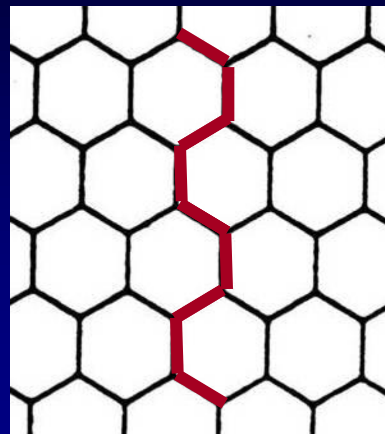
(Hsieh et al, 2008)

D-dimensional diamond

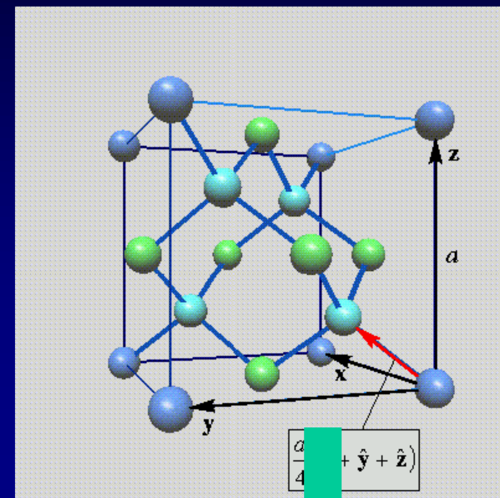
1D (Se, Te)



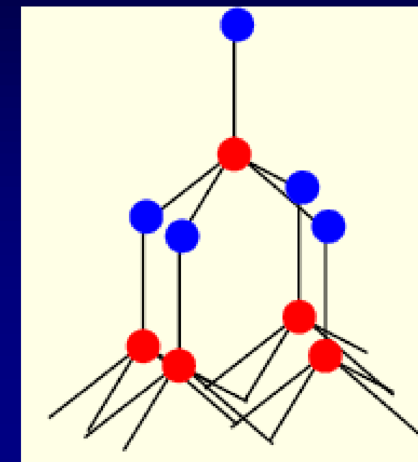
2D (graphite)

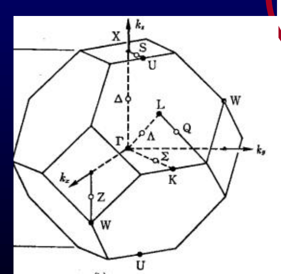


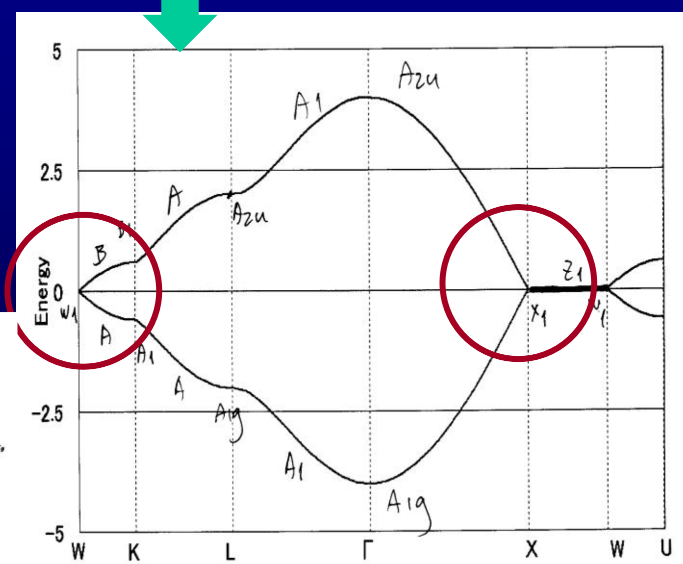
3D (diamond)



4D (Creutz 2008)



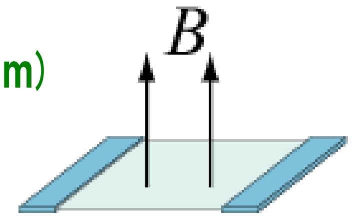




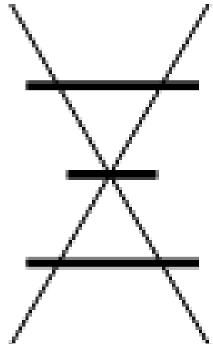
Wider Hall effects in Dirac systems

1. QHE

(Geim; Kim)



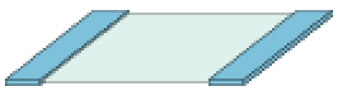
graphene



Landau level

2. Spin Hall effect in topological insulators

(Kane & Mele, 2005)

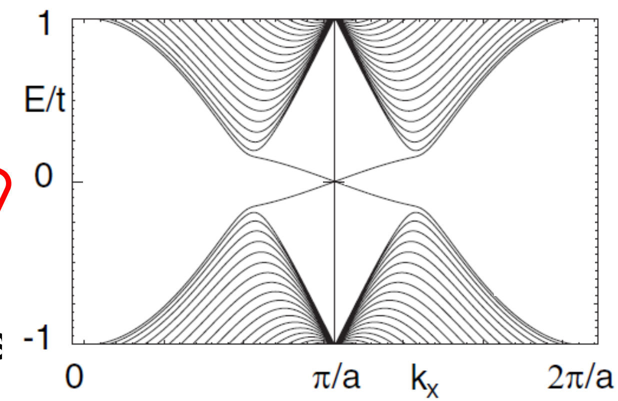


+ spin-orbit

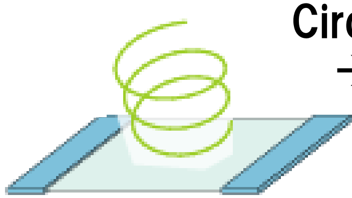
$$H = \begin{pmatrix} m_{\uparrow,\downarrow} & k_x - ik_y \\ k_x + ik_y & -m_{\uparrow,\downarrow} \end{pmatrix}$$

mass term ← spin-orbit

Spin-orbit too small in graphene; rather, HgTe systems



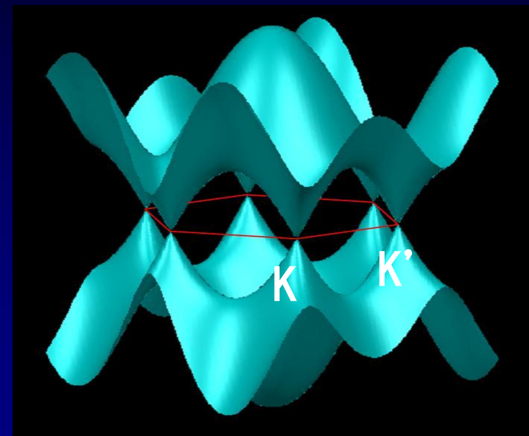
3. Photovoltaic Hall effect



Circularly-polarised light
→ breaks time-reversal

(Oka & Aoki, 2009)

Wave propagation in graphene

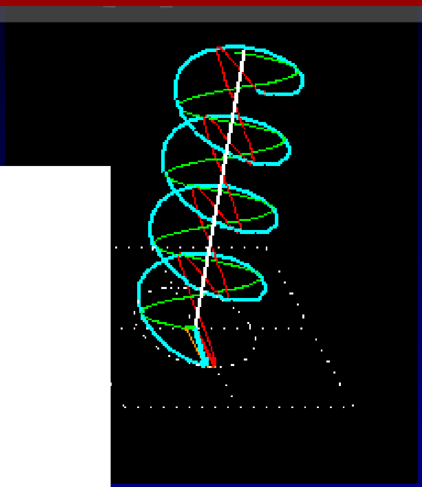


$B = 0$

time evolution in a honeycomb lattice

Wave propagation in a circularly polarised light

honeycomb lattice
+ circularly polarised light （B = 0）



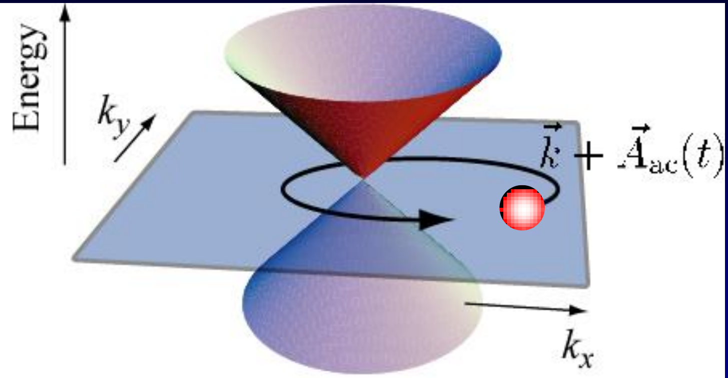
B = 0

© Oka 2009

Wavefunctions evolve more slowly, $R(t) \sim t^{1/2}$

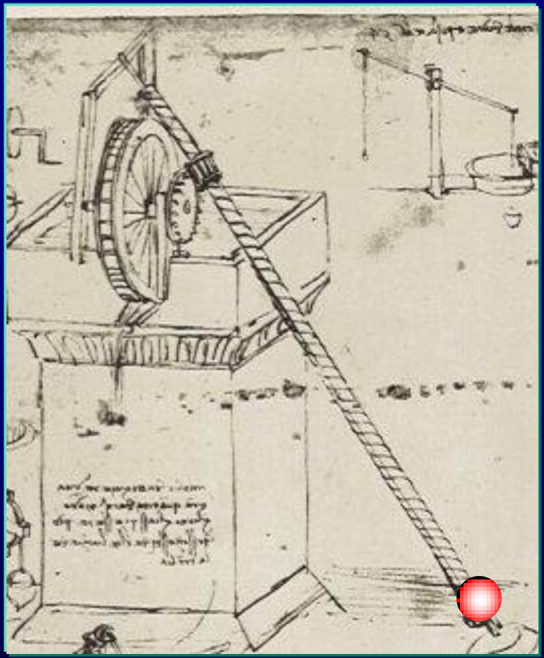
Dynamical mass opens

DC response in AC fields – Geometric phase

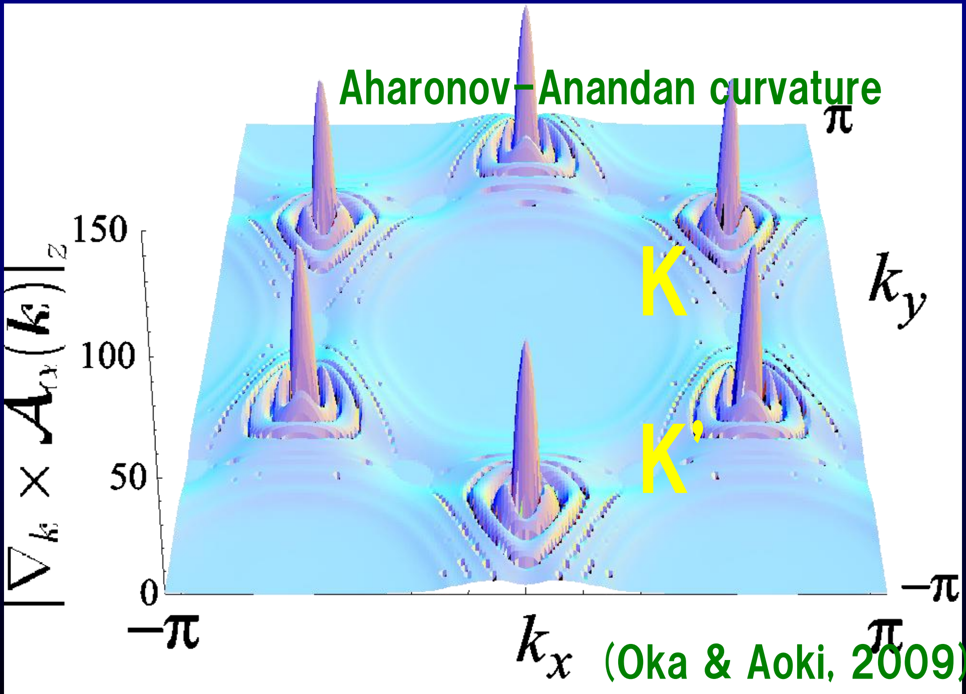


ac field → *k*-point encircles the Dirac points

→ Aharonov-Anandan phase
→ Non-adiabatic charge pumping
→ Photovoltaic Hall effect (Oka & Aoki 09)

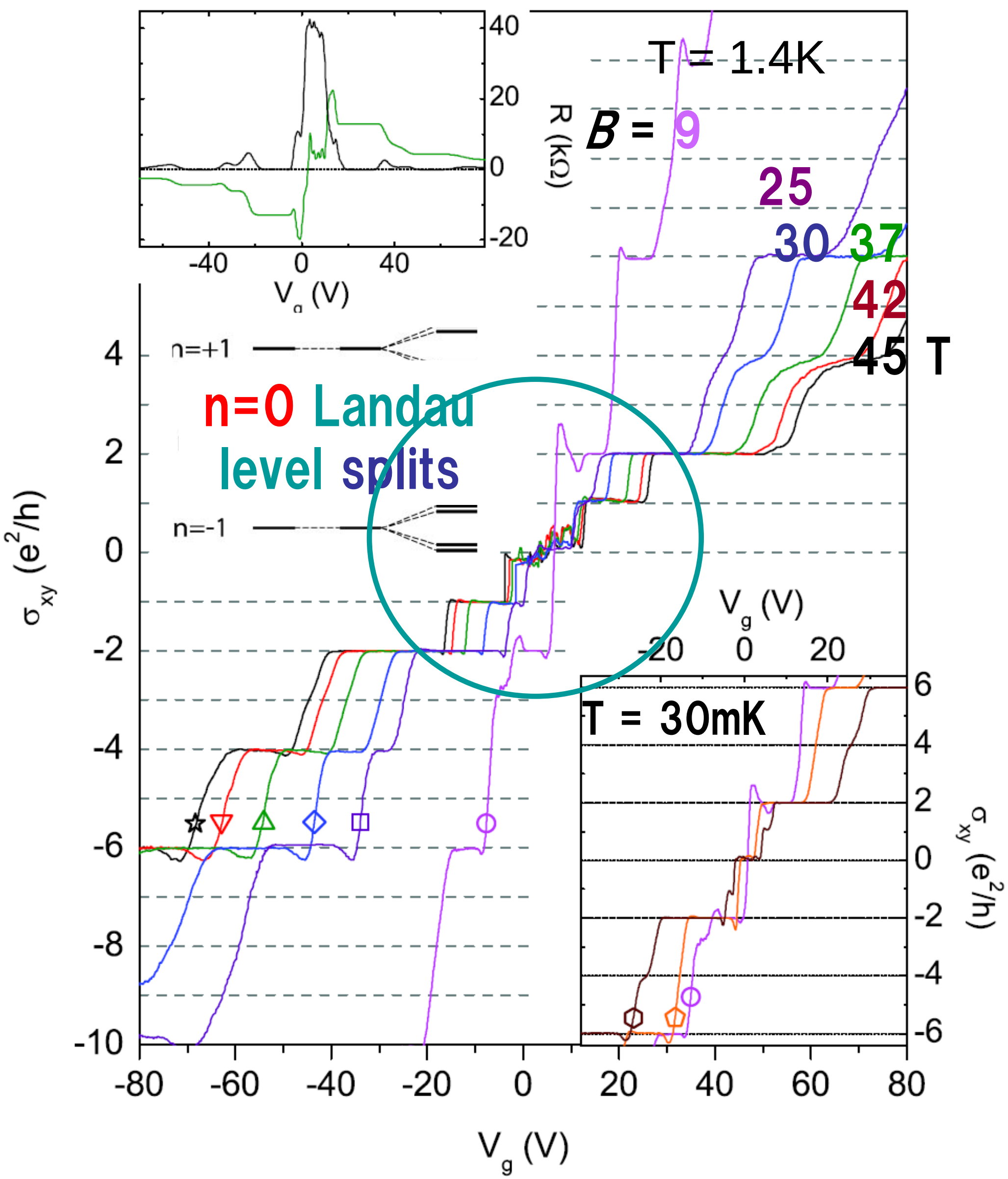


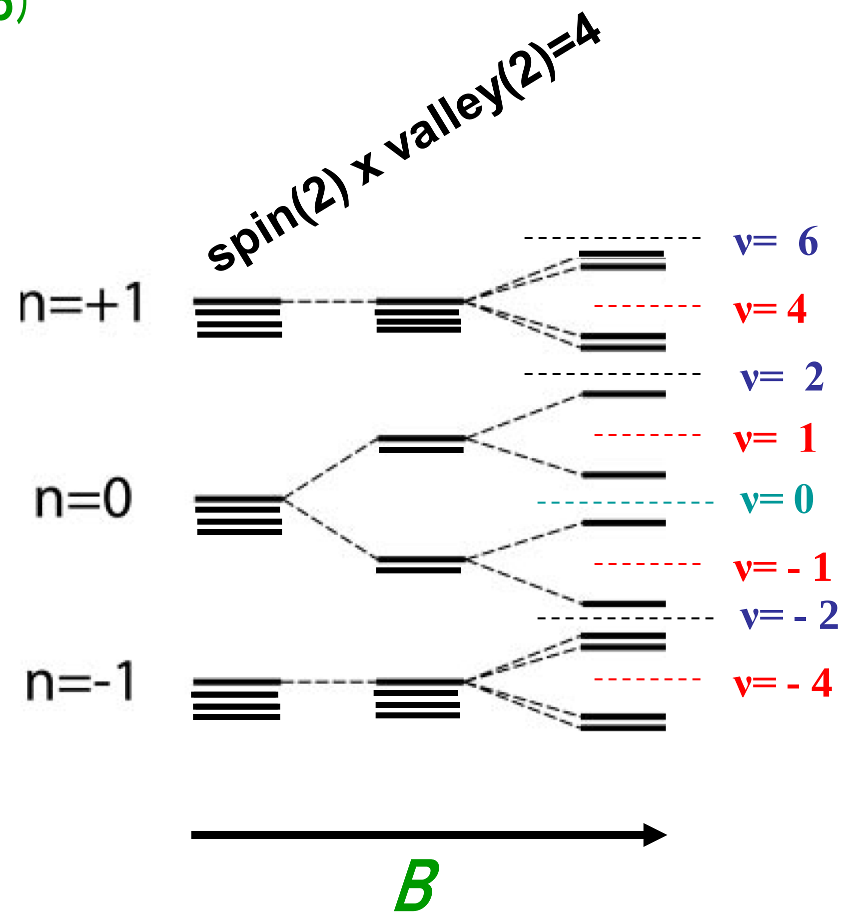
Can Berry`s phase still be defined in non-equilibrium?
--- Yes, Aharonov-Anandan phase (1987)



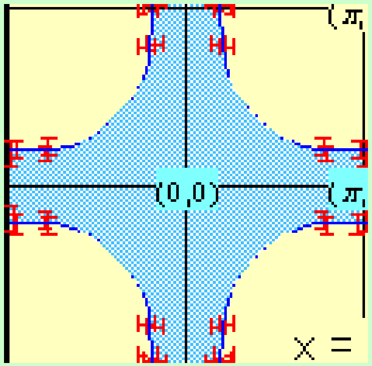
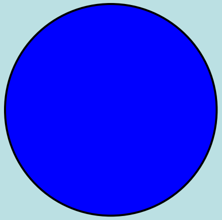
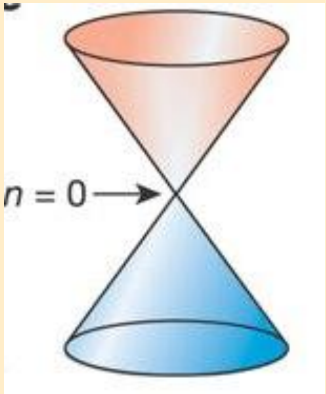
Splitting of graphene Landau levels

(Zhang et al, PRL 2006)





Correlated electron systems:

HTC	FQHE	graphene
Coulomb anisotropic	gauge field isotropic	
		

Mechanisms for split Landau levels in graphene

* Excitonic gap (Gusynin et al, 2006)

* FQHE (Apalkov & Chakraborty, 2006)

* SU(4)-breaking (Nomura & MacDonald, 2007)

* Peierls distortion (Fuchs & Lederer, 2007)

* Bond order (Hatsugai et al, 2007)

B = 0, Coulomb interaction

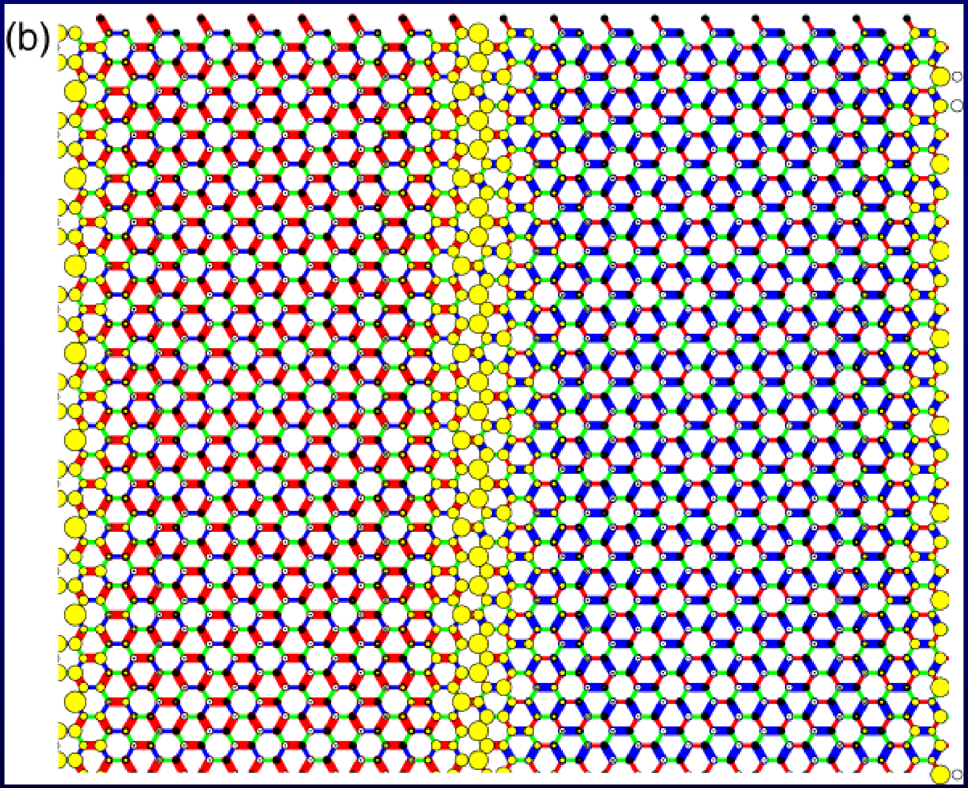
* Exciton condensate (~ chiral condensate in QCD)

(1/N + RG: Son 2007;
RG: Herbut 2009;
Schwinger-Dyson: Khveshchenko 2009;
Monte Carlo: Hands 2008, Drut 2009)

Topological quantum liquid ?

(Hatsugai et al, 2007)

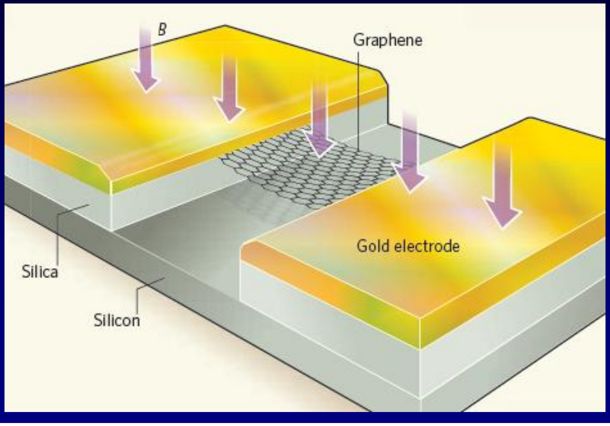
bond-order a la Affleck-Marston 1988 $\langle c_a^\dagger c_b \rangle$

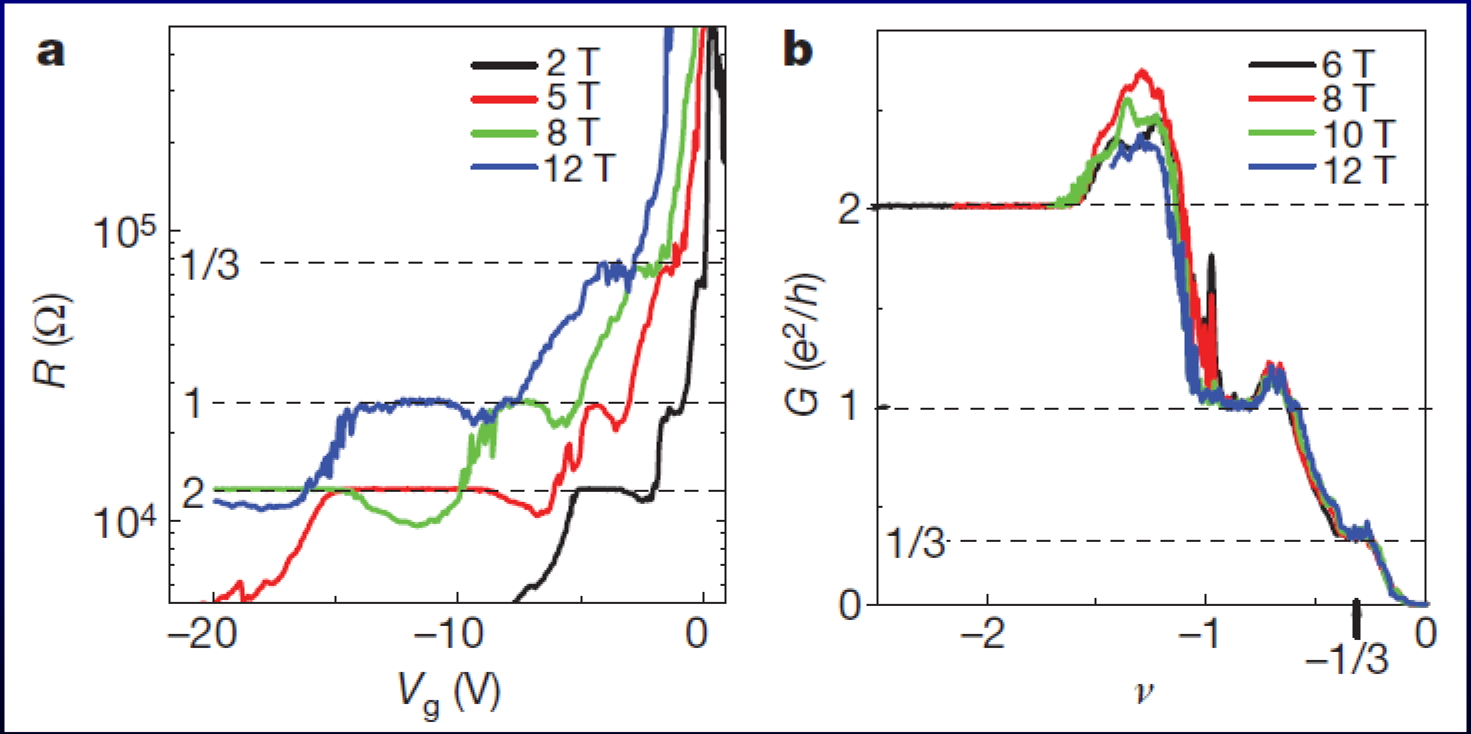


→ string net condensation → quantum liquid ?

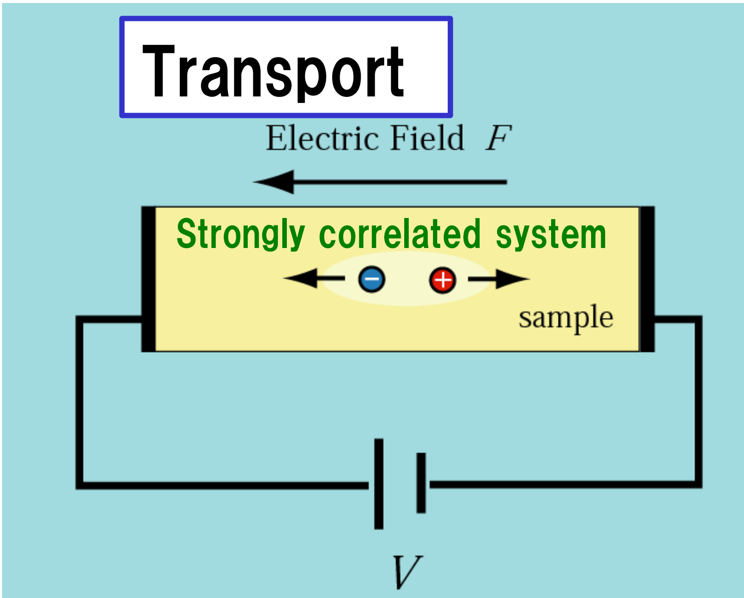
FQHE in graphene

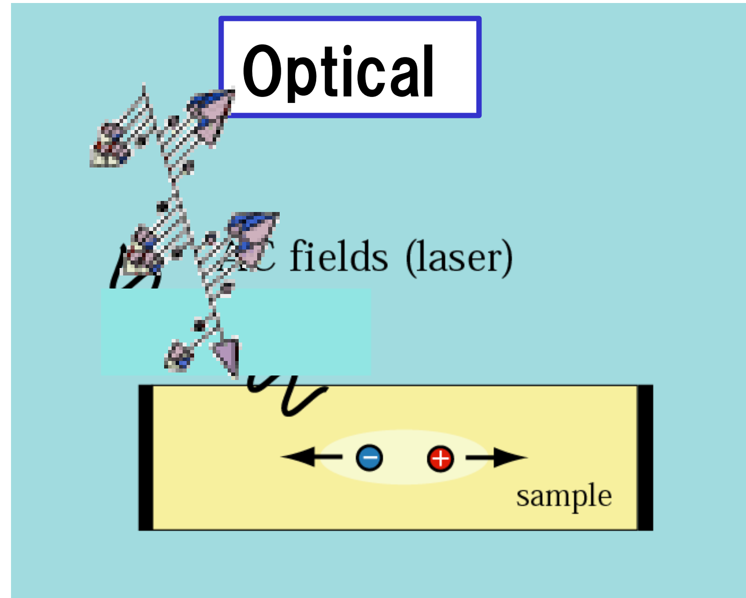
(Suspended graphene: Du et al; Bolotin et al, 2009)

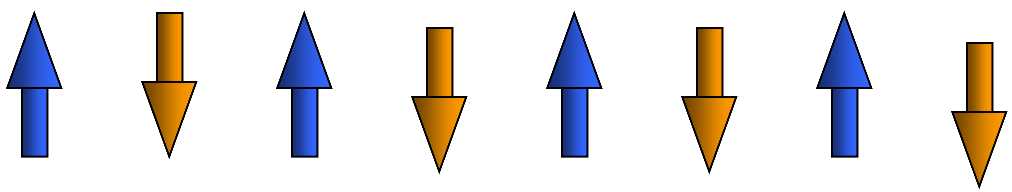




Out of equilibrium



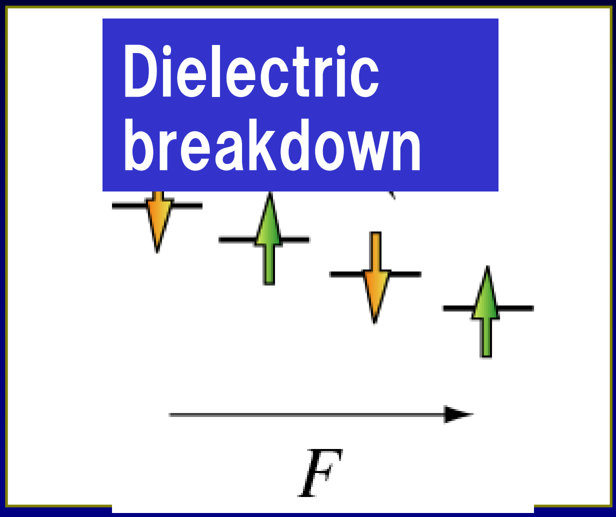


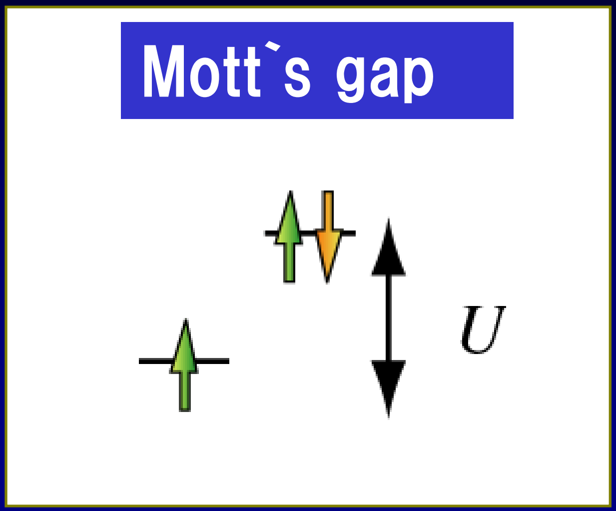


Production of carriers in crossing the phase boundaries:

Schwinger mechanism for QED vacuum decay

* Mott insulator - metal (Oka, 2003; 2005)
* Superfluid - Mott insulator (Sondhi et al, 2005)





Non-adiabatic quantum tunnelling

$$J \propto p \sim e^{-\alpha \Delta(U)^2/F}$$

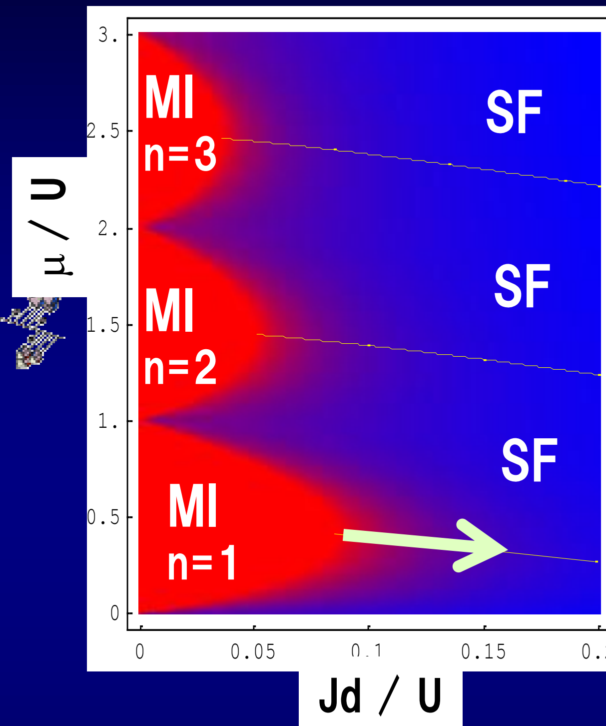
Mott transition

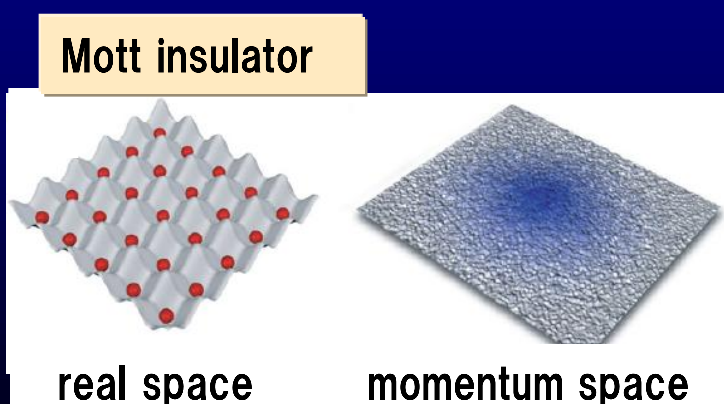
$$\Delta \sim \sqrt{tU} \exp(-2\pi t/U)$$

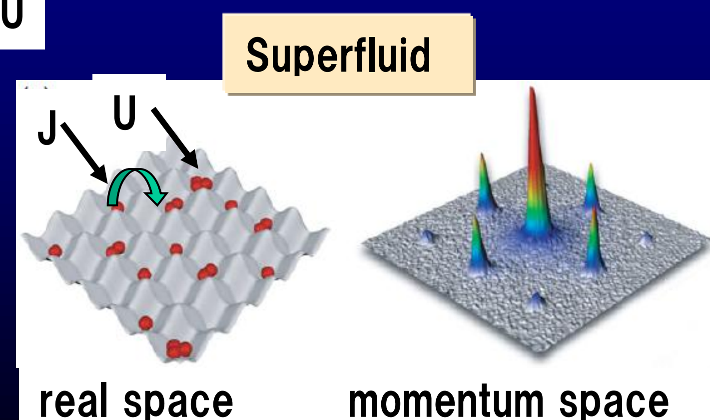
Two non-perturbative effects

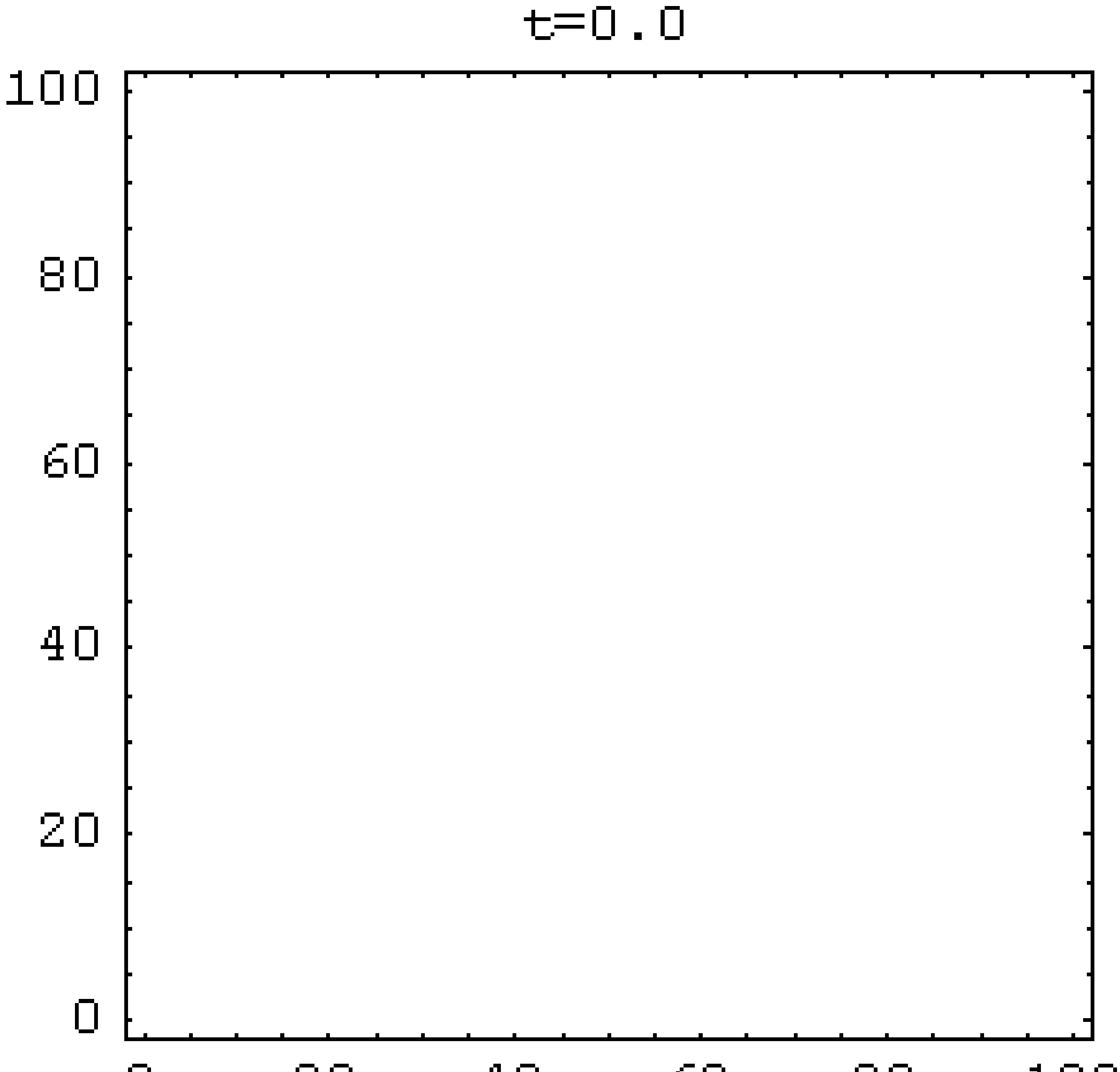
(Oka et al, 2003; 2005)

Bose-Hubbard for cold atoms in optical lattices – an ideal playground for nonequilibrium





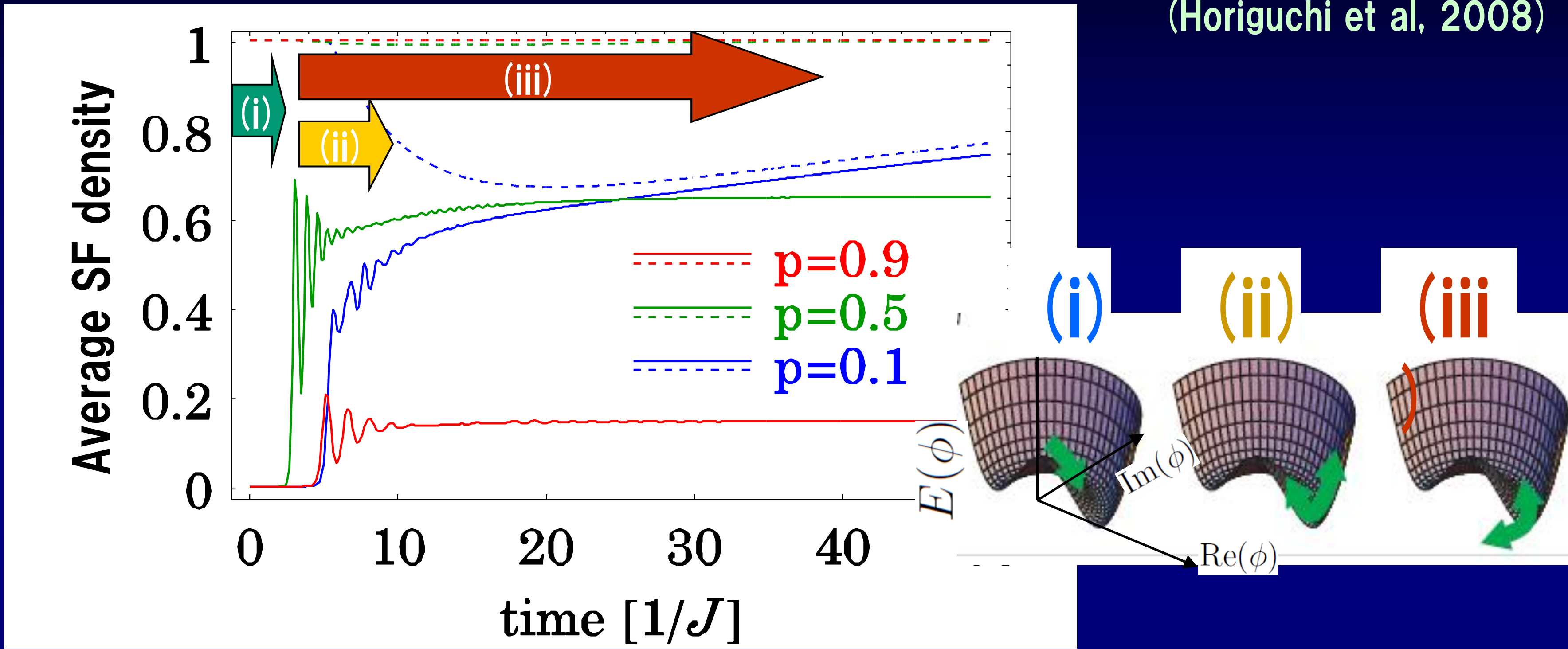




(Horiguchi et al, 2008)

Kibble-Zurek for Mott→superfluid

(Horiguchi et al, 2008)



continuous symmetry broken in the condensate $\phi=|\phi|e^{i\theta}$
→ topological defects, Kibble-Zurek mechanism

Summary

interaction

+0 SC, M, QHE, … ∞

dimensionality

1 2D 3D ∞

equilibrium non-equilibrium