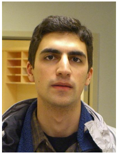
# A unification of photons, electrons, and gravitons under qbit models

Xiao-Gang Wen, MIT

- Quantum ether: photons and electrons from a rotor model
  – Phys. Rev. B73, 035122 (2006)
- Emergence of helicity $\pm 2$ modes (gravitons) from qbit models
  – arXiv:0907.1203



M. Levin

Z.-C. Gu

# Seven basic assumptions

- The current physical theory explain a very wide range of phenomena from some simple "starting points".
  It unifies everything into seven fundamental assumptions – seven wonders of our universe:
  (1) Locality.
  (2) Identical particles.
  (3) Gauge interactions.
  (4) Fermi statistics.
  (5) Chiral fermions. ($SU(2)$ only couples to left-hand fermions)
  (6) Lorentz invariance.
  (7) Gravity.

## Seven basic assumptions

- The current physical theory explain a very wide range of phenomena from some simple "starting points".
  It unifies everything into seven fundamental assumptions – seven wonders of our universe:
  (1) Locality.
  (2) Identical particles.
  (3) Gauge interactions.
  (4) Fermi statistics.
  (5) Chiral fermions. ($SU(2)$ only couples to left-hand fermions)
  (6) Lorentz invariance.
  (7) Gravity.

  **Can we unify further to obtain all seven wonders from a single simple structure? Can we obtain the 6 other assumptions from the Locality alone?**

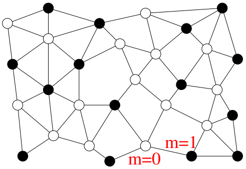
# Seven basic assumptions

- The current physical theory explain a very wide range of phenomena from some simple "starting points".
  It unifies everything into seven fundamental assumptions – seven wonders of our universe:
  (1) Locality.
  (2) Identical particles.
  (3) Gauge interactions.
  (4) Fermi statistics.
  (5) Chiral fermions. ($SU(2)$ only couples to left-hand fermions)
  (6) Lorentz invariance.
  (7) Gravity.

  **Can we unify further to obtain all seven wonders from a single simple structure? Can we obtain the 6 other assumptions from the Locality alone?**
- **Everything has to come from something**

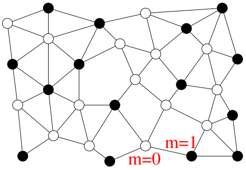
# Emergence approach – everything from qbits

- One type of fundamental building blocks for everything: **qbits**
- Space = collection of $10^{183}$ qbits. No qbits, no space
- Empty space (vacuum) = ground state of qbits: $\Phi_0(\{m_i\})$.
- "Elementary" particles = collective excitations (such as topological defects, collective waves) above the ground states: $\Phi(\{m_i\})$.



# Emergence approach – everything from qbits

- One type of fundamental building blocks for everything: **qbits**
- Space = collection of $10^{183}$ qbits. No qbits, no space
- Empty space (vacuum) = ground state of qbits: $\Phi_0(\{m_i\})$.
- "Elementary" particles = collective excitations (such as topological defects, collective waves) above the ground states: $\Phi(\{m_i\})$.



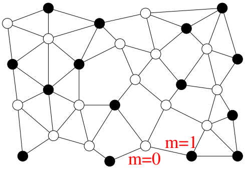

**Originate from organization (order)**

To understand the 6 other wonders from qbit model

- The issue is not *"what are the elementary building blocks"*.
  The elementary building block is known: **qbits**
- The issue is *"how the qbits are organized"*.
  The organizations (orders) of qbits = **origin of the 6 wonders**

# Emergence approach – everything from qbits

- One type of fundamental building blocks for everything: **qbits**
- Space = collection of $10^{183}$ qbits. No qbits, no space
- Empty space (vacuum) = ground state of qbits: $\Phi_0(\{m_i\})$.
- "Elementary" particles = collective excitations (such as topological defects, collective waves) above the ground states: $\Phi(\{m_i\})$.




**Originate from organization (order)**

To understand the 6 other wonders from qbit model

- The issue is not *"what are the elementary building blocks"*.
  The elementary building block is known: **qbits**
- The issue is *"how the qbits are organized"*.
  The organizations (orders) of qbits = **origin of the 6 wonders**
- **Different orders of qbits correspond to different phases, and different universes with different "elementary" particles**

# Can the six wonders emerge from an organization

- Old picture of phases and phase transitions:
  All orders are described by symmetry breaking.

- The symmetry breaking states can only give rise to bosonic collective excitations described by bosonic field $\sim$ order parameters.
  But no gauge bosons and fermions.

# Can the six wonders emerge from an organization

- Old picture of phases and phase transitions:
  All orders are described by symmetry breaking.

- The symmetry breaking states can only give rise to bosonic collective excitations described by bosonic field $\sim$ order parameters. But no gauge bosons and fermions.

  *How to get gauge bosons and fermions from qbit models?*

# Can the six wonders emerge from an organization

- Old picture of phases and phase transitions:
  All orders are described by symmetry breaking.

- The symmetry breaking states can only give rise to bosonic collective excitations described by bosonic field $\sim$ order parameters. But no gauge bosons and fermions.

  *How to get gauge bosons and fermions from qbit models?*

- The symmetry breaking states are "trivial" unentangled states:
  $|\text{symm. breaking}\rangle = |\uparrow\rangle \otimes |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes ...$
- Unentangled direct-product states are very special states. They are not most general quantum many-qbit states.

  Maybe gauge bosons and fermions can emerge from more general highly entangled many-qbit quantum states.

# New states of matter with long range entanglements exist

**Gapped states** (topological order Wen 1989):

- Many fractional quantum Hall states. Tsui, Stormer, Gossard 1982
- Many superconducting states ($p + ip$, $d + id$, ...) Read, Green, 2000
- Many Mott-insulators = gapped spin liquids. Wen etal 89; Moessner, Sondhi 03

# New states of matter with long range entanglements exist

**Gapped states** (topological order Wen 1989):

- Many fractional quantum Hall states. Tsui, Stormer, Gossard 1982
- Many superconducting states ($p + ip$, $d + id$, ...) Read, Green, 2000
- Many Mott-insulators = gapped spin liquids. Wen etal 89; Moessner, Sondhi 03

**Symmetry protected topological order**:

- Haldane phase of spin-1 chain (2 kinds protected by $P$). Haldane 1982
- Topological insulators (2 kinds by $T$). Kane, Mele 2005; Bernevig etal 2006
- Topological superconductors (2 kinds by $T$, Roy 06; Qi etal 09; Sato etal 09. 256 kinds by $T_{x,y}$ Kou, Wen 09).

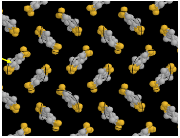
# New states of matter with long range entanglements exist

**Gapped states** (topological order Wen 1989):

- Many fractional quantum Hall states. Tsui, Stormer, Gossard 1982
- Many superconducting states ($p + ip$, $d + id$, ...) Read, Green, 2000
- Many Mott-insulators = gapped spin liquids. Wen etal 89; Moessner, Sondhi 03

**Symmetry protected topological order**:

- Haldane phase of spin-1 chain (2 kinds protected by $P$). Haldane 1982
- Topological insulators (2 kinds by $T$). Kane, Mele 2005; Bernevig etal 2006
- Topological superconductors (2 kinds by $T$, Roy 06; Qi etal 09; Sato etal 09. 256 kinds by $T_{x,y}$ Kou, Wen 09).



**Gapless states** (quantum order):

- Algebraic spin liquids ($ZnCu_3(OH)_6Cl_2$, Y. Lee, 06 $\kappa$-$(ET)_2X$ Kanoda, 03)
- High Tc superconductors (?) Barskaran, Zou, Anderson 87; Marston, Affleck 89

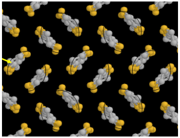
# New states of matter with long range entanglements exist

**Gapped states** (topological order Wen 1989):

- Many fractional quantum Hall states. Tsui, Stormer, Gossard 1982
- Many superconducting states ($p + ip$, $d + id$, ...) Read, Green, 2000
- Many Mott-insulators = gapped spin liquids. Wen etal 89; Moessner, Sondhi 03

**Symmetry protected topological order**:

- Haldane phase of spin-1 chain (2 kinds protected by $P$). Haldane 1982
- Topological insulators (2 kinds by $T$). Kane, Mele 2005; Bernevig etal 2006
- Topological superconductors (2 kinds by $T$, Roy 06; Qi etal 09; Sato etal 09. 256 kinds by $T_{x,y}$ Kou, Wen 09).



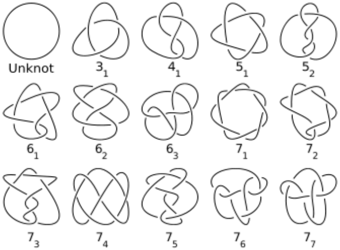
**Gapless states** (quantum order):

- Algebraic spin liquids ($ZnCu_3(OH)_6Cl_2$, Y. Lee, 06 $\kappa$-$(ET)_2X$ Kanoda, 03)
- High Tc superconductors (?) Barskaran, Zou, Anderson 87; Marston, Affleck 89
- **Can gauge bosons and fermions emerge from the new qbit states with long range entanglements?**

# Long range entanglement $\rightarrow$ fermions and gauge bosons

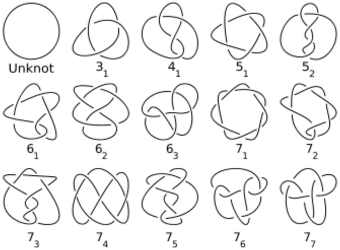
# Long range entanglement → fermions and gauge bosons

- Lord Kelvin's ele. particles = knotted strings of ether Kelvin 1867



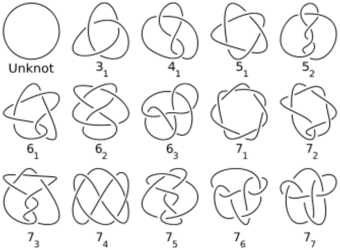
# Long range entanglement → fermions and gauge bosons

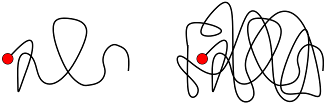
- Lord Kelvin's ele. particles = knotted strings of ether Kelvin 1867
  cannot be fermions. (Knots are unstable against local destruction.)



# Long range entanglement → fermions and gauge bosons

- Lord Kelvin's ele. particles = knotted strings of ether Kelvin 1867
  cannot be fermions. (Knots are unstable against local destruction.)



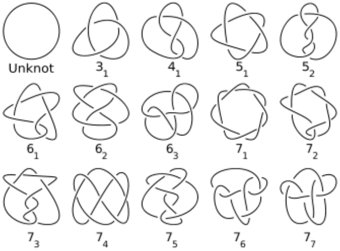



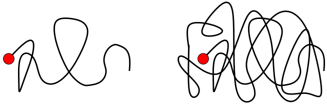
String-net picture of ele. particles (not knots but ends of strings):

- Electons/Quarks = ends of strings → produce Fermi statistics.

# Long range entanglement → fermions and gauge bosons

- Lord Kelvin's ele. particles = knotted strings of ether Kelvin 1867
  cannot be fermions. (Knots are unstable against local destruction.)



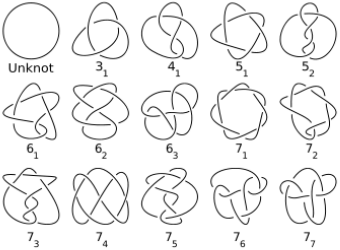



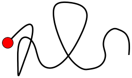
String-net picture of ele. particles
(which unifies fermions and gauge bosons Levin & Wen 03):

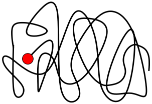
- Electons/Quarks = ends of strings → produce Fermi statistics.
  Light = fluctuations of strings (density wave of strings).

# Long range entanglement → fermions and gauge bosons

- Lord Kelvin's ele. particles = knotted strings of ether Kelvin 1867
  cannot be fermions. (Knots are unstable against local destruction.)





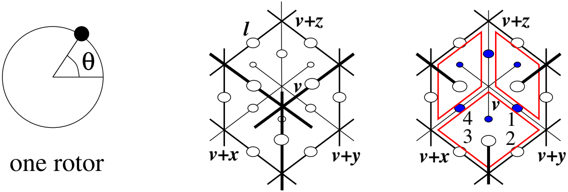
String-net picture of ele. particles
(which unifies fermions and gauge bosons Levin & Wen 03):

- Electons/Quarks = ends of strings → produce Fermi statistics.
  Light = fluctuations of strings (density wave of strings).
- **String-net order in qbit model unifies light, electrons ... ...**

$|\text{string-net ordered}\rangle = \sum |\text{loops or string-nets}\rangle \to$ long range entangled

# A quantum rotor model on cubic lattice

A concrete model that has string-net order: Motrunich & Senthil 02, Wen 03



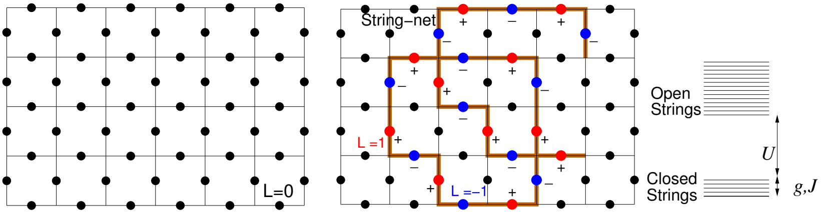

A rotor $\theta_i$ on every link of the cubic lattice:

$$H = U \sum_{\mathbf{v}} Q_{\mathbf{v}}^2 - g \sum_{\mathbf{p}} (B_{\mathbf{p}} + h.c.) + J \sum_{\mathbf{l}} (L_{\mathbf{l}})^2$$

$$Q_{\mathbf{v}} = \sum_{\mathbf{l} \text{ next to } \mathbf{v}} L_{\mathbf{l}}, \qquad B_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^-$$

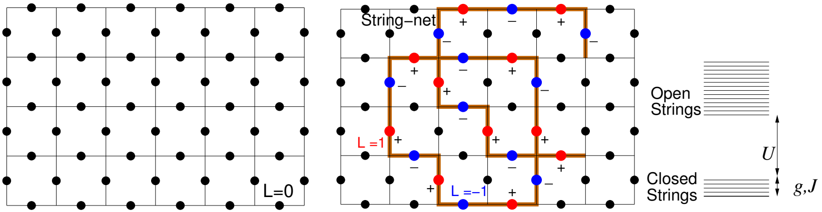
$L = -i\partial_\theta$: the angular momentum of the rotor
$L^\pm = e^{\pm i\theta}$: the raising/lowering operators of $L$

# String-net liquid

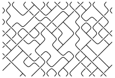



- the $UQ_{\mathbf{v}}^2$-term $\to$ closed strings. Open ends cost energy.
- $J(L_{\mathbf{l}})^2$-term $\to$ string tension
- the $gB_{\mathbf{p}}$-term $\to$ strings can fluctuate

# String-net liquid




- the $UQ_{\mathbf{v}}^2$-term $\to$ closed strings. Open ends cost energy.
- $J(L_{\mathbf{l}})^2$-term $\to$ string tension
- the $gB_{\mathbf{p}}$-term $\to$ strings can fluctuate
- The ground state of the rotor Hamiltonian $H$ when $U \gg g \gg J$

$$|\text{String-net liquid}\rangle = \sum_{\text{all closed string conf.}} \Big|$$  $$\Big\rangle$$

- The string-net liquid is a new state of quantum matter with long range entanglement

# String-net condensation

- Boson condensed state

$$\langle \text{Boson condensed} | \; \phi \; | \text{Boson condensed} \rangle \neq 0$$

where $\phi$ is a boson creation operator.

- String condensed state

$$\langle \text{String condensed} | \; W \; | \text{String condensed} \rangle \neq 0$$

where $W$ is the string creation operator:

$$W = ... L^{+}_{\mathbf{l}_1} L^{-}_{\mathbf{l}_2} L^{+}_{\mathbf{l}_3} L^{-}_{\mathbf{l}_4} ...$$

- The closed string operator $W_{\text{closed}}$ cost no energy $[H, W_{\text{closed}}] = 0$.
- The open string operator $W_{\text{open}}$ create a pair of quasiparticles.
  Open string operator $\to$ properties of quasiparticles

# 1) Maxwell waves in string-net liquid – a cartoon approach

Zero-point fluctuations in ground state    Waves in string-net liquid

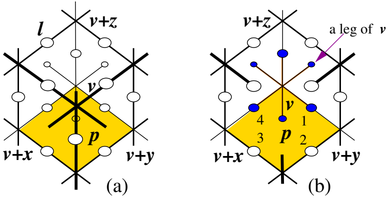
- "String density" $\mathbf{E}(\mathbf{r}, t)$ satisfies $\partial \cdot \mathbf{E} = 0$
- String density wave satisfies $\dot{\mathbf{E}} = \partial \times \mathbf{B}, \quad \dot{\mathbf{B}} = -\partial \times \mathbf{E}.$

# 1) Maxwell waves in string-net liquid – a cartoon approach

Zero-point fluctuations in ground state     Waves in string-net liquid

- "String density" $\mathbf{E}(\mathbf{r}, t)$ satisfies $\partial \cdot \mathbf{E} = 0$
- String density wave satisfies $\dot{\mathbf{E}} = \partial \times \mathbf{B}, \quad \dot{\mathbf{B}} = -\partial \times \mathbf{E}$.
- End of strings = source of $\mathbf{E}$ = gauge charges
- "Vortices" in string liquid = magnetic monopoles

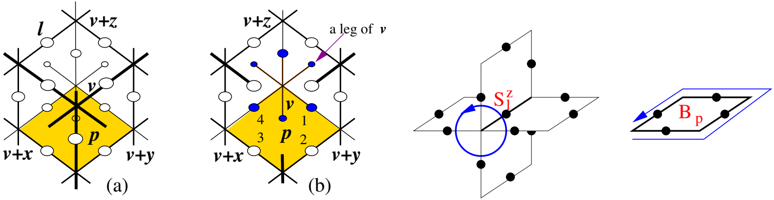
# 2) Emergence of Maxwell equation – E.O.M approach




$$H = U\sum_{\mathbf{v}} Q_{\mathbf{v}}^2 - g\sum_{\mathbf{p}}(B_{\mathbf{p}} + h.c.) + J\sum_{\mathbf{l}}(L_{\mathbf{l}})^2, \quad B_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^-$$

Key: dynamics of low energy closed-string states.

# 2) Emergence of Maxwell equation – E.O.M approach




$$H = U\sum_{\mathbf{v}} Q_{\mathbf{v}}^2 - g\sum_{\mathbf{p}}(B_{\mathbf{p}} + h.c.) + J\sum_{\mathbf{l}}(L_{\mathbf{l}})^2, \quad B_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^-$$

Key: dynamics of low energy closed-string states.

The operators $B_{\mathbf{p}}$ and $L_{\mathbf{l}}$ act within the closed-string subspace:

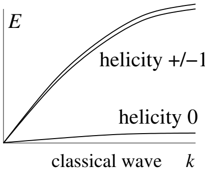
$$\partial_t \langle L_{\mathbf{l}}\rangle = \langle i[H, L_{\mathbf{l}}]\rangle \sim i\langle \sum_{a=1,..,4} B_{\mathbf{p}_a} - h.c.\rangle \quad \to \quad \dot{\mathbf{E}} = \partial \times \mathbf{B}$$

$$\partial_t \langle B_{\mathbf{p}}\rangle = \langle i[H, B_{\mathbf{p}}]\rangle \sim i\langle \sum_{a=1,..,4} L_{\mathbf{l}_a} B_{\mathbf{p}}\rangle, \quad \to \quad \dot{\mathbf{B}} = \partial \times \mathbf{E}$$

$$\langle B_{\mathbf{p}}\rangle = e^{i\mathbf{B}\cdot\mathbf{n}_{\mathbf{p}}}, \quad \langle L_{\mathbf{l}}\rangle = \mathbf{E}\cdot\mathbf{n}_{\mathbf{l}}$$

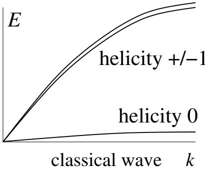
# 3) Semi-classical/quantum-freeze approach

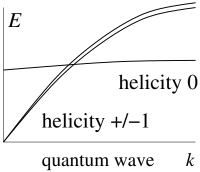
- Each cube has three rotors on the links in the $x$-, $y$-, and $z$-directions. The three $\theta$'s form the three component of a vector field $\mathbf{A} = (\theta^x, \theta^y, \theta^z)$.
- If we treat the rotor system as a classical system, the classical equation of motion is determined from the phase-space Lagrangian $\mathcal{L} = \sum L_{\mathbf{i}}\dot{\theta}_{\mathbf{i}} - H(L_{\mathbf{i}}, \theta_{\mathbf{i}})$
- Dispersions of three modes is designed to have the following form



# 3) Semi-classical/quantum-freeze approach

- Each cube has three rotors on the links in the $x$-, $y$-, and $z$-directions. The three $\theta$'s form the three component of a vector field $\mathbf{A} = (\theta^x, \theta^y, \theta^z)$.
- If we treat the rotor system as a classical system, the classical equation of motion is determined from the phase-space Lagrangian $\mathcal{L} = \sum L_\mathbf{l}\dot{\theta}_\mathbf{l} - H(L_\mathbf{l}, \theta_\mathbf{l})$
- Dispersions of three modes is designed to have the following form




- Quantum fluctuations: When $g \gg J$, the fluctuations $\delta\theta_{\pm 1} \ll 2\pi$ and $\delta\theta_0 \gg 2\pi$. The helicity-$0$ mode is gapped in quantum theory, and the helicity-$\pm 1$ modes remain gapless.

# Origin of Fermi statistics

**What is the statistics of string ends?**

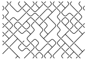
- A pair of string ends is bosonic, since
  A pair of string ends = an open string

# Origin of Fermi statistics

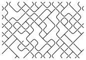
**What is the statistics of string ends?**

- A pair of string ends is bosonic, since
  A pair of string ends $=$ an open string

  *The statistics of a single end of strings is determined by the form of string-net condensation.*

- For string-net condensed state $|\Phi\rangle = \sum_{\text{all conf.}} \Big|$  $\Big\rangle$

  The end of strings are bosons.
  The the ends of string in the above spin-1 model are bosons.

- For string-net condensed state $|\Phi\rangle = \sum_{\text{all conf.}} \pm \Big|$  $\Big\rangle$

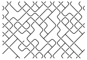
  The end of strings are fermion.

# Origin of Fermi statistics

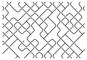
**What is the statistics of string ends?**

- A pair of string ends is bosonic, since
  A pair of string ends $=$ an open string

  *The statistics of a single end of strings is determined by the form of string-net condensation.*

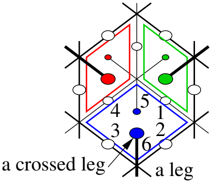
- For string-net condensed state $|\Phi\rangle = \sum_{\text{all conf.}} \Big|$  $\Big\rangle$

  The end of strings are bosons.
  The the ends of string in the above spin-1 model are bosons.

- For string-net condensed state $|\Phi\rangle = \sum_{\text{all conf.}} \pm \Big|$  $\Big\rangle$

  The end of strings are fermion.

  **String-net condensation provides a way to produce Fermi statistics from local qbit models.**




Just add a little twist

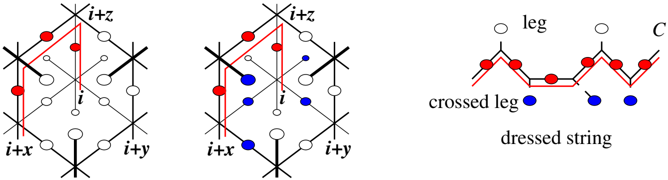
- Twisted-string model: Levin & Wen 04

$$\tilde{H} = U \sum_{\mathbf{v}} Q_{\mathbf{v}} - g \sum_{\mathbf{p}} \tilde{B}_{\mathbf{p}} + J \sum_{\mathbf{l}} (L_{\mathbf{l}})^2, \qquad \tilde{B}_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^- (-1)^{L_5 + L_6}$$

The sign change in the string hopping operator produces the sign change in the string wave function → fermionic string ends

# Statistics of the ends of strings from the string operators

- Closed-string operators $W$ are defined through $[W, H] = 0$ in $J = 0$ limit. Strings cost no energy and is unobservable.




- In the untwisted model – untwisted-string operator

$$L^{+}_{\mathbf{I}_1} L^{-}_{\mathbf{I}_2} L^{+}_{\mathbf{I}_3} L^{-}_{\mathbf{I}_4} \cdots$$

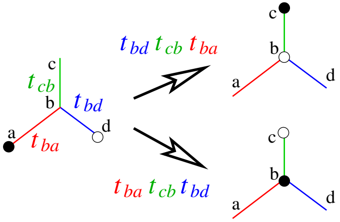
- In the twisted model – twisted-string operator

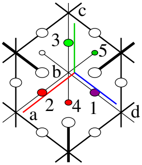
$$(L^{+}_{\mathbf{I}_1} L^{-}_{\mathbf{I}_2} L^{+}_{\mathbf{I}_3} L^{-}_{\mathbf{I}_4} \cdots) \prod_{\mathbf{I} \text{ on crossed legs of } C} (-1)^{L_{\mathbf{I}}}$$

- A pair of string ends is created by an open string operator. Their statistics can be calculated from the open string operator.

# Statistics of string-ends from the alg. of string operators

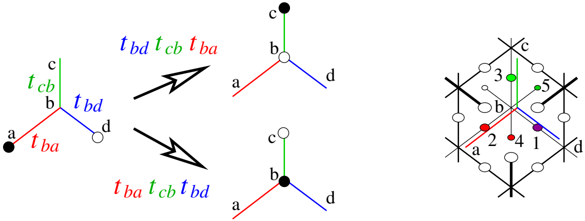
- An open string operator is a hopping operator of the string-ends.
- The statistics is determined by particle hopping operators Levin&Wen 03:



# Statistics of string-ends from the alg. of string operators

- An open string operator is a hopping operator of the string-ends.
- The statistics is determined by particle hopping operators Levin&Wen 03:




**Algebra of open string operator determine the statistics**

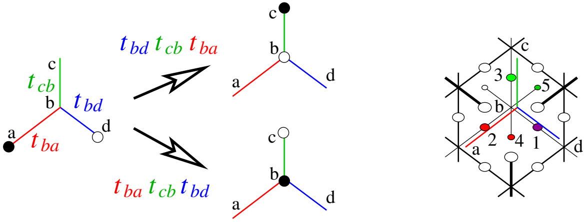
- For untwisted model: $t_{ba} = L_2^+$, $t_{cb} = L_3^-$, $t_{bd} = L_1^+$

  We find $t_{bd} t_{cb} t_{ba} = t_{ba} t_{cb} t_{bd}$

  $\rightarrow$ **The ends of untwisted-string are bosons**

# Statistics of string-ends from the alg. of string operators

- An open string operator is a hopping operator of the string-ends.
- The statistics is determined by particle hopping operators Levin&Wen 03:




**Algebra of open string operator determine the statistics**

- For untwisted model: $t_{ba} = L_2^+$, $t_{cb} = L_3^-$, $t_{bd} = L_1^+$

  We find $t_{bd}t_{cb}t_{ba} = t_{ba}t_{cb}t_{bd}$

  $\to$ **The ends of untwisted-string are bosons**
- For twisted model: $t_{ba} = (-)^{L_4+L_1} L_2^+$, $t_{cb} = (-)^{L_5} L_3^-$, $t_{bd} = L_1^+$

  We find $t_{bd}t_{cb}t_{ba} = -t_{ba}t_{cb}t_{bd}$

  $\to$ **The ends of twisted-string are fermions**

# String-net liquid produces and unifies three wonders

0-qbits = no string state. Strings = lines of 1-qbits.
String-net ordered state = a superposition of string states

**How much can we get from string-net order?**

- The collective motion of string-nets (fluctuation of quantum entanglements) give rise to gauge bosons ($U(1) \times SU(2) \times SU(3)$, as well as other gauge groups) Wen 02, Levin & Wen 04
- Ends of strings (topological excitations) give rise to spin-1/2 fermions – the matter (leptons and quarks) Levin & Wen 03
- Three of the seven wonders

*(2) Identical particles.*
*(3) Gauge interactions.*
*(4) Fermi statistics.*

can emerge form a qbit model, if our vacuum is a string-net liquid.

**String-net unifies gauge interaction and Fermi statistics**

# Can qbit model also unifies gravity?

Can gravitons emerge from a qbit model?

**What is quantum gravity and what is graviton?**

# Can qbit model also unifies gravity?

Can gravitons emerge from a qbit model?

**What is quantum gravity and what is graviton?**

Gravitons = Helicity $\pm 2$ modes, but
Helicity $\pm 2$ modes $\neq$ Gravitons

# Can qbit model also unifies gravity?

Can gravitons emerge from a qbit model?

**What is quantum gravity and what is graviton?**

Gravitons = Helicity $\pm 2$ modes, but
Helicity $\pm 2$ modes $\neq$ Gravitons

Here we will use a very strict definition:

- Gravitons = helicity $\pm 2$ modes **as the only gapless excitations**

# Can qbit model also unifies gravity?

Can gravitons emerge from a qbit model?

**What is quantum gravity and what is graviton?**

Gravitons = Helicity $\pm 2$ modes, but
Helicity $\pm 2$ modes $\neq$ Gravitons

Here we will use a very strict definition:

- Gravitons = helicity $\pm 2$ modes **as the only gapless excitations**
  A field theory of symmetric tensor,
  $\mathcal{L} = (\dot{a}_{ij})^2 - (\partial a_{ij})^2 \;\sim\; \mathcal{L}_{\text{phase-space}} = \mathcal{E}^{ij}\dot{a}_{ij} - (\mathcal{E}^{ij})^2 - (\partial a_{ij})^2$,
  has helicity $0, 0, \pm 1, \pm 2$ modes,
  here the helicity $\pm 2$ modes are not gravitons.

# Can qbit model also unifies gravity?

Can gravitons emerge from a qbit model?

**What is quantum gravity and what is graviton?**

Gravitons = Helicity $\pm 2$ modes, but
Helicity $\pm 2$ modes $\neq$ Gravitons

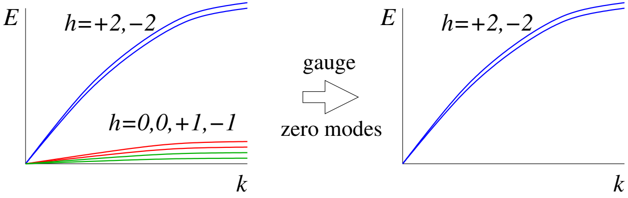
Here we will use a very strict definition:

- Gravitons = helicity $\pm 2$ modes **as the only gapless excitations**
  A field theory of symmetric tensor,
  $\mathcal{L} = (\dot{a}_{ij})^2 - (\partial a_{ij})^2 \sim \mathcal{L}_{\text{phase-space}} = \mathcal{E}^{ij}\dot{a}_{ij} - (\mathcal{E}^{ij})^2 - (\partial a_{ij})^2$,
  has helicity $0, 0, \pm 1, \pm 2$ modes,
  here the helicity $\pm 2$ modes are not gravitons.

  *It is very hard to construct a well defined local quantum model (ie a lattice Hamiltonian) with graviton (as defined above).*
  Maybe we do not have any such model (?)
  $\rightarrow$ no well defined theory of quantum gravity yet.
- Here we will try to construct local lattice models with gravitons.

# Field theory considerations Gu & Wen, arXiv:0907.1203

- Start with a field theory of symmetric tensor described by a phase-space Lagrangian $\mathcal{L}(\mathcal{E}^{ij}, a_{ij})$
- Remove the helicity $0, 0, \pm 1$ modes by constraints:




- Vector constraint to remove helicity $0, \pm 1$ modes:
  We set a combination of $\mathcal{E}^{ij}$ to zero: $\pi^j = \partial_i \mathcal{E}^{ij} = 0$.
  The corresponding canonical conjugate of $\pi^i$ is $f_i$:
  $\delta a_{ij} = \partial_i f_j + \partial_j f_i$. The Lagrangian $\mathcal{L}(\mathcal{E}^{ij}, a_{ij})$ must not contain $f_i$:
  $\mathcal{L}(\mathcal{E}^{ij}, a_{ij} + \partial_i f_j + \partial_j f_i) = \mathcal{L}(\mathcal{E}^{ij}, a_{ij})$. $\to$ constraint-gauge pair:

$$\partial_i \mathcal{E}^{ij} = 0, \qquad \mathrm{e}^{\mathrm{i} \int f_j \partial_i \hat{\mathcal{E}}^{ij}} : \; a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i.$$

# Gauge invariant phase-space Lagrangian

- Gauge invariant field strength: $R^{ij} = \epsilon^{imk}\epsilon^{jln}\partial_m\partial_l a_{nk}$
- Gauge invariant phase-space Lagrangian:

$$\mathcal{L}(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}(\mathcal{E}^{ij})^2 - \frac{g}{2}R^{ij}R^{ij}$$

  which has helicity $0, \pm 2$ modes with $\omega \sim k^2$ dispersion.
- $\to$ $\omega \sim k^2$ pseudo-gravity.

# Gauge invariant phase-space Lagrangian

- Gauge invariant field strength: $R^{ij} = \epsilon^{imk}\epsilon^{jln}\partial_m\partial_l a_{nk}$
- Gauge invariant phase-space Lagrangian:

$$\mathcal{L}(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}(\mathcal{E}^{ij})^2 - \frac{g}{2}R^{ij}R^{ij}$$

which has helicity $0, \pm 2$ modes with $\omega \sim k^2$ dispersion.

- $\to$ $\omega \sim k^2$ pseudo-gravity.

**Remove the remaining helicity 0 mode**

- Scaler constraint to remove the remaining helicity 0 mode: We set a combination of $R^{ij}$ to zero: $\pi^0 = R^{ii} = 0$. The corresponding canonical conjugate of $\pi^0$ is $f_0$: $\delta\mathcal{E}_{ij} = (\delta_{ij}\partial^2 - \partial_i\partial_j)f_0$. The Lagrangian $\mathcal{L}(\mathcal{E}^{ij}, a_{ij})$ must not contain $f_0$: $\mathcal{L}(\mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i\partial_j)f_0, a_{ij}) = \mathcal{L}(\mathcal{E}^{ij}, a_{ij})$.
  $\to$ constraint-gauge pair:

$$R^{ii} = 0, \qquad \mathrm{e}^{i\int f_0 \hat{R}^{ii}}: \ \mathcal{E}^{ij} \to \mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i\partial_j)f_0,$$

# $\omega \sim k^3$ gravity

- New gauge invariant field strength: $C^i_j = \epsilon^{imn}\partial_m \left(\mathcal{E}^{nj} - \frac{1}{2}\delta_{nj}\mathcal{E}^{ll}\right)$
- Gauge invariant phase-space Lagrangian:

$$\mathcal{L}_L(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2} C^i_j C^j_i - \frac{g}{2} R^{ij} R^{ij} = \mathcal{E}^{ij}\partial_0 a_{ij} - \mathcal{H}_L$$

  which has helicity $\pm 2$ modes only, with $\omega \sim k^3$ dispersion.
- Local gauge invariance: the Hamiltonian density $\mathcal{H}_N$ is invariant under the above gauge transformations. (Maxwell type)
- Such a local gauge invariance protects the $\omega \sim k^3$ dispersion. ($\mathcal{L}_L$ is already the lowest order Lagrangian.)

# $\omega \sim k$ gravity Gu & Wen, arXiv:gr-qc/0606100

$$\mathcal{L}_N(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}[(\mathcal{E}^{ij})^2 - \frac{1}{2}(\mathcal{E}^{ii})^2] - \frac{g}{2} a_{ij} R^{ij} = \mathcal{E}^{ij}\partial_0 a_{ij} - \mathcal{H}_N$$

with the same gauge transformations and constraints:

$$a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i, \qquad \partial_i \mathcal{E}^{ij} = 0$$
$$\mathcal{E}^{ij} \to \mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i\partial_j) f_0, \qquad R^{ii} = 0,$$

which has helicity $\pm 2$ modes only, with $\omega \sim k$ dispersion.

- Non-local gauge invariance: the Hamiltonian $H = \int d^3x\, \mathcal{H}_N$ is invariant under the above gauge transformations, but the Hamiltonian density is not: $\mathcal{H}_N \to \mathcal{H}_N + \partial F$. (Chern-Simons type)
- $\omega \sim k$ dispersion requires a *non-local* gauge invariance: the Hamiltonian density to transforms as $\mathcal{H}_N \to \mathcal{H}_N + \partial F$.

# Relation to Einstein gravity

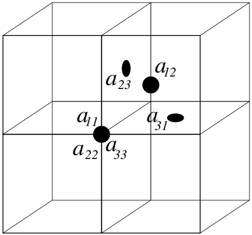
- If we introduce $a_{00}$ and $a_{0i}$ as Lagrangian multipliers to enforce the vector and the scaler constraints, we can rewrite the $\omega \sim k$ model as

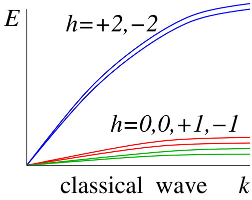
$$\mathcal{L} = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}\left[(\mathcal{E}^{ij})^2 - \frac{1}{2}(\mathcal{E}^{ii})^2\right] - \frac{g}{2}a_{ij}R^{ij} + 2a_{0i}\partial_j\mathcal{E}^{ij} + a_{00}(\partial^2 a_{ii} - \partial_i\partial_j a_{ij})$$

- After integrating out $\mathcal{E}^{ij}$, we find that the above action is exactly the linearized Einstein action around a flat space-time: $\delta g_{\mu\nu} \sim a_{\mu\nu}$.
- The gauge transformations $f_i(x^i), f_0(x^i)$ in space are enlarged to gauge transformations in space-time $f_i(x^i, t), f_0(x^i, t)$ $\to$ linearized diffeomorphism of space-time.

# Putting $\mathcal{H}_L = \frac{J}{2} C^i_j C^j_i + \frac{g}{2} R^{ij} R^{ij}$ on lattice ( $\omega \sim k^3$ model)

- Lattice model: each vertex has three real variables $a^{11}, a^{22}, a^{33}$ with their canonical conjugate $\mathcal{E}^{11}, \mathcal{E}^{22}, \mathcal{E}^{33}$ .
  Each face has one real variable $(a_{ij}, \mathcal{E}^{ij})$, $ij = 12, 23, 31$.
- L-type qbit model ( $U_{1,2}$ terms to enforce the constraints):
  $$L_L = \sum \mathcal{E}^{ij} \dot{a}_{ij} - \sum [\mathcal{H}_L + U_1 (\partial_i \mathcal{E}^{ij})^2 + U_2 (R^{ii})^2]$$
- Total six modes with helicity $0, 0, \pm 1, \pm 2$

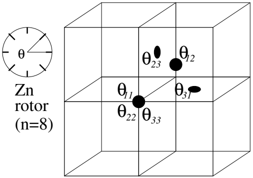


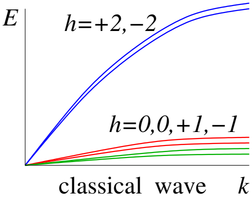




Even in large $U_1, U_2$ limit, the helicity $0, 0, \pm 1$ modes remain gapless! (Both as classical model and quantum model)

# Gap the helicity $0, 0, \pm 1$ modes Gu & Wen, arXiv:0907.1203

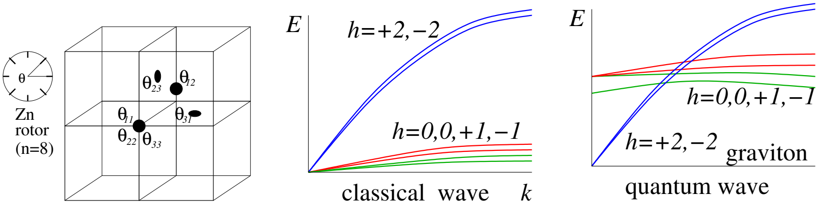
- Compactify $a_{ij} = \alpha\theta_{ij}$ : $\theta_{ij} \sim \theta_{ij} + 2\pi$
- Discretize $a_{ij}$ and $\theta_{ij}$: $\theta_{ij} = 2\pi/n_G \times$ int.
- The canonical conjugate of $\theta^{ij}$, $L^{ij} \sim \mathcal{E}^{ij}$ is also compactified and discretized: $L^{ij} \sim L^{ij} + n_G$, $L^{ij} =$ int.
- We know that if we treat $\theta_{ij}, L^{ij}$ in our lattice model $L_L$ as continuous classical fields, there will be total six gapless modes with helicity $0, 0, \pm 1, \pm 2$



# Gap the helicity $0, 0, \pm 1$ modes Gu & Wen, arXiv:0907.1203

- Compactify $a_{ij} = \alpha\theta_{ij}$ : $\theta_{ij} \sim \theta_{ij} + 2\pi$
- Discretize $a_{ij}$ and $\theta_{ij}$: $\theta_{ij} = 2\pi/n_G \times$ int.
- The canonical conjugate of $\theta^{ij}$, $L^{ij} \sim \mathcal{E}^{ij}$ is also compactified and discretized: $L^{ij} \sim L^{ij} + n_G$, $L^{ij} =$ int.
- We know that if we treat $\theta_{ij}, L^{ij}$ in our lattice model $L_L$ as continuous classical fields, there will be total six gapless modes with helicity $0, 0, \pm 1, \pm 2$




- After include quantum effects, the helicity $0, 0, \pm 1$ modes are gapped!

# Why the helicity $0, 0, \pm 1$ modes are gapped?

Consider the helicity $0, \pm 1$ modes described by the canonical conjugate pair $(\pi^i, f_i)$: $\pi^i = \partial_j L^{ij}$, $\delta\theta_{ij} = \partial_i f_j + \partial_j f_i$

- Still treat $(\pi^i, f_i)$ as continuous classical fields but consider the quantum fluctuations of $(\pi^i, f_i)$ with wave vector $k \sim 1$:

$$\mathcal{H} = U(\pi^i)^2 + g'(f_i)^2, \quad U = \text{large}, \quad g' = \text{small} = 0$$

The above oscillator-like Hamiltonian gives us $\delta\pi^i \sim (g'/U)^{1/4}$ and $\delta f_i \sim (U/g')^{1/4}$.

- The fluctuations of $f_i$ is much larger than the compactification radius of $f_i$: $\delta f_i \gg 2\pi$
The fluctuations of $\pi^i$ is much less than the discreteness of $\pi^i$: $\delta\pi^i \ll 1$
*Classical results cannot be trusted.*
*The quantum freeze of $\pi^i$ $\to$ modes $(\pi^i, f_i)$ are gapped.*

# Why the helicity $\pm 2$ modes are not gapped?

Consider the helicity $\pm 2$ modes described by the canonical conjugate pair $(L^{\pm}, \theta_{\pm})$.

- Still treat $(L^{\pm}, \theta_{\pm})$ as continuous classical fields but consider the quantum fluctuations of $(L^{\pm}, \theta_{\pm})$: $\mathcal{H} = J(L^{\pm})^2 + g(\theta_{\pm})^2,$.

# Why the helicity $\pm 2$ modes are not gapped?

Consider the helicity $\pm 2$ modes described by the canonical conjugate pair $(L^{\pm}, \theta_{\pm})$.

- Still treat $(L^{\pm}, \theta_{\pm})$ as continuous classical fields but consider the quantum fluctuations of $(L^{\pm}, \theta_{\pm})$: $\mathcal{H} = J(L^{\pm})^2 + g(\theta_{\pm})^2,$. The above oscillator-like Hamiltonian gives us $\delta L^{\pm} \sim (g/J)^{1/4}$ and $\delta\theta_{\pm} \sim (J/g)^{1/4}$.

# Why the helicity $\pm 2$ modes are not gapped?

Consider the helicity $\pm 2$ modes described by the canonical conjugate pair $(L^{\pm}, \theta_{\pm})$.

- Still treat $(L^{\pm}, \theta_{\pm})$ as continuous classical fields but consider the quantum fluctuations of $(L^{\pm}, \theta_{\pm})$: $\mathcal{H} = J(L^{\pm})^2 + g(\theta_{\pm})^2,$.
  The above oscillator-like Hamiltonian gives us $\delta L^{\pm} \sim (g/J)^{1/4}$ and $\delta\theta_{\pm} \sim (J/g)^{1/4}$.
- Choose the coupling constants in the qbit model to satisfy $g/J \sim n_G^2$
  The fluctuations of $\theta_{\pm} \sim \sqrt{1/n_G}$ is much less than the compactification radius of $\theta_{\pm}$: $\delta\theta_{\pm} \ll 2\pi$, but much bigger than the discreteness of $\theta_{\pm}$: $\delta\theta_{\pm} \gg 2\pi/n_G$.
  The fluctuations of $L^{\pm} \sim \sqrt{n_G}$ is much bigger than the discreteness of $L^{\pm}$: $\delta L^{\pm} \gg 1$, but much less than the compactification radius of $L^{\pm}$: $\delta L^{\pm} \ll n_G$.

# Why the helicity $\pm 2$ modes are not gapped?

Consider the helicity $\pm 2$ modes described by the canonical conjugate pair $(L^\pm, \theta_\pm)$.

- Still treat $(L^\pm, \theta_\pm)$ as continuous classical fields but consider the quantum fluctuations of $(L^\pm, \theta_\pm)$: $\mathcal{H} = J(L^\pm)^2 + g(\theta_\pm)^2,$.
  The above oscillator-like Hamiltonian gives us
  $\delta L^\pm \sim (g/J)^{1/4}$ and $\delta\theta_\pm \sim (J/g)^{1/4}$.
- Choose the coupling constants in the qbit model to satisfy
  $g/J \sim n_G^2$
  The fluctuations of $\theta_\pm \sim \sqrt{1/n_G}$ is much less than the compactification radius of $\theta_\pm$: $\delta\theta_\pm \ll 2\pi$, but much bigger than the discreteness of $\theta_\pm$: $\delta\theta_\pm \gg 2\pi/n_G$.
  The fluctuations of $L^\pm \sim \sqrt{n_G}$ is much bigger than the discreteness of $L^\pm$: $\delta L^\pm \gg 1$, but much less than the compactification radius of $L^\pm$: $\delta L^\pm \ll n_G$.

*The helicity $\pm 2$ modes $(L^\pm, \theta_\pm)$ are semiclassical, and the classical result of gaplessness can be trusted.*

# The emergence of $\omega \sim k^3$ gravity from qbit model

- The L-type qbit model on lattice produces the $\omega \sim k^3$ gravitons described by the following low energy effective Lagrangian

$$\mathcal{L}_L(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}C^i_j C^i_j - \frac{g}{2}R^{ij}R^{ij}$$

$$C^i_j = \epsilon^{imn}\partial_m\left(\mathcal{E}^{nj} - \frac{1}{2}\delta_{nj}\mathcal{E}^{ll}\right), \quad R^{ij} = \epsilon^{imk}\epsilon^{jln}\partial_m\partial_l a_{nk}$$

with the following emergent gauge transformation and constraints

$$a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i, \qquad \partial_i \mathcal{E}^{ij} = 0$$

$$\mathcal{E}^{ij} \to \mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i\partial_j)f_0, \qquad R^{ii} = 0$$

- The only gapless excitations are helicity $\pm 2$ modes with $\omega \sim k^3$ dispersion.
- The result is obtained by a controlled semiclassical approximation and is reliable.

# The gaplessness of emergent gauge bosons and gravitons

- Gaplessness of Goldstone bosons in symmetry-breaking states is protected by the symmetry of the underlying Hamiltonian.

  **What protect the gaplessness of emergent gauge bosons and gravitons in qbit models?**

# The gaplessness of emergent gauge bosons and gravitons

- Gaplessness of Goldstone bosons in symmetry-breaking states is protected by the symmetry of the underlying Hamiltonian.

  **What protect the gaplessness of emergent gauge bosons and gravitons in qbit models?**

- The gaplessness of emergent gauge bosons and gravitons do not need any protection
  (assuming their self interaction is irrelevant at low energies).
  Any local perturbations to the underlying Hamiltonian cannot break the emergent local gauge symmetry and cannot gap the emergent gauge bosons and gravitons. Hastings & Wen 05

# The gaplessness of emergent gauge bosons and gravitons

- Gaplessness of Goldstone bosons in symmetry-breaking states is protected by the symmetry of the underlying Hamiltonian.

  **What protect the gaplessness of emergent gauge bosons and gravitons in qbit models?**

- The gaplessness of emergent gauge bosons and gravitons do not need any protection
  (assuming their self interaction is irrelevant at low energies).
  Any local perturbations to the underlying Hamiltonian cannot break the emergent local gauge symmetry and cannot gap the emergent gauge bosons and gravitons. Hastings & Wen 05

- **The $\omega \sim k^3$ dispersion is also protected** by the local gauge invariance of the Hamiltonian density $\mathcal{H}_L$. Local perturbations cannot gap the $\omega \sim k^3$ gravitons and cannot turn the $\omega \sim k^3$ gravity to the $\omega \sim k$ gravity.

# The emergence of $\omega \sim k$ (Einstein) gravity

- N-type qbit model: Put $\mathcal{H}_N + U_1(\partial_i \mathcal{E}^{ij})^2 + U_2(R^{ii})^2$ on lattice.
- Under the similar semiclassical approach at quadratic order we can obtain $\omega \propto k$ gravitons, described by low energy effective Lagrangian (in phase space):

$$\mathcal{L}_N(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}[(\mathcal{E}^{ij})^2 - \frac{1}{2}(\mathcal{E}^{ii})^2] - \frac{g}{2} a_{ij} R^{ij}$$

with the same gauge transformations and constraints:

$$a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i, \qquad \partial_i \mathcal{E}^{ij} = 0$$
$$\mathcal{E}^{ij} \to \mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i \partial_j) f_0, \qquad R^{ii} = 0$$

- But for the N-type model, the strong fluctuating helicity $0, 0, \pm 1$ modes couple to weak fluctuating $\pm 2$ modes at quartic order and beyond. The semi-classical result is unreliable.
- **The Einstein equation may emerge at low energies from the N-type model at linear level (need to be confirmed).**

# Summary

- Gravitons = helicity $\pm 2$ modes **as the only gapless excitations**
- Compactifying and discretizing the metric tensor $h_{ij} = g_{ij} - \delta_{ij}$ are very important in obtaining a quantum theory of gravity.
- Two kinds of gravity: the $\omega \sim k^3$ gravity and the $\omega \sim k$ gravity, both emerge as stable phases in some qbit models.
- In those gravity phases, the gaplessness of helicity $\pm 2$ gravitons is topologically protected: stable against *any* perturbations that do not break translation symmetry.
- The $\omega \sim k^3$ gravity emerges from the L-type model (reliable). The $\omega \sim k$ gravity emerges from the N-type model (not reliable). Need numerical calculations on the N-type qbit model, to confirm the emergence of the $\omega \sim k$ gravity.
- The N-type qbit model may be a quantum theory of gravity (at least at linear level).

# A new paradigm of many-body quantum physics

which connects cond. matter physics, particle physics, superstring theory, quantum gravity, quantum information, and mathematics.