Topological Phases in Noncentrosymmetric Superconductors and Superfluids (mainly in 2D or quasi-2D)

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Collaboration with

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Outline

1. Introduction -- What is a topological phase ?

- gapless edge states associated with topological number
  c.f. quantum Hall effect, quantum spin Hall effect

- fractionalization of quasiparticles
  e.g. non-Abelian statistics

2. What is a noncentrosymmetric (NC) superconductor ?

3. Topological phases in odd-parity P-wave NC superconductors

4. Topological phases in even-parity S-wave NC superconductors
   and superfluids of cold atoms

Introduction - What is a topological phase ?

Topological Phase of Matter (Wen, Wen-Niu,Read-Moore, Nayak-Wilczek,
Fradkin et al., Kitaev, D.H.Lee et al.,)

Novel quantum many-body state
characterized not by local order parameter
(e.g. magnetic order, superconductivity,.....)

but by *topologically non-trivial structure*

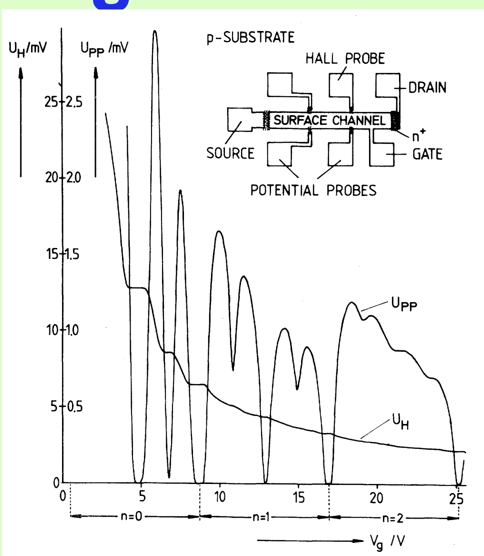
not in real space but in many-body Hilbert space !

Topological order

It emerges due to many-body interaction.

Examples of topological phase in condensed matter systems

Quantum Hall effect in 2D DEG in high magnetic fields

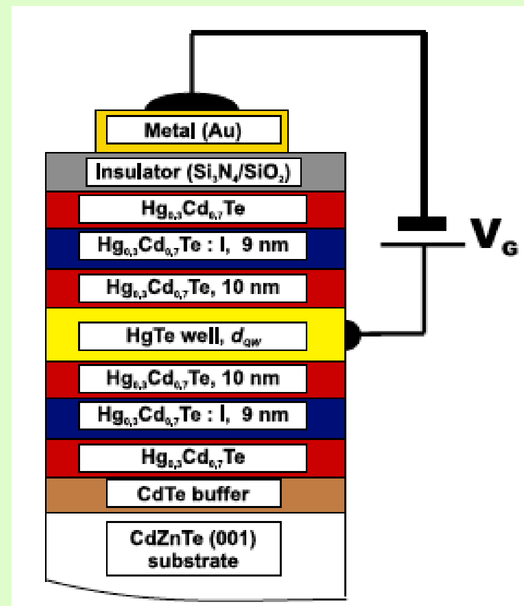


(von Klitzing et al.)

(Tsui et al.)

Abelian and non-Abelian top. order

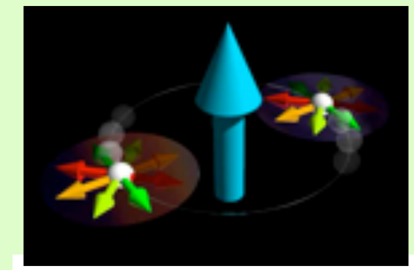
Quantum spin Hall effect in topological insulator

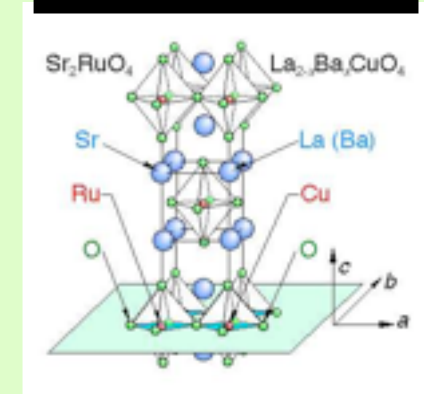


(Zhang et al.) (Kane and Mele)

Z_2 top. order

Chiral p+ip superconductors (Sr_2RuO_4 ?)





(Y. Maeno et al.)

non-Abelian top. order

Also, spin liquid state of quantum spin systems (Wen,Kitaev)

noncentrosymmetric superconductors and superfluids ....

Introduction

Three important features characterizing topological phases

(i) *Bulk excitation gap between the ground state and excited states*

(ii) *gapless edge states* at the boundaries of the system

(iii) In 2D, *fractionalization* of quasiparticles

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Three important features characterizing topological phases

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Topological phase is the ground state separated from non-topological excited state by an excitation gap. ensure the stability of top. phase

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Bulk excitation gap between the ground state and excited states

Quantum Hall state

Integer QHE : Gap between the Landau levels

Fractional QHE : Coulomb gap

Superconductor (Superfluid)

BCS gap $\Delta_k = \langle c^\dagger_{\boldsymbol{k}\sigma} c^\dagger_{-\boldsymbol{k}\sigma'} \rangle$ ***without nodes***

***s*-wave state** $\Delta_k = \Delta$

***p+ip*-wave state** $\Delta_k = \Delta(k_x + ik_y)$

BW phase of *p*-wave state

~~*d*-wave~~

nodes of gap $\Delta_k = 0$

for some *k*-points

e.g. ***d*-wave SC**

$\Delta_k = \Delta_0(\cos k_x - \cos k_y)$

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stable against perturbations such as disorder (topologically-protected)

→ associated with transport phenomena
quantum (spin) Hall effect

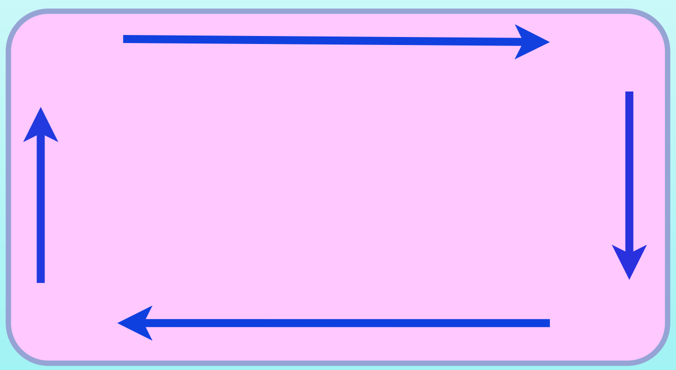
(Thouless, Kohmoto, Wen, Niu, Hatsugai)
(Kane,Mele, Bernevig, Hughes, Zhang)

(iii) In 2D, *fractionalization* of quasiparticles

Gapless edge states characterizing topological phasess

Case with broken time-reversal symmetry

Quantum Hall state



chiral edge states (origin of Integer QHE)

$$\sigma_{xy} = \frac{e^2}{h} n_c$$

first Chern number **= # of edge modes**

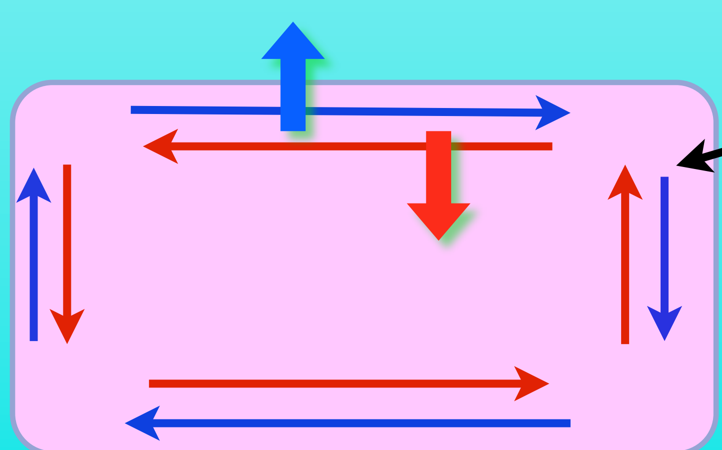
$$n_c = \int \frac{d\boldsymbol{k}}{4\pi} i\epsilon_{\mu\nu} \langle \partial_{k_\mu} \psi | \partial_{k_\nu} \psi \rangle$$

(Thouless,Kohmoto,Nightingale,den Nijs; Hatsugai)

Time-reversal invariant case **(Chern number = 0)**

Quantum spin Hall state (topological insulator)

(Kane,Mele, Bernevig,Hughes,Zhang,)



helical edge states

zero charge current, nonzero spin current

origin of quantum spin Hall effect

Z_2 invariant *(Kane,Mele)*

parity of spin-resolved first Chern number

takes only two values (trivial or non-trivial)

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(iii) In 2D, *fractionalization* of quasiparticles

e.g. fractional charge, spin, (c.f. FQHE)

fractional statistics (Abelian topological order)

Majorana fermion (half of conventional fermion)

non-Abelian statistics (non-Abelian topological order)

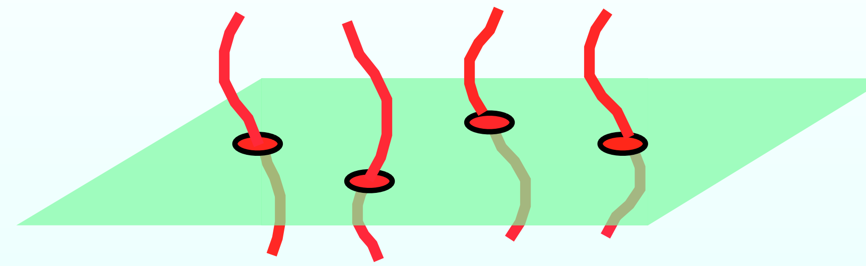
(Read,Moore,Green,Nayak,Wilczek,Ivanov,Lee,Zhang,Xing)

Non-Abelian statistics of vortices in *spinless* p_x+ip_y superconductors

(Read and Green 2000, Ivanov 2001, Stone and Chung 2006)

Non-Abelian anyon: vortex of SC order

$$\Delta = \Delta_0 e^{i\phi}$$



★There is *a zero energy mode* in a vortex core of p+ip SC. (Kopnin-Salomaa)

⟺ **conventional s-wave SC**

$$E_{\rm core} \sim \Delta^2/E_F$$ **No zero mode exists**

★The zero energy mode is a *Majorana fermion* !!

Bogoliubov quasiparticle $\gamma^\dagger = \int d\boldsymbol{r}[u_E(\boldsymbol{r})c^\dagger(\boldsymbol{r}) + v_E(\boldsymbol{r})c(\boldsymbol{r})]$ → $\boxed{\gamma^\dagger = \gamma}$

Because of particle-hole symmetry of BCS Hamiltonian $\Gamma\hat{\mathcal{H}}\Gamma = -\hat{\mathcal{H}}^*$ $\Gamma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

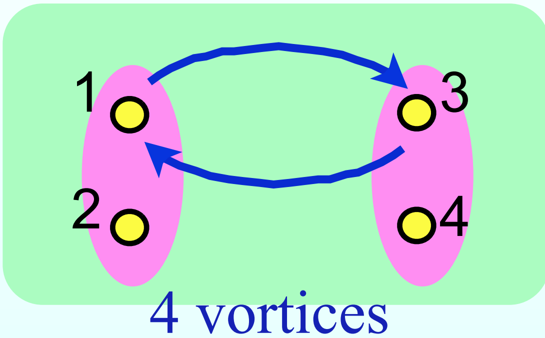
if $\hat{\mathcal{H}}\psi = E\psi$ then $\hat{\mathcal{H}}\Gamma\psi^* = -E\Gamma\psi^*$ $\psi^T = (u, v)$

→ If there is only one independent zero energy solution of Bogoliubov-de-Gennes eq.

$$\psi = \Gamma\psi^* \rightarrow \gamma^\dagger = \gamma$$

A vortex with a Majorana mode obeys the non-Abelian statistics

e.g.



topological degeneracy of Majorana fermion states

$$\psi_1 = (\gamma_1 + i\gamma_2)/\sqrt{2} \qquad \psi_2 = (\gamma_3 + i\gamma_4)/\sqrt{2}$$

$$|n_1, n_2\rangle = (\psi_1^\dagger)^{n_1}(\psi_2^\dagger)^{n_2}|0,0\rangle \qquad n_i = 1 \text{ or } 0$$ **(occupied or unoccupied)**

two-fold degenerate states $(|1,1\rangle\ ,\ |0,0\rangle)$ **or** $(|1,0\rangle\ ,|0,1\rangle)$

Exchange of 1 and 3 $\gamma_1 \longrightarrow \gamma_3 \qquad \gamma_3 \longrightarrow -\gamma_1$ **phase winding due to vortex**

$$\tau_{31} = e^{\frac{\pi}{4}\gamma_3\gamma_1} \qquad \tau_{31}\gamma_1\tau_{31}^\dagger = \gamma_3 \qquad \tau_{31}\gamma_3\tau_{31}^\dagger = -\gamma_1$$

$$\tau_{31}|1,1\rangle = \frac{1}{\sqrt{2}}(|1,1\rangle + |0,0\rangle) \qquad \tau_{31}|0,1\rangle = \frac{1}{\sqrt{2}}(-|1,0\rangle + |0,1\rangle)$$

$$\tau_{31}|0,0\rangle = \frac{1}{\sqrt{2}}(-|1,1\rangle + |0,0\rangle) \qquad \tau_{31}|1,0\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle + |0,1\rangle)$$

→ **unitary transformation in the degenerate G.S. space**

Not commutative ! ***Non-Abelian statistics emerges !***

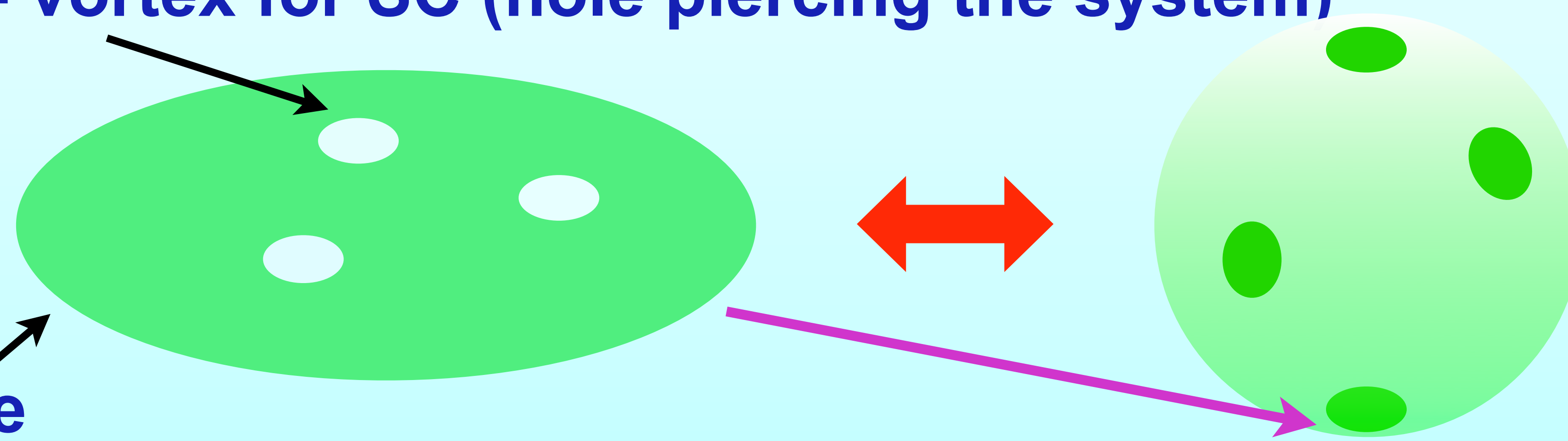
Low energy effective theory for non-Abelian topological phase: Topological field theory and conformal field theory

Low energy theory for bulk : level-k SU(2) Chern-Simons theory

Quasiparticle = vortex for SC (hole piercing the system)

topologically equivalent

Edge



For edge and holes(quasiparticles) : $SU(2)_k$ Wess-Zumino-Witten CFT

central charge: $c = \underline{1} + \underline{\dfrac{2(k-1)}{k+2}}$

U(1) charge part
(fractional charge for FQHE)

for superconductors, this part is absent, because quasiparticles are the superpositions of particle and hole

Z_k parafermion CFT, *non-Abelian part*

for k=2 $\quad \dfrac{2(k-1)}{k+2} = \dfrac{1}{2}$
Ising CFT
(Z_2 parafermion = Majorana fermion)
Case of SC

Topological Phases in Noncentrosymetric (NC) Superconductors and Superfluids

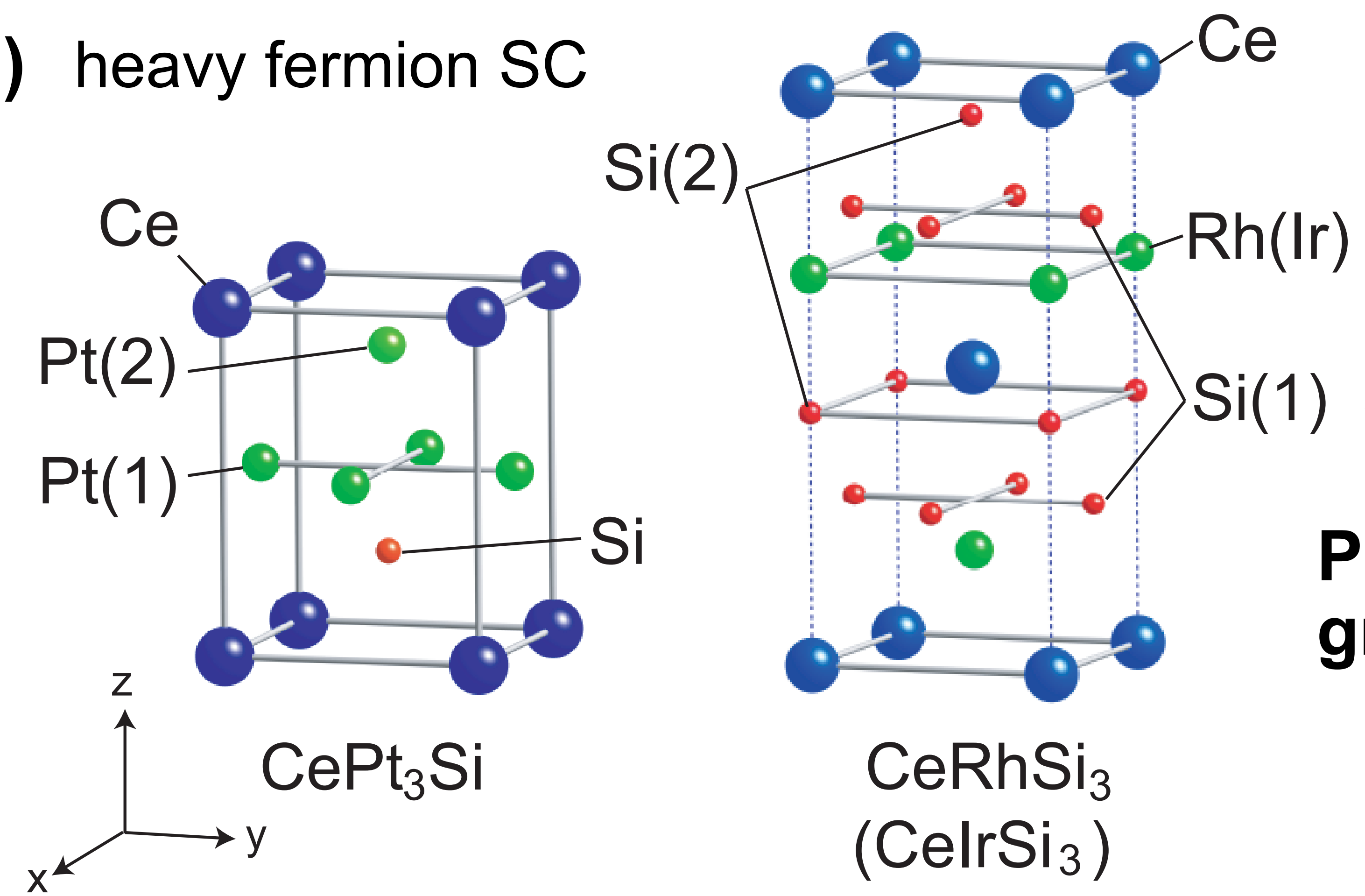
Non-centrosymmetric Superconductors

Superconductors with no inversion symmetry in their crystal structure

$CePt_3Si$, UIr, $CeRhSi_3$, $CeIrSi_3$, $Cd_2Re_2O_7$, Li_2Pt_3B, Li_2Pd_3B, ...

Bauer et al. (2004), Kimura et al. (2005), Sugitani et al. (2006), ...

Eg.) heavy fermion SC



Inversion symmetry breaking

Potential gradient

Asymmetric potential gradient

$$\vec{\nabla} V \parallel (001)$$

Anti-symmetric Spin-orbit interaction (Rashba interaction)

$$(\vec{p} \times \vec{\nabla} V) \cdot \vec{\sigma}$$

Broken inversion sym.

$$\vec{p} \rightarrow -\vec{p}$$

$$(\vec{p} \perp \vec{\nabla} V)$$

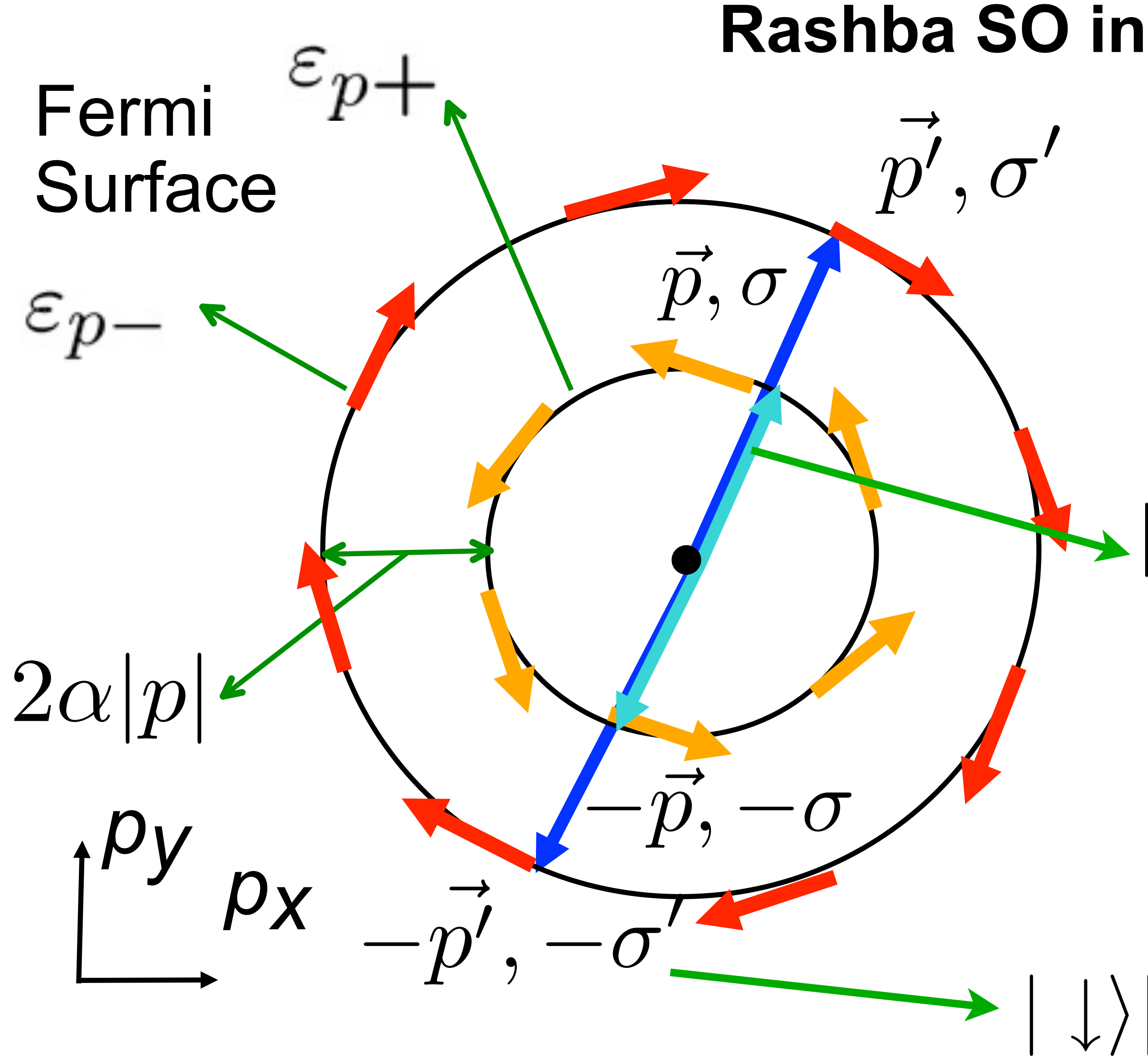
Broken Spin inversion sym.

$$\sigma \rightarrow -\sigma$$

Non-centrosymmetric Superconductors (contd.)

Edelstein,JETP68,1244(1989) ; Gor'kov-Rashba(2001); Yip(2002),Frigeri et al.(2004)

Parity non-conserved → Mixture of spin singlet and triplet states



Rashba SO int.

$$E_{p\sigma\sigma'} = \varepsilon_p + \underline{\alpha(\vec{p}\times\vec{n})\cdot\vec{\sigma}_{\sigma\sigma'}}$$

$\vec{n} = (0,0,1)$

"Zeeman energy" depending on $\vec{p}$

$$\Rightarrow \varepsilon_{p\pm} = \varepsilon_p \pm \alpha|p|$$

$$|\uparrow\rangle|\downarrow\rangle = \frac{1}{2}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle) \quad \textbf{singlet}$$

$$+\frac{1}{2}(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)$$

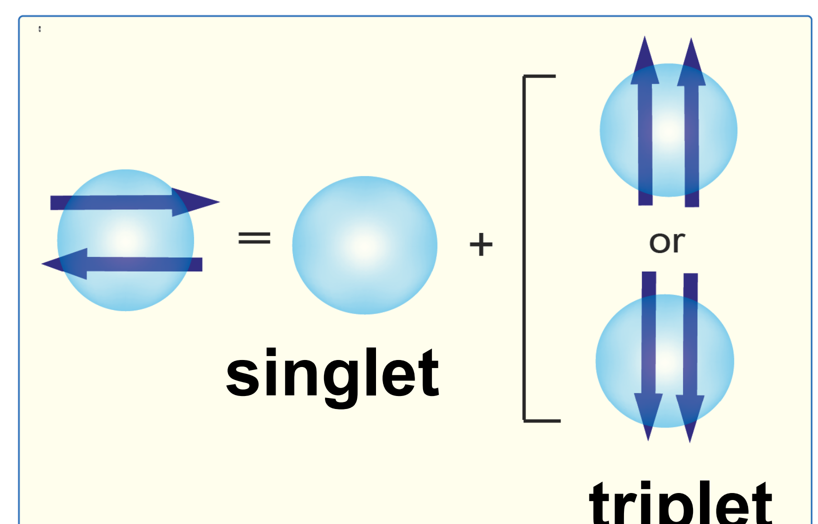
triplet with $S_{\text{in-plane}} = 0$

$|\downarrow\rangle|\uparrow\rangle$

Superconducting gap

$$\hat{\Delta}(p) = \Delta_s(p) i\sigma_y + \Delta_t(p)\underline{(\vec{p}\times\vec{n})}\cdot\vec{\sigma} i\sigma_y$$

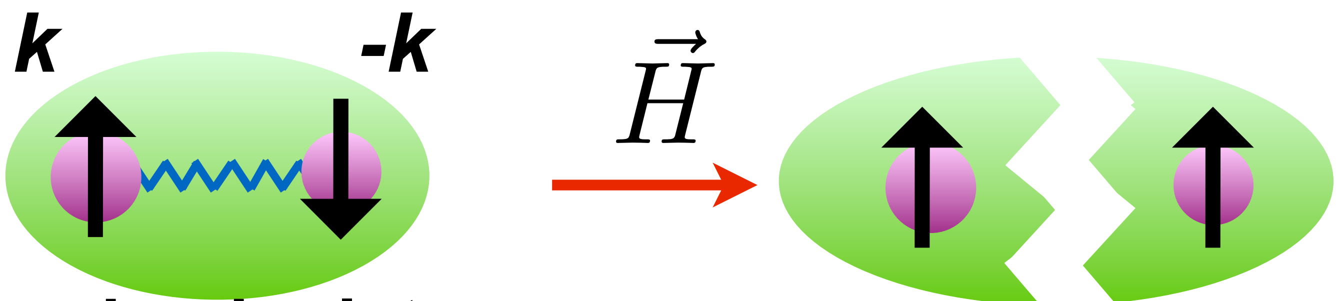
d-vector is constrained by the SO int.



Suppression of conventional superconductivity with inversion symmetry due to magnetic fields

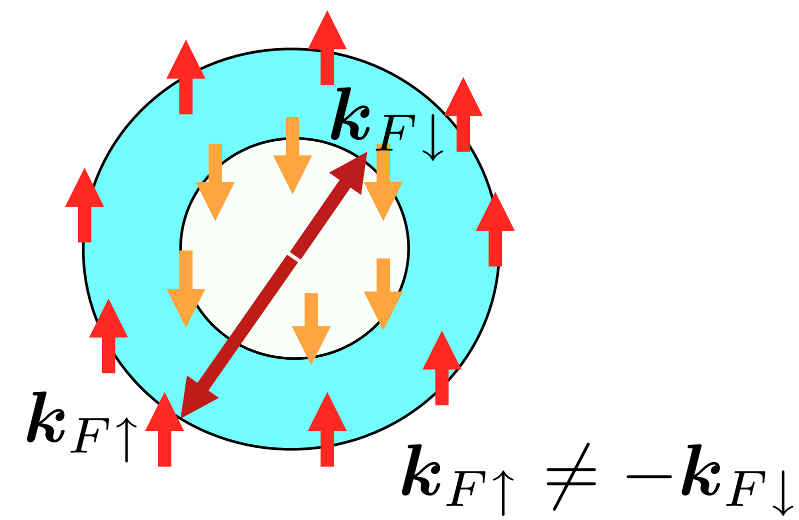
(I) Zeeman effect breaks spin-singlet Cooper pairs

(Pauli depairing effect)



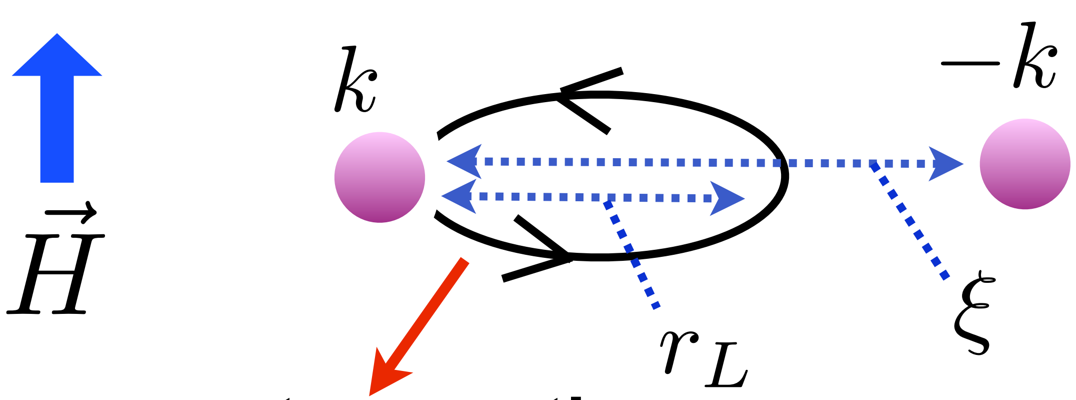
spin-singlet Cooper pair

Zeeman-split Fermi surface



(c.f. spin-triplet Cooper pairs are not destroyed by this effect)

(II) Lorentz force breaks Cooper pairs (Orbital depairing effect)



cycrotron motion of electron

ξ **: coherence length for SC**

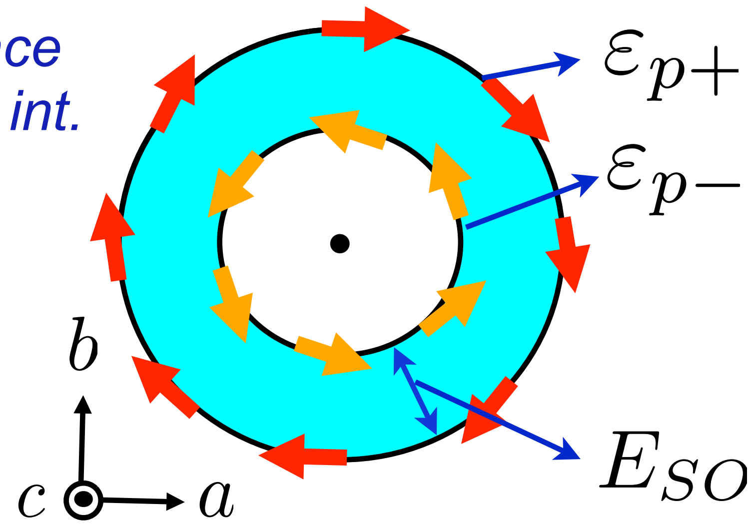
r_L **: Larmor radius**

For $\xi > r_L$

Cooper pairs are destroyed

In the case of NCS, Pauli depairing effect is strongly suppressed !!

SO split Fermi Surface for Rashba-type SO int.



for $H_z \parallel c$

$$\chi_N = \chi_{VV} = -\sum_p \frac{f(\varepsilon_{p+}) - f(\varepsilon_{p-})}{\varepsilon_{p+} - \varepsilon_{p-}}$$

orbital term

H_P **: Pauli limiting field**

For $H > H_P$, **SC is destroyed by the Pauli depairing effect**

Energy balance between the normal state and the SC state

$$\frac{\chi_N H_P^2}{2} = \frac{\chi_S H_P^2}{2} + \frac{N(0)\Delta^2}{2}$$

For $E_{SO} \gg \Delta$ $\quad \chi_N \approx \chi_S \quad \rightarrow H_P \rightarrow \infty$

Large orbital susceptibility suppresses Pauli depairing effect !!

Not due to parity-mixing !

even for pure spin-singlet case, $H_P \rightarrow \infty$ (Frigeri et al. 2004, S.F. 2007)

2011年3月4日金曜日

Suppression of Pauli depairing effect due to antisymmetric SO interaction depends on the direction of applied magnetic fields

anti-symmetric SO int. $\mathcal{H}_{SO} = \lambda \boldsymbol{\mathcal{L}}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}$

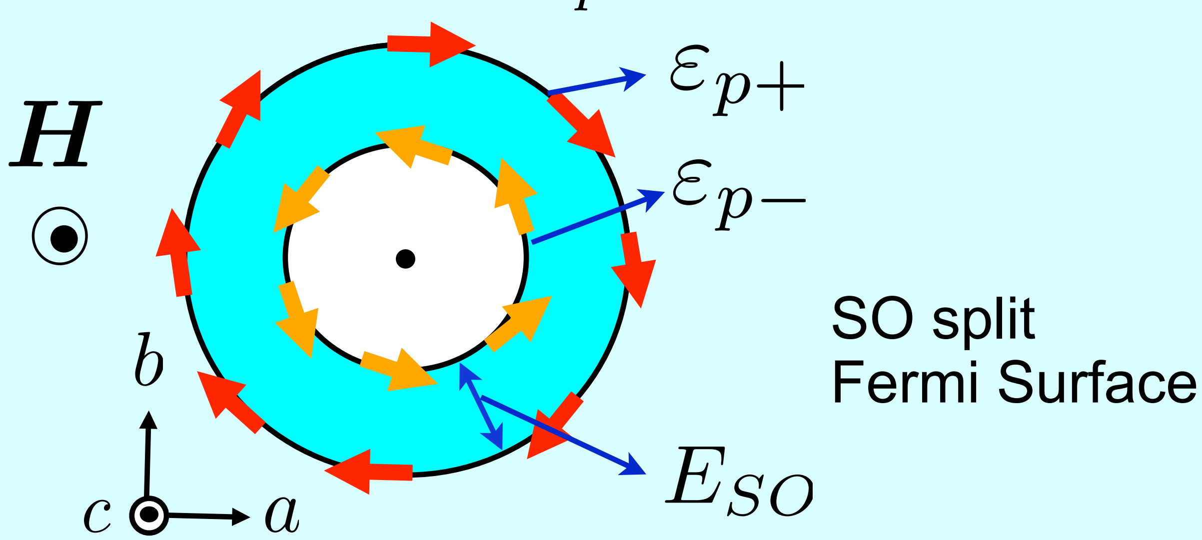
For magnetic field satisfying $\boldsymbol{\mathcal{L}}(\boldsymbol{k}) \cdot \boldsymbol{H} = 0$, Pauli depairing effect is suppressed

Rashba SO interaction: $\boldsymbol{\mathcal{L}}(\boldsymbol{k}) = (k_y, -k_x, 0)$

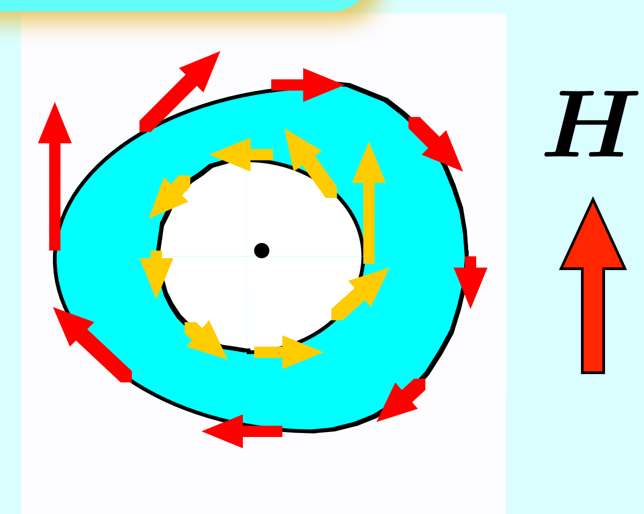
$\boldsymbol{H} \parallel$ z-axis

orbital term

$$\chi_{\parallel} = \chi_{VV} = -\sum_p \frac{f(\varepsilon_{p+}) - f(\varepsilon_{p-})}{\varepsilon_{p+} - \varepsilon_{p-}}$$



$\boldsymbol{H} \perp$ z-axis



asymmetric deformation of Fermi surface raises Pauli depairing effect

Topological phases in *P*-wave pairing state of noncentrosymmetric superconductors

[M. Sato and S.F., Phys. Rev.B79, 094504 (2009)]

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In the case with $\mathcal{T}$-symmetry

Rashba p-wave SC ⟷ ***(p+ip SC)+(p-ip SC)***

adiabatic deformation of Hamiltonian

Z_2 topological order
(quantum spin Hall effect state)

(c.f. Schnyder,Ryu,Furusaki,Ludwig,(2008), Qi, Hughes, Raghu, Zhang(2009))

In the case with broken $\mathcal{T}$-symmetry

Rashba p-wave SC ⟷ ***p+ip SC***

adiabatic deformation of Hamiltonian

(Read-Moore state of FQHE)

Non-abelian topological order

Topological Order in *P(+s)* wave NCS

Model: 2D Rashba superconductors with s+p-wave gap symmetry

Hamiltonian (defined on 2D square lattice)

$$\mathcal{H} = -t \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle,\sigma} c^{\dagger}_{\boldsymbol{i}\sigma} c_{\boldsymbol{j}\sigma} - \mu \sum_{\boldsymbol{i},\sigma} c^{\dagger}_{\boldsymbol{i}\sigma} c_{\boldsymbol{i}\sigma} - \mu_{\mathrm{B}} H_z \sum_{\boldsymbol{i},\sigma,\sigma'} (\sigma_z)_{\sigma\sigma'} c^{\dagger}_{\boldsymbol{i}\sigma} c_{\boldsymbol{i}\sigma'}$$

$$- i\frac{\alpha}{2} \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle,\sigma,\sigma'} (\boldsymbol{\sigma}_{\sigma\sigma'} \times \hat{\boldsymbol{r}}_{\boldsymbol{ij}})_z c^{\dagger}_{\boldsymbol{i}\sigma} c_{\boldsymbol{j}\sigma'}$$

← **Rashba SO interaction**

$$+\Delta_{\mathrm{s}} \sum_{\boldsymbol{i}} (c^{\dagger}_{\boldsymbol{i}\uparrow} c^{\dagger}_{\boldsymbol{i}\downarrow} + \mathrm{h.c.})$$

← **s-wave pairing**

$$-\frac{1}{2}\Delta_{\mathrm{t}} \sum_{\boldsymbol{i}} (c^{\dagger}_{\boldsymbol{i}\uparrow} c^{\dagger}_{\boldsymbol{i}+\hat{x}\uparrow} + c^{\dagger}_{\boldsymbol{i}\downarrow} c^{\dagger}_{\boldsymbol{i}+\hat{x}\downarrow} - i c^{\dagger}_{\boldsymbol{i}\uparrow} c^{\dagger}_{\boldsymbol{i}+\hat{y}\uparrow} + i c^{\dagger}_{\boldsymbol{i}\downarrow} c^{\dagger}_{\boldsymbol{i}+\hat{y}\downarrow} + \mathrm{h.c.}),$$

← **p-wave pairing**

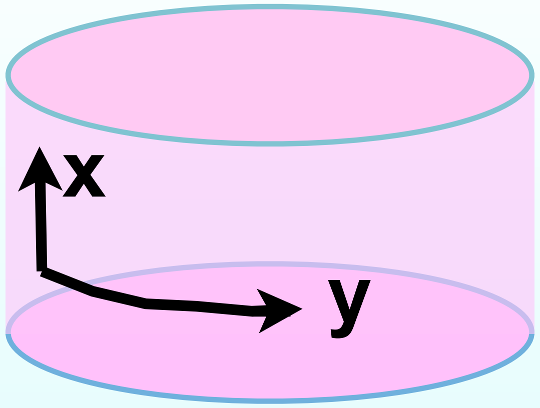
***d*-vector of the p-wave pairing satisfies**

$$\boldsymbol{d} \propto (\sin k_y, -\sin k_x, 0)$$ **required by the Rashba SO int.**

Rashba SO int.

$$\mathcal{H}_{\mathrm{SO}} = \alpha \sum_{k} \boldsymbol{g_k} \cdot \boldsymbol{\sigma}_{\sigma\sigma'} c^{\dagger}_{k\sigma} c_{k\sigma'} \qquad \boldsymbol{g_k} = (\sin k_y, -\sin k_x, 0)$$

Energy spectrum of 2D Rashba SC: gapless edge states



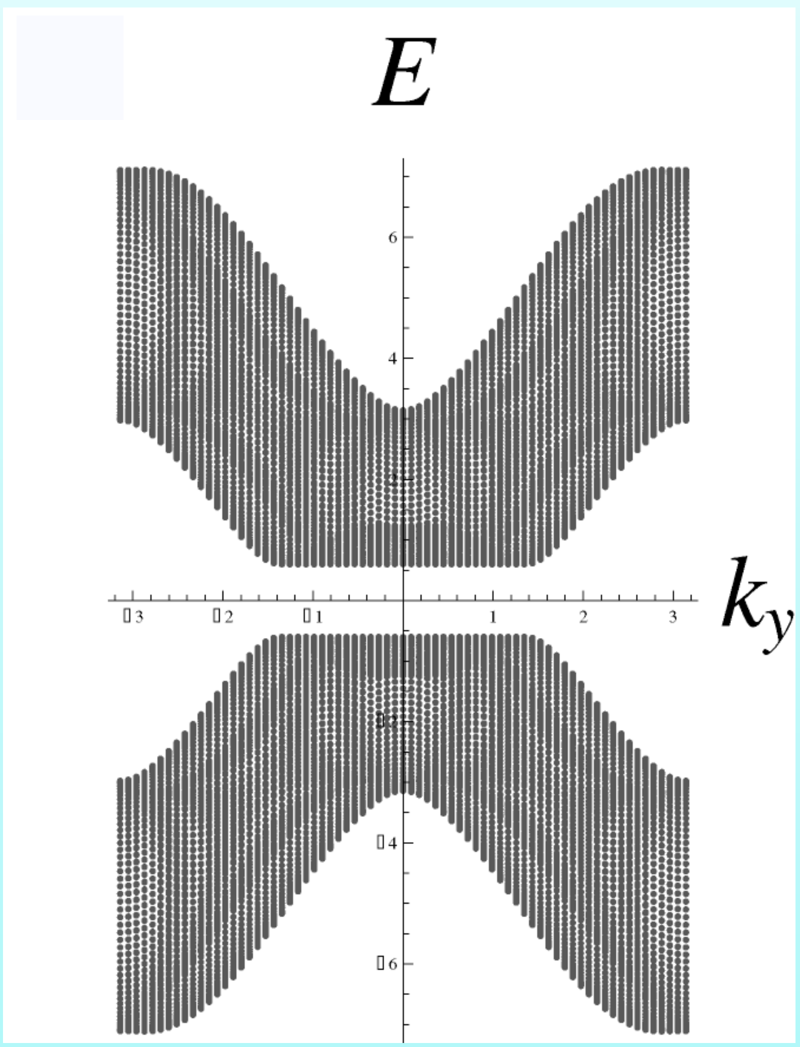
open boundary condition for x-direction

periodic boundary condition for y-direction

Note that bulk SC gap exists $\Delta_t + \Delta_s$, $|\Delta_t - \Delta_s|$

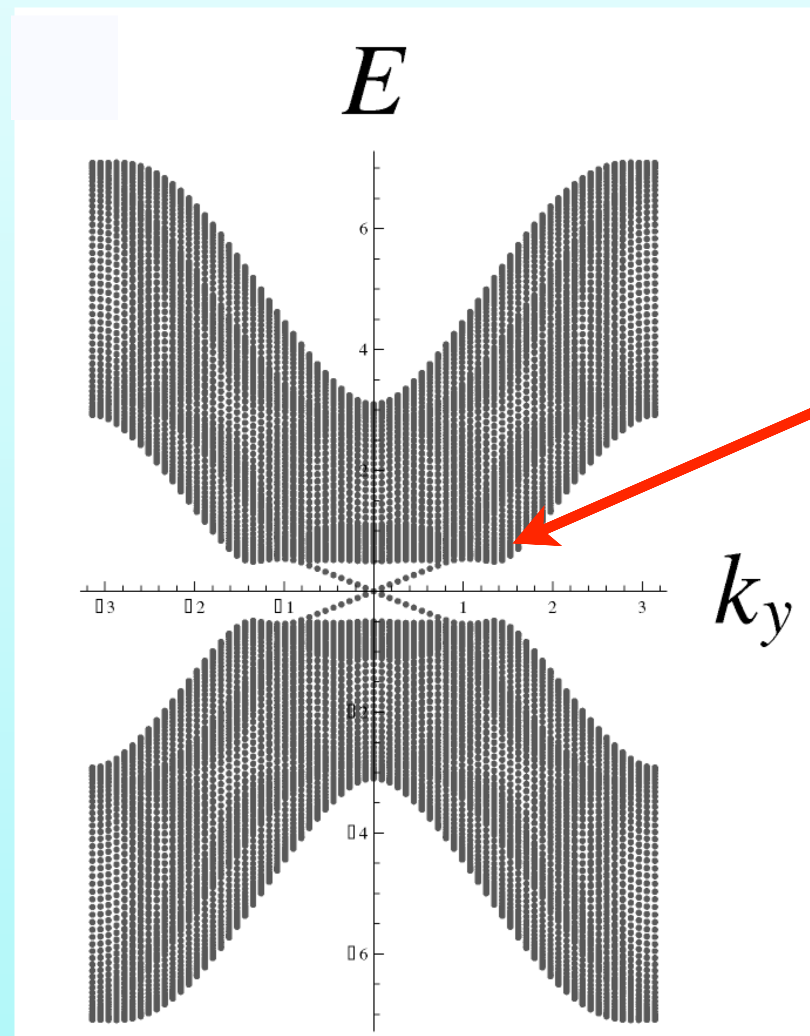
Case without magnetic fields (with $\mathcal{T}$-symmetry)

No gapless edge modes



pure s-wave gap

$\Delta_t = 0 \qquad \Delta_s = 0.6$



p-wave gap dominated

$\Delta_t = 0.6 \qquad \Delta_s = 0.1.$

gapless edge modes

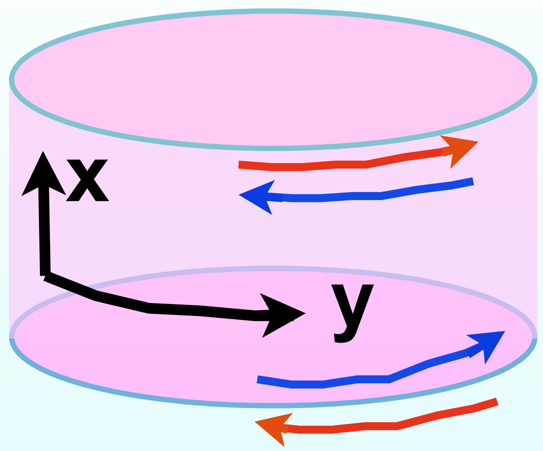
4 modes (left-moving, right-moving X 2 bands)

Chern # = 0

Z_2 invariant = nontrivial

(M. Sato, 2009)

Energy spectrum of 2D Rashba SC: gapless edge states



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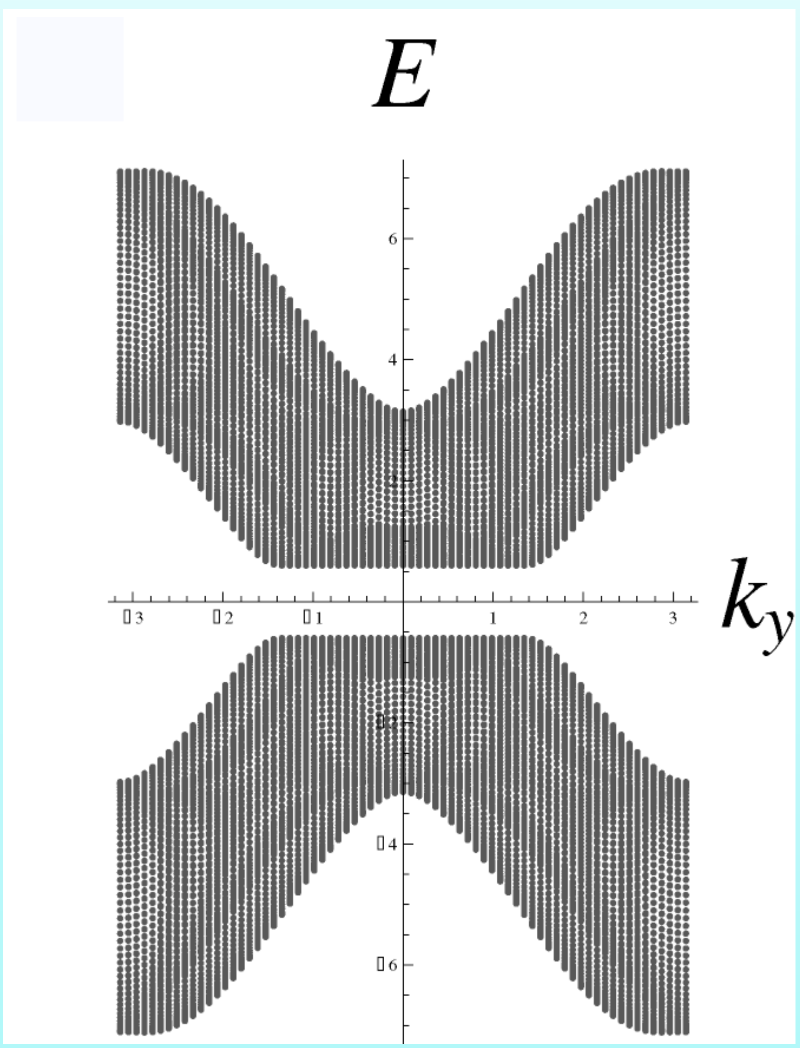
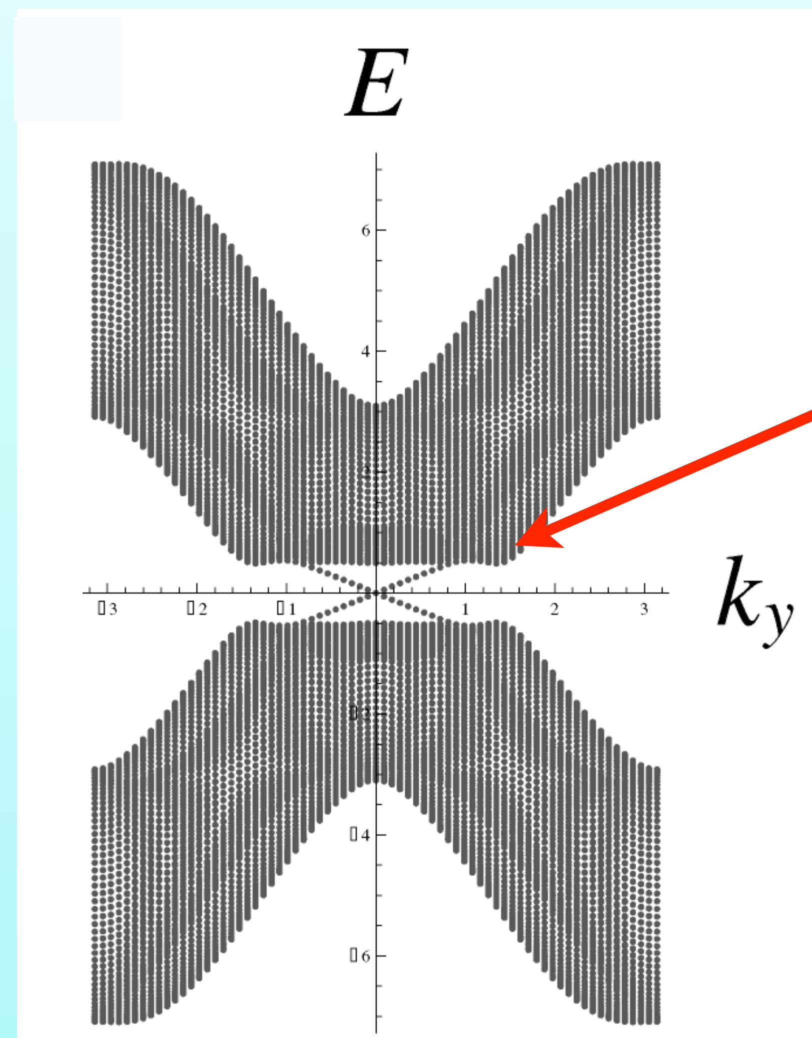


pure s-wave gap

$\Delta_t = 0 \qquad \Delta_s = 0.6$



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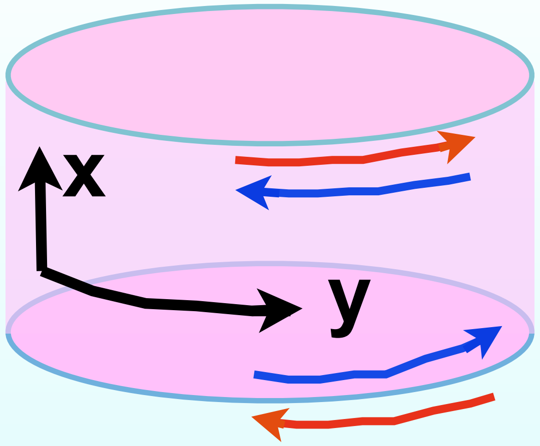

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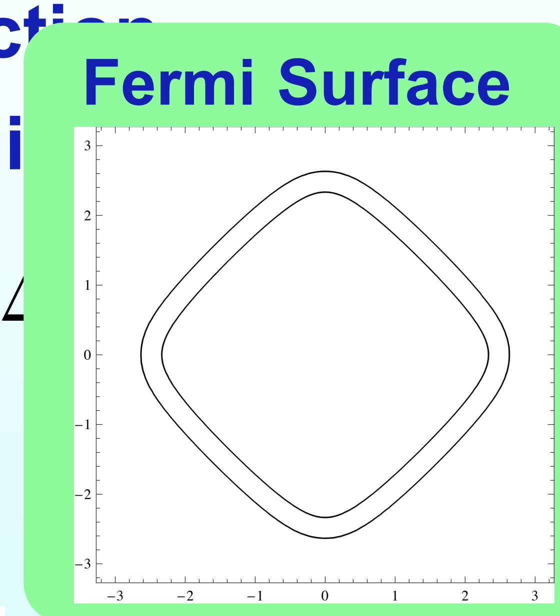

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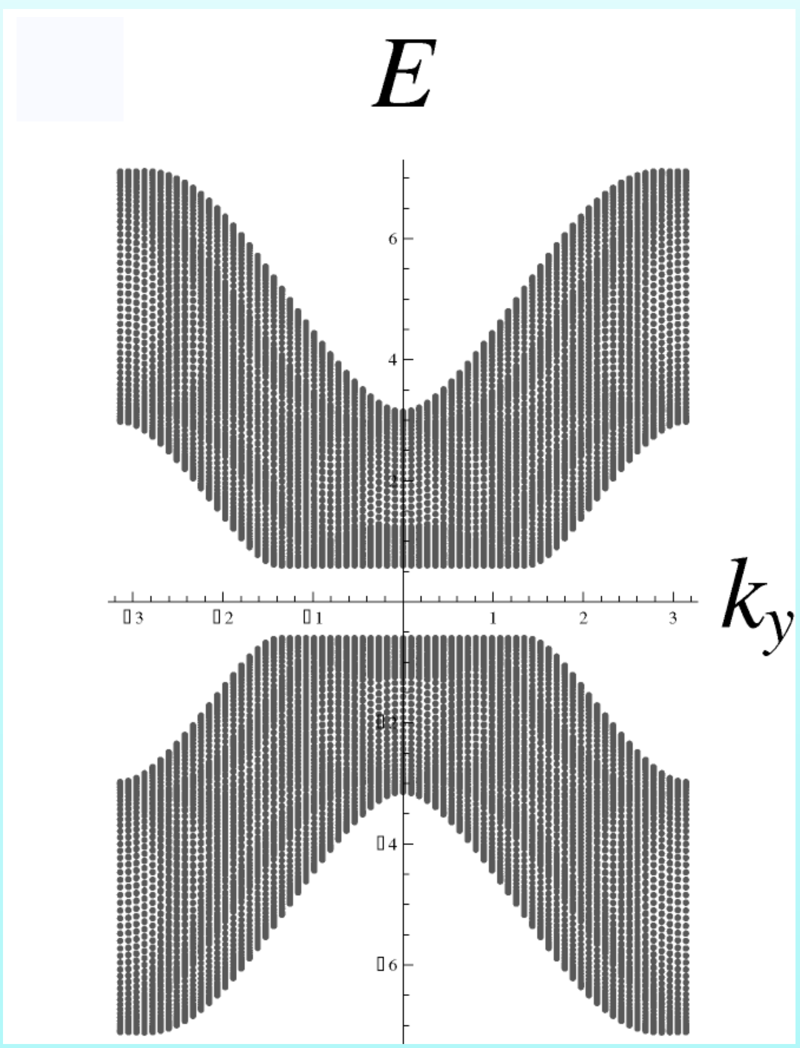
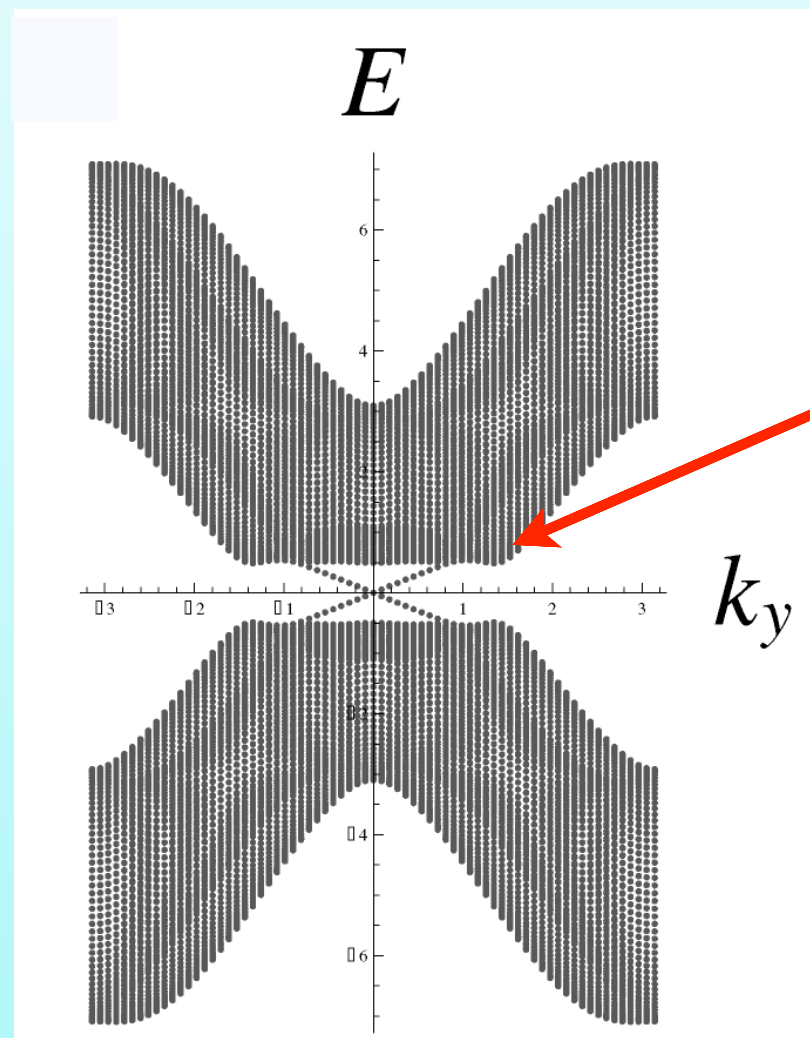
No gapless edge modes



pure s-wave gap

$\Delta_t = 0 \qquad \Delta_s = 0.6$



p-wave gap dominated

$\Delta_t = 0.6 \qquad \Delta_s = 0.1.$

gapless edge modes

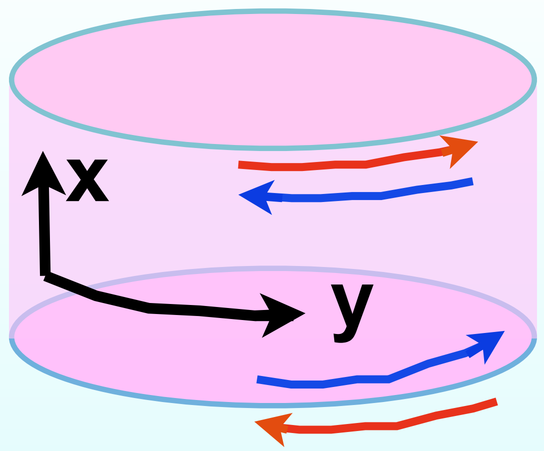
4 modes (left-moving, right-moving X 2 bands)

Chern # = 0

Z_2 invariant = nontrivial

(M. Sato, 2009)

Energy spectrum of 2D Rashba SC: gapless edge states



open boundary condition for x-directi

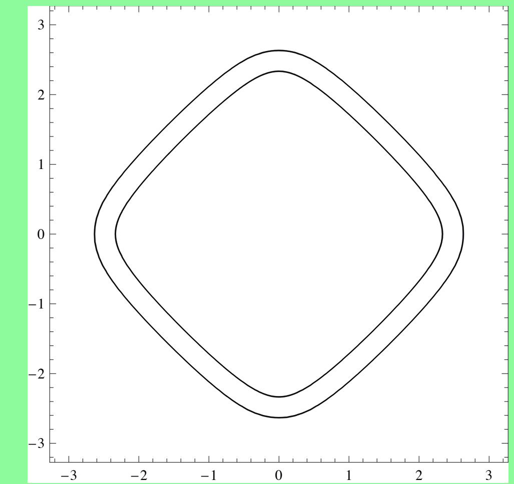
periodic boundary condition for y-di

N

Same class as
Z_2 topological Insulator

Case without magnet

Fermi Surface



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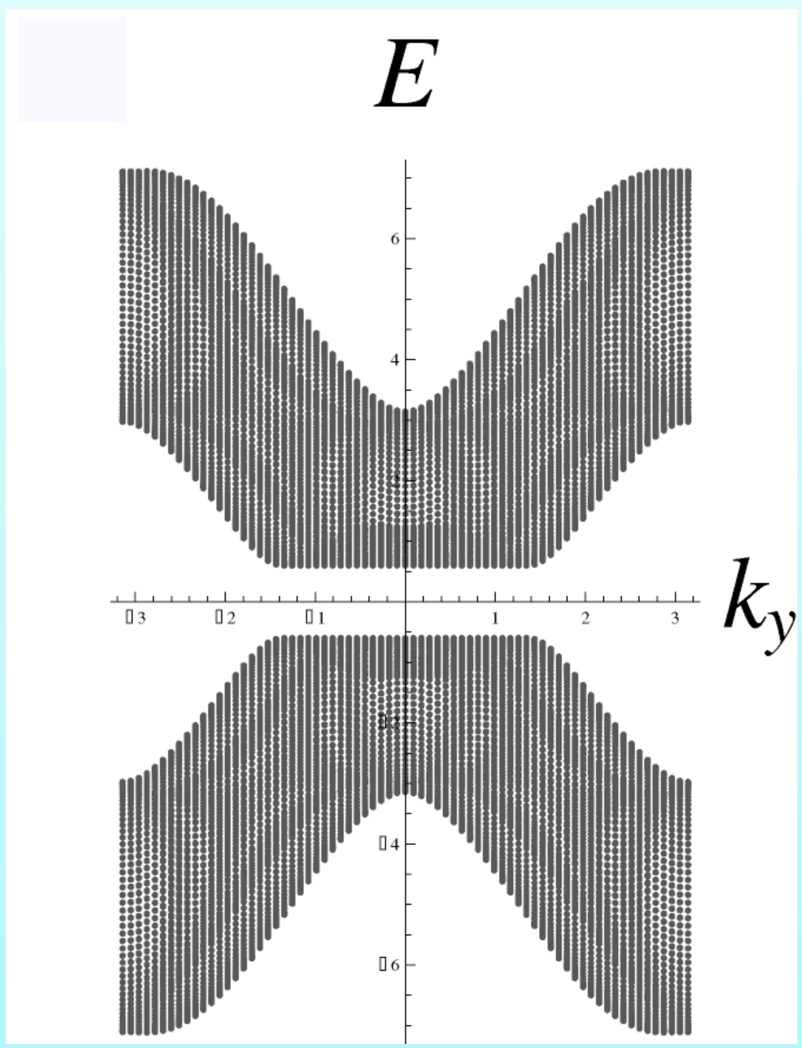

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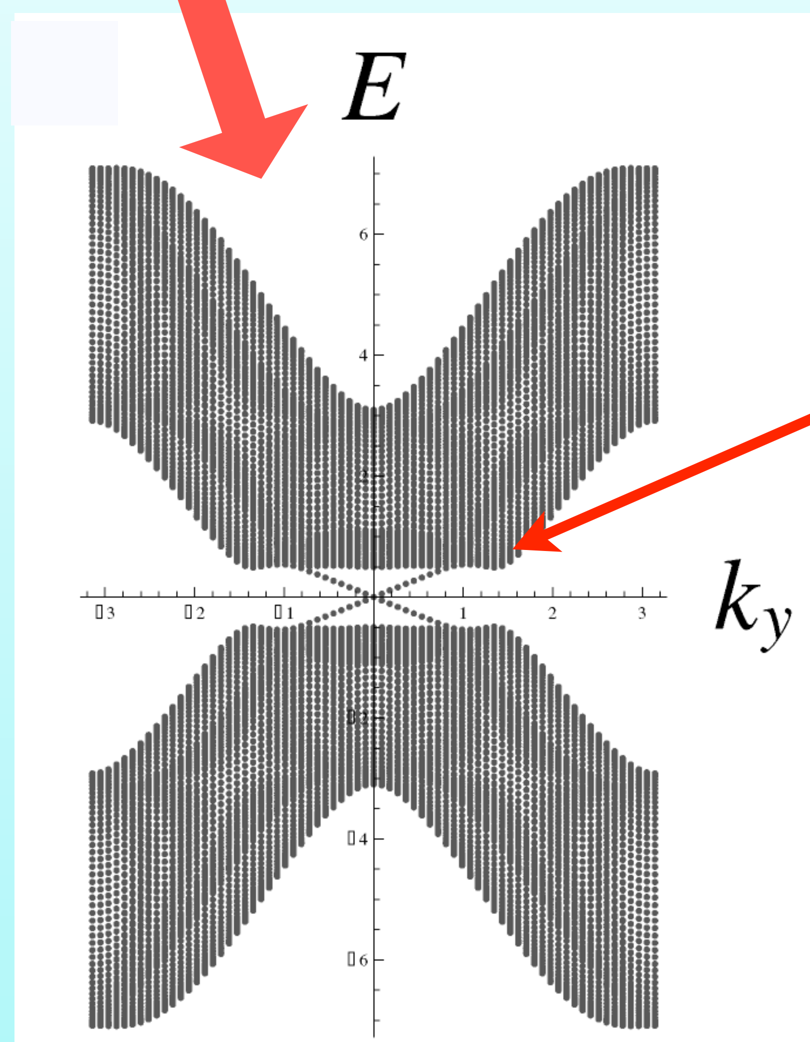

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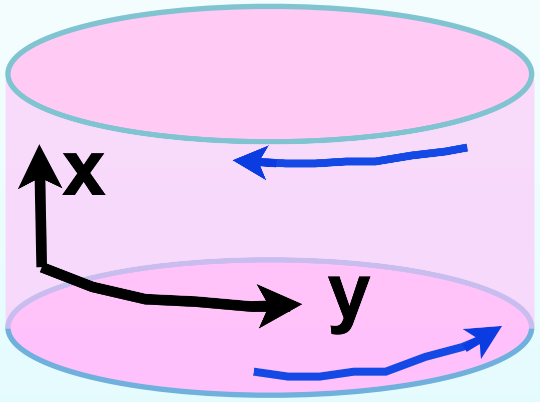
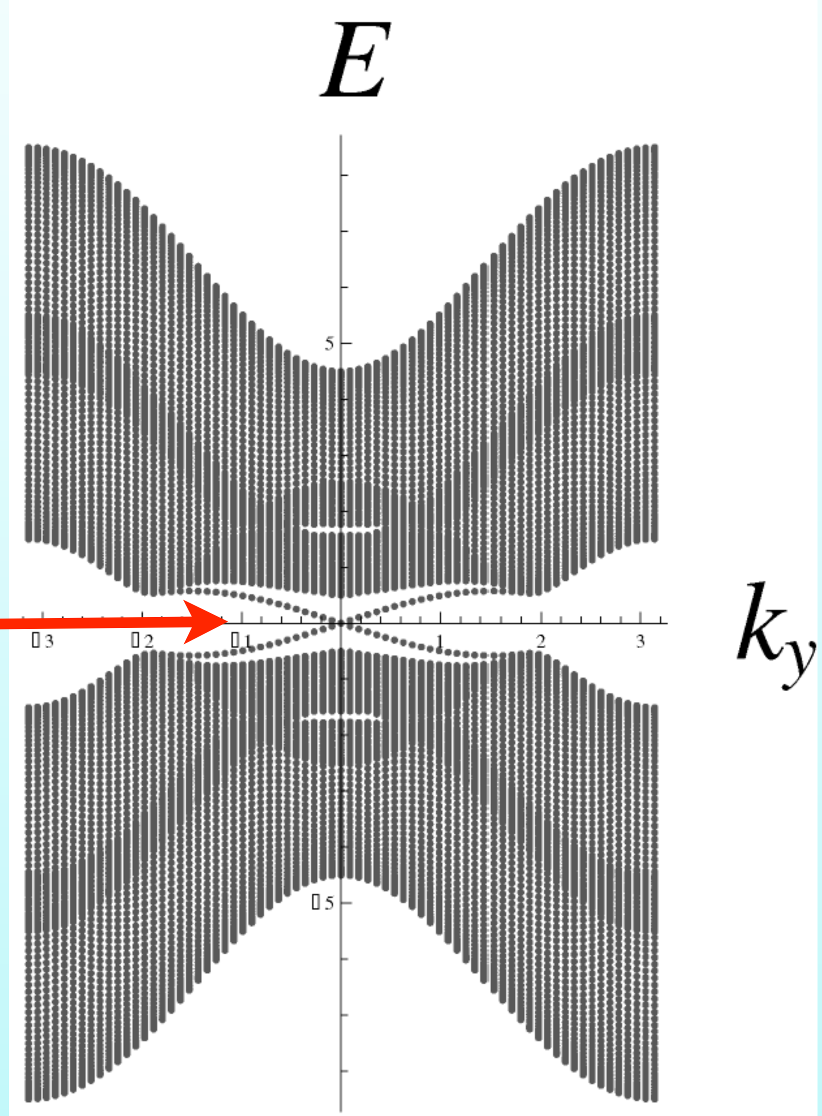
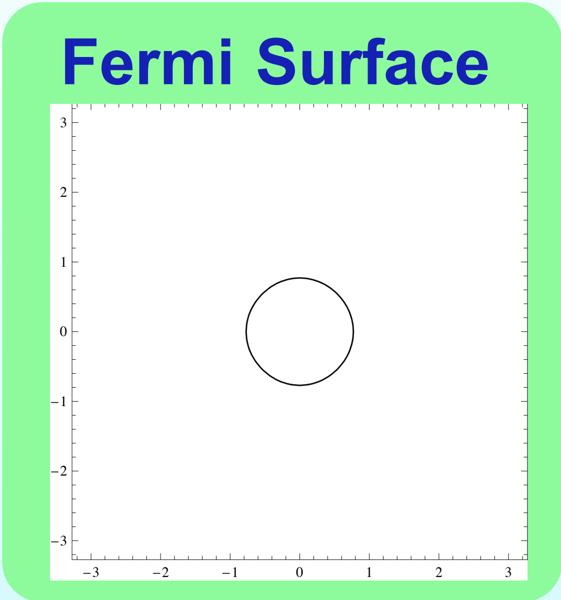
Case with magnetic fields (broken $\mathcal{T}$-symmetry)

Case with sufficently strong Zeeman fields, where there is only one band crossing Fermi level







2 gapless edge modes (left-moving, right-moving on different sides of boundaries)

Majorana Fermion mode

p-wave gap dominated

$$\Delta_t = 0.6 \qquad \Delta_s = 0.1.$$

Chern # = 1

Z_2 invariant =can not be defined

Same class as quantum Hall effect state

Majorana zero energy states in vortex cores of NCS

vortex with single vorticity $\Delta \rightarrow \Delta e^{i\phi}$

Purely p-wave case

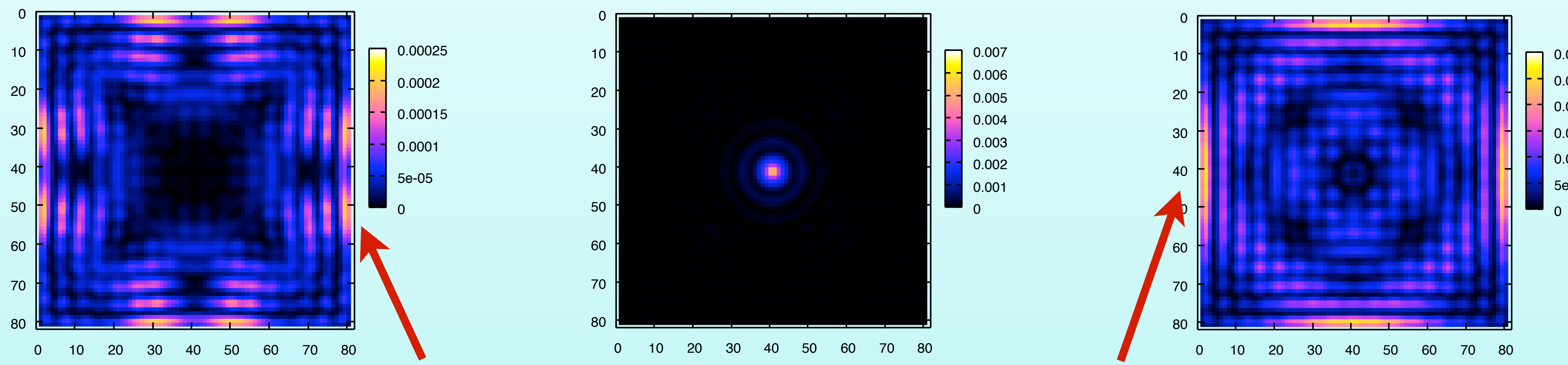
$\Delta_s = 0 \quad \Delta_p = 0.05t \quad \alpha = 0.5t \quad \frac{\Delta_p^2}{E_F} \sim 0.005$

Numerical analysis of BdG eqs.

DOS profile of quasiparticles

identified with zero energy modes

$\varepsilon = 4.87 \times 10^{-4}$ $\quad$ $\varepsilon = 7.67 \times 10^{-4}$ $\quad$ $\varepsilon = 1.02 \times 10^{-3}$



low energy edge modes also exist

As long as $\Delta_p > \Delta_s$ holds, Majorana modes in vortex core appear

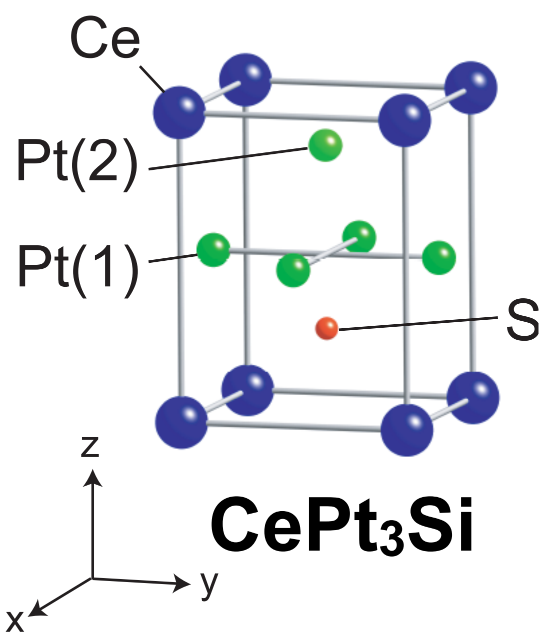
(c.f. Lu, Yip)

The existence of Majorana zero energy modes is also confirmed by the extension of the Jackiw-Rebbi's index theorem *(c.f. Tewari et al.)*

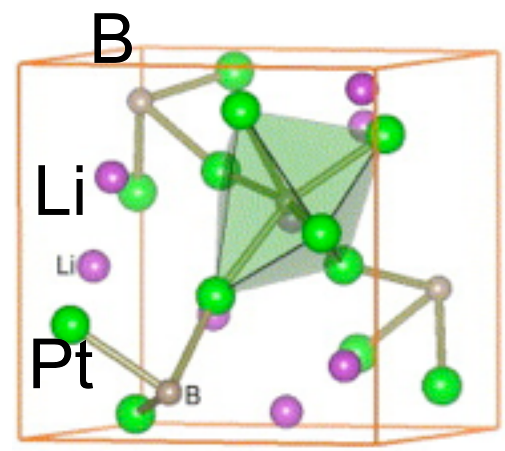
However, the first excitation energy $\sim \Delta^2/E_F$ → very low temperature (~μK) is required to realize the Majorana state !

Where can we find topological phases in *p*-wave NC SC ?

p-wave pairing dominated state in NCSC ?



$CePt_3Si$



Li_2Pt_3B

$$\mathcal{H}_{SO} = \lambda \boldsymbol{k} \cdot \boldsymbol{\sigma} \ \ ?$$

$$\Delta(\boldsymbol{k}) = \Delta_0 \boldsymbol{k} \cdot \boldsymbol{\sigma} i\sigma_2 \ ?$$

(like BW phase)

TRI top. SC ?

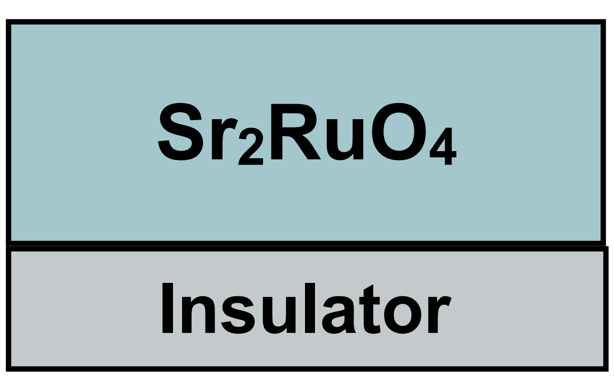
However, there are nodes of SC gap (gapless) due to s+p wave state

cubic structure

fully gapped or gapless ?

experimental data are controversial

TRI topological SC in heterostructure which consists of Sr_2RuO_4



bulk Sr_2RuO_4 : chiral *p+ip* SC without TRI

However

Rashba SO int
due to broken inv. sym.

TRI recovered
(*p+ip*)+(*p-ip*) SC
TRI top. SC

(Tada, Kawakami, Fujimoto, New J. Phys.(2009))

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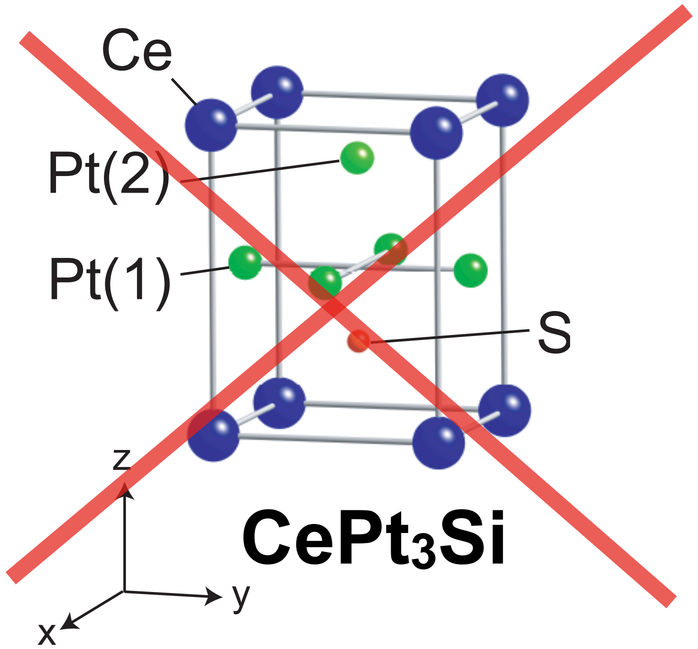

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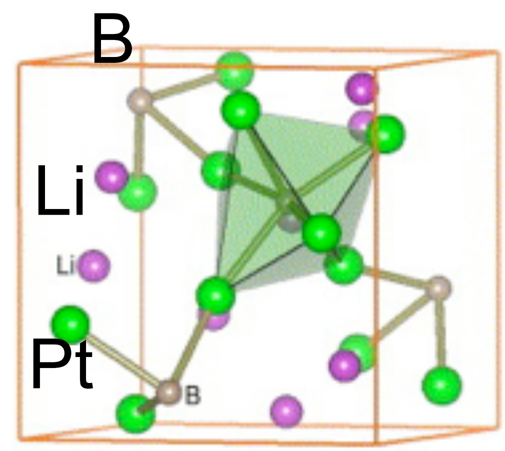

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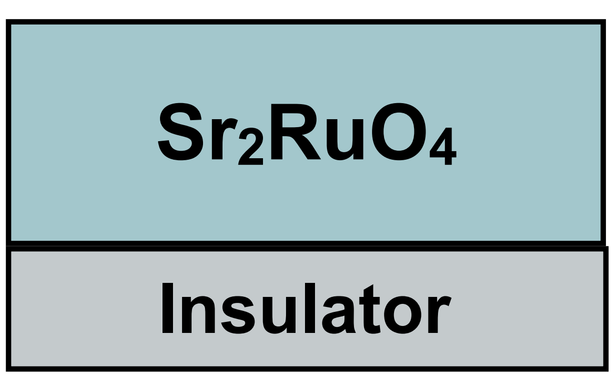

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Summary for *P*(+*s*)-wave Rashba superconductors

Topological phases realize in Rashba *P*(+*s*) wave SC with $\Delta_p > \Delta_s$ and with no nodes of SC gap

In the case with no magnetic fields ($\mathcal{T}$-invariant),

Rashba p-waveSC ⟷ ***Z_2 topological insulator (quantum spin Hall effect state)***

(c.f. Schnyder,Ryu,Furusaki,Ludwig,Qi,Hughes,Raghu,Zhang)

In the case with magnetic fields (broken $\mathcal{T}$-symmetry)

For Rashba p-wave SC, non-Abelian topological order appears

Rashba p-wave SC ⟷ ***Read-Moore state of FQHE***

(c.f. *p+ip SC* ⟷ ***Read-Moore state of FQHE*)**

Non-Abelian topological order in *S*-wave pairing state

[M. Sato, Y. Takahashi, and S.F.,Phys. Rev. Lett.103, 020401 (2009)]

Preliminary ....

Zero modes of vortex in s-wave pairing state of Dirac fermions

"in the n-vortex sector there are |n| isolated, linearly independent, zero energy bound states, which are eigenstates of a particle conjugation transformation"

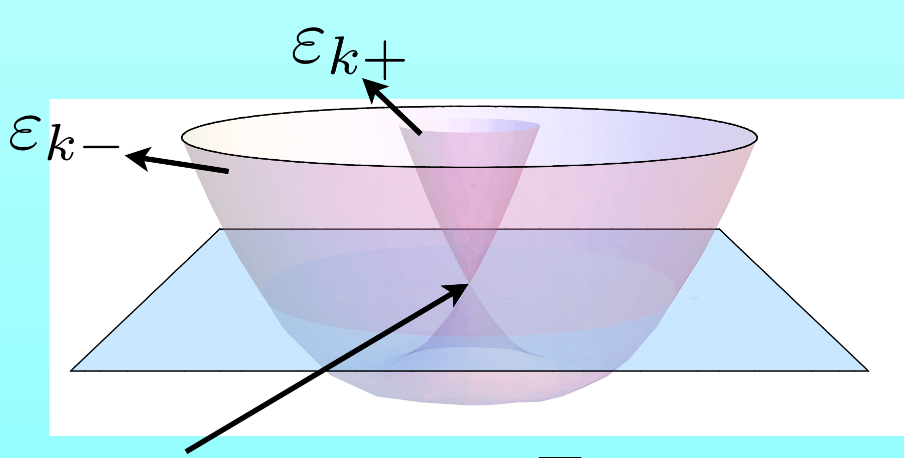
Jackiw-Rossi, Nucl. Phys. B190, 681 (1981)

→ **one Majorana fermion mode in the vortex with the vorticity n=1**

(c.f. At an interface between Z_2 topological insulator and s-wave SC, odd number of massless Dirac cone exists, Fu and Kane, PRL100, 096407 (2008))

In NC superconductors, there are Dirac cones in the vicinity of time-reversal invariant *k*-points in the Brillouin zone.

Energy band split by Rashba SO int.



Dirac cone at the Γ point

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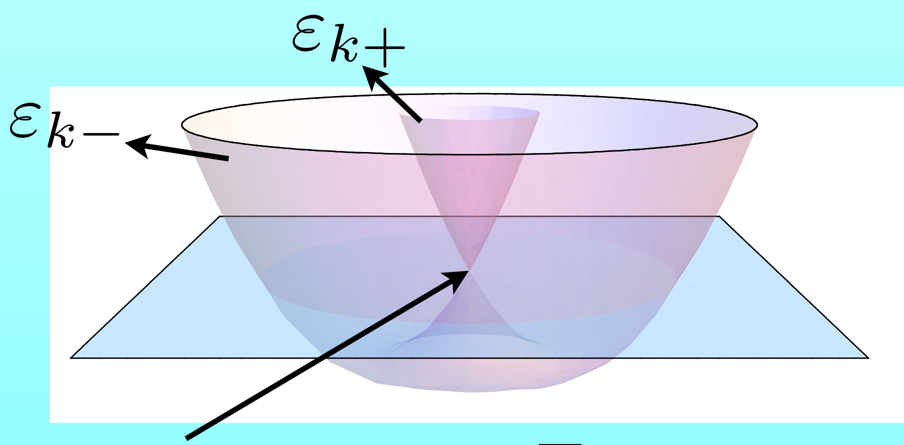

Dirac cone at the Γ point

However, s-wave NC superconductors for zero magnetic field are topologically trivial !

- ***there is no edge state***
- ***Z_2 trivial***
- ***zero Chern number for small magnetic fields***

Even number of Dirac cone. Also, other non-Dirac-like bands give rise to mass gap in the vortex state and edge states

Nevertheless !!

Non-Abelian Topological order and Majorana Fermions realizes in S-Wave Rashba superconductors (superfluids), provided that there is a large magnetic field $\mu_{\mathrm{B}}H > \Delta$

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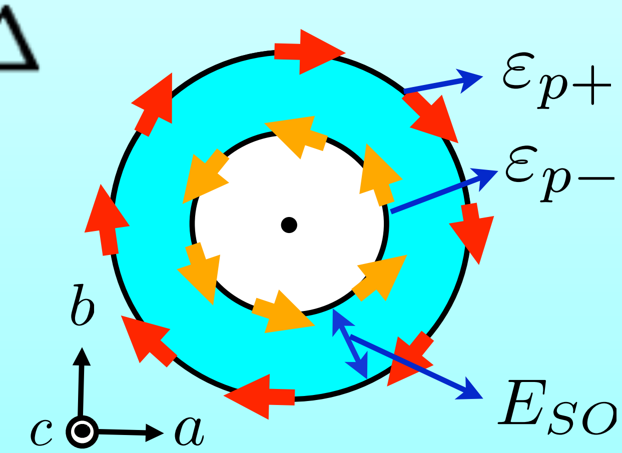
possible !

for $H_z \parallel c$

$$\chi_N \approx \chi_S$$



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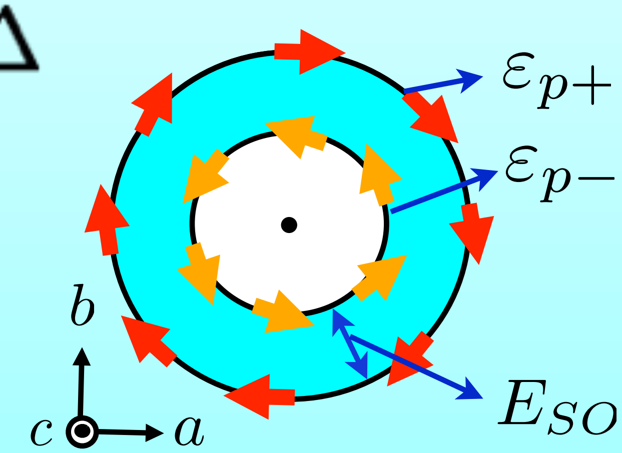

Orbital limiting field

$$\mu_B H_{c2}^{\mathrm{orb}} \sim \frac{1}{z} \cdot \frac{\Delta}{E_F} \cdot \Delta$$

If mass enhancement

$$\frac{1}{z} \sim -\frac{\partial \Sigma}{\partial \varepsilon} \sim 100$$

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2011年3月4日金曜日

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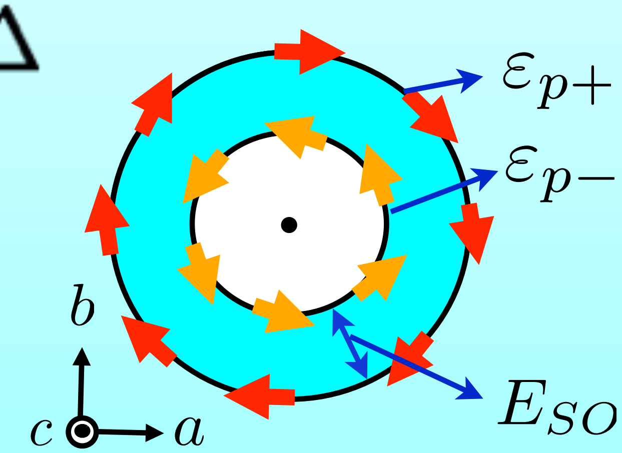

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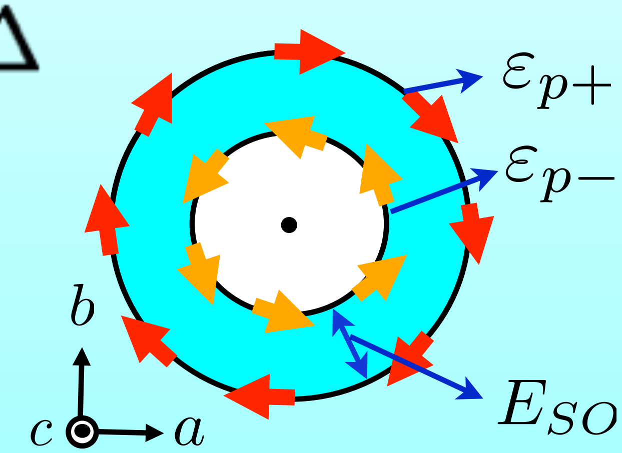

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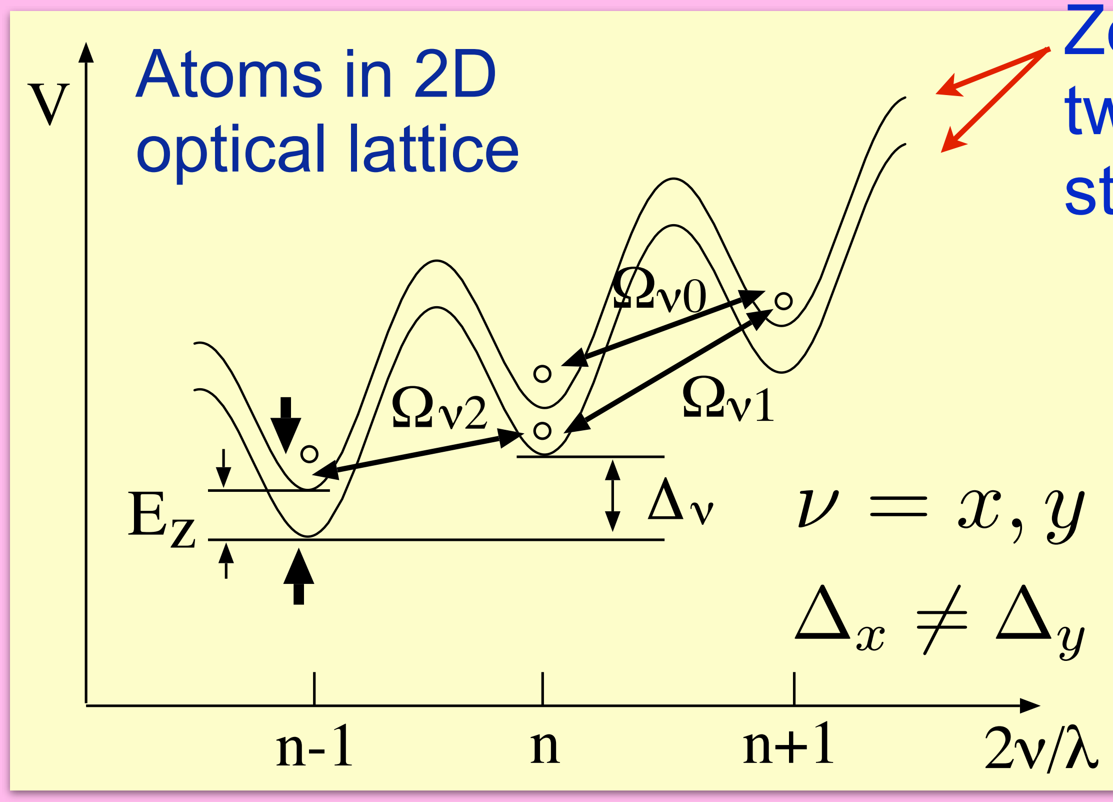

Zeeman split two hyperfine states

(Osterloh et al., PRL95, 010403; Ruseckas et al., PRL95, 010404; Sato et al., PRL103, 0204101)

$$\Omega_{\nu 1}: |n, \uparrow\rangle \longleftrightarrow |n+1, \downarrow\rangle$$

$$\Omega_{\nu 2}: |n, \downarrow\rangle \longleftrightarrow |n+1, \uparrow\rangle$$

$$\Omega_{x2} = |\Omega_{x2}| e^{ik_z z} \qquad \Omega_{x2} = -\Omega_{x1}$$

$$\Omega_{y2} = -i\Omega_{x1} \qquad \Omega_{y1} = -\Omega_{y2}$$

Rashba SO int. $\mathcal{H}_{\rm SO} = \sum_i [\lambda_x (c^\dagger_{i-\hat{x}\downarrow} c_{i\uparrow} - c^\dagger_{i+\hat{x}\downarrow} c_{i\uparrow}) + i\lambda_y (c^\dagger_{i-\hat{y}\downarrow} c_{i\uparrow} - c^\dagger_{i+\hat{y}\downarrow} c_{i\uparrow}) + {\rm h.c.}]$

$$\lambda_\nu = c_\nu \int d\boldsymbol{r} \psi^*_\downarrow(\boldsymbol{r} - \boldsymbol{r}_{i-\hat{\mu}}) \Omega_{\nu 2}(\boldsymbol{r}) \psi_\uparrow(\boldsymbol{r} - \boldsymbol{r}_i) \qquad c_x = 1 \qquad c_y = -i$$

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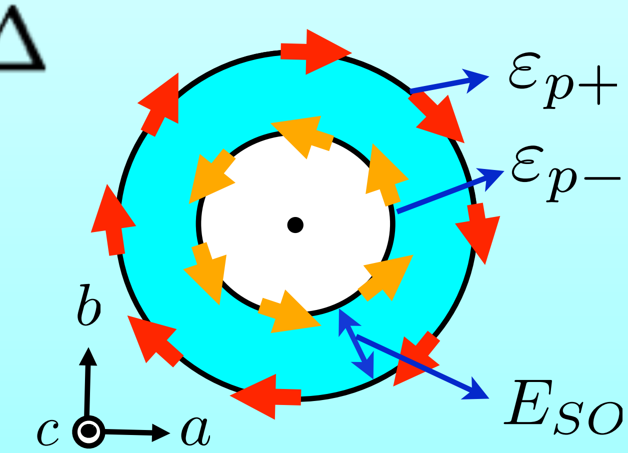

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Model Hamiltonian

$$\mathcal{H} = \mathcal{H}_{\rm kin} + \mathcal{H}_{\rm SO} + \mathcal{H}_{\rm s}$$

$$\begin{aligned}\mathcal{H}_{\rm kin} &= -t\sum_{i\sigma}\sum_{\hat\mu=\hat x,\hat y}(c^\dagger_{i+\hat\mu\sigma}c_{i\sigma} + c^\dagger_{i-\hat\mu\sigma}c_{i\sigma}) - \mu\sum_{i\sigma}c^\dagger_{i\sigma}c_{i\sigma} \\ &\quad -h\sum_i(c^\dagger_{i\uparrow}c_{i\uparrow} - c^\dagger_{i\downarrow}c_{i\downarrow})\end{aligned}$$

kinetic energy term

Zeeman term

$$\mathcal{H}_{\rm SO} = -\lambda\sum_i[(c^\dagger_{i-\hat x\downarrow}c_{i\uparrow} - c^\dagger_{i+\hat x\downarrow}c_{i\uparrow}) + i(c^\dagger_{i-\hat y\downarrow}c_{i\uparrow} - c^\dagger_{i+\hat y\downarrow}c_{i\uparrow}) + \text{h.c.}]$$

Rashba SO int.

$$\mathcal{H}_{\rm s} = -\sum_i\psi_{\rm s}(c^\dagger_{i\uparrow}c^\dagger_{i\downarrow} + \text{h.c.})$$

s-wave pairing term

Key observation:

Duality relation between s-wave Rashba SC and p-wave SC

$$\mathcal{H} = \frac{1}{2}\sum_{\boldsymbol{k}} \left(c^\dagger_{\boldsymbol{k}}, c_{-\boldsymbol{k}}\right) \mathcal{H}(\boldsymbol{k}) \begin{pmatrix} c_{\boldsymbol{k}} \\ c^\dagger_{-\boldsymbol{k}} \end{pmatrix} \qquad c^\dagger_{\boldsymbol{k}} = \frac{1}{\sqrt{V}}\sum_i e^{i\boldsymbol{k}i}(c^\dagger_{i\uparrow}, c^\dagger_{i\downarrow})$$

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \epsilon_{\boldsymbol{k}} - h\sigma_z + \boldsymbol{g}_{\boldsymbol{k}}\cdot\boldsymbol{\sigma} & i\psi_{\mathrm{s}}\sigma_y \\ -i\psi_{\mathrm{s}}\sigma_y & -\epsilon_{\boldsymbol{k}} + h\sigma_z + \boldsymbol{g}_{\boldsymbol{k}}\cdot\boldsymbol{\sigma}^* \end{pmatrix}$$

$$\epsilon_{\boldsymbol{k}} = -2t(\cos k_x + \cos k_y) - \mu$$

$$\boldsymbol{g}_{\boldsymbol{k}} = 2\lambda(\sin k_y, -\sin k_x, 0)$$

Dual Hamiltonian

$$\mathcal{H}^{\mathrm{D}}(\boldsymbol{k}) = D\mathcal{H}(\boldsymbol{k})D^\dagger \qquad D = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i\sigma_y \\ i\sigma_y & 1 \end{pmatrix}$$

independent of $\boldsymbol{k}$ and $\boldsymbol{r}$

not affect topological order

$$\mathcal{H}^{\mathrm{D}}(\boldsymbol{k}) = \begin{pmatrix} \psi_{\mathrm{s}} - h\sigma_z & -i\epsilon_{\boldsymbol{k}}\sigma_y - \underline{i\boldsymbol{g}_{\boldsymbol{k}}\cdot\boldsymbol{\sigma}\sigma_y} \\ i\epsilon_{\boldsymbol{k}}\sigma_y + \underline{i\boldsymbol{g}_{\boldsymbol{k}}\sigma_y\boldsymbol{\sigma}} & -\psi_{\mathrm{s}} + h\sigma_z \end{pmatrix}$$

similar to p-wave SC gap !

For $\boldsymbol{k} \sim 0$

$$\mathcal{H}^{\mathrm{D}}(\boldsymbol{k}) \to \begin{pmatrix} \psi_s - h\sigma_z & -i\boldsymbol{g}_{\boldsymbol{k}}\cdot\boldsymbol{\sigma}\sigma_y \\ \underline{i\boldsymbol{g}_{\boldsymbol{k}}\sigma_y\boldsymbol{\sigma}} & -\psi_s + h\sigma_z \end{pmatrix}$$

(c.f. Read and Green (2000))

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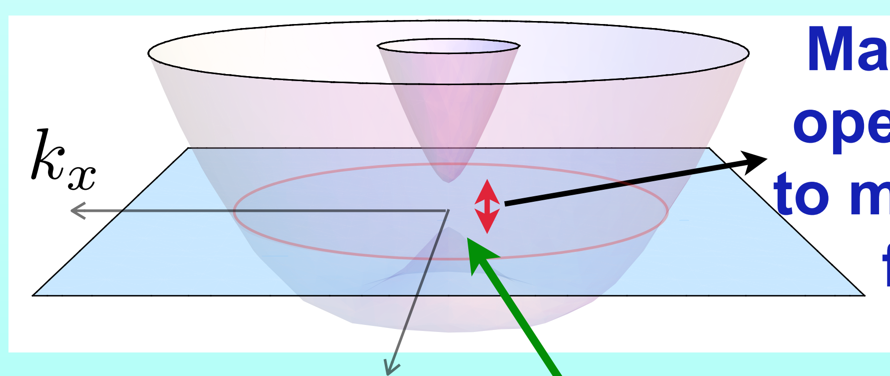
$$\mathcal{H}^{\mathrm{D}}(\boldsymbol{k}) \rightarrow \begin{pmatrix} \psi_s - h\sigma_z & -i\boldsymbol{g_k}\cdot\boldsymbol{\sigma}\sigma_y \\ i\boldsymbol{g_k}\sigma_y\boldsymbol{\sigma} & -\psi_s + h\sigma_z \end{pmatrix} \qquad \boldsymbol{g_k} = 2\lambda(\sin k_y, -\sin k_x, 0)$$

similar to p+ip-wave SC

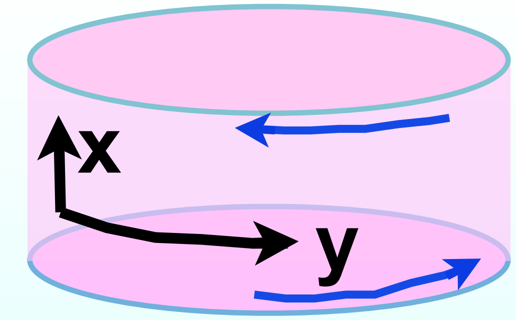
For $\psi_{\mathrm{s}}^2 + \epsilon(0,0)^2 < h^2 < \psi_{\mathrm{s}}^2 + \epsilon(\pi,0)^2$ ($\epsilon(k_x,k_y) = \epsilon_{\boldsymbol{k}}$)

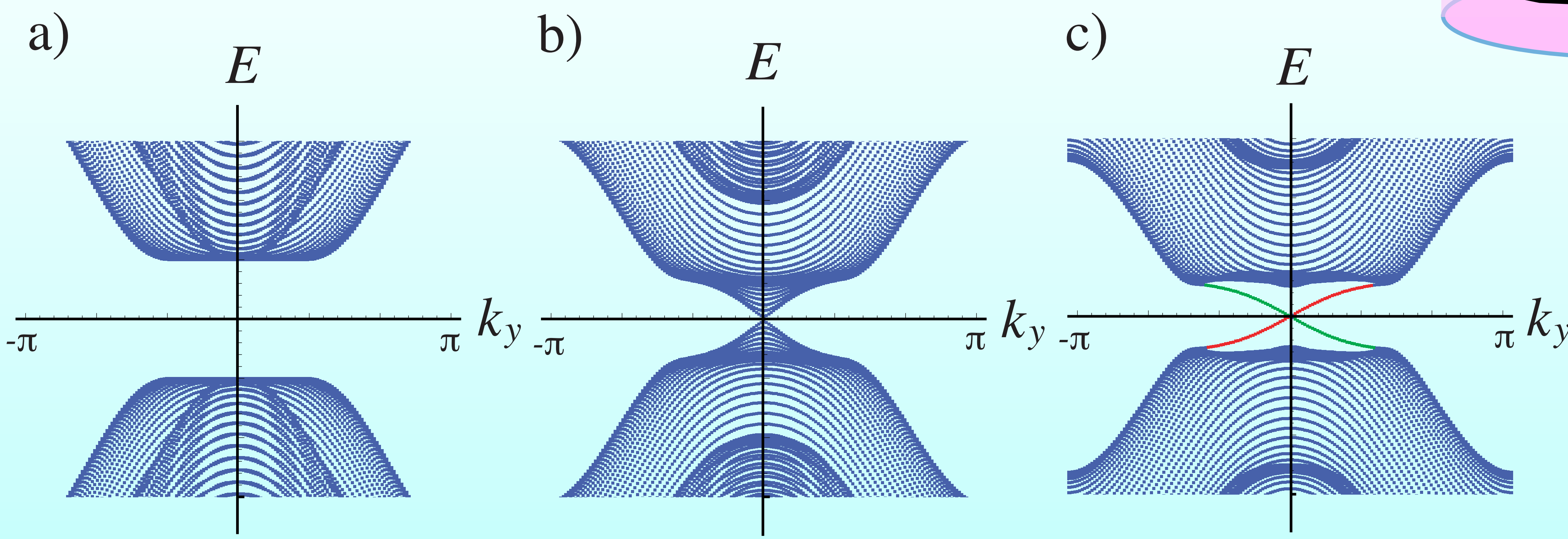
Non-Abelian topological order appears like chiral p+ip SC !

Energy band



Gapless edge states





$h = 0$

$h = \sqrt{\psi_s^2 + \epsilon(0,0)^2}$

$\sqrt{\psi_s^2 + \epsilon(0,0)^2} < h < \sqrt{\psi_s^2 + \epsilon(\pi,0)^2}$

For $\psi_s^2 + \epsilon(0,0)^2 < h^2 < \psi_s^2 + \epsilon(\pi,0)^2$

Chiral gapless edge state appears like chiral p+ip SC !

energy dispersion

$E_k^{\mathrm{edge}} = vk$ $\quad$ $k \to 0$ **limit** $\quad$ **Majorana fermion !**

Chern number

For $\psi_s^2 + \epsilon(0,0)^2 < h^2 < \psi_s^2 + \epsilon(\pi,0)^2$

nonzero first Chern number $\mathcal{Q} = 1$

For $\boldsymbol{k} \sim 0$ $\quad \mathcal{H}^{\mathrm{D}}$ decouples into two parts

$$\mathcal{H}_{+}^{\mathrm{D}} = \begin{pmatrix} \psi_s - h & 2\lambda(k_y - ik_x) \\ 2\lambda(k_y + ik_x) & -\psi_s + h \end{pmatrix} = \boldsymbol{B} \cdot \boldsymbol{\sigma}$$

$$\boldsymbol{B} = (2\lambda k_y, 2\lambda k_x, \psi_s - h)$$

monopole charge 1 at

$$(k_x, k_y, h) = (0, 0, \psi_s)$$

$$(\ \boldsymbol{B} = 0\ )$$

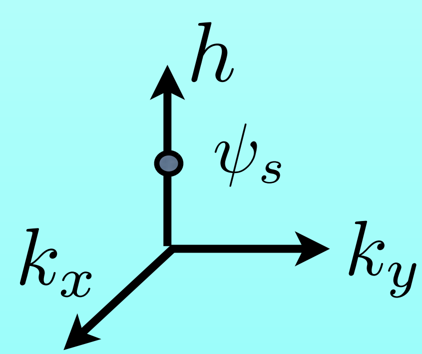
$$(\ h > 0 \quad \psi_s > 0\ )$$

$$\mathcal{H}_{-}^{\mathrm{D}} = \begin{pmatrix} \psi_s + h & 2\lambda(-k_y - ik_x) \\ 2\lambda(-k_y + ik_x) & -\psi_s - h \end{pmatrix}$$

As h ***increases,***

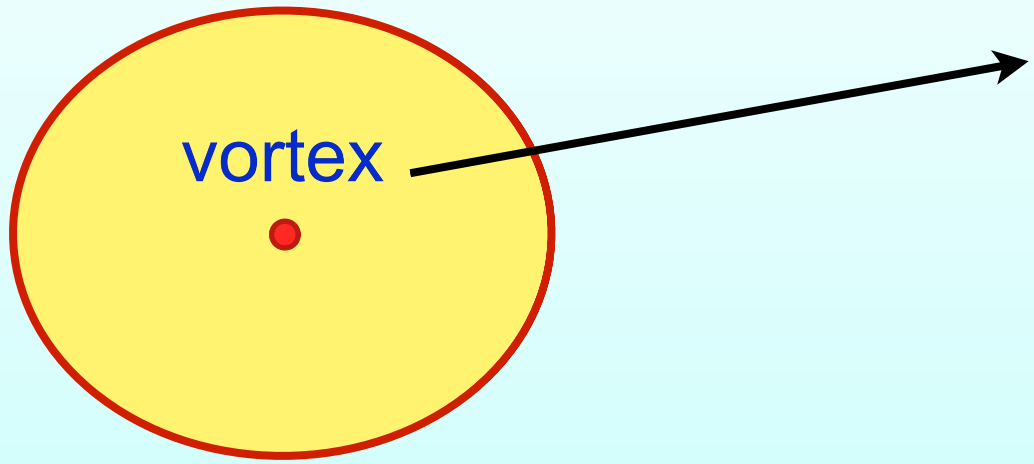
first Chern number (total flux penetrating a spherical surface in the (k_x, k_y, h) ***space) changes from 0 to 1,***

at $h = \psi_s$



Majorana zero energy mode in a vortex = Non-Abelian anyon

Majorana modes can not exist isolately. They exist only as pairs.



single Majorana edge mode

When vorticity = odd# ,

a Majorana zero energy mode

c.f. Tewari, Sau, Das Sarma, Annals. Phys.325,219 (2010)

Index theorem a la Jackiw-Rebbi's argument for zero energy mode

Majorana zero energy mode in a vortex = Non-Abelian anyon

Dual Hamiltonian For $\boldsymbol{k} \sim 0$ (near the massive Dirac cone)

$$\mathcal{H}^D(\boldsymbol{k}) \xrightarrow[\text{decoupled}]{\text{neglecting } \sim O(\boldsymbol{k}^2)}$$

$$\mathcal{H}^{\mathrm{D}}_{+} = \begin{pmatrix} \psi_s - h & 2\lambda(\hat{k}_y - i\hat{k}_x) \\ 2\lambda(\hat{k}_y + i\hat{k}_x) & -\psi_s + h \end{pmatrix}$$

$$\mathcal{H}^{\mathrm{D}}_{-} = \begin{pmatrix} \psi_s + h & 2\lambda(-\hat{k}_y - i\hat{k}_x) \\ 2\lambda(-\hat{k}_y + i\hat{k}_x) & -\psi_s - h \end{pmatrix}$$

(assumed that $\varepsilon_{\boldsymbol{k}=0} = 0$)

$\hat{k}_x = -i\partial_x \quad \hat{k}_y = -i\partial_y$

Bogoliubov-de-Gennes equation for the case with a vortex of “p-wave gap” $2\lambda(\pm k_y + ik_x)$

$$\mathcal{H}^{\mathrm{D}}_{+}\phi_{+} = \varepsilon\phi_{+} \qquad \mathcal{H}^{\mathrm{D}}_{-}\phi_{-} = \varepsilon\phi_{-}$$

$\phi_{\pm}(\theta + 2\pi) = -\phi_{\pm}(\theta)$ θ : polar angle around the vortex line

We found the zero energy Majorana mode for both $\mathcal{H}^{\mathrm{D}}_{+}$ and $\mathcal{H}^{\mathrm{D}}_{-}$

However !   ***For*** $\psi_s > h$

these zero mode solutions are killed by corrections $\sim O(\boldsymbol{k}^2)$

Majorana zero energy mode in a vortex = Non-Abelian anyon

In contrast ! *For* $\psi_s < h$

the zero energy Majorana solution for $\mathcal{H}_+^{\rm D}\phi_+ = \varepsilon\phi_+$ *survives !*

There is only one zero energy solution.

quasiparticle field $\gamma^\dagger = \int d\boldsymbol{r}[u_0\psi_+^\dagger + v_0\psi_+]$

$$\phi_+^T = (u_0(\boldsymbol{r}), v_0(\boldsymbol{r}))$$

$$u_0(\boldsymbol{r}) = i(re^{i\theta})^{-1/2}\underline{e^{-(h-\psi_s)\frac{r}{2\lambda}}} \qquad v_0(\boldsymbol{r}) = -i(re^{-i\theta})^{-1/2}\underline{e^{-(h-\psi_s)\frac{r}{2\lambda}}}$$

Majorana fermion ! $\gamma^\dagger = \gamma$

Note that the vortex of the Rashba SO int. !!

From BdG equation for $\mathcal{H}^D(\boldsymbol{k})$ with a vortex of "p-wave gap" $2\lambda(\pm k_y + ik_x)$

We obtain Majorana zero energy fermion mode
<u>in the vortex of SO int.</u>

⟷

After a singular gauge transformation,

$$\tilde{c}^\dagger_{\uparrow i} = c^\dagger_{\uparrow i} e^{i\phi_i} \qquad \tilde{c}^\dagger_{\downarrow i} = c^\dagger_{\downarrow i}$$

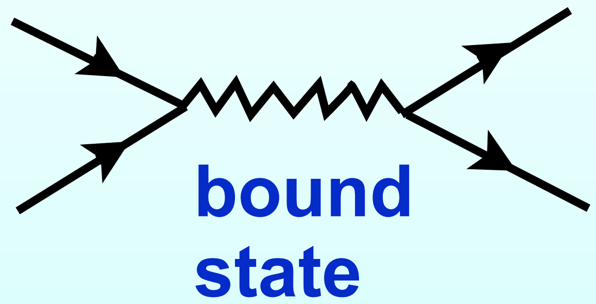
$$\lambda e^{i\phi} c^\dagger_{\uparrow i} c_{\downarrow j} \to \lambda \tilde{c}^\dagger_{\uparrow i} \tilde{c}_{\downarrow j} \qquad \Delta c^\dagger_{\uparrow i} c^\dagger_{\downarrow i} \to \Delta e^{-i\phi} \tilde{c}^\dagger_{\uparrow i} \tilde{c}^\dagger_{\downarrow i}$$

Majorana fermion <u>in the vortex of SC order</u> !

(c.f. Sau et al. arXiv:0907.2239)

How to realize in ultracold fermionic atoms ?

In ultracold fermionic atoms, it is possible to realize s-wave BCS pairing state with high T_c via the Feshbach resonance *(Bourdel et al., PRL93, 050401 (2004))*



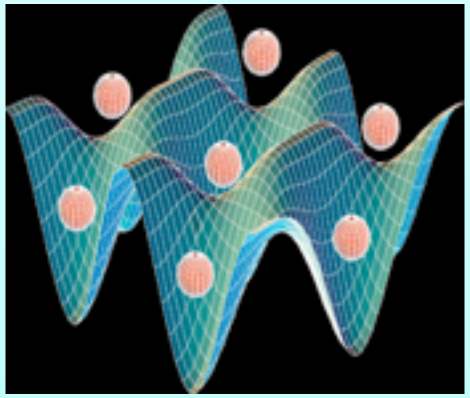
$$V_{\text{eff}} \sim \frac{g^2}{B - B_0}$$

Superfluidity of ^{6}Li in optical trap

(Chin et al., Nature 443, 961 (2006))

very high transition temperature

$$T_c \sim E_F$$



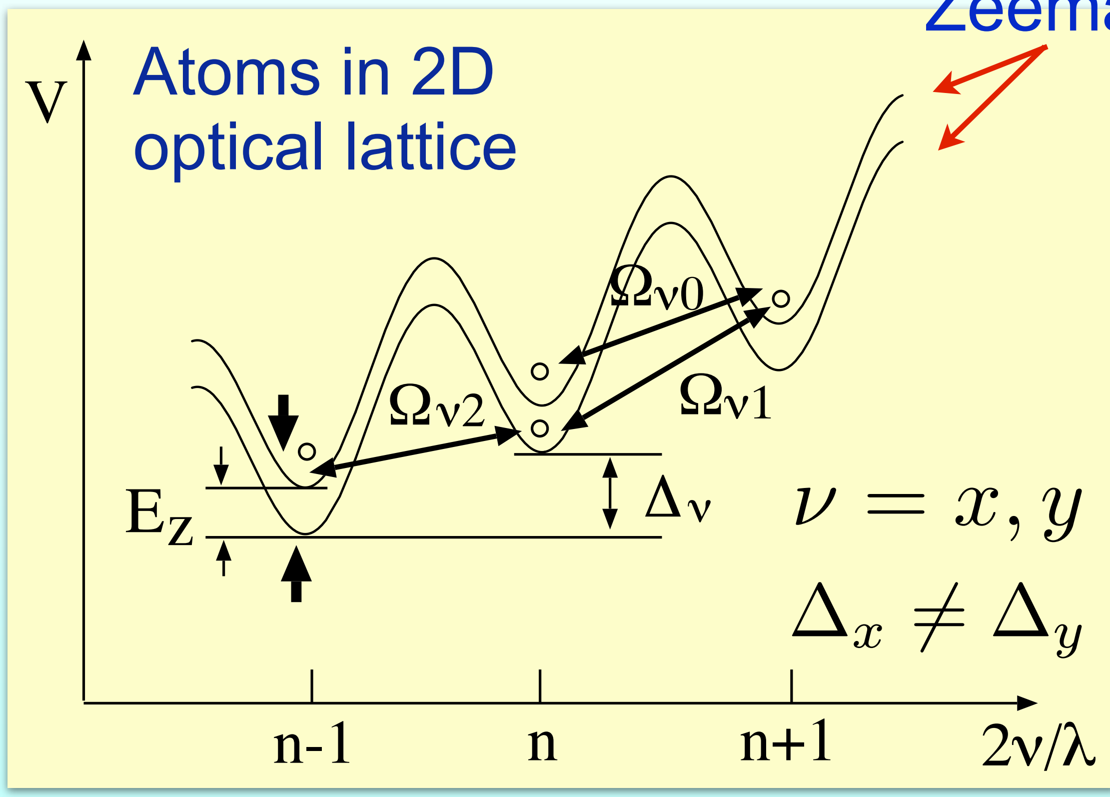
c.f. It is still technically difficult to realize p+ip wave pairing state via the p-wave Feshbach resonance because of the failure of cooling.

(Inada et al. PRL101, 100401 (2008))

s-wave pairing state via the s-wave Feshbach resonance is more advantageous than p-wave pairing state via the p-wave Feshbach resonance for the realization of the non-Abelian topological order

How to realize in ultracold fermionic atoms ?

Scheme for the Rashba SO interaction in cold atoms



Zeeman split two hyperfine states

Rabi frequencies of lasers

$$\Omega_{\nu 1}: |n,\uparrow\rangle \longleftrightarrow |n+1,\downarrow\rangle$$

$$\Omega_{\nu 2}: |n,\downarrow\rangle \longleftrightarrow |n+1,\uparrow\rangle$$

$$\Omega_{x2} = |\Omega_{x2}|e^{ik_z z} \qquad \Omega_{x2} = -\Omega_{x1}$$

$$\Omega_{y2} = -i\Omega_{x1} \qquad \Omega_{y1} = -\Omega_{y2}$$

Rashba SO int. $\mathcal{H}_{\mathrm{SO}} = \sum_i [\lambda_x(c^\dagger_{i-\hat{x}\downarrow}c_{i\uparrow} - c^\dagger_{i+\hat{x}\downarrow}c_{i\uparrow}) + i\lambda_y(c^\dagger_{i-\hat{y}\downarrow}c_{i\uparrow} - c^\dagger_{i+\hat{y}\downarrow}c_{i\uparrow}) + \mathrm{h.c.}]$

$$\lambda_\nu = c_\nu \int d\boldsymbol{r}\psi^*_\downarrow(\boldsymbol{r}-\boldsymbol{r}_{i-\hat{\mu}})\Omega_{\nu 2}(\boldsymbol{r})\psi_\uparrow(\boldsymbol{r}-\boldsymbol{r}_i) \qquad c_x = 1 \qquad c_y = -i$$

How to generate a vortex of the SO interaction ?

When lasers have angular momentum (Laguerre-Gaussian beam), vortex of the SO int. is introduced

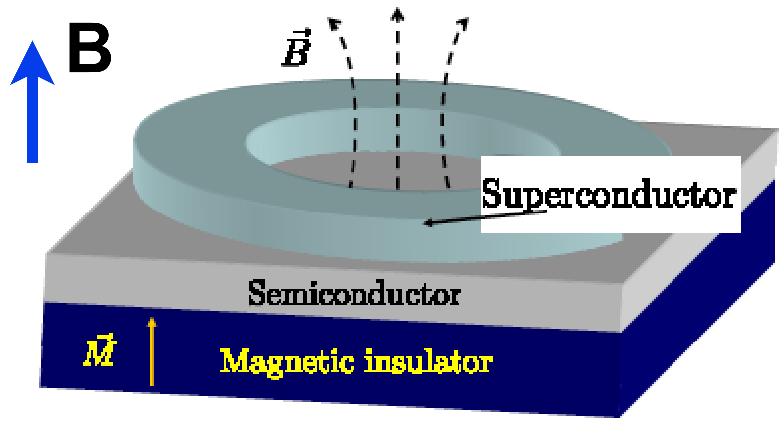
$$\lambda_\nu \to \lambda_\nu e^{im\phi}$$

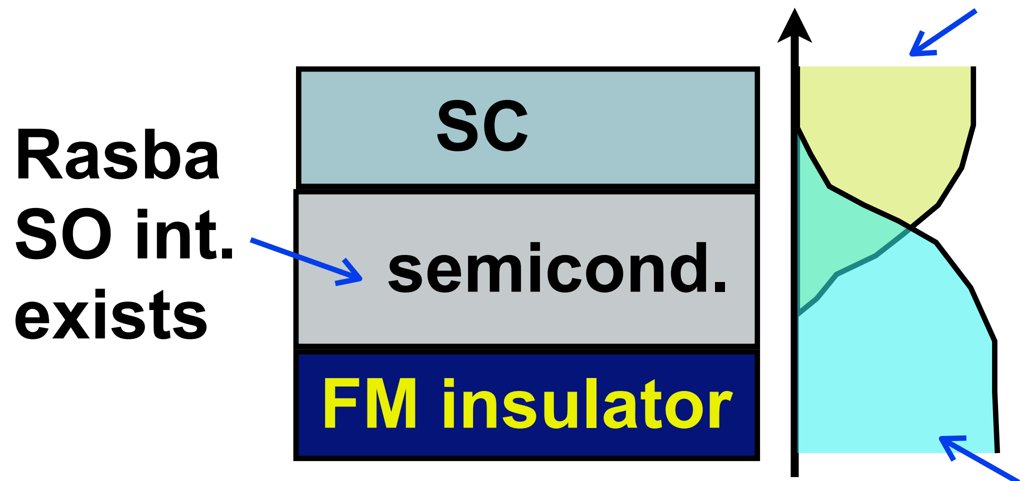
Other proposals for non-Abelian topological order in s-wave NC SC

Key idea ········ How to suppress the destruction of SC due to strong magnetic fields $\mu_{\rm B} H > \Delta$

(i) semiconductor heterostructure

Sau, Lutchyn, Tewari, and Das Sarma, arXiv:0907.2239

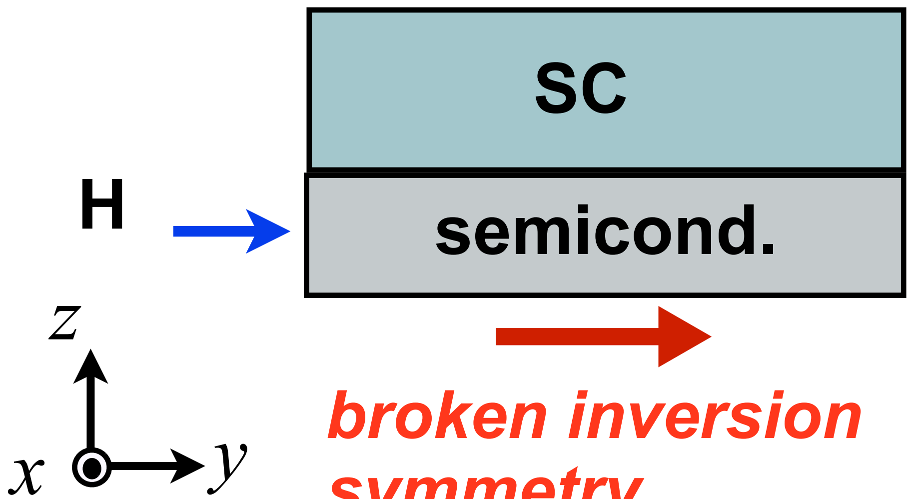




There is a region where $\mu_{\rm B} H > \Delta$ holds

(ii) semiconductor heterostructure with Rashba and Dresselhaus SO int.

J. Alicea, arXiv: 0912.2115



For magnetic fields parallel to the thin film, orbital depairing effect is negligible

But, for usual Rashba term, $\lambda_R (\boldsymbol{k} \times \boldsymbol{n}_z) \cdot \boldsymbol{\sigma}$

Pauli depairing effect is not suppressed.

Combining Rashba and Dresselhaus int.:

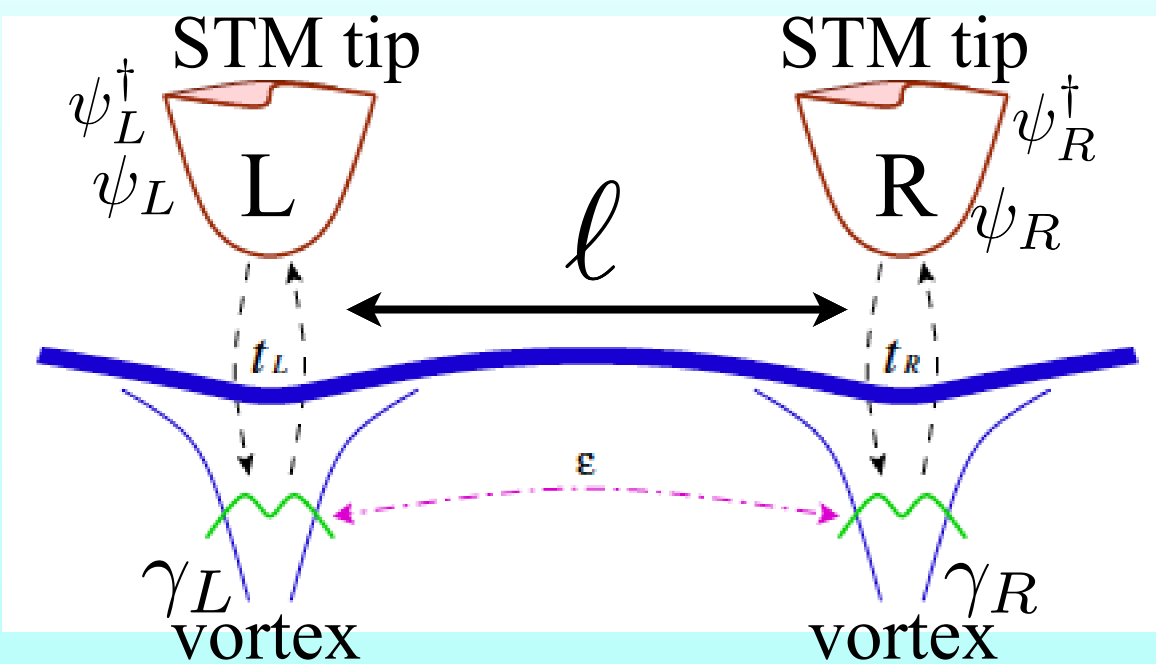
$$\mathcal{H}_{\rm SO} = \lambda (k_z \sigma_x - k_x \sigma_z)$$

Pauli depairing effect is suppressed

How to detect Majorana fermions?

detect non-locality (teleportation)

(Semenoff and Sodano, J. Phys. B40,1479 (2007), Bolech and Demler, PRL98, 237002 (2007), Tewari,Zhang,Das Sarma,Nayak,Lee, PRL100, 027001 (2008),Nilsson,Akhmerov,Beenakker,PRL101,120403(2008))



from Bolech and Demler

tunneling between L and R mediated via Majorana states

$$\mathcal{H}_t = t_L i\gamma_L(\psi_L^\dagger + \psi_L) + t_R(\psi_R^\dagger - \psi_R)\gamma_R$$

tunneling probability does not depend on the distance ℓ

teleportation !

Also, applicable to cold atoms *(Tewari et al.,PRL100, 027001(2008))*

Laser-induced tunneling between normal state and SC state *(Torma and Zoller,PRL85, 487 (2000))* **can be used for this experiment**

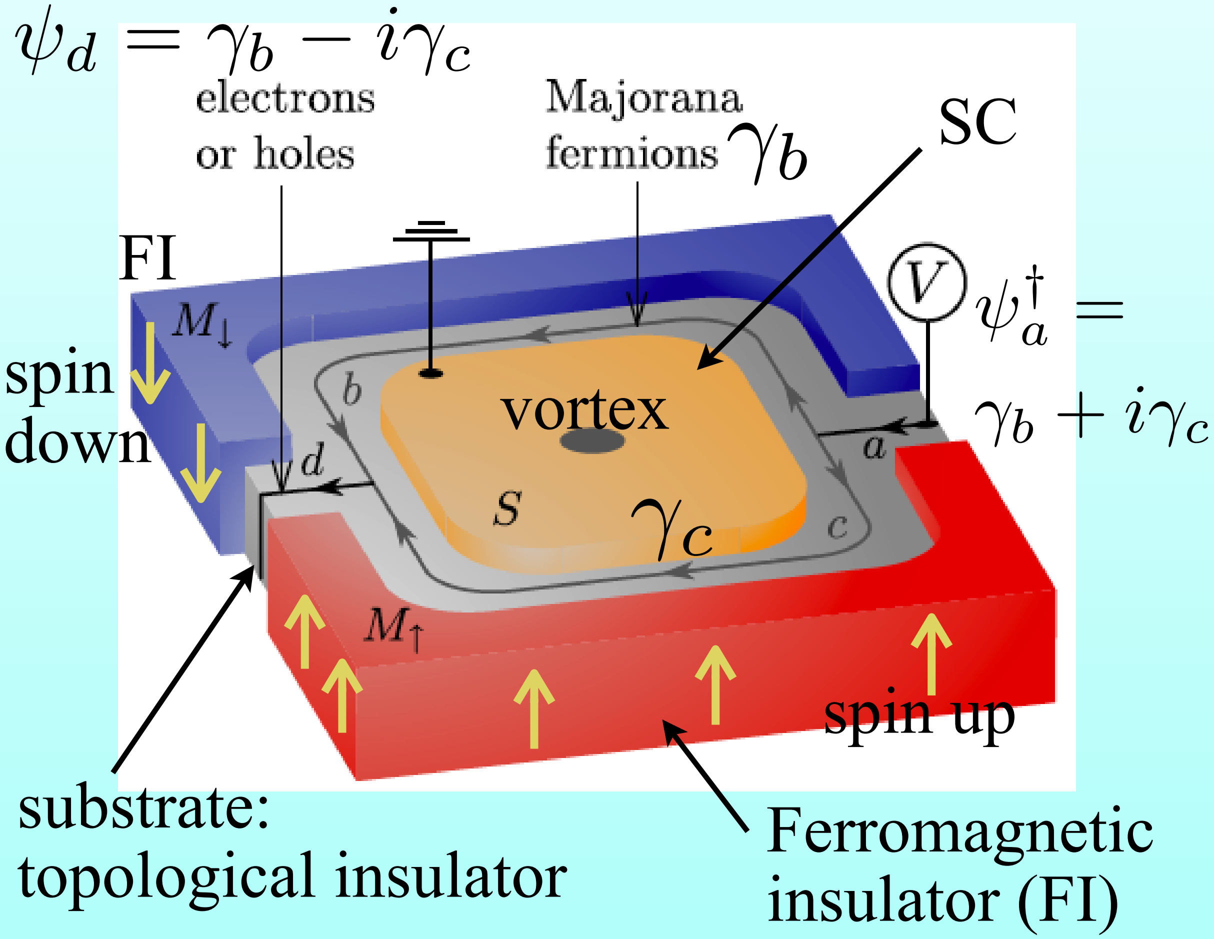
How to detect Majorana fermions?

Converting a pair of Majorana fermions to a charged fermion

(Akhmerov, Nilsson, Beenakker, PRL102, 216404(2009), Fu and Kane, PRL102, 216403 (2009))

interface between superconductor (SC) and Z_2 topological insulator

$$\psi_d^\dagger = \gamma_b + i\gamma_c$$
$$\psi_d = \gamma_b - i\gamma_c$$



(from Akhmerov et al.)

→ **Majorana fermions in vortex and edge**

current flows along edge give rise
the braiding of Majorana fermions

(i) When vorticity even

no Majorana modes on edge

gapped edge mode $\quad E_g \sim O\left(\dfrac{1}{\text{perimeter}}\right)$

(ii) When vorticity odd

$$\gamma_b \rightarrow \gamma_b \qquad \gamma_c \rightarrow -\gamma_c$$

conductance:
(zero temperature)

$$G = \begin{cases} 0 & \textbf{for vorticity even} \\ 2e^2/h & \textbf{for vorticity odd} \end{cases}$$

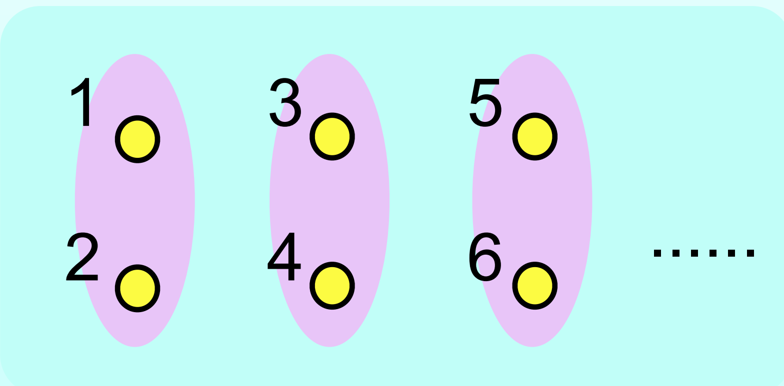
also, applicable to noncentrosymmetric s-wave superconductors

How to detect Majorana fermions?

detect ground state degeneracy (topological entanglement entropy) via bulk thermodynamic measurement

(Cooper and Stern, PRL102, 176807(2009))

For $2N$ vortices,



$$\psi_i^\dagger = \frac{1}{2}(\gamma_{2i-1} - i\gamma_{2i}) \qquad n_i = \psi_i^\dagger \psi_i = 0 \;\; \text{or} \;\; 1$$

degeneracy : $2^N \times \frac{1}{2} = 2^{N-1}$

In SC state, change of $\sum_i n_i$ must be even.

Entropy per a vortex : $s = \frac{1}{2N} k_{\mathrm{B}} \ln 2^{N-1} \approx k_{\mathrm{B}} \frac{1}{2} \ln 2$

$\sqrt{2}$: quantum dimension of non-Abelian anyon

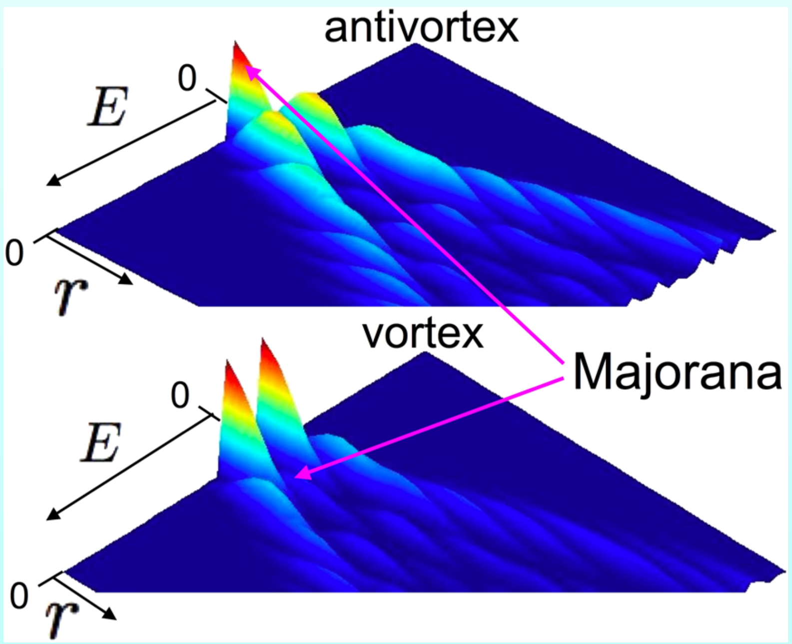
precise specific heat measurement at sufficiently low temperature may probe the topological entropy

How to detect Majorana fermions?

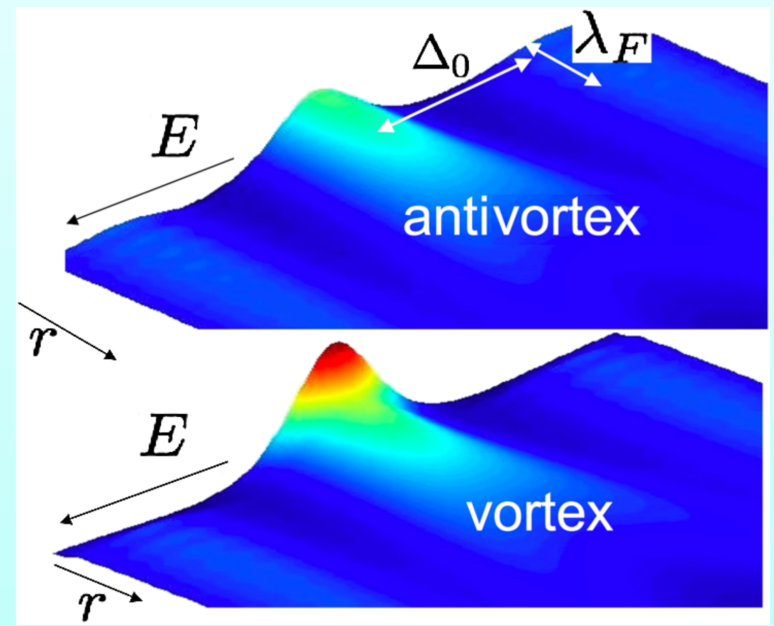
Tunnel spectroscopy observing difference between a vortex core and an antivortex core

(Kraus, Auerbach, Fertig, and Simon, PRL101, 267002(2008))

Chiral p+ip SC



Dos at zero temperature near the vortex



Tunnel conductance at

$$T = 0.15\Delta \gg \Delta^2/E_F$$

Signature remains even at relatively high temperatures

Difference between vortex and anti-vortex is a characteristic of Majorana modes associated with chiral nature of the superconducting order parameter

$$p_x + ip_y$$

Summary

Non-Abelian topological order realize even for *s-wave* pairing state with the Rashba SO interaction for the large Zeeman field $\mu_{\mathrm{B}} H > \Delta$

In vortex cores and boundary edges of the system, Majorana fermion modes which obey the non-Abelian statistics appears.

Possible realization in

- **heavy fermion NC SC**

  strong electron correlation suppress orbital depairing effect

  however,

  unfortunately, heavy fermion NC SCs known so far have gap-node

- **ultracold fermionic atoms with a fictitious "SO interaction" generated by laser fields**

(ref. M. Sato, Y. Takahashi, and S.F.,Phys. Rev. Lett.103, 020401 (2009))