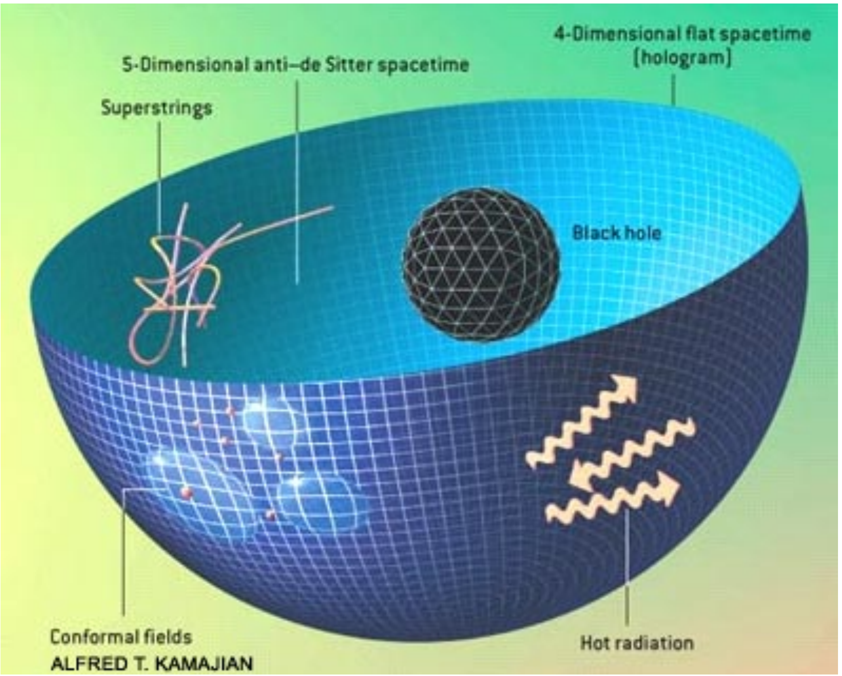
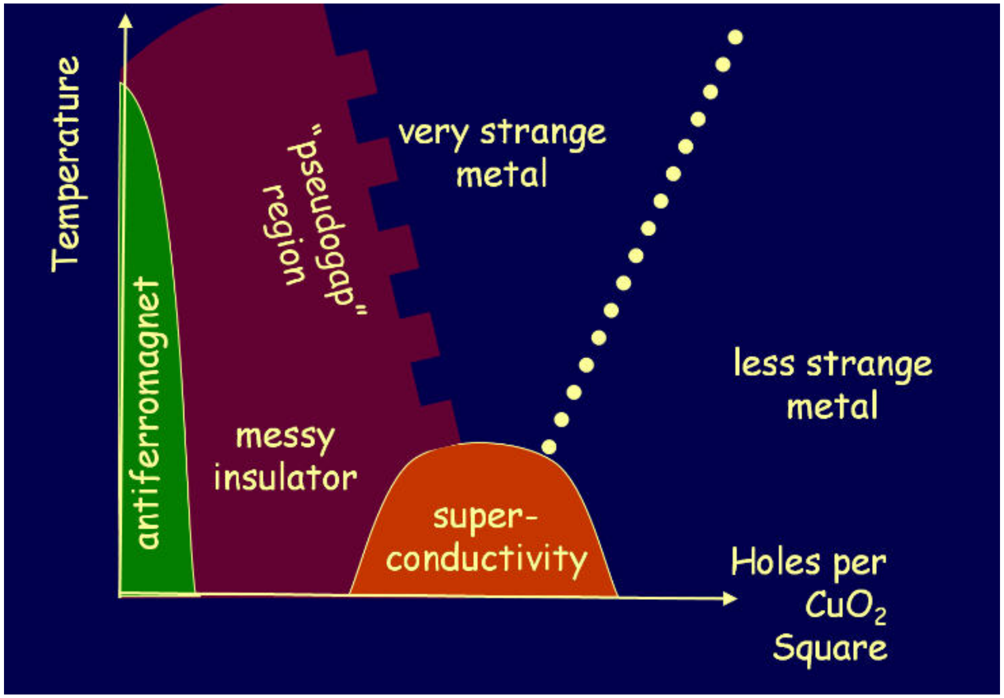
From black holes to strange metals:

many-body physics through a gravitational lens

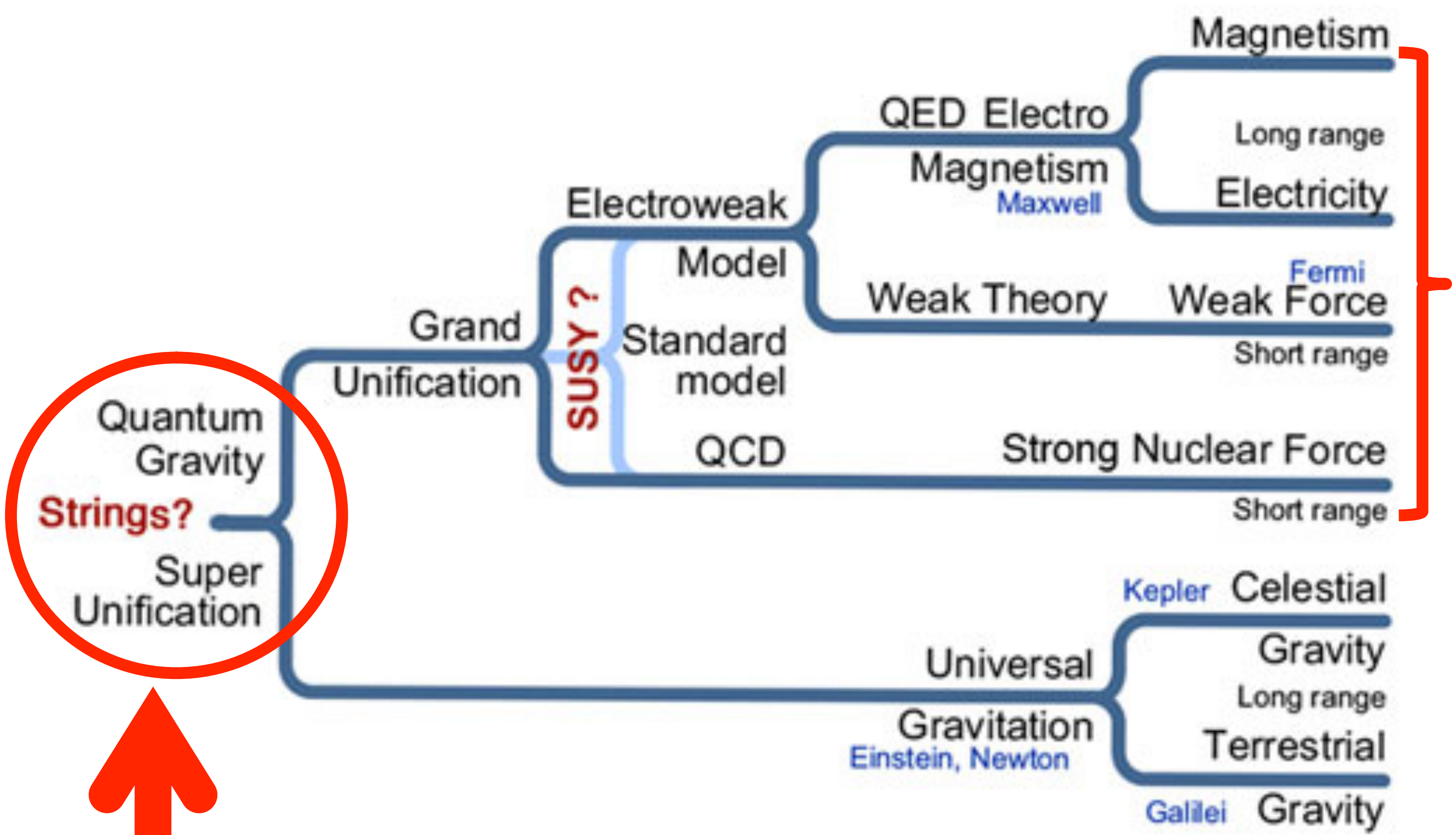
Hong Liu

Massachusetts Institute of Technology





Unification of Physics



Many-body Physics:

Condensed Matter

QCD at finite temperature and density

....

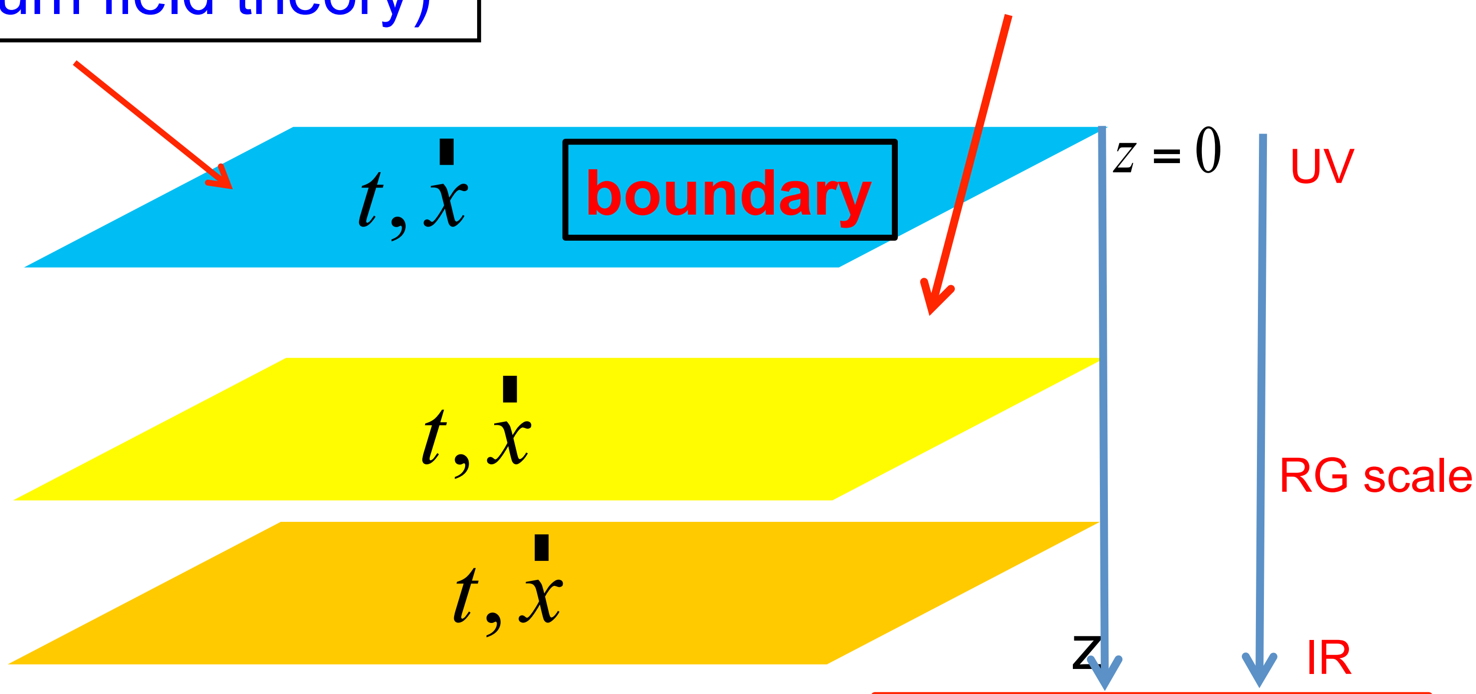
Gauge/gravity duality

Maldacena, Polyakov , Gubser, Klebanov, Witten …..

Many-body system
without gravity in
d spacetime dimension
(Quantum field theory)

=

String theory,
quantum gravity in
d+1 dimensions



UV/IR connection

Gauge/gravity duality

String theory,
quantum gravity in
d+1 dimensions

=

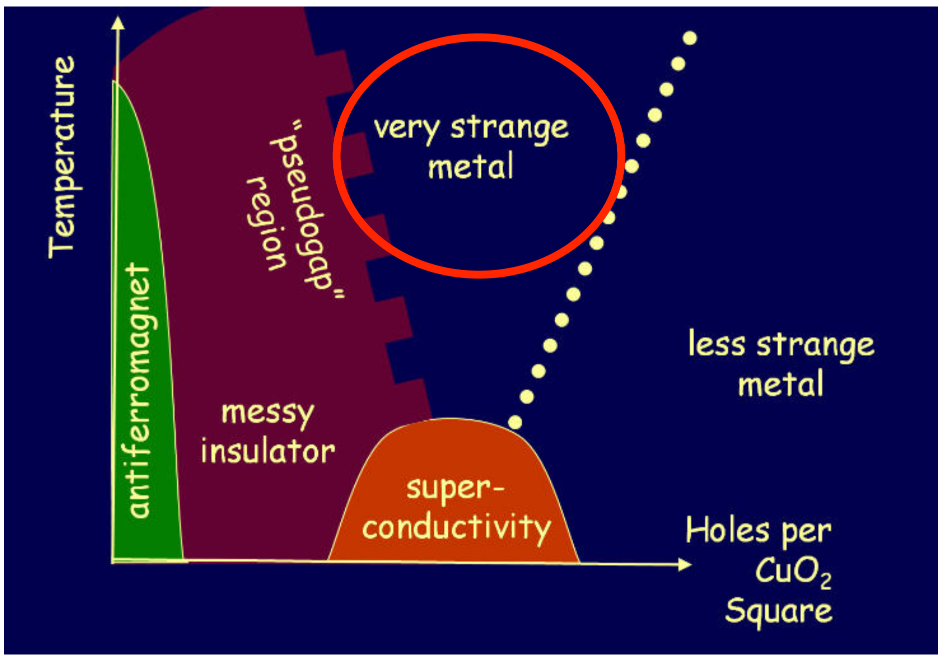
Many-body system
without gravity in
d spacetime dimension
(Quantum field theory)

Weakly curved,
classical gravity

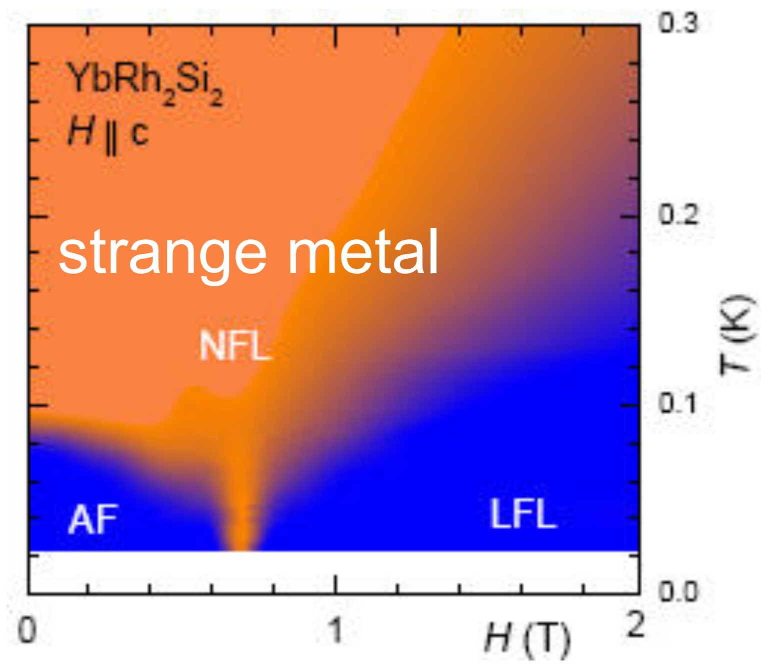
Strongly coupled,
Large N

We would like to use this framework to study strongly correlated fermionic systems at a finite density, in particular those near a quantum critical point.

Strongly correlated fermionic systems at finite density



High Tc cuprates



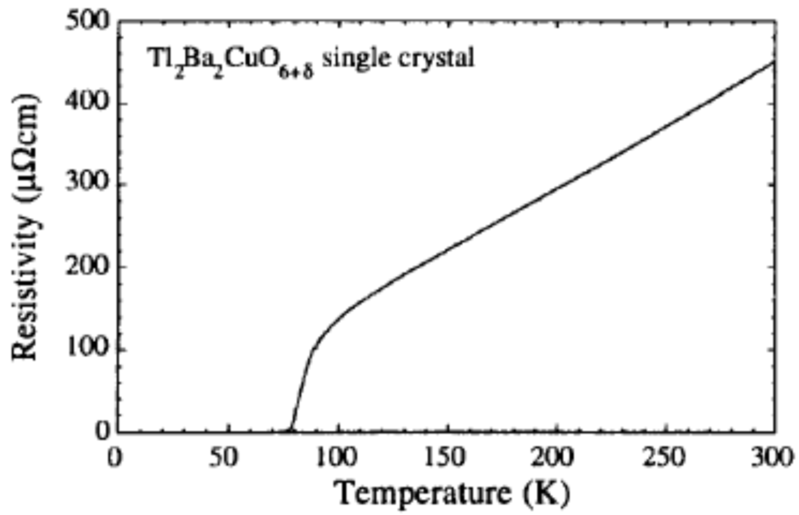
Quantum phase transitions of heavy fermion metals

The quasi-particle residue Z should be zero on the Fermi surface.

Strange metals (I): linear resistivity

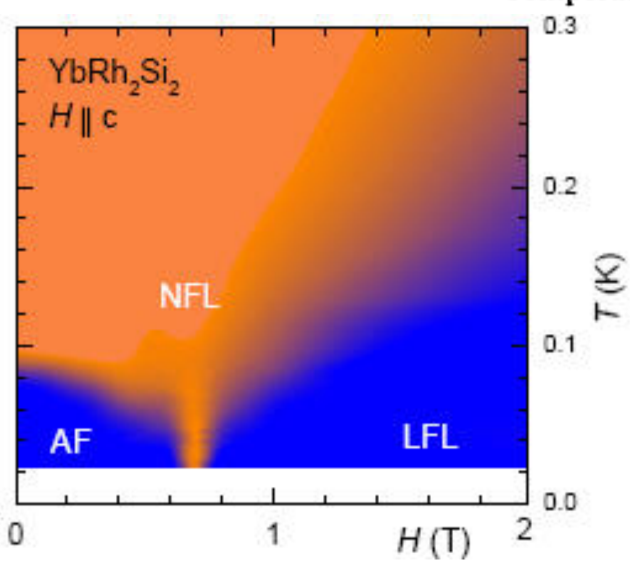
Resistivity linear in temperature:

Mackenzie 97



In sharp contrast with that of a Fermi Liquid:

$$\rho = \rho_0 + cT^2$$

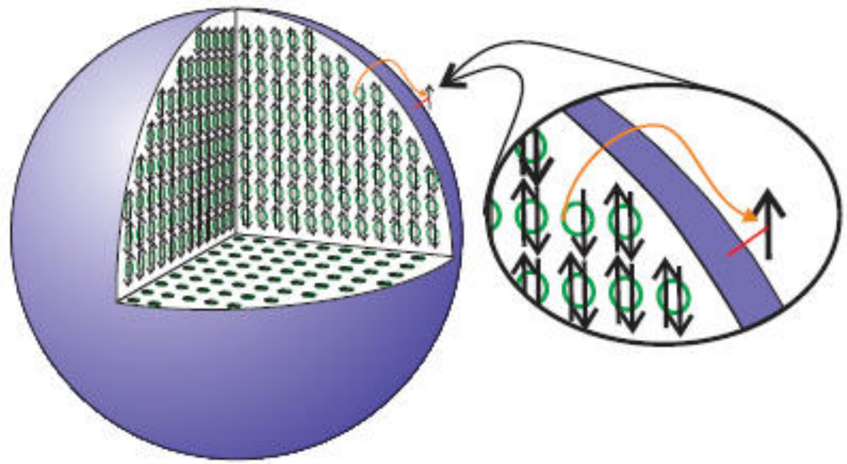


Simple, robust, universal behavior, yet resisted explanation for more than 20 years !

Strange metals (II): Fermi surface without quasi-particles

In the Fermi liquid theory, transports are carried out by quasi-particles (excitations on the Fermi surface).

Quasi-particle decay rate $\boxed{\Gamma \propto \omega^2}$



$$\rho = \rho_0 + cT^2$$

Strange metals:

Quasi-particle picture breaks down (from transports and others)

sharp Fermi surface (photoemission experiments)

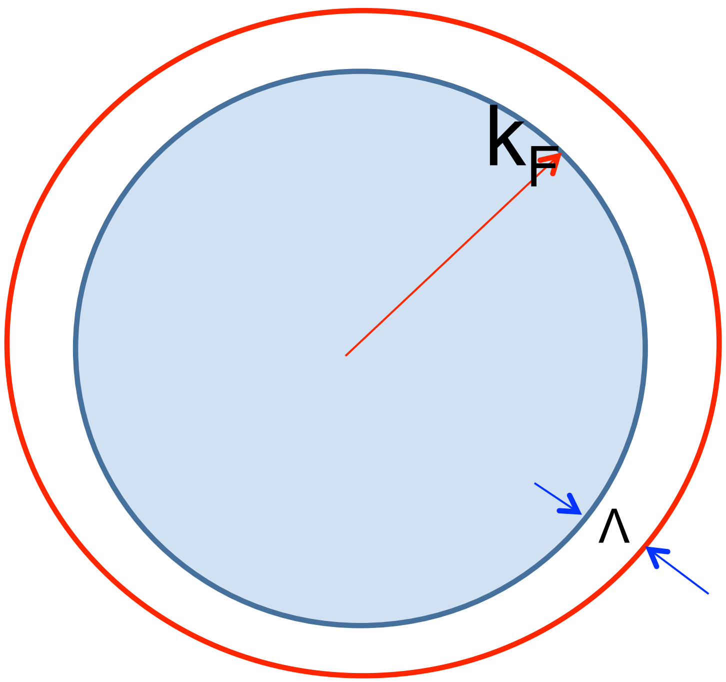
``Marginal Fermi liquid'' Quasi-particle decay rate $\boxed{\Gamma \propto \omega}$

(Varma et al)

RG perspective

Landau Fermi Liquid: free fermion fixed point of the RG toward the Fermi surface.

Shankar, Polchinski
Benfatto, Gallavotti



Λ : RG scale

strange metals:
likely controlled by some interacting fixed points.

What is the organization principle?

Need to develop a proper language to think about such fixed points

Summary

Strange metals and other non-Fermi liquids:

no systematic theoretical understanding of their properties

not clear what are organizing principles

Strongly correlated systems, famous theoretical challenge

Numerical: fermionic sign problem (NP-hard) Troyer. Wiese (04)

Important:

high Tc cuprates

Quantum phase transitions

....

More generally: a glimpse of possibly rich dynamical phenomena from an interacting fermionic system.

How can gravity approach help?

- Find exactly solvable models which exhibit similar phenomena to strange metals.

- Find organizing principles behind different phenomena.

Steps:

1. Construct a gravity description of a metallic state of finite density of strongly interacting fermions at T=0.

2. Does the system possess a Fermi surface? Properties of small excitations?

3. Study charge transport properties such as resistivity.

Based on:

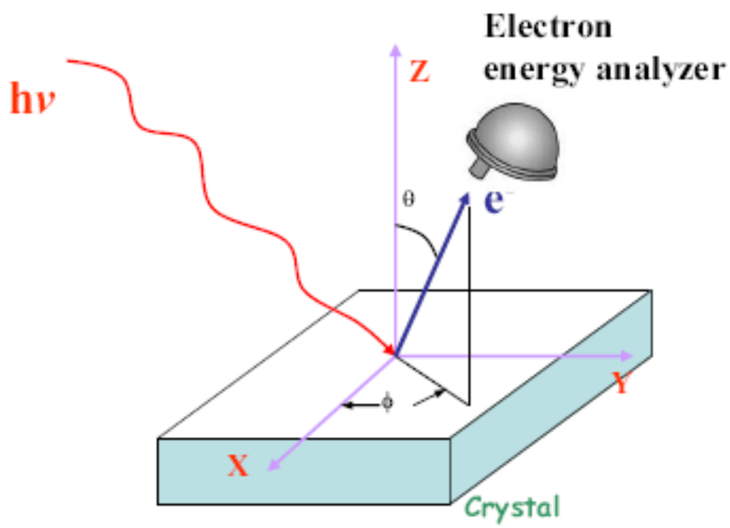
HL, John McGreevy, David Vegh, 0903.2477
Tom Faulkner, HL, JM, DV, 0907.2694
TF, Nabil Iqbal, HL, JM, DV, to appear

Sung-Sik Lee, 0809.3402
Cubrovic, Zaanen, Schalm, 0904.1933

Other related work:

Albash and Johnson, 0907.5406; Basu, He, Mukherjee, Shieh, 0908.1436, Denef, Hartnoll and Sachdev, 0908.1788; Hartnoll, Hofman, 0912.0008; S-J. Rey, 0911.5295; Chen, Kao, Wen, 0911.2821; Faulkner et al, 0911.3402; Gubser, Rocha, Talavera 0911.3632; Faulkner and Polchinski, 1001.5049, ………

Signature of Fermi surfaces (I)



ARPES

spectral function

$$A(\omega,k) \equiv \frac{1}{\pi}\operatorname{Im} G_R(\omega,k)$$

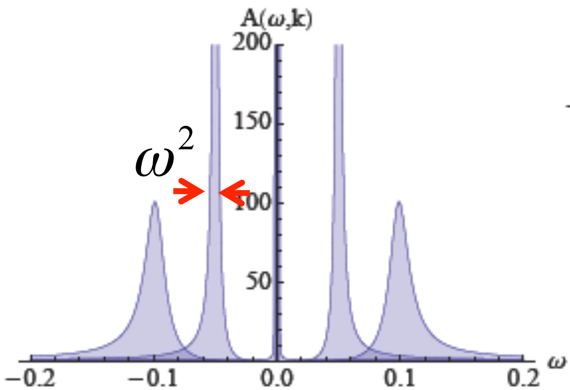
$$G_R(t,\vec{x}) = i\theta(t)\langle\{\psi(t,\vec{x}),\psi(0,0)\}\rangle$$

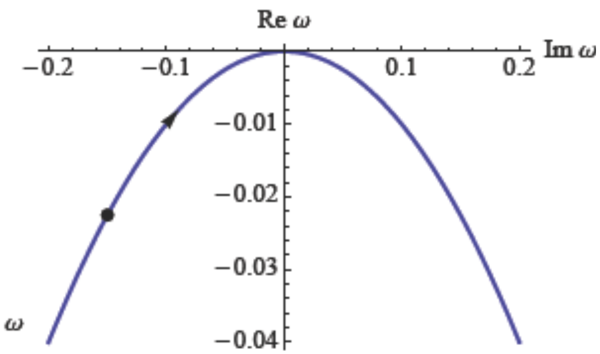
Ψ : electron operator

Fermi surface: nonanalytic behavior behavior in A(ω,k) near some shell in momentum space.

Signature of Fermi surfaces (II)

Fermi liquids: stable quasi-particles at FS





$$G_R(\omega, \vec{k}) = \frac{Z}{\omega - v_F k_\perp + i\Gamma}$$

Quasi-particle decay rate $\Gamma \propto \omega^2$

Z: quasi-particle weight

\`\`Marginal Fermi liquid'' for cuprates (Varma, Littlewood, Schmitt-Rink, Abrahams, Ruckenstein 1989)

$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \log \omega + c_1 \omega}$$

$\tilde{c}_1$: real

c_1 : complex

Quasi-particle decay rate $\Gamma \propto \omega$ weight vanishes as $\frac{1}{|\log \omega|}$

Fermi surfaces from AdS/CFT?

Start with your favorite field theories with a gravity dual:

D=3+1: N=4 super-Yang-Mills theory

D=2+1: ABJM

Non-Abelian gauge fields coupled to scalars and fermions. Gauge group: SU(N)

Take a U(1) global symmetry. Put the system at a finite chemical potential for this U(1).

This generates a metallic (finite density) state: does it have a Fermi surface?

Not obvious: since both scalars and fermions carry the same U(1) charge.

At strong coupling: dual gravity should tell us.

"Photoemission experiments" on black holes (I)

Field theory at finite chemical potential at T=0	$\Longleftrightarrow$	An extremal charged black hole

Gravity description : large N limit

charge density: $O(N^2)$

We want to compute:

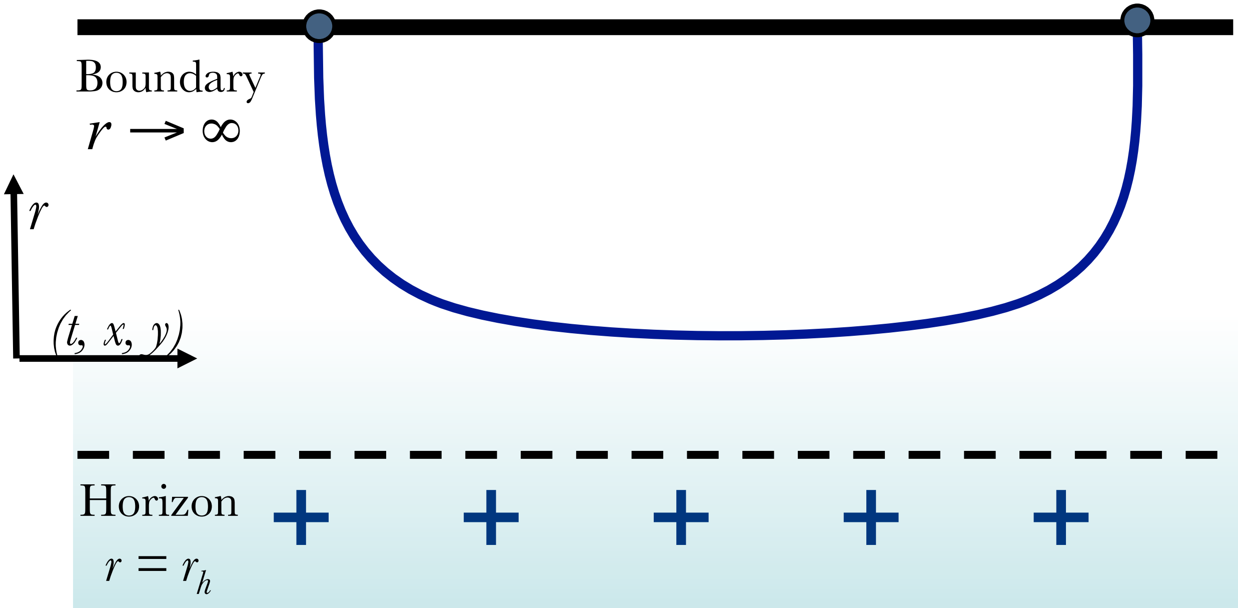
O: some fermionic operator

$$G_R(t,\vec{x}) = i\theta(t)\langle\{\mathcal{O}(t,\vec{x}),\mathcal{O}(0,0)\}\rangle \qquad A(\omega,\vec{k}) = \mathrm{Im}\, G_R(\omega,\vec{k})$$

Recall:

O	$\Longleftrightarrow$	Φ	(bulk spinor field)
Δ	$\Longleftrightarrow$	m	
q	$\Longleftrightarrow$	q	

"Photoemission experiments" on black holes (II)



S-S Lee

Solving Dirac equation for Φ, extracting boundary values

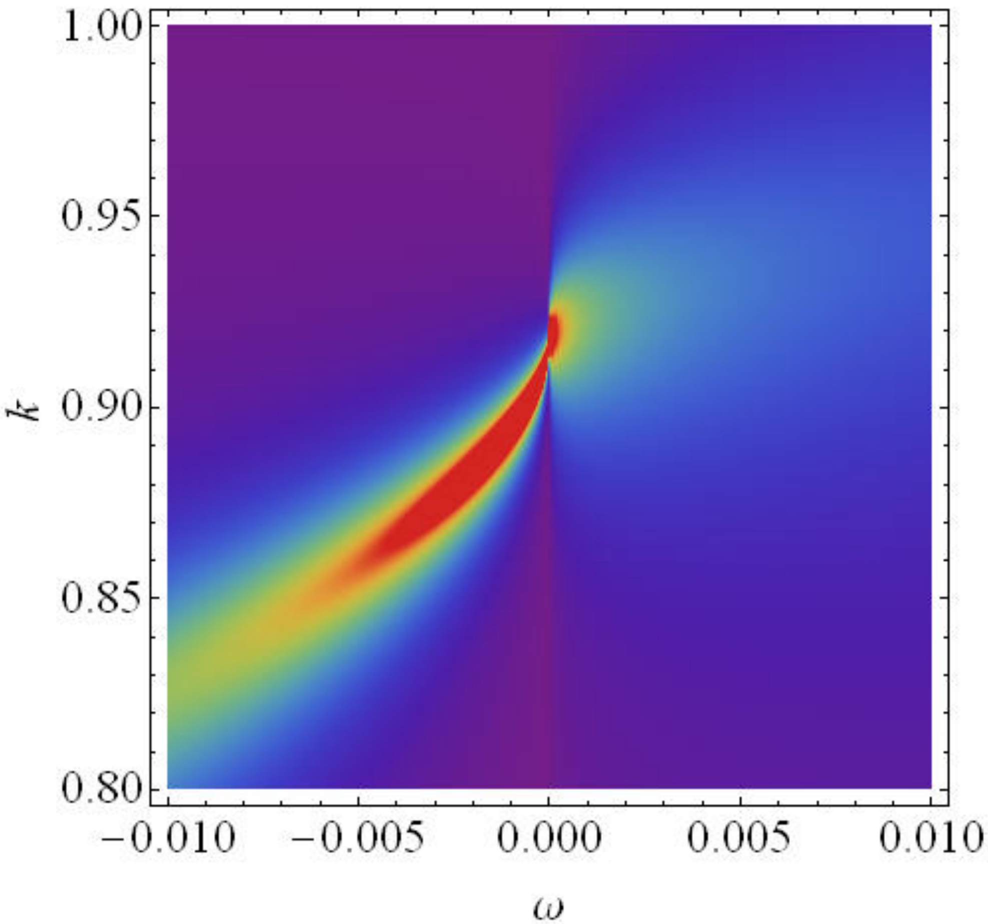
Universality of 2-point functions: (controlled by Dirac equation)

do not depend on which specific theory and operator we use.
Results will only depend on charge q and dimension m .

Will now use q and m as input parameters

Fermi surface

HL, McGreevy, Vegh
Cubrovic, Zaanen, Schalm



Features:

- Dispersion not linear in k

$$\omega \sim k_\perp^z \qquad z = 2.09$$

- Scaling behavior: $\alpha = 1$

$$G_R(\lambda k_\perp, \lambda^z \omega) = \lambda^{-\alpha} G_R(k_\perp, \omega)$$

- Width $\Gamma \sim \omega$
- Particle-hole asymmetry

for q=1, m=0

$k_F \approx 0.918528499$ (in units of chemical potential)

Clearly non-Fermi liquids !

What is this non-Fermi liquid composed of?

What controls the exponent?

What controls the Fermi momentum

What exponents are allowed?

Gravity (geometry) will tell us!

Important:

None of the leading order (in 1/N) thermodynamical or transport properties of the system will be sensitive to the presence of the Fermi surface.

k_F is of order O(1), which implies a charge density of order O(1).

The total charge density is $O(N^2)$

Thus the charge density associated with the Fermi surface is only a tiny bit of the whole system.

Physical picture (from gravity side)
and
Analytic understanding

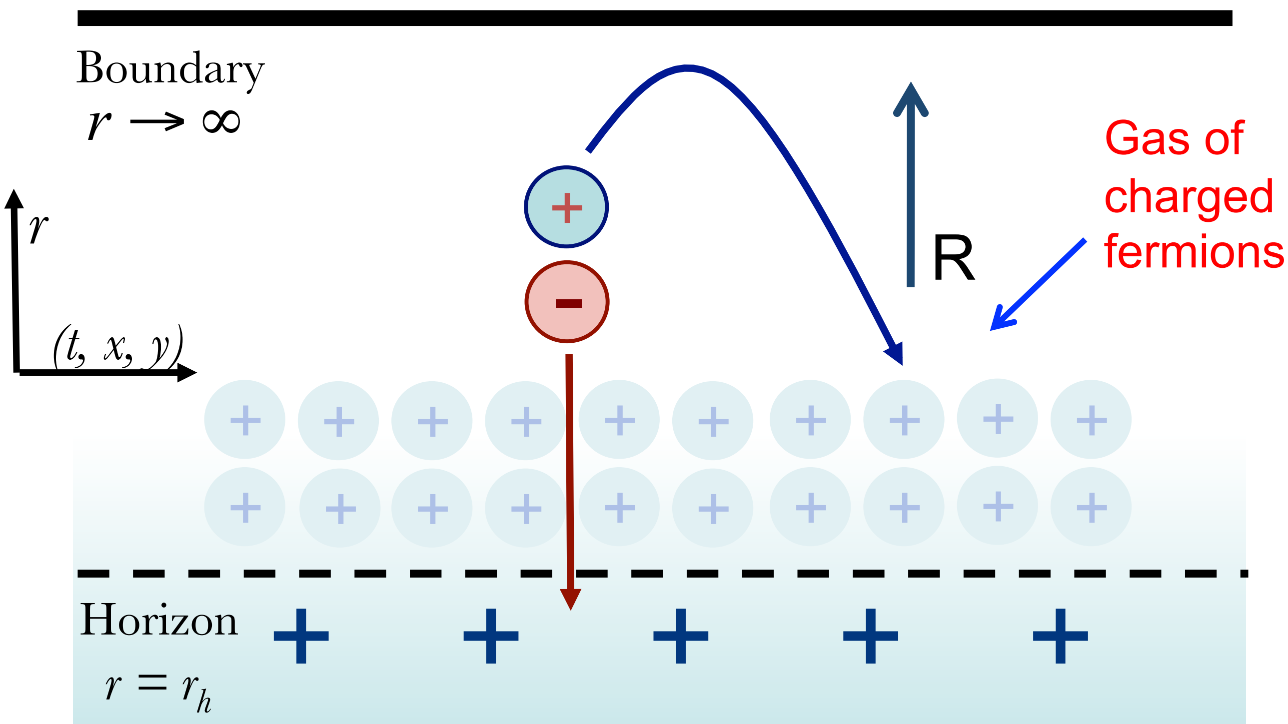
Why only O(1) density? Where does it come from?

What controls the scaling behavior?

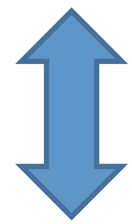
Gravity interpretation of the Fermi surface and quasi-particles?

Finite fermion density

Now consider a charged fermionic field outside the black hole:
(whose excitations correspond to fermionic excitations of the boundary system)



bulk charged fermion gas



a finite O(1) density of fermions in a strongly interacting boundary theory

Fermion: reflection probability R < 1 , leading to an equilibrium

Scalar: R > 1 (superradiance) , will grow and condense.

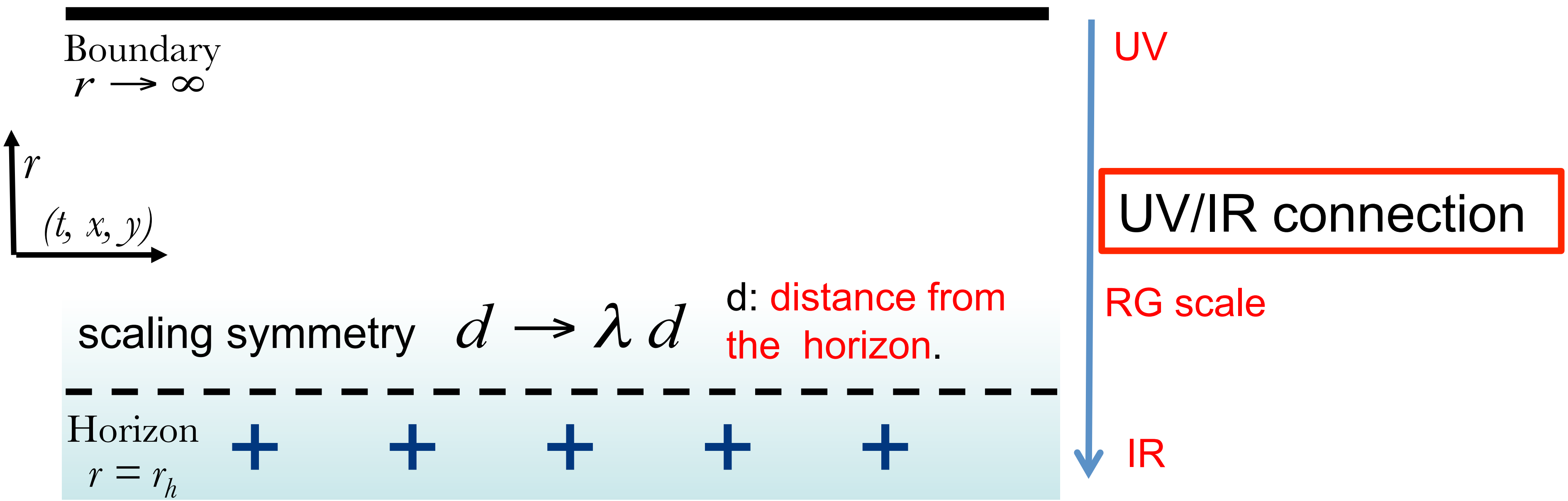
In boundary theory:

$$\mathcal{O}(\epsilon)^\dagger \mathcal{O}(0) \sim \frac{1}{\epsilon^{2\Delta}} + \frac{c(\Delta)}{N^2}\frac{qJ^0}{\epsilon^{2\Delta-2}} + \cdots$$

$$\Rightarrow \quad n_{\mathcal{O}} = \left\langle \mathcal{O}(\epsilon)^\dagger \mathcal{O}(0) \right\rangle \sim \frac{1}{\epsilon^{2\Delta}} + \frac{c(\Delta)}{N^2}\frac{q\langle J^0\rangle}{\epsilon^{2\Delta-2}} + \cdots$$

Emergent scaling symmetry

Faulkner, HL, McGreevy,Vegh



Near horizon geometry $\mathrm{AdS}_2 \times \mathbb{R}^2$

At low energies, there is an emergent scaling symmetry.

The boundary theory is controlled by an infrared CFT dual to AdS_2.

Power of geometry!

An emergent IR CFT

At low frequencies, the parent theory at finite density should be controlled by an emergent IR CFT !

Gravity in the AdS_2 region $\Longleftrightarrow$ a (0+1)-d CFT

CFT is only in the time direction, spatial directions become labels.

Each operator will develop new scaling dimensions in the IR.

AdS_2 gravity $\Rightarrow$ Operator dimensions, correlation functions

IR scaling dimensions for $O_{\vec{k}}$

$$\delta_{\vec{k}} = \frac{1}{2} + \nu_{\vec{k}}, \quad \nu_{\vec{k}} = \frac{1}{\sqrt{6}}\sqrt{m^2 + k^2 - \frac{q^2}{2}}$$

IR correlation functions for $O_{\vec{k}}$

$$\mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k}$$

(c(k) explicitly known)

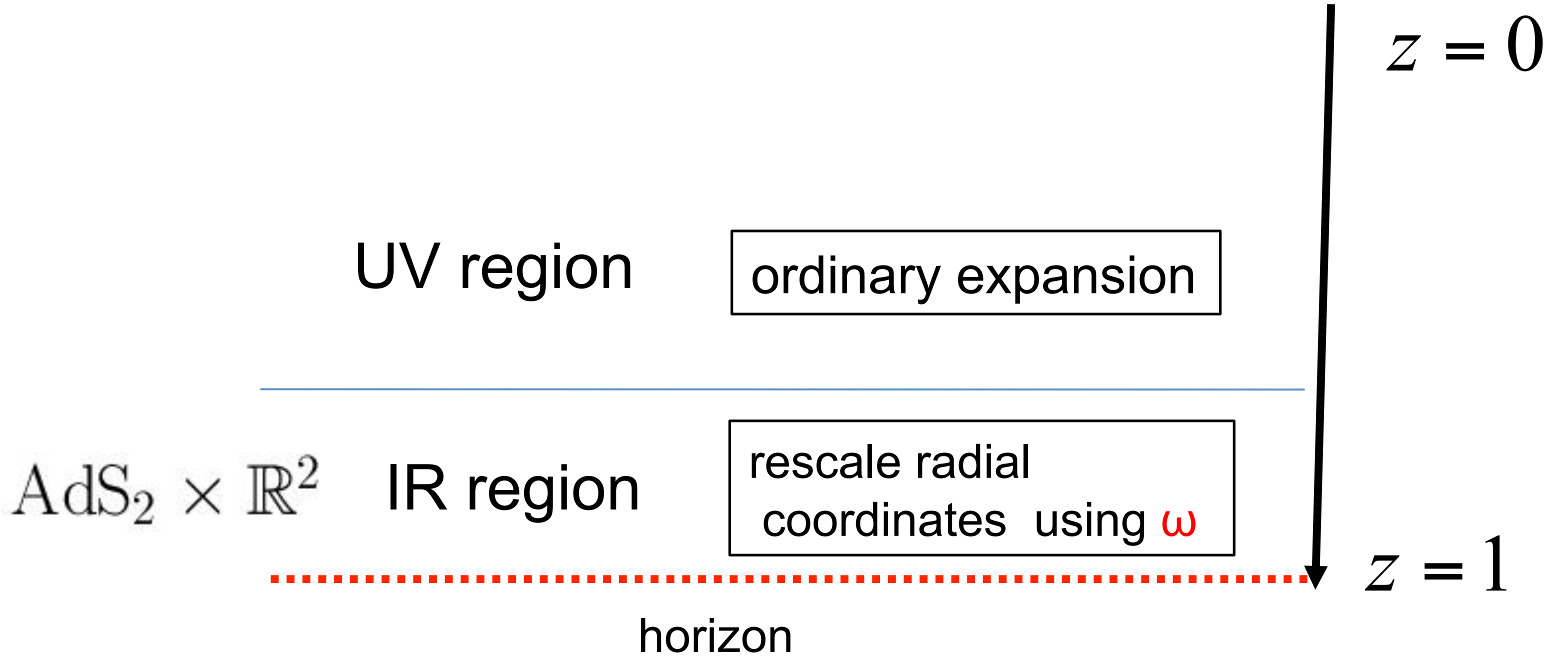
This insight now allows us to obtain analytically the low frequency behavior of the retarded function for the full theory

$$G_R(\omega, \vec{k})$$

in terms of quantities obtained from AdS_2 regions

$$\mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k}$$

Small frequency expansions



1. Separate the BH spacetime into two regions: IR , UV

2. Perform small ω expansions in each region separately

3. Match them at the overlapping region.

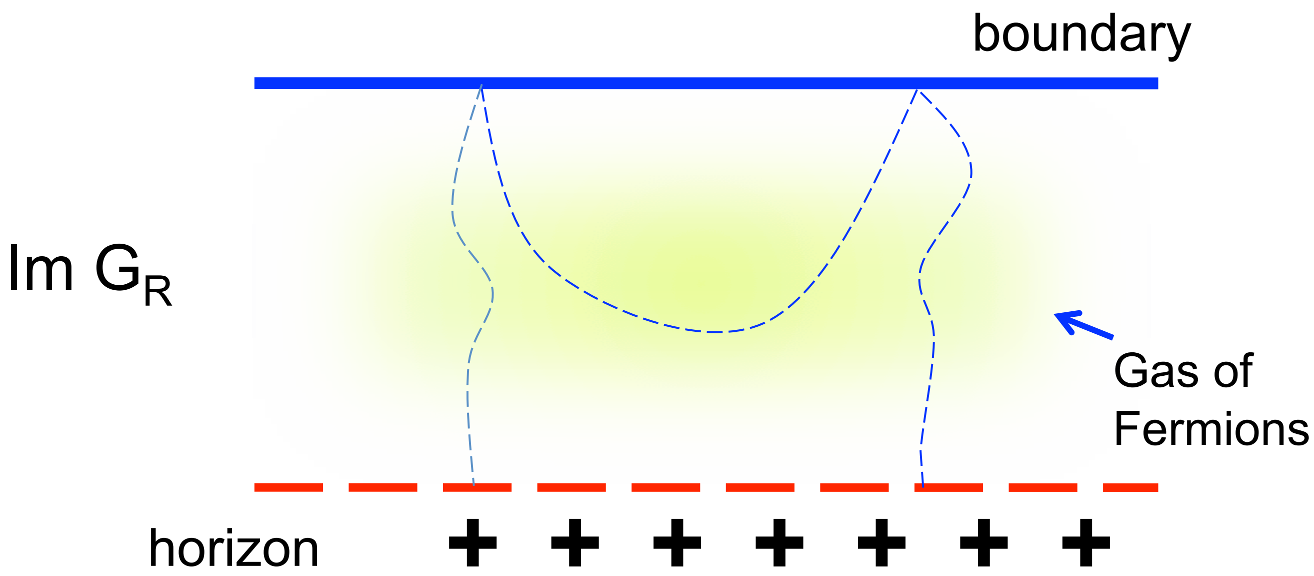
Reminiscent of the standard RG picture

Generic k: no quasi-particles

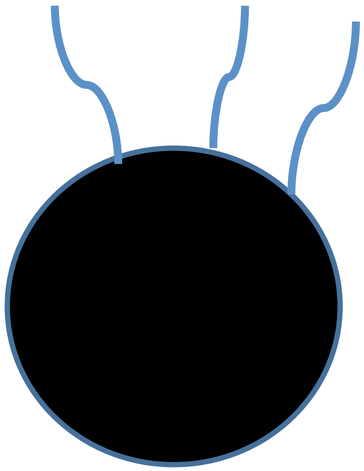
$$G_R(\omega, k) = f_0(k) + f_1(k)\omega + f_2(k)\mathcal{G}_k(\omega) + \cdots$$

UV: analytic and real

Non-analytic behavior and dissipation are controlled by the IR CFT.



Fermionic black hole hair and Fermi surface



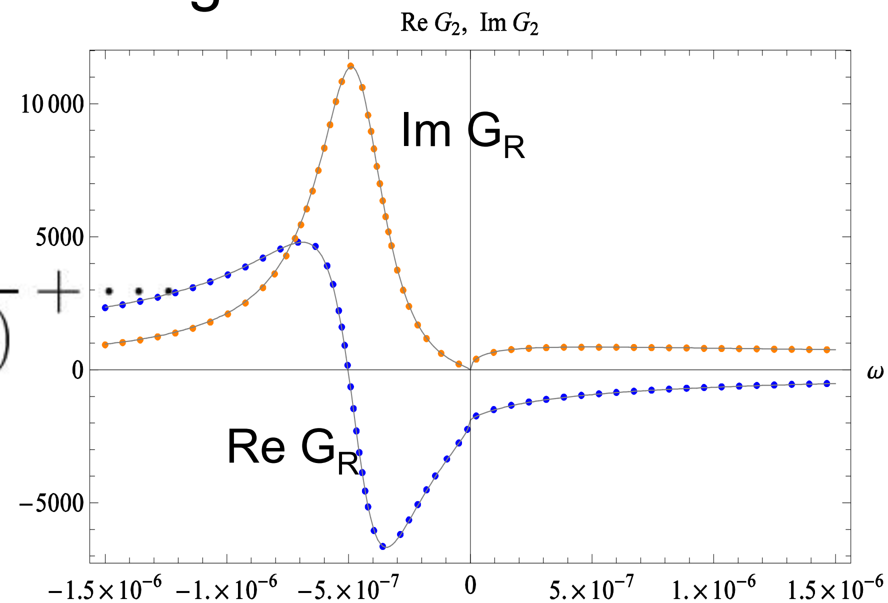
Surprise: an extremal AdS charged black hole admits fermionic hair of nonzero momentum !

At such a k_F, the retarded function is singular at $\omega = 0$

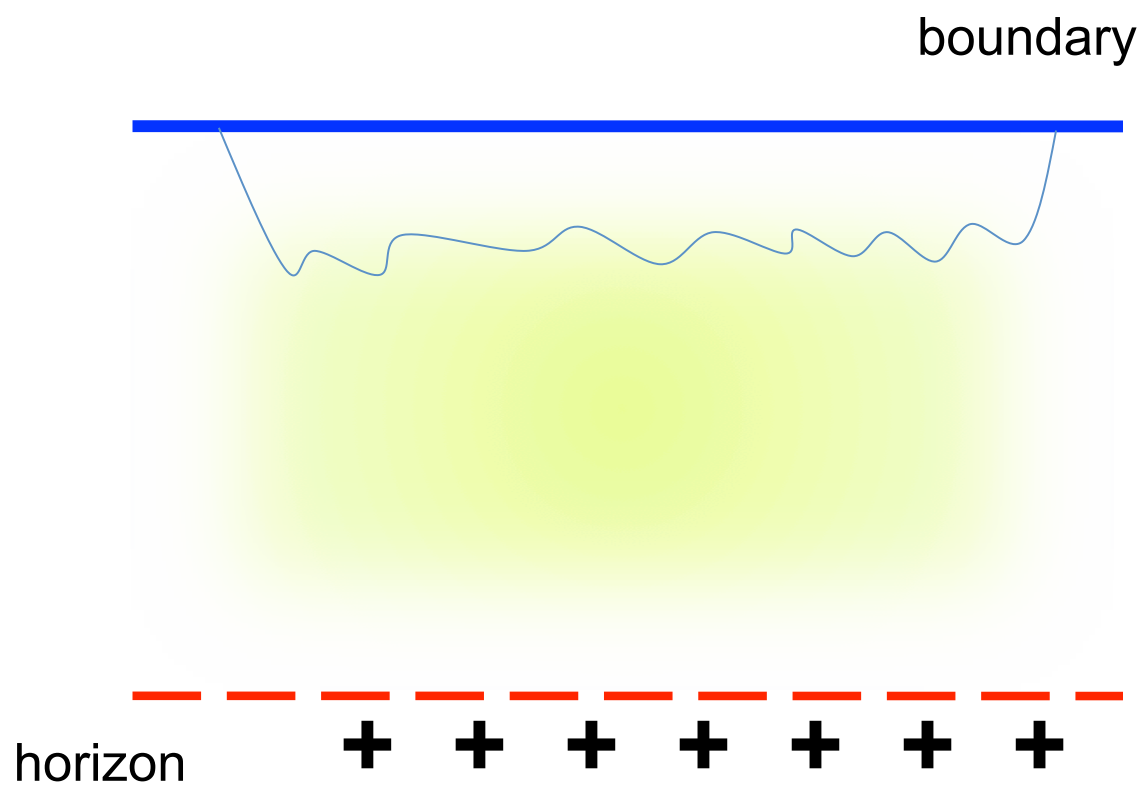
then near k_F, at small ω

$$G_R(k,\omega) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2\,\mathcal{G}_{k_F}(\omega)} + \cdots$$

$$\mathcal{G}_{k_F}(\omega) = c(k_F)\omega^{2\nu_{k_F}}$$



Quasi-particle-like excitations



Properties of the quasi-particle pole

$$G_R(k,\omega) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 c(k_F)\omega^{2\nu_{k_F}}} + \cdots$$

competition

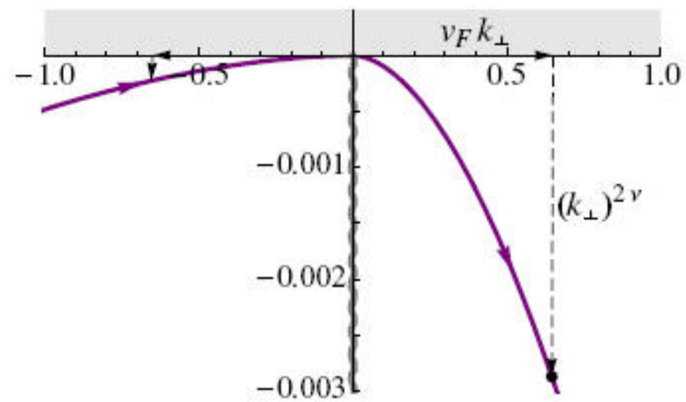
Scaling behavior around the Fermi surface controlled by dimension ν (evaluated at Fermi momentum)

$$\nu_{\vec{k}_F} = \frac{1}{\sqrt{6}}\sqrt{m^2 + k_F^2 - \frac{q^2}{2}}$$

Will treat ν_{k_F} as a tunable parameter

(k_F: controlled by UV physics)

For $\nu_{k_F} > \frac{1}{2}$ pole $\omega = \omega_* - i\Gamma$



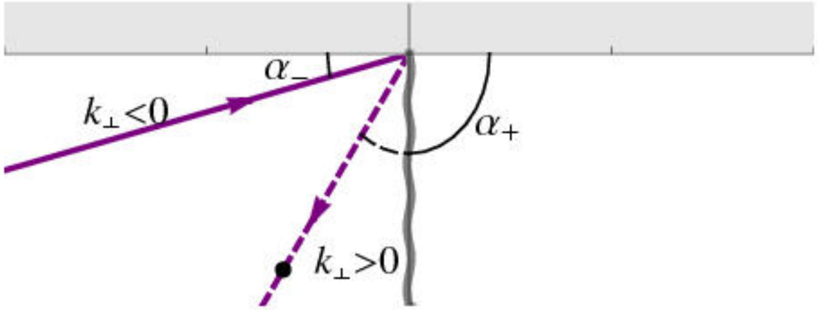
$$\omega_*(k) = v_F k_\perp + \cdots \,, \qquad \frac{\Gamma(k)}{\omega_*(k)} \propto k_\perp^{2\nu_{k_F}-1} \to 0, \quad Z = h_1 v_F$$

Linear dispersion relation, the quasi-particle becomes stable approaching the Fermi surface,
non-vanishing residue at the Fermi surface.

Quasi-particle picture applies, like in Fermi liquids.

But $\Gamma \propto \omega^{2\nu_k}$

For $\nu_{k_F} < 1/2$ pole $\omega = \omega_* - i\Gamma$



$$\omega_*(k) \sim k_\perp^z, \quad z = \frac{1}{2\nu_{k_F}} > 1, \quad \frac{\Gamma(k)}{\omega_*(k)} = \text{const}$$

Imaginary part is always comparable to the real part (quasi-particle never stable)

$$Z \propto k_\perp^{\frac{1-2\nu_{k_F}}{2\nu_{k_F}}} \to 0, \quad k_\perp \to 0$$

Residue of the pole vanishes at the Fermi surface

Fermi surface without sharp quasi-particles !

Marginal Fermi liquid

For $\nu_{k_F} = \frac{1}{2}$

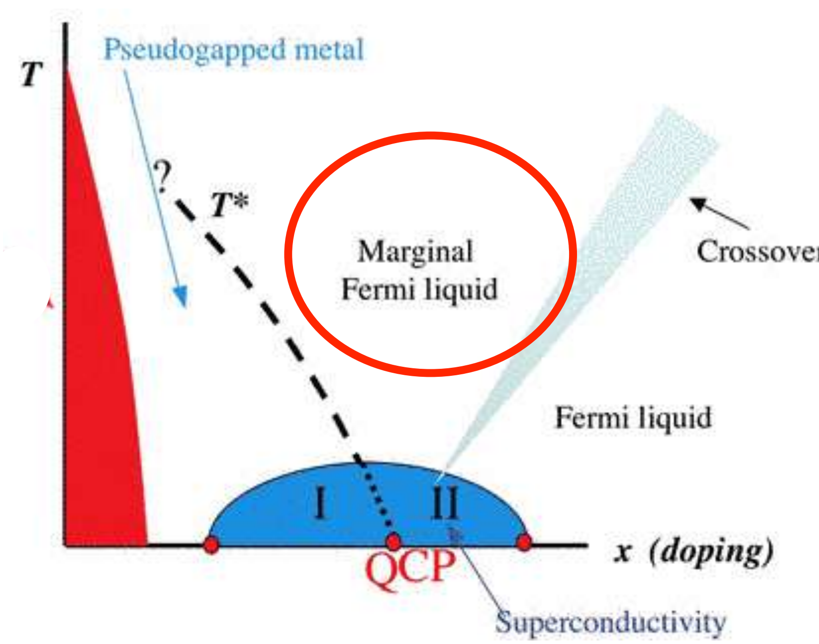
$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \log \omega + c_1 \omega}$$

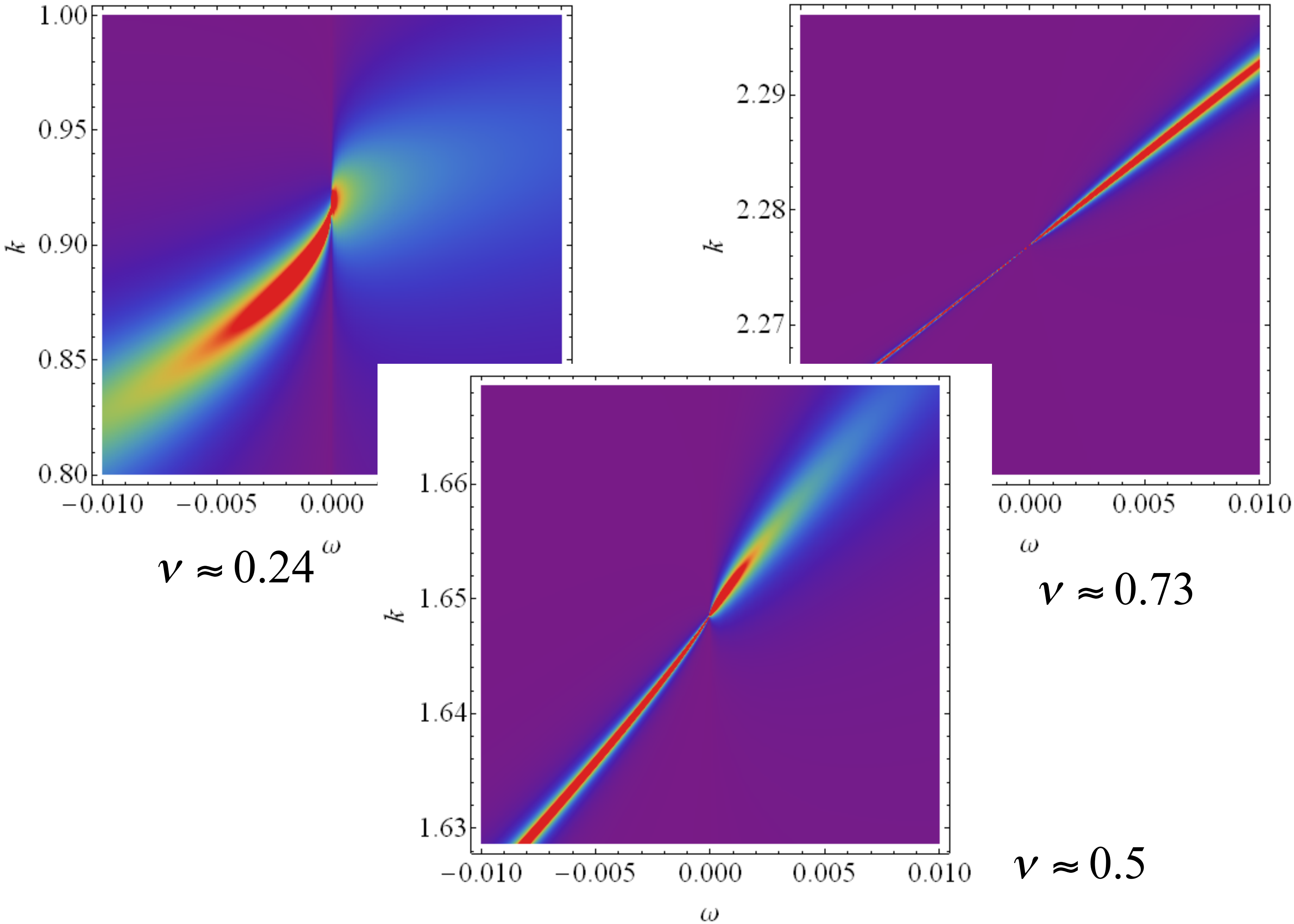
$\tilde{c}_1$: real

c_1 : complex

Precisely that for
``Marginal Fermi liquid''
proposed on phenomenological
ground for high Tc cuprates
near optimal doping.

Varma, Littlewood, Schmitt-Rink,
Abrahams, Ruckenstein (89)





1.00
0.95
0.90
0.85
0.80
k
−0.010
−0.005
0.000
ω
$\nu \approx 0.24$
2.29
2.28
2.27
k
0.000
0.005
0.010
ω
$\nu \approx 0.73$
1.66
1.65
1.64
1.63
k
−0.010
−0.005
0.000
0.005
0.010
ω
$\nu \approx 0.5$

Summary

Operator dimensions in the IR CFT $\Longleftrightarrow$ Scaling exponents near the Fermi surface

Depending on values of m and q, we can have

Fermi surface with almost stable quasi-particles

Fermi surface without quasi-particles

Marginal Fermi liquid for high Tc cuprates arises

for $\nu_{k_F} = \frac{1}{2}$ (O_{k_F} is a marginal operator in the IR CFT)

Self-energy is analytic in k: reminiscent of local quantum criticality and a Fermi surface coupled to a gapless boson.

We have shown that:

Marginal Fermi liquid for high Tc cuprates arises

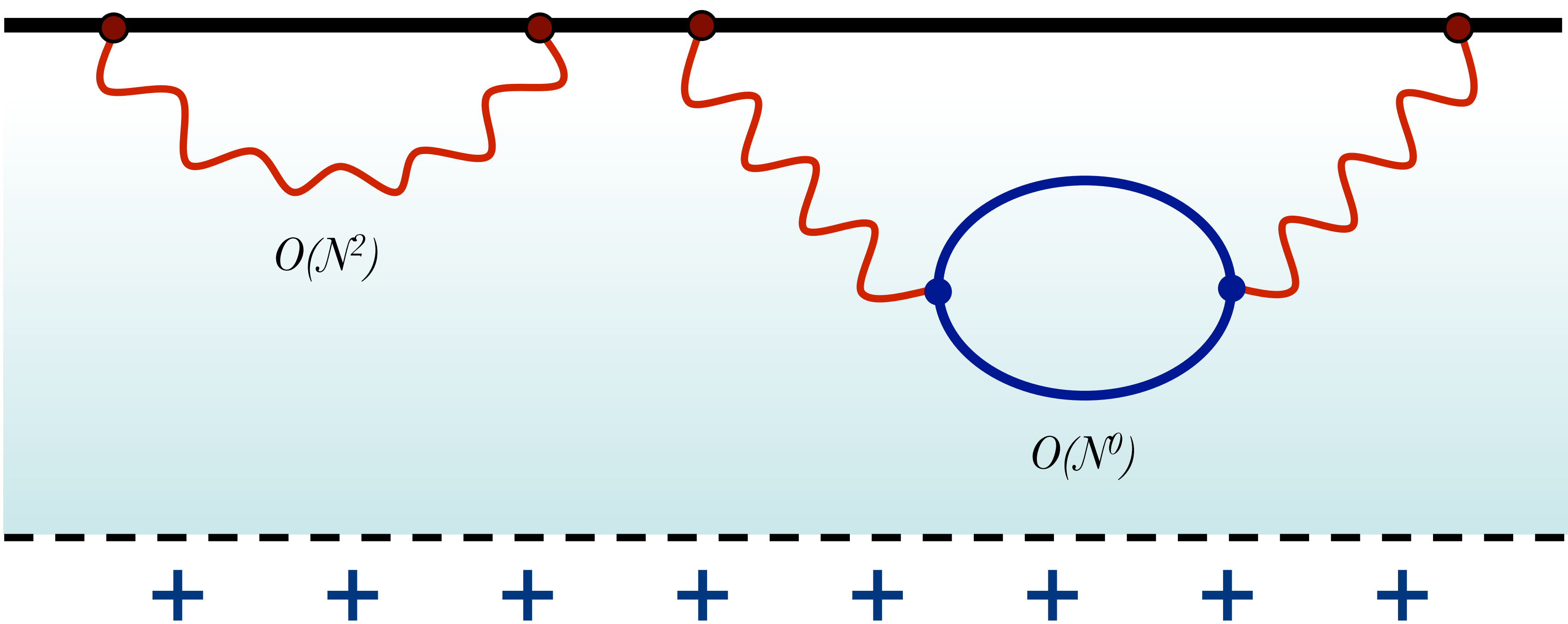
for $\nu_{k_F} = \frac{1}{2}$

What about resistivity?

Conductivity

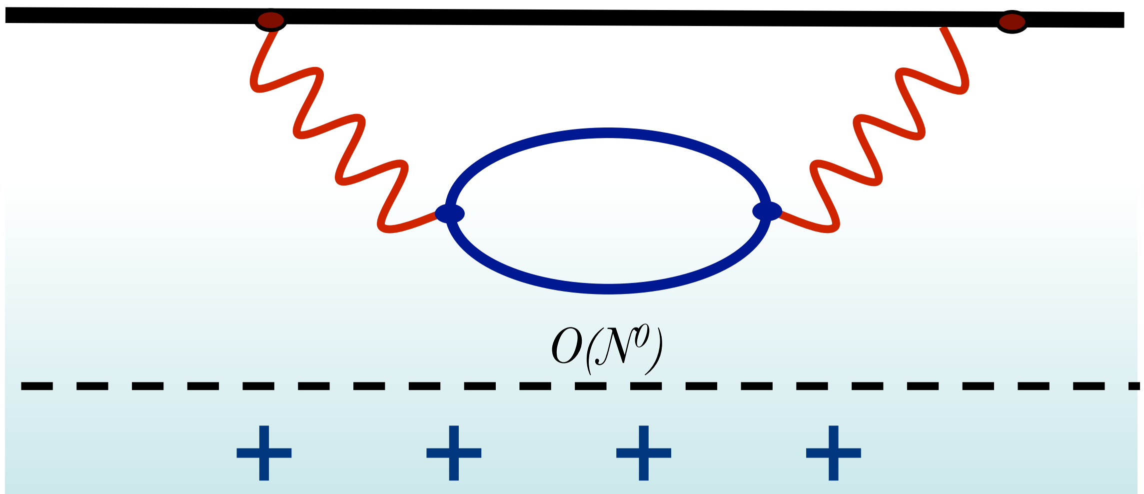
$$\sigma(\omega) = \frac{1}{i\omega}\langle J_x(\omega) J_x(-\omega)\rangle_{\text{retarded}}$$

$$J_x \Leftrightarrow A_x$$



Conductivity from fermions

Faulkner, Iqbal, HL, McGreevy,Vegh
to appear

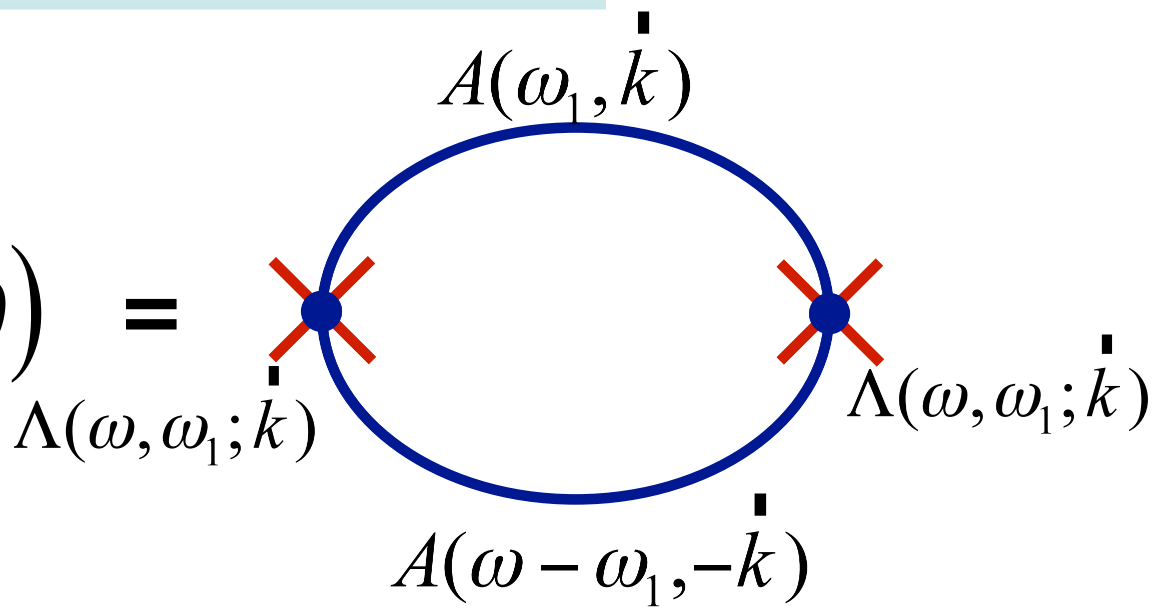


One-loop calculation in gravity:

many subtleties and potential pitfalls ……

after an epic calculation

$$\sigma(\omega) =$$



In the low temperature limit, the contribution near the Fermi surface dominates, for which

$$\Lambda(\omega,\omega_1;\vec{k}) \sim \mathrm{O}(1) \qquad \Longrightarrow$$

$$\sigma_{FS} \propto T^{-\alpha} \qquad \text{with} \qquad \alpha = 2\nu_{k_F}$$

For marginal fermi liquid (relevant for cuprates) $\nu_{k_F} = \dfrac{1}{2}$

$$\sigma_{FS} \propto T^{-1}$$ leading to linear resistivity !

The precise prefactor can also be calculated (in progress)

Optical conductivity

$$\nu_{k_F} < \frac{1}{2}: \ \sigma(\omega) = T^{-2\nu_{k_F}} F_1\left(\frac{\omega}{T}\right) \Rightarrow a(i\omega)^{-2\nu_{k_F}}, \quad T \ll \omega \ll \mu$$

Scaling form, no quasi-particle

$\nu_{k_F} > \frac{1}{2}$ quasi-particle lifetime $\Gamma^{-1} \sim T^{-2\nu_{k_F}} \gg T^{-1}$

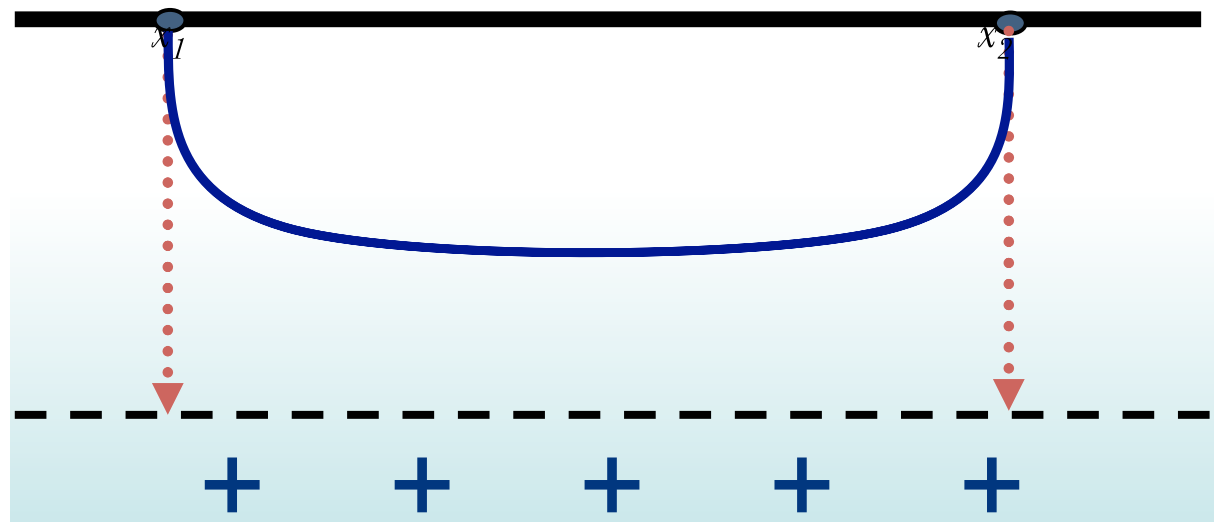
$$\sigma(\omega) \sim \begin{cases} \frac{\omega_p^2}{\frac{1}{\tau} - i\omega} & \omega \sim \tau^{-1} \sim \Gamma \\ \frac{i\omega_p^2}{\omega} + b(i\omega)^{2\nu_{k_F}-2} & T \ll \omega \ll \mu \end{cases}$$

$$\nu_{k_F} = \frac{1}{2}: \ \sigma(\omega) = T^{-1} F_2\left(\frac{\omega}{T}, \log\frac{T}{\mu}\right)$$

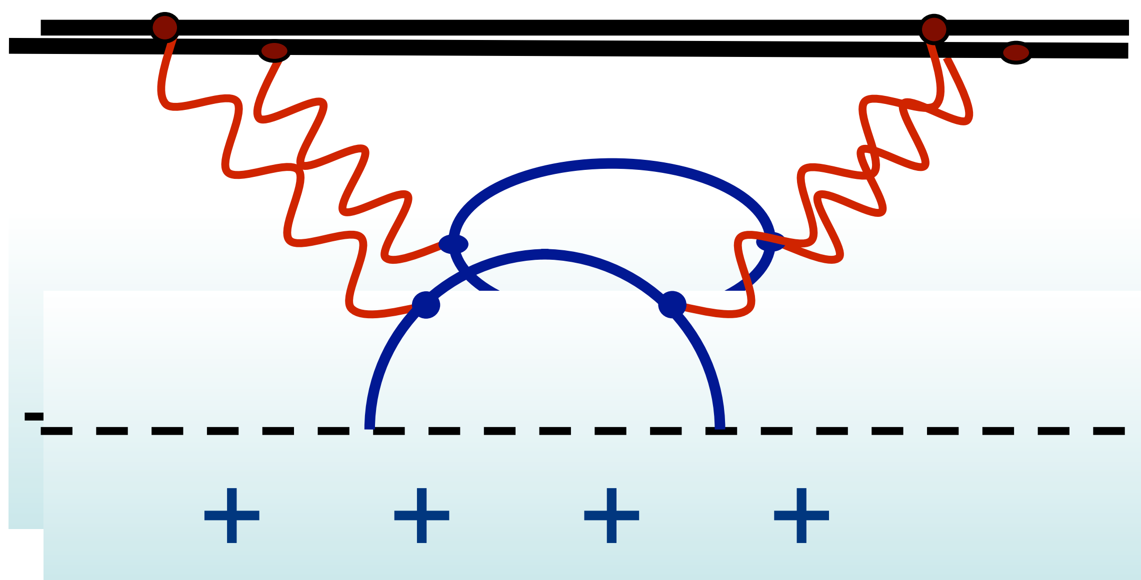
$$\sigma(\omega) \propto \frac{i}{\omega}\left(\frac{1}{\log\frac{\omega}{\mu}} + \frac{1}{(\log\frac{\omega}{\mu})^2}\frac{1+i\pi}{2}\right) + \cdots, \quad \omega \gg T$$

Bulk physical picture

Single-particle decay rate



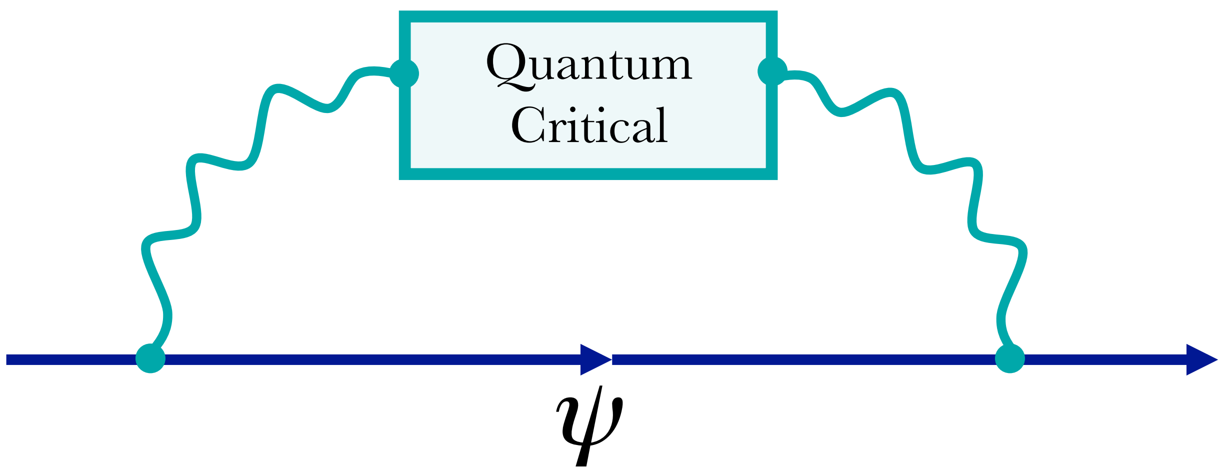
Decay of a current



Overall Physical picture

IR CFT with nontrivial scaling only in temporal directions

Gravity: AdS_2

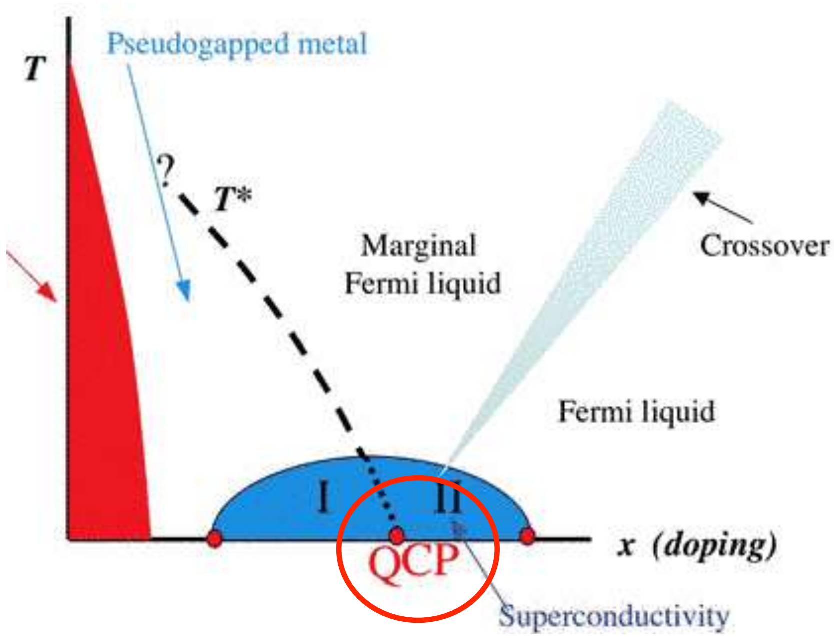


Faulkner, HL, McGreevy, Vegh; Faulkner Polchinksi

Some perspective

I have talked about two aspects of the gravity example at $\nu_{k_F} = \frac{1}{2}$ which matches perfectly with high Tc cuprates.

→ a good laboratory for studying many other questions related to high Tc or other materials



Could it be that our IR CFT lie in the same universality class of the (conjectured) quantum critical point for high Tc cuprates?

Other generalizations and future directions:

0. Other observables like specific heat, density correlation functions ……

1. Turn on a magnetic field, Hall conductivity, FQHE ….

Albash and Johnson; Basu, He, Mukherjee, Shieh;
Denef, Hartnoll and Sachdev; Hartnoll, Hofman

2. Couple fermions to a superconducting state
a spin density date (AFM)

Chen, Kao, Wen; Faulkner et al; Gubser, Rocha, Talavera

3. Pairing instability of non-Fermi liquids

4. Phase transition from FL to NFL

…………

Superconducting instability

The gravity duals of field theory systems we consider generically have light charged scalars and thus suffers from superconducting instabilities with condensation of charged scalars.

(Gubser, Hartnoll, Herzog, Horowitz; Roberts Horowitz; Denef, Hartnoll ……)

At T=0, the IR CFT of this talk could be generically hidden behind a superconducting state, a situation not dissimilar to high Tc cuprates or many heavy fermion materials.

From gravity: hard to find a quantum critical point without light charged scalars (which then want to condense).

This could be a general lesson.

Thank You

Some additional materials

Imaginary exponent

$$\mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k} \qquad \delta_k = \frac{1}{2} + \nu_k$$

$\nu_k = -i\lambda_k$ is pure imaginary for small enough k when

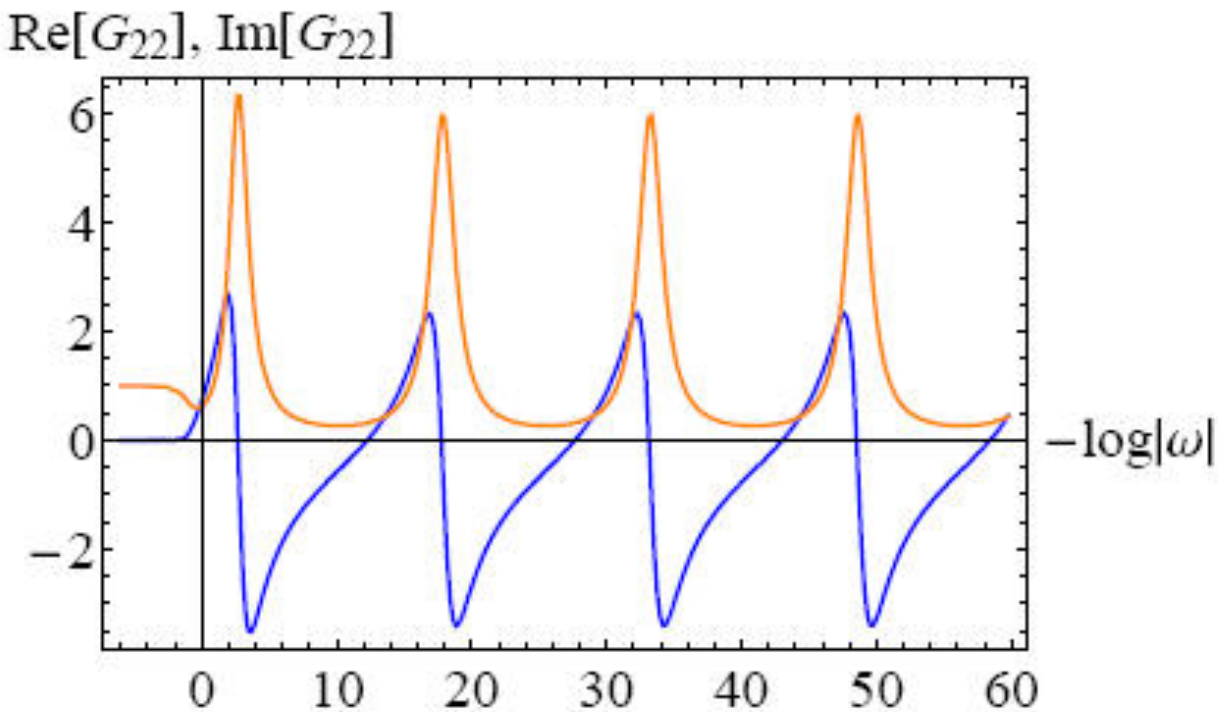
$$\Delta < \frac{|q|}{\sqrt{2}} + \frac{d}{2}$$

$$G_R(\omega, k) \approx \frac{b_+^{(0)} + b_-^{(0)} c(k)\omega^{-2i\lambda_k}}{a_+^{(0)} + a_-^{(0)} c(k)\omega^{-2i\lambda_k}} + O(\omega)$$

Note: no instability

Log-periodic behavior

This leads to a discrete scaling symmetry and



Conditions for Fermi surface

For what values of q and Δ, are Fermi surfaces allowed? i.e. when fermionic hair exists

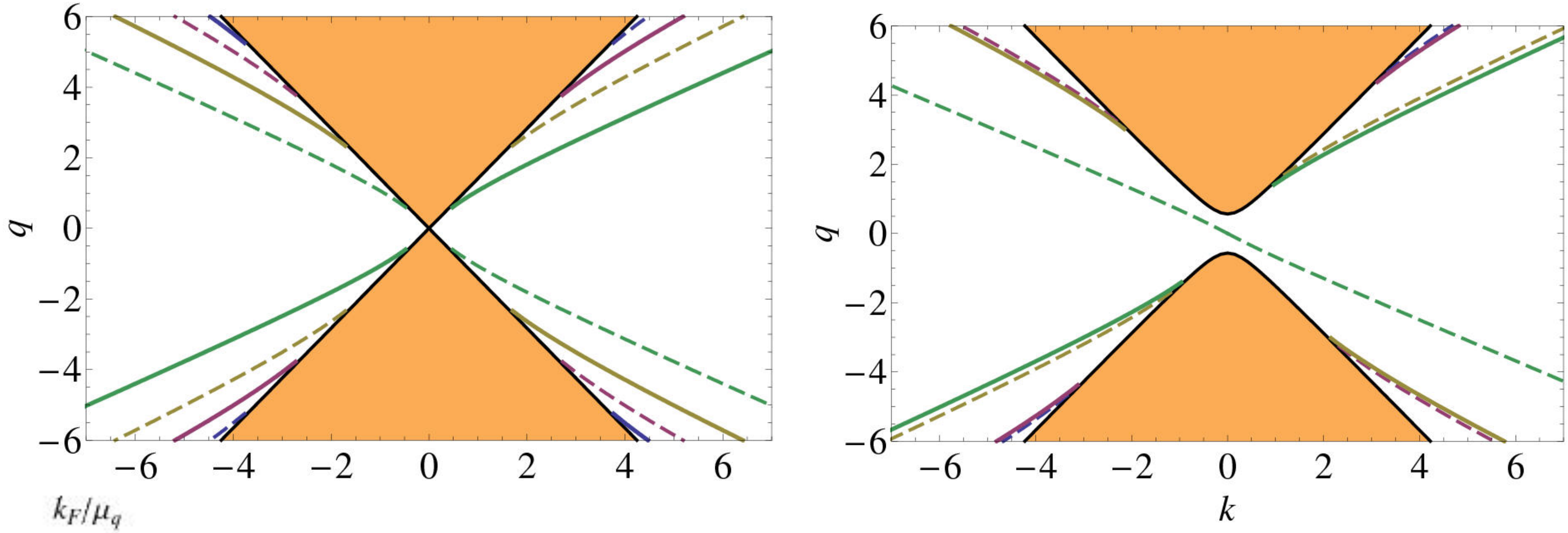
$$\Delta < \frac{|q|}{\sqrt{3}} + \frac{d}{2}$$

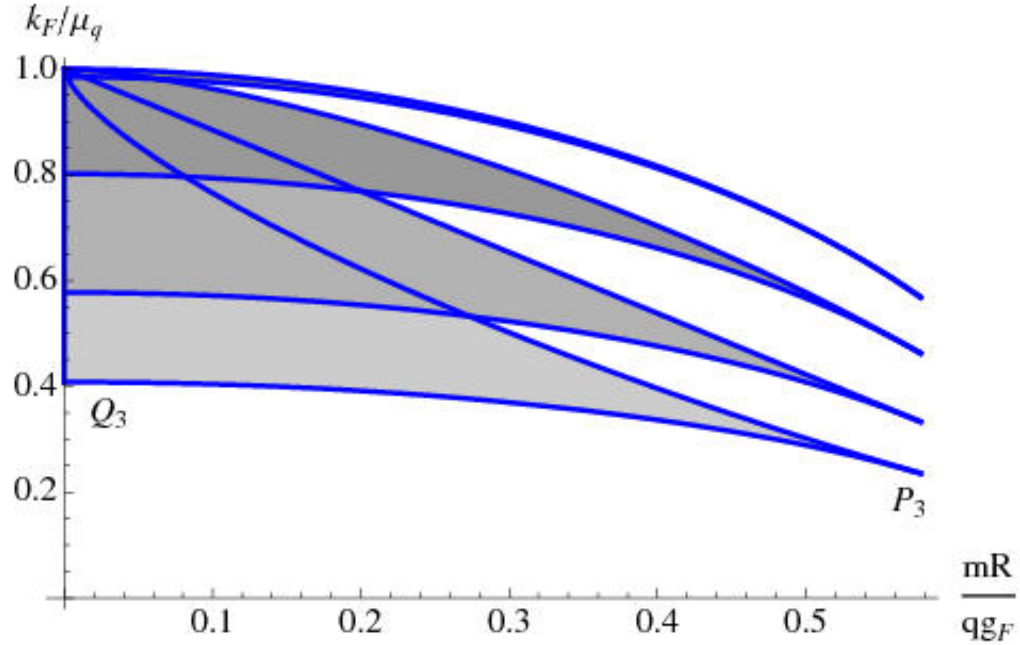
It always lies inside the region which allows log-periodic behavior

$$\Delta < \frac{|q|}{\sqrt{2}} + \frac{d}{2}$$

Except for $$\frac{d-1}{2} < \Delta < \frac{d}{2} - \frac{|q|}{\sqrt{2}}$$ (alternative quantization)

How does k_F depend on q and Δ ?

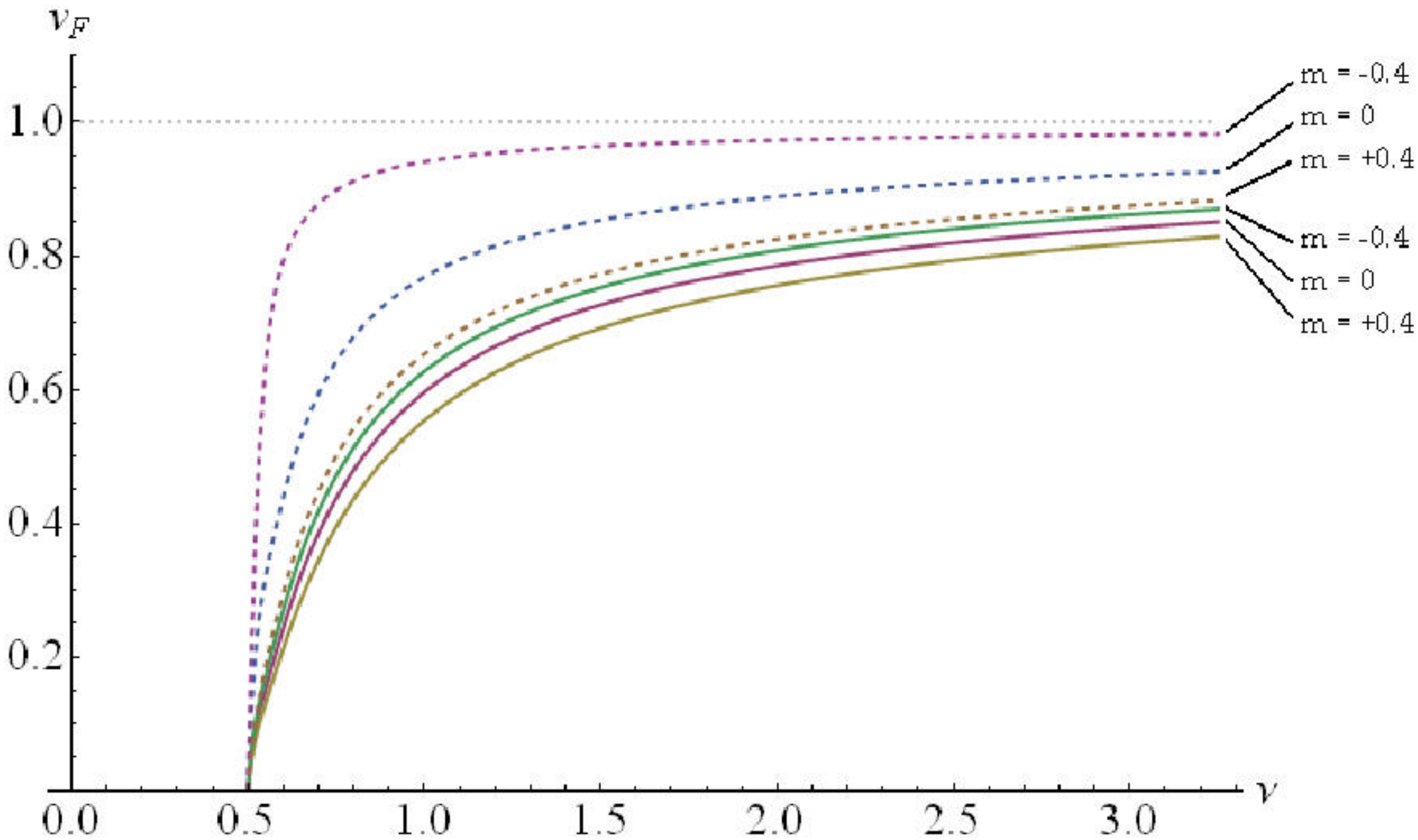




For fixed Δ, k_F increases with q.

For fixed q, k_F decreases with Δ.

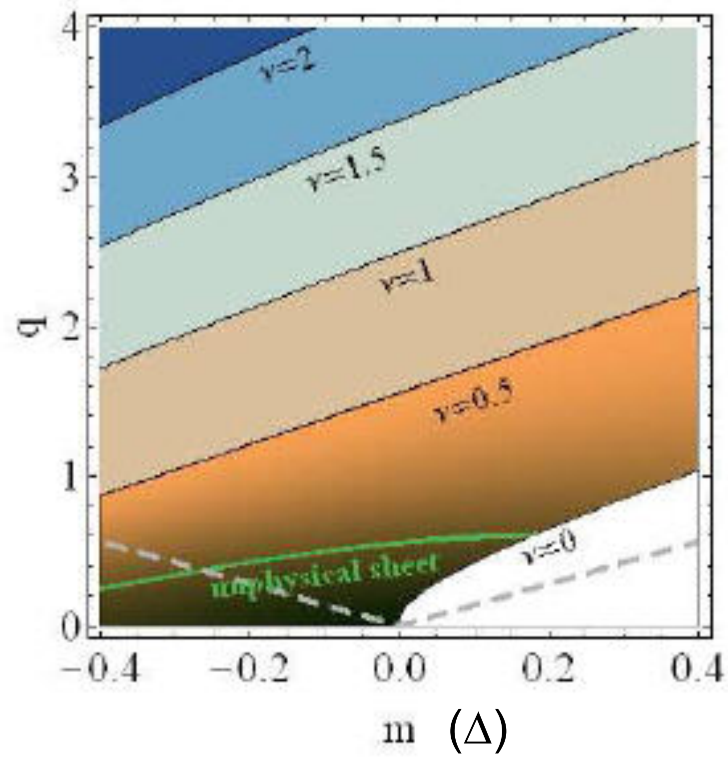
UV data: Fermi Velocity

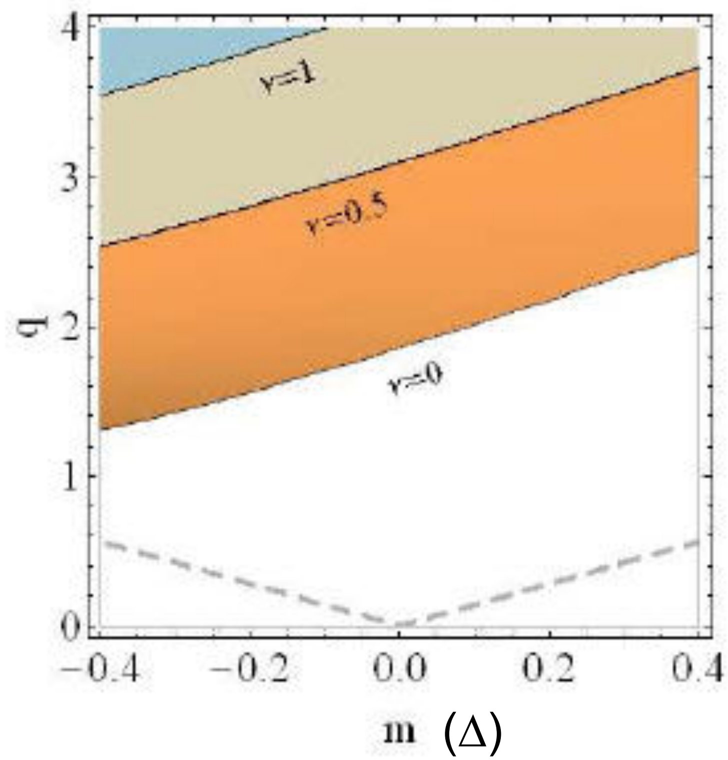


Fermi velocity goes to zero as the marginal limit is approached, so does the residue.

Landscape of exponents

$$\delta_{\vec{k}} = \frac{1}{2} + \nu_{\vec{k}}, \quad \nu_{\vec{k}} = \frac{1}{\sqrt{6}}\sqrt{m^2 + k^2 - \frac{q^2}{2}}$$





Small frequency expansion

$$G_R(\omega, k) = \frac{b_+(\omega, k) + \mathcal{G}_k(\omega) b_-(\omega, k)}{a_+(\omega, k) + \mathcal{G}_k(\omega) a_-(\omega, k)}$$

$G_k(\omega)$: retarded function for O_k in the IR CFT, depending only on the AdS_2 region. (IR data)

(generically) non-analytic in ω and complex (dissipative)

$a_\pm, b_\pm$: from solving the Dirac equation in the UV region

Real, analytic in ω and k, expressed in power series of ω.

(UV data)