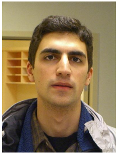
A unification of photons, electrons, and gravitons under qbit models

Xiao-Gang Wen, MIT

- Quantum ether: photons and electrons from a rotor model
  – Phys. Rev. B73, 035122 (2006)
- Emergence of helicity ± 2 modes (gravitons) from qbit models
  – arXiv:0907.1203



M. Levin

Z.-C. Gu

Seven basic assumptions

- The current physical theory explain a very wide range of phenomena from some simple "starting points".
  It unifies everything into seven fundamental assumptions – seven wonders of our universe:
  (1) Locality.
  (2) Identical particles.
  (3) Gauge interactions.
  (4) Fermi statistics.
  (5) Chiral fermions. ($SU(2)$ only couples to left-hand fermions)
  (6) Lorentz invariance.
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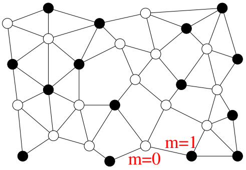
  Can we unify further to obtain all seven wonders from a single simple structure? Can we obtain the 6 other assumptions from the Locality alone?
- **Everything has to come from something**

Emergence approach – everything from qbits

- One type of fundamental building blocks for everything: **qbits**
- Space = collection of 10^{183} qbits. No qbits, no space
- Empty space (vacuum) = ground state of qbits: $\Phi_0(\{m_i\})$.
- "Elementary" particles = collective excitations (such as topological defects, collective waves) above the ground states: $\Phi(\{m_i\})$.



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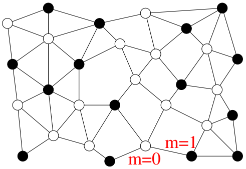

Originate from organization (order)

To understand the 6 other wonders from qbit model

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- The issue is *"how the qbits are organized"*.
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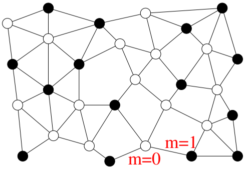

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- **Different orders of qbits correspond to different phases, and different universes with different "elementary" particles**

Can the six wonders emerge from an organization

- Old picture of phases and phase transitions:
  All orders are described by symmetry breaking.

- The symmetry breaking states can only give rise to bosonic collective excitations described by bosonic field $\sim$ order parameters.
  But no gauge bosons and fermions.

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- The symmetry breaking states are "trivial" unentangled states:
  $|\text{symm. breaking}\rangle = |\uparrow\rangle \otimes |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes \ldots$
- Unentangled direct-product states are very special states. They are not most general quantum many-qbit states.

  Maybe gauge bosons and fermions can emerge from more general highly entangled many-qbit quantum states.

New states of matter with long range entanglements exist

Gapped states (topological order Wen 1989):

- Many fractional quantum Hall states. Tsui, Stormer, Gossard 1982
- Many superconducting states ($p + ip$, $d + id$, ...) Read, Green, 2000
- Many Mott-insulators = gapped spin liquids. Wen etal 89; Moessner, Sondhi 03

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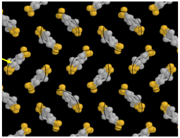

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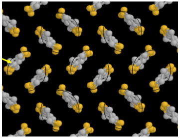

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- **Can gauge bosons and fermions emerge from the new qbit states with long range entanglements?**

Long range entanglement → fermions and gauge bosons

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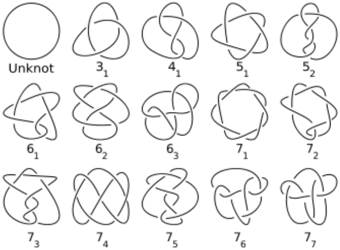

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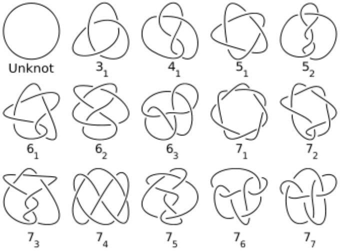

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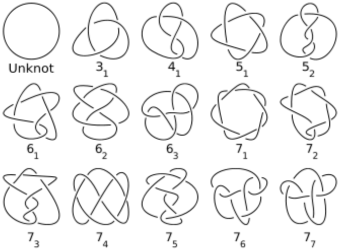

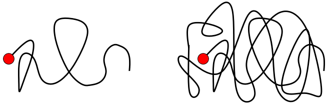

String-net picture of ele. particles (not knots but ends of strings):

- Electons/Quarks = ends of strings → produce Fermi statistics.

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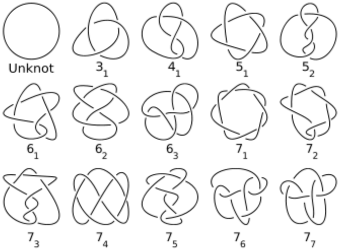

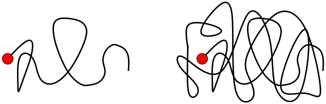

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- Electons/Quarks = ends of strings → produce Fermi statistics.
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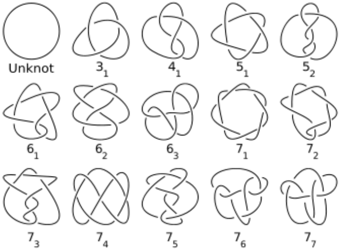

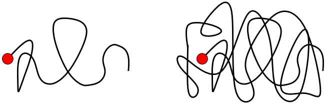

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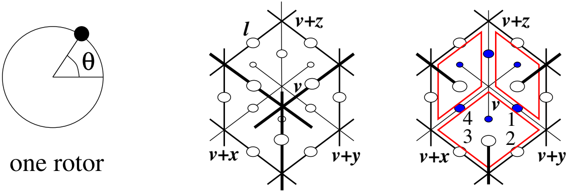
- **String-net order in qbit model unifies light, electrons ... ...**

$|\text{string-net ordered}\rangle = \sum |\text{loops or string-nets}\rangle \to$ long range entangled

A quantum rotor model on cubic lattice

A concrete model that has string-net order: Motrunich & Senthil 02, Wen 03



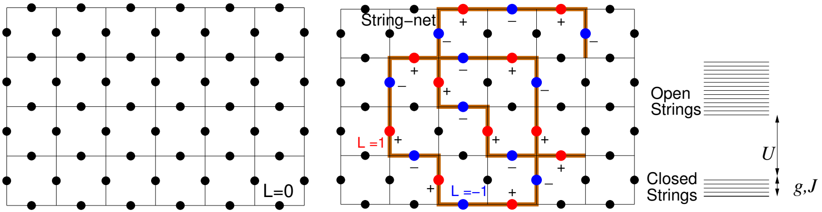
A rotor θ_i on every link of the cubic lattice:

$$H = U \sum_{\mathbf{v}} Q_{\mathbf{v}}^2 - g \sum_{\mathbf{p}} (B_{\mathbf{p}} + h.c.) + J \sum_{\mathbf{l}} (L_{\mathbf{l}})^2$$

$$Q_{\mathbf{v}} = \sum_{\mathbf{l} \text{ next to } \mathbf{v}} L_{\mathbf{l}}, \qquad B_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^-$$

$L = -i\partial_\theta$: the angular momentum of the rotor
$L^\pm = e^{\pm i\theta}$: the raising/lowering operators of L

String-net liquid



- the $UQ_{\mathbf{v}}^2$-term → closed strings. Open ends cost energy.
- $J(L_{\mathbf{l}})^2$-term → string tension
- the $gB_{\mathbf{p}}$-term → strings can fluctuate

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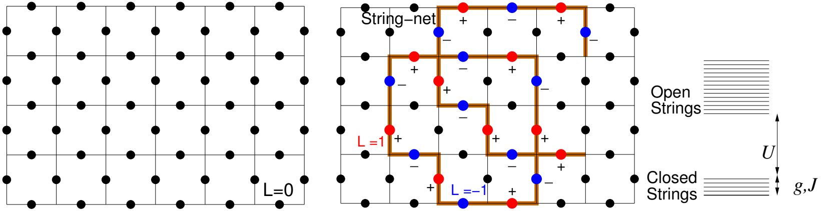
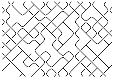


- the $UQ_{\mathbf{v}}^2$-term $\to$ closed strings. Open ends cost energy.
- $J(L_{\mathbf{l}})^2$-term $\to$ string tension
- the $gB_{\mathbf{p}}$-term $\to$ strings can fluctuate
- The ground state of the rotor Hamiltonian H when $U \gg g \gg J$

$$|\text{String-net liquid}\rangle = \sum_{\text{all closed string conf.}} \Big|$$  $$\Big\rangle$$

- The string-net liquid is a new state of quantum matter with long range entanglement

String-net condensation

- Boson condensed state

$$\langle \text{Boson condensed} |\ \phi\ | \text{Boson condensed} \rangle \neq 0$$

where ϕ is a boson creation operator.

- String condensed state

$$\langle \text{String condensed} |\ W\ | \text{String condensed} \rangle \neq 0$$

where W is the string creation operator:

$$W = \ldots L^{+}_{\mathbf{l}_1} L^{-}_{\mathbf{l}_2} L^{+}_{\mathbf{l}_3} L^{-}_{\mathbf{l}_4} \ldots$$

- The closed string operator W_{closed} cost no energy $[H, W_{\text{closed}}] = 0$.
- The open string operator W_{open} create a pair of quasiparticles.
  Open string operator $\to$ properties of quasiparticles

1) Maxwell waves in string-net liquid – a cartoon approach

Zero-point fluctuations in ground state   Waves in string-net liquid

- "String density" $\mathbf{E}(\mathbf{r}, t)$ satisfies $\partial \cdot \mathbf{E} = 0$
- String density wave satisfies $\dot{\mathbf{E}} = \partial \times \mathbf{B}, \quad \dot{\mathbf{B}} = -\partial \times \mathbf{E}$.

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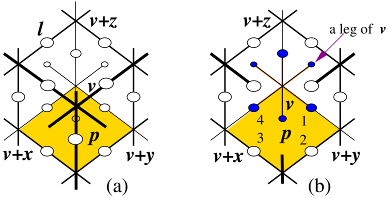
- End of strings = source of $\mathbf{E}$ = gauge charges
- "Vortices" in string liquid = magnetic monopoles

2) Emergence of Maxwell equation – E.O.M approach



$$H = U\sum_{\mathbf{v}} Q_{\mathbf{v}}^2 - g\sum_{\mathbf{p}}(B_{\mathbf{p}} + h.c.) + J\sum_{\mathbf{l}}(L_{\mathbf{l}})^2, \quad B_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^-$$

Key: dynamics of low energy closed-string states.

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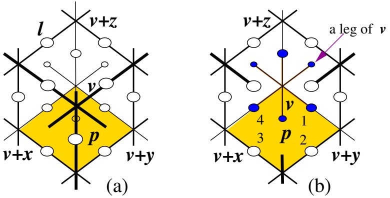

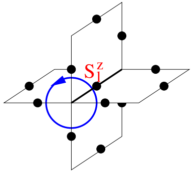

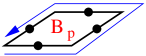

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Key: dynamics of low energy closed-string states.

The operators $B_{\mathbf{p}}$ and $L_{\mathbf{l}}$ act within the closed-string subspace:

$$\partial_t \langle L_{\mathbf{l}}\rangle = \langle i[H, L_{\mathbf{l}}]\rangle \sim i\langle \sum_{a=1,..,4} B_{\mathbf{p}_a} - h.c.\rangle \quad \to \quad \dot{\mathbf{E}} = \partial \times \mathbf{B}$$

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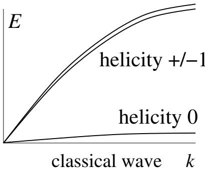
$$\langle B_{\mathbf{p}}\rangle = e^{i\mathbf{B}\cdot\mathbf{n}_{\mathbf{p}}}, \quad \langle L_{\mathbf{l}}\rangle = \mathbf{E}\cdot\mathbf{n}_{\mathbf{l}}$$

3) Semi-classical/quantum-freeze approach

- Each cube has three rotors on the links in the x-, y-, and z-directions. The three θ's form the three component of a vector field $\mathbf{A} = (\theta^x, \theta^y, \theta^z)$.
- If we treat the rotor system as a classical system, the classical equation of motion is determined from the phase-space Lagrangian $\mathcal{L} = \sum L_{\mathbf{l}} \dot{\theta}_{\mathbf{l}} - H(L_{\mathbf{l}}, \theta_{\mathbf{l}})$
- Dispersions of three modes is designed to have the following form



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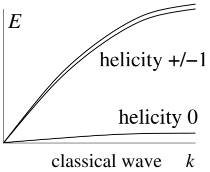

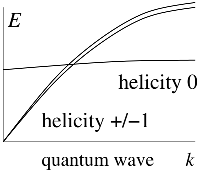

- Quantum fluctuations: When $g \gg J$, the fluctuations $\delta\theta_{\pm 1} \ll 2\pi$ and $\delta\theta_0 \gg 2\pi$. The helicity-0 mode is gapped in quantum theory, and the helicity-± 1 modes remain gapless.

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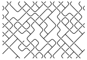
- For string-net condensed state $|\Phi\rangle = \sum_{\text{all conf.}} \Big|$  $\Big\rangle$

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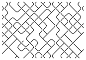
  The the ends of string in the above spin-1 model are bosons.

- For string-net condensed state $|\Phi\rangle = \sum_{\text{all conf.}} \pm \Big|$  $\Big\rangle$

  The end of strings are fermion.

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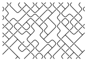
- For string-net condensed state $|\Phi\rangle = \sum_{\text{all conf.}} \Big|$  $\Big\rangle$
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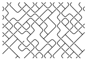
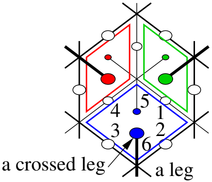
- For string-net condensed state $|\Phi\rangle = \sum_{\text{all conf.}} \pm \Big|$  $\Big\rangle$
  The end of strings are fermion.
  String-net condensation provides a way to produce Fermi statistics from local qbit models.

Twisted rotor model with emergent fermions



Just add a little twist

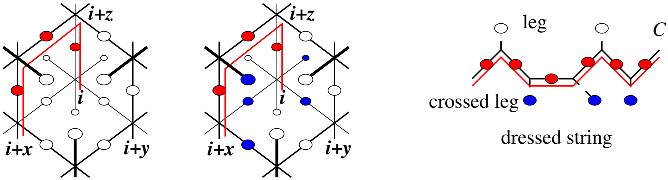
- Twisted-string model: Levin & Wen 04

$$\tilde{H} = U \sum_{\mathbf{v}} Q_{\mathbf{v}} - g \sum_{\mathbf{p}} \tilde{B}_{\mathbf{p}} + J \sum_{\mathbf{l}} (L_{\mathbf{l}})^2, \qquad \tilde{B}_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^- (-1)^{L_5 + L_6}$$

The sign change in the string hopping operator produces the sign change in the string wave function → fermionic string ends

Statistics of the ends of strings from the string operators

- Closed-string operators W are defined through $[W, H] = 0$ in $J = 0$ limit. Strings cost no energy and is unobservable.



- In the untwisted model – untwisted-string operator

$$L^+_{\mathbf{l}_1} L^-_{\mathbf{l}_2} L^+_{\mathbf{l}_3} L^-_{\mathbf{l}_4} \cdots$$

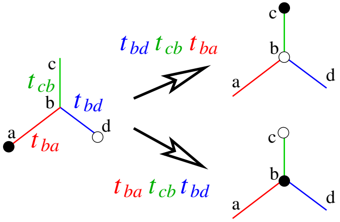
- In the twisted model – twisted-string operator

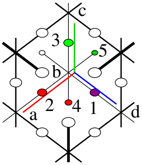
$$(L^+_{\mathbf{l}_1} L^-_{\mathbf{l}_2} L^+_{\mathbf{l}_3} L^-_{\mathbf{l}_4} \cdots) \prod_{\mathbf{l} \text{ on crossed legs of } C} (-1)^{L_\mathbf{l}}$$

- A pair of string ends is created by an open string operator. Their statistics can be calculated from the open string operator.

Statistics of string-ends from the alg. of string operators

- An open string operator is a hopping operator of the string-ends.
- The statistics is determined by particle hopping operators Levin&Wen 03:





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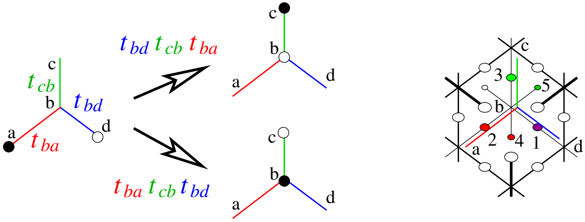

Algebra of open string operator determine the statistics

- For untwisted model: $t_{ba} = L_2^+$, $t_{cb} = L_3^-$, $t_{bd} = L_1^+$

  We find $t_{bd} t_{cb} t_{ba} = t_{ba} t_{cb} t_{bd}$

  $\to$ **The ends of untwisted-string are bosons**

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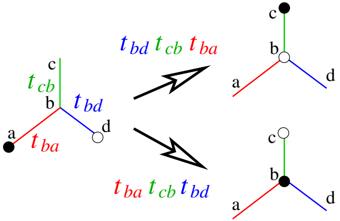

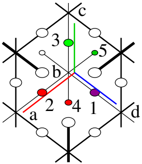

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  $\to$ **The ends of untwisted-string are bosons**
- For twisted model: $t_{ba} = (-)^{L_4+L_1}L_2^+$, $t_{cb} = (-)^{L_5}L_3^-$, $t_{bd} = L_1^+$
  We find $t_{bd}t_{cb}t_{ba} = -t_{ba}t_{cb}t_{bd}$
  $\to$ **The ends of twisted-string are fermions**

String-net liquid produces and unifies three wonders

0-qbits = no string state. Strings = lines of 1-qbits.
String-net ordered state = a superposition of string states

How much can we get from string-net order?

- The collective motion of string-nets (fluctuation of quantum entanglements) give rise to gauge bosons ($U(1) \times SU(2) \times SU(3)$, as well as other gauge groups) Wen 02, Levin & Wen 04
- Ends of strings (topological excitations) give rise to spin-1/2 fermions – the matter (leptons and quarks) Levin & Wen 03
- Three of the seven wonders

(2) Identical particles.
(3) Gauge interactions.
(4) Fermi statistics.

can emerge form a qbit model, if our vacuum is a string-net liquid.

String-net unifies gauge interaction and Fermi statistics

Can qbit model also unifies gravity?

Can gravitons emerge from a qbit model?

What is quantum gravity and what is graviton?

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What is quantum gravity and what is graviton?

Gravitons = Helicity ± 2 modes, but
Helicity ± 2 modes $\neq$ Gravitons

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Here we will use a very strict definition:

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  A field theory of symmetric tensor,
  $\mathcal{L} = (\dot{a}_{ij})^2 - (\partial a_{ij})^2 \ \sim \ \mathcal{L}_{\text{phase-space}} = \mathcal{E}^{ij}\dot{a}_{ij} - (\mathcal{E}^{ij})^2 - (\partial a_{ij})^2$,
  has helicity $0, 0, \pm 1, \pm 2$ modes,
  here the helicity ± 2 modes are not gravitons.

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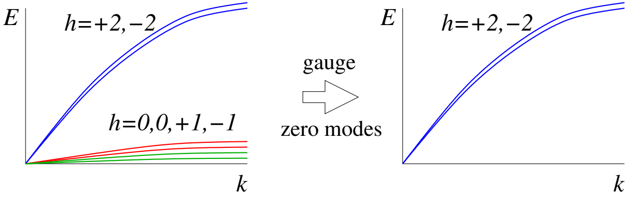
  It is very hard to construct a well defined local quantum model (ie a lattice Hamiltonian) with graviton (as defined above).
  Maybe we do not have any such model (?)
  $\rightarrow$ no well defined theory of quantum gravity yet.
- Here we will try to construct local lattice models with gravitons.

Field theory considerations Gu & Wen, arXiv:0907.1203

- Start with a field theory of symmetric tensor described by a phase-space Lagrangian $\mathcal{L}(\mathcal{E}^{ij}, a_{ij})$
- Remove the helicity $0, 0, \pm 1$ modes by constraints:



- Vector constraint to remove helicity $0, \pm 1$ modes:
  We set a combination of $\mathcal{E}^{ij}$ to zero: $\pi^j = \partial_i \mathcal{E}^{ij} = 0$.
  The corresponding canonical conjugate of π^i is f_i:
  $\delta a_{ij} = \partial_i f_j + \partial_j f_i$. The Lagrangian $\mathcal{L}(\mathcal{E}^{ij}, a_{ij})$ must not contain f_i:
  $\mathcal{L}(\mathcal{E}^{ij}, a_{ij} + \partial_i f_j + \partial_j f_i) = \mathcal{L}(\mathcal{E}^{ij}, a_{ij})$. → constraint-gauge pair:

$$\partial_i \mathcal{E}^{ij} = 0, \qquad \mathrm{e}^{\mathrm{i} \int f_j \partial_i \hat{\mathcal{E}}^{ij}} : \; a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i.$$

Gauge invariant phase-space Lagrangian

- Gauge invariant field strength: $R^{ij} = \epsilon^{imk}\epsilon^{jln}\partial_m\partial_l a_{nk}$
- Gauge invariant phase-space Lagrangian:

$$\mathcal{L}(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}(\mathcal{E}^{ij})^2 - \frac{g}{2}R^{ij}R^{ij}$$

which has helicity $0, \pm 2$ modes with $\omega \sim k^2$ dispersion.

- $\to$ $\omega \sim k^2$ pseudo-gravity.

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Remove the remaining helicity 0 mode

- Scaler constraint to remove the remaining helicity 0 mode:
We set a combination of R^{ij} to zero: $\pi^0 = R^{ii} = 0$.
The corresponding canonical conjugate of π^0 is f_0:
$\delta\mathcal{E}_{ij} = (\delta_{ij}\partial^2 - \partial_i\partial_j)f_0$. The Lagrangian $\mathcal{L}(\mathcal{E}^{ij}, a_{ij})$ must not contain f_0: $\mathcal{L}(\mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i\partial_j)f_0, a_{ij}) = \mathcal{L}(\mathcal{E}^{ij}, a_{ij})$.
$\to$ constraint-gauge pair:

$$R^{ii} = 0, \qquad \mathrm{e}^{i\int f_0 \hat{R}^{ii}}:\ \mathcal{E}^{ij} \to \mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i\partial_j)f_0,$$

$\omega \sim k^3$ gravity

- New gauge invariant field strength: $C^i_j = \epsilon^{imn}\partial_m \left(\mathcal{E}^{nj} - \frac{1}{2}\delta_{nj}\mathcal{E}^{ll}\right)$
- Gauge invariant phase-space Lagrangian:

$$\mathcal{L}_L(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}C^i_j C^j_i - \frac{g}{2}R^{ij}R^{ij} = \mathcal{E}^{ij}\partial_0 a_{ij} - \mathcal{H}_L$$

  which has helicity ± 2 modes only, with $\omega \sim k^3$ dispersion.
- Local gauge invariance: the Hamiltonian density $\mathcal{H}_N$ is invariant under the above gauge transformations. (Maxwell type)
- Such a local gauge invariance protects the $\omega \sim k^3$ dispersion. ($\mathcal{L}_L$ is already the lowest order Lagrangian.)

$\omega \sim k$ gravity Gu & Wen, arXiv:gr-qc/0606100

$$\mathcal{L}_N(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}[(\mathcal{E}^{ij})^2 - \frac{1}{2}(\mathcal{E}^{ii})^2] - \frac{g}{2}a_{ij}R^{ij} = \mathcal{E}^{ij}\partial_0 a_{ij} - \mathcal{H}_N$$

with the same gauge transformations and constraints:

$$a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i, \qquad \partial_i \mathcal{E}^{ij} = 0$$
$$\mathcal{E}^{ij} \to \mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i\partial_j) f_0, \qquad R^{ii} = 0,$$

which has helicity ± 2 modes only, with $\omega \sim k$ dispersion.

- Non-local gauge invariance: the Hamiltonian $H = \int d^3x\, \mathcal{H}_N$ is invariant under the above gauge transformations, but the Hamiltonian density is not: $\mathcal{H}_N \to \mathcal{H}_N + \partial F$. (Chern-Simons type)
- $\omega \sim k$ dispersion requires a *non-local* gauge invariance: the Hamiltonian density to transforms as $\mathcal{H}_N \to \mathcal{H}_N + \partial F$.

Relation to Einstein gravity

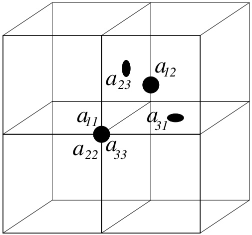
- If we introduce a_{00} and a_{0i} as Lagrangian multipliers to enforce the vector and the scaler constraints, we can rewrite the $\omega \sim k$ model as

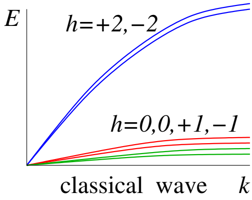
$$\mathcal{L} = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}\left[(\mathcal{E}^{ij})^2 - \frac{1}{2}(\mathcal{E}^{ii})^2\right] - \frac{g}{2}a_{ij}R^{ij} + 2a_{0i}\partial_j\mathcal{E}^{ij} + a_{00}(\partial^2 a_{ii} - \partial_i\partial_j a_{ij})$$

- After integrating out $\mathcal{E}^{ij}$, we find that the above action is exactly the linearized Einstein action around a flat space-time: $\delta g_{\mu\nu} \sim a_{\mu\nu}$.
- The gauge transformations $f_i(x^i), f_0(x^i)$ in space are enlarged to gauge transformations in space-time $f_i(x^i,t), f_0(x^i,t)$ → linearized diffeomorphism of space-time.

Putting $\mathcal{H}_L = \frac{J}{2} C^i_j C^j_i + \frac{g}{2} R^{ij} R^{ij}$ on lattice ($\omega \sim k^3$ model)

- Lattice model: each vertex has three real variables a^{11}, a^{22}, a^{33} with their canonical conjugate $\mathcal{E}^{11}, \mathcal{E}^{22}, \mathcal{E}^{33}$.
  Each face has one real variable $(a_{ij}, \mathcal{E}^{ij})$, $ij = 12, 23, 31$.
- L-type qbit model ($U_{1,2}$ terms to enforce the constraints):
  $$L_L = \sum \mathcal{E}^{ij} \dot{a}_{ij} - \sum [\mathcal{H}_L + U_1(\partial_i \mathcal{E}^{ij})^2 + U_2(R^{ii})^2]$$
- Total six modes with helicity $0, 0, \pm 1, \pm 2$

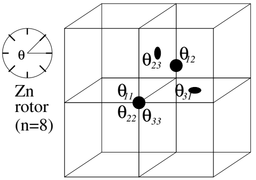


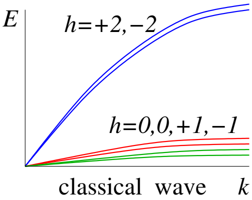


Even in large U_1, U_2 limit, the helicity $0, 0, \pm 1$ modes remain gapless! (Both as classical model and quantum model)

Gap the helicity $0, 0, \pm 1$ modes Gu & Wen, arXiv:0907.1203

- Compactify $a_{ij} = \alpha\theta_{ij}$: $\theta_{ij} \sim \theta_{ij} + 2\pi$
- Discretize a_{ij} and θ_{ij}: $\theta_{ij} = 2\pi/n_G \times$ int.
- The canonical conjugate of θ^{ij}, $L^{ij} \sim \mathcal{E}^{ij}$ is also compactified and discretized: $L^{ij} \sim L^{ij} + n_G$, $L^{ij} =$ int.
- We know that if we treat θ_{ij}, L^{ij} in our lattice model L_L as continuous classical fields, there will be total six gapless modes with helicity $0, 0, \pm 1, \pm 2$





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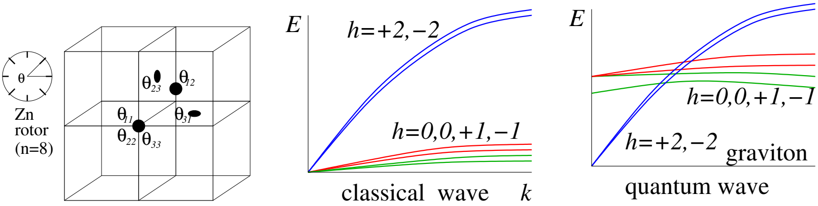

- After include quantum effects, the helicity $0, 0, \pm 1$ modes are gapped!

Why the helicity $0, 0, \pm 1$ modes are gapped?

Consider the helicity $0, \pm 1$ modes described by the canonical conjugate pair (π^i, f_i): $\pi^i = \partial_j L^{ij}$, $\delta\theta_{ij} = \partial_i f_j + \partial_j f_i$

- Still treat (π^i, f_i) as continuous classical fields but consider the quantum fluctuations of (π^i, f_i) with wave vector $k \sim 1$:

$$\mathcal{H} = U(\pi^i)^2 + g'(f_i)^2, \quad U = \text{large}, \quad g' = \text{small} = 0$$

The above oscillator-like Hamiltonian gives us $\delta\pi^i \sim (g'/U)^{1/4}$ and $\delta f_i \sim (U/g')^{1/4}$.

- The fluctuations of f_i is much larger than the compactification radius of f_i: $\delta f_i \gg 2\pi$

  The fluctuations of π^i is much less than the discreteness of π^i: $\delta\pi^i \ll 1$

  Classical results cannot be trusted.

  The quantum freeze of π^i → modes (π^i, f_i) are gapped.

Why the helicity ± 2 modes are not gapped?

Consider the helicity ± 2 modes described by the canonical conjugate pair $(L^{\pm}, \theta_{\pm})$.

- Still treat $(L^{\pm}, \theta_{\pm})$ as continuous classical fields but consider the quantum fluctuations of $(L^{\pm}, \theta_{\pm})$: $\mathcal{H} = J(L^{\pm})^2 + g(\theta_{\pm})^2,$.

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  $\delta L^\pm \sim (g/J)^{1/4}$ and $\delta\theta_\pm \sim (J/g)^{1/4}$.
- Choose the coupling constants in the qbit model to satisfy
  $g/J \sim n_G^2$
  The fluctuations of $\theta_\pm \sim \sqrt{1/n_G}$ is much less than the compactification radius of $\theta_\pm$: $\delta\theta_\pm \ll 2\pi$, but much bigger than the discreteness of $\theta_\pm$: $\delta\theta_\pm \gg 2\pi/n_G$.
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The helicity ± 2 modes $(L^{\pm}, \theta_{\pm})$ are semiclassical, and the classical result of gaplessness can be trusted.

The emergence of $\omega \sim k^3$ gravity from qbit model

- The L-type qbit model on lattice produces the $\omega \sim k^3$ gravitons described by the following low energy effective Lagrangian

$$\mathcal{L}_L(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2} C^i_j C^i_j - \frac{g}{2} R^{ij} R^{ij}$$

$$C^i_j = \epsilon^{imn}\partial_m \left(\mathcal{E}^{nj} - \frac{1}{2}\delta_{nj}\mathcal{E}^{ll}\right), \quad R^{ij} = \epsilon^{imk}\epsilon^{jln}\partial_m\partial_l a_{nk}$$

with the following emergent gauge transformation and constraints

$$a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i, \qquad \partial_i \mathcal{E}^{ij} = 0$$

$$\mathcal{E}^{ij} \to \mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i\partial_j) f_0, \qquad R^{ii} = 0$$

- The only gapless excitations are helicity ± 2 modes with $\omega \sim k^3$ dispersion.
- The result is obtained by a controlled semiclassical approximation and is reliable.

The gaplessness of emergent gauge bosons and gravitons

- Gaplessness of Goldstone bosons in symmetry-breaking states is protected by the symmetry of the underlying Hamiltonian.

  What protect the gaplessness of emergent gauge bosons and gravitons in qbit models?

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  Any local perturbations to the underlying Hamiltonian cannot break the emergent local gauge symmetry and cannot gap the emergent gauge bosons and gravitons. Hastings & Wen 05

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- **The $\omega \sim k^3$ dispersion is also protected** by the local gauge invariance of the Hamiltonian density $\mathcal{H}_L$. Local perturbations cannot gap the $\omega \sim k^3$ gravitons and cannot turn the $\omega \sim k^3$ gravity to the $\omega \sim k$ gravity.

The emergence of $\omega \sim k$ (Einstein) gravity

- N-type qbit model: Put $\mathcal{H}_N + U_1(\partial_i \mathcal{E}^{ij})^2 + U_2(R^{ii})^2$ on lattice.
- Under the similar semiclassical approach at quadratic order we can obtain $\omega \propto k$ gravitons, described by low energy effective Lagrangian (in phase space):

$$\mathcal{L}_N(\mathcal{E}^{ij}, a_{ij}) = \mathcal{E}^{ij}\partial_0 a_{ij} - \frac{J}{2}[(\mathcal{E}^{ij})^2 - \frac{1}{2}(\mathcal{E}^{ii})^2] - \frac{g}{2} a_{ij} R^{ij}$$

with the same gauge transformations and constraints:

$$a_{ij} \to a_{ij} + \partial_i f_j + \partial_j f_i, \qquad \partial_i \mathcal{E}^{ij} = 0$$
$$\mathcal{E}^{ij} \to \mathcal{E}^{ij} - (\delta_{ij}\partial^2 - \partial_i \partial_j) f_0, \qquad R^{ii} = 0$$

- But for the N-type model, the strong fluctuating helicity $0, 0, \pm 1$ modes couple to weak fluctuating ± 2 modes at quartic order and beyond. The semi-classical result is unreliable.
- **The Einstein equation may emerge at low energies from the N-type model at linear level (need to be confirmed).**

Summary

- Gravitons = helicity ± 2 modes **as the only gapless excitations**
- Compactifying and discretizing the metric tensor $h_{ij} = g_{ij} - \delta_{ij}$ are very important in obtaining a quantum theory of gravity.
- Two kinds of gravity: the $\omega \sim k^3$ gravity and the $\omega \sim k$ gravity, both emerge as stable phases in some qbit models.
- In those gravity phases, the gaplessness of helicity ± 2 gravitons is topologically protected: stable against *any* perturbations that do not break translation symmetry.
- The $\omega \sim k^3$ gravity emerges from the L-type model (reliable). The $\omega \sim k$ gravity emerges from the N-type model (not reliable). Need numerical calculations on the N-type qbit model, to confirm the emergence of the $\omega \sim k$ gravity.
- The N-type qbit model may be a quantum theory of gravity (at least at linear level).

A new paradigm of many-body quantum physics

which connects cond. matter physics, particle physics, superstring theory, quantum gravity, quantum information, and mathematics.