10th Feb. 2010
@ IPMU focus week
"Condensed matter physics meets high energy physics"

Holography and Nuclear Physics

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arXiv/1002.???? N.Iizuka, P.Yi, KH
arXiv/1002.???? N.Iizuka, KH

(arXiv/0809.3141,0910.2303 KH
arXiv/0901.4449 T.Sakai, S.Sugimoto, KH)

Nuclear physics

How can we describe multi-nucleon (=nucleus) system?

Serious problem

Elementary particles : quarks and gluons in QCD

Nucleon : bound state of 3 quarks via strong coupling

Nuclear properties from strongly coupled QCD?

Our solution

Strongly coupled QCD (large N_c) → AdS/CFT

→ Matrix quantum mechanics of multi-nucleon system

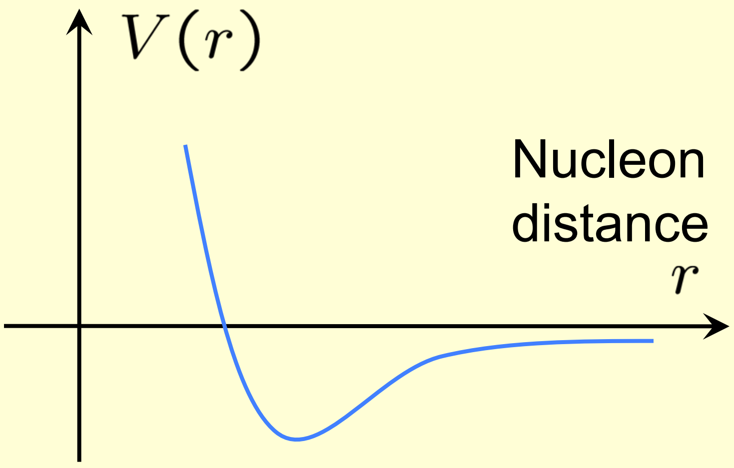
→ Derivation of nuclear force at short distance

Formulating a concrete question

Nuclear force

Short range :
Strong repulsion

Long range :
Pion exchange



Importance of repulsive core

- Nucleon scattering
- Nuclear density saturation

$$R \sim 1.18 A^{1/3} [\mathrm{fm}]$$

- Supernova explosion

EoS at high density?

- Neutron star

Core structure?

- QCD phase diagram

Phases at high density?

Deriving nuclear force at short distance from QCD

AdS/CFT derives "Matrix model for multi-nucleons"

$$S = \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt\, {\rm tr}_k \Bigg[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot\alpha} D_0 w_{\dot\alpha i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot\alpha} w_{\dot\alpha i} - \frac{3^6\pi^2}{4\lambda^2 M_{\rm KK}^4} \left(\vec{D}\right)^2 + i\vec{D}\cdot\vec{\tau}^{\dot\alpha}{}_{\dot\beta} \bar{X}^{\dot\beta\alpha} X_{\alpha\dot\alpha} + i\vec{D}\cdot\vec{\tau}^{\dot\alpha}{}_{\dot\beta} \bar{w}_i^{\dot\beta} w_{\dot\alpha i} \Bigg] + 4N_c \int dt\, {\rm tr}_k A_0$$

Quantum mechanics with bosonic matrix variables.

- X^M $(M = 1, 2, 3, 4)$: k x k Hermitian matrix
  - Eigenvalues of $X^{M=1,2,3}$ → location of k nucleons
- $A_0,\ \vec{D}$: k x k auxiliary Hermitian matrix
- w_i $(i = 1, \cdots, N_f)$: k x N_f complex matrix

Size, spin/isospin of nucleon (baryon)

According to AdS/CFT, this describes <u>k nucleon system</u> in large N_c QCD with N_f kinds of quarks at strong coupling

$(\lambda = g_{\rm QCD}^2 N_c \sim 14,\ N_c = 3,\ M_{\rm KK} = 949$ [MeV]$)$

Exercise : Single nucleon

After integrating out auxiliary fields $A_0, \vec{D},$ hamiltonian is

$$
\begin{aligned}
H &\equiv P_i^{\dot\alpha}\dot{w}_i^{\dot\alpha} + \bar{P}_{\dot\alpha}^i \dot{\bar{w}}_{\dot\alpha}^i - L \\
&= \frac{\lambda N_c M_{\rm KK}}{54\pi}\Bigg[\partial_0 \bar{w}_i^{\dot\alpha}\partial_0 w_{\dot\alpha}^i + \frac{1}{6}M_{\rm KK}^2 \bar{w}_i^{\dot\alpha} w_{\dot\alpha}^i \\
&\qquad + \frac{\lambda^2 M_{\rm KK}^4}{3^6\pi^2}\left[4w_1^i(w_2^i)^* w_2^j (w_1^j)^* \right. \\
&\qquad\qquad \left. +(w_1^i(w_1^i)^*)^2 + (w_2^i(w_2^i)^*)^2 - 2w_1^i(w_1^i)^* w_2^j(w_2^j)^*\right] \\
&\qquad + \frac{1}{4\bar{w}_i^{\dot\alpha} w_{\dot\alpha}^i}\left(\left(\frac{216\pi}{\lambda M_{\rm KK}}\right)^2 + \left(\bar{w}_i^{\dot\alpha}\partial_0 w_{\dot\alpha}^i - \partial_0 \bar{w}_i^{\dot\alpha} w_{\dot\alpha}^i\right)^2\right)\Bigg].
\end{aligned}
$$

Minimizing large λ term gives $w_{\dot\alpha}^{i=1} = \begin{pmatrix}\rho \\ 0\end{pmatrix},\ w_{\dot\alpha}^{i=2} = \begin{pmatrix}0 \\ \rho\end{pmatrix}$

$$
H = \frac{\lambda N_c M_{\rm KK}}{54\pi}\left[\left(\frac{54\pi}{\lambda M_{\rm KK}}\right)^2 \frac{2}{\rho^2} + \frac{1}{3}M_{\rm KK}^2\rho^2\right] \Rightarrow \rho = \frac{2^{3/4}3^{7/4}\sqrt{\pi}}{\sqrt{\lambda}M_{\rm KK}}
$$

Quantization → Baryon mass spectrum

The quantum mechanics is almost equal to what was obtained in instanton (soliton) approach in holographic QCD

[Hata, Sakai, Sugimoto, Yamato hep-th/0701280]

X^4 : Harmonic oscillator

$$\left(w^{i=1}, w^{i=2}\right) = \rho\, U^{i}_{\ \dot{\alpha}} \ (U \in \mathrm{SU}(2))$$

: Harmonic oscillator + SU(2) rotation (spin / isospin)

➔ Energy spectrum : $l = I/2 = J/2$

$$M = M_0 + \sqrt{\frac{(l+1)^2}{6} + 2N_c^2} + \frac{2(n_\rho + n_Z) + 2}{\sqrt{6}}$$

⇨ $M_\Delta - M_{\mathrm{P/N}} = 0.20 M_{\mathrm{KK}} \sim 2 \times 10^2$ [MeV]

(Experiments : $292\ [\mathrm{MeV}]$)

Nuclear force, repulsive core

Let us work for the *k*=2 case (2 nucleons).

Minimizing large λ term gives

$$X^M = \tau^3 \frac{r^M}{2} + \tau^1 Y^M \qquad Y^M \equiv \frac{\rho_1\rho_2}{4(r_P)^2}\mathrm{tr}\left[\bar{\sigma}_M r_N \sigma_N \left((U^{(1)})^\dagger U^{(2)} - (U^{(2)})^\dagger U^{(1)}\right)\right]$$

$$w_{\dot{\alpha}i}^{A=1} = U_{\dot{\alpha}i}^{(A=1)}\rho_1, \quad w_{\dot{\alpha}i}^{A=2} = U_{\dot{\alpha}i}^{(A=2)}\rho_2$$

After integrating out auxiliary fields $A_0, \vec{D},$ the potential is

$$V = \frac{27\pi N_c}{4\lambda M_{\mathrm{KK}}}\frac{1}{(r_M)^2}\left(8 + 6\left(\mathrm{tr}\left[U^{(1)\dagger}U^{(2)}\right]\right)^2 + \left(-4 + 5\left(\mathrm{tr}\left[U^{(1)\dagger}U^{(2)}\right]\right)^2\right)\left(\frac{\rho_2^2}{\rho_1^2} + \frac{\rho_1^2}{\rho_2^2}\right)\right)$$
$$+\frac{\lambda N_c}{162\pi}M_{\mathrm{KK}}^3\frac{\rho^4}{(r_i^2)^2}\left(r_j\ \mathrm{tr}\left[i\tau_j\left(U^{(1)}\right)^\dagger U^{(2)}\right]\right)^2$$

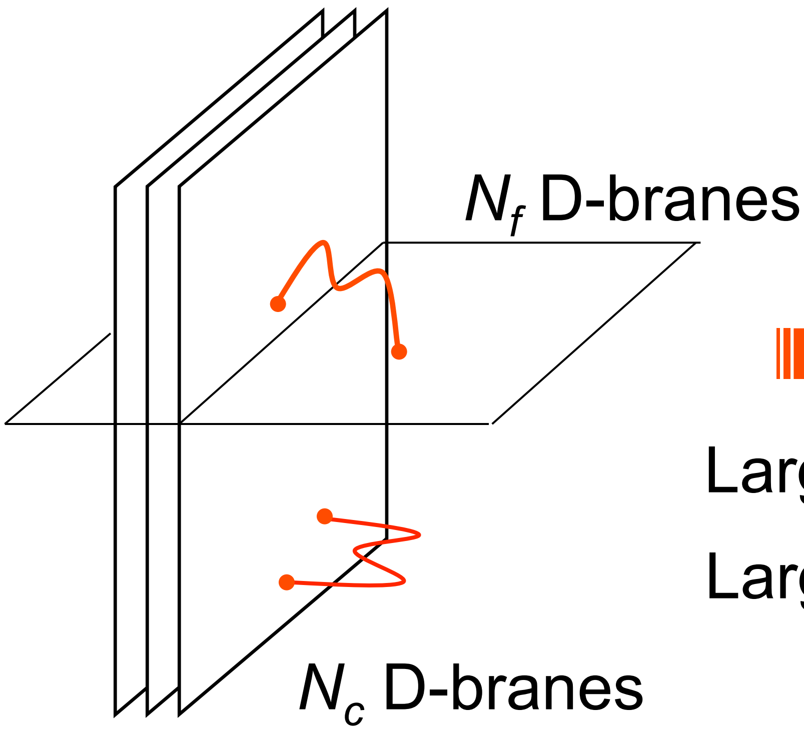
This is positive, and scales as $1/r^2$ ➔ Repulsive core

The final result is similar to that in soliton approach

[Sakai, Sugimoto, KH, 0901.4449]

Derivation of our matrix model

QCD



Sakai-Sugimoto model

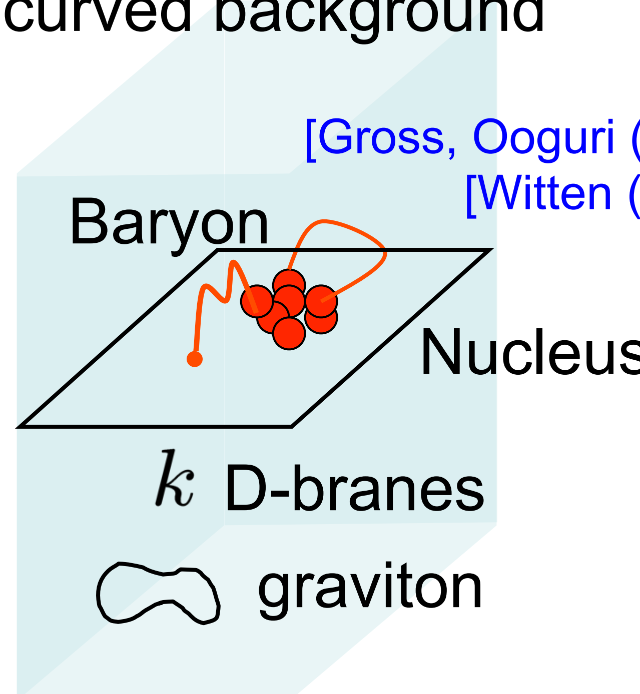
[Sakai, Sugimoto (04)]

Large N_c

Large λ

String theory
in curved background

[Gross, Ooguri (98)]
[Witten (98)]



Theory on *k* D4-branes
on the flavor D8-branes
in Witten's curved geometry
||
Our matrix model

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Future directions

Long range nuclear force (pion exchanges)

Large *k* limit and heavy nuclei

arXiv/0809.3141,0910.2303 KH

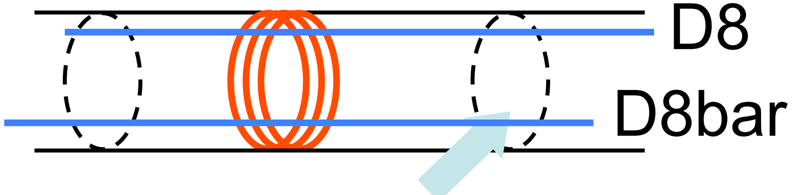
Infinite *k* and finite density system?

……

Derivation of the matrix model

5d YM on $S^1 \times R^4$

N_C D4

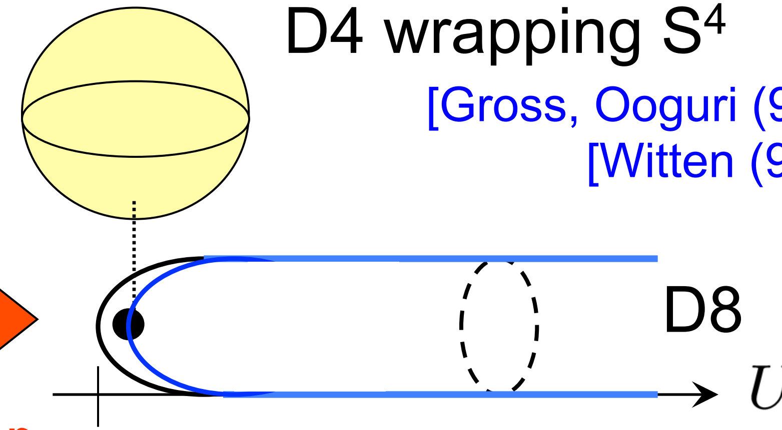


radius : $1/M_{\mathrm{KK}}$

Near horizon

Baryon vertex :
D4 wrapping S^4

[Gross, Ooguri (98)]
[Witten (98)]



Witten's geometry

At intersections of D8/D4,
chiral quarks appear

4d QCD at low energy

[Sakai, Sugimoto (04)]

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Our matrix model