Quantum anomalies in hydrodynamics

Dam T. Son (INT, University of Washington)

Ref.: DTS, Piotr Surówka, arXiv:0906.5044

Plan of the talk

- Hydrodynamics as a low-energy effective theory
- Relativistic hydrodynamics
- Triangle anomaly: a new hydrodynamic effect

Place of hydrodynamics in theoretical physics

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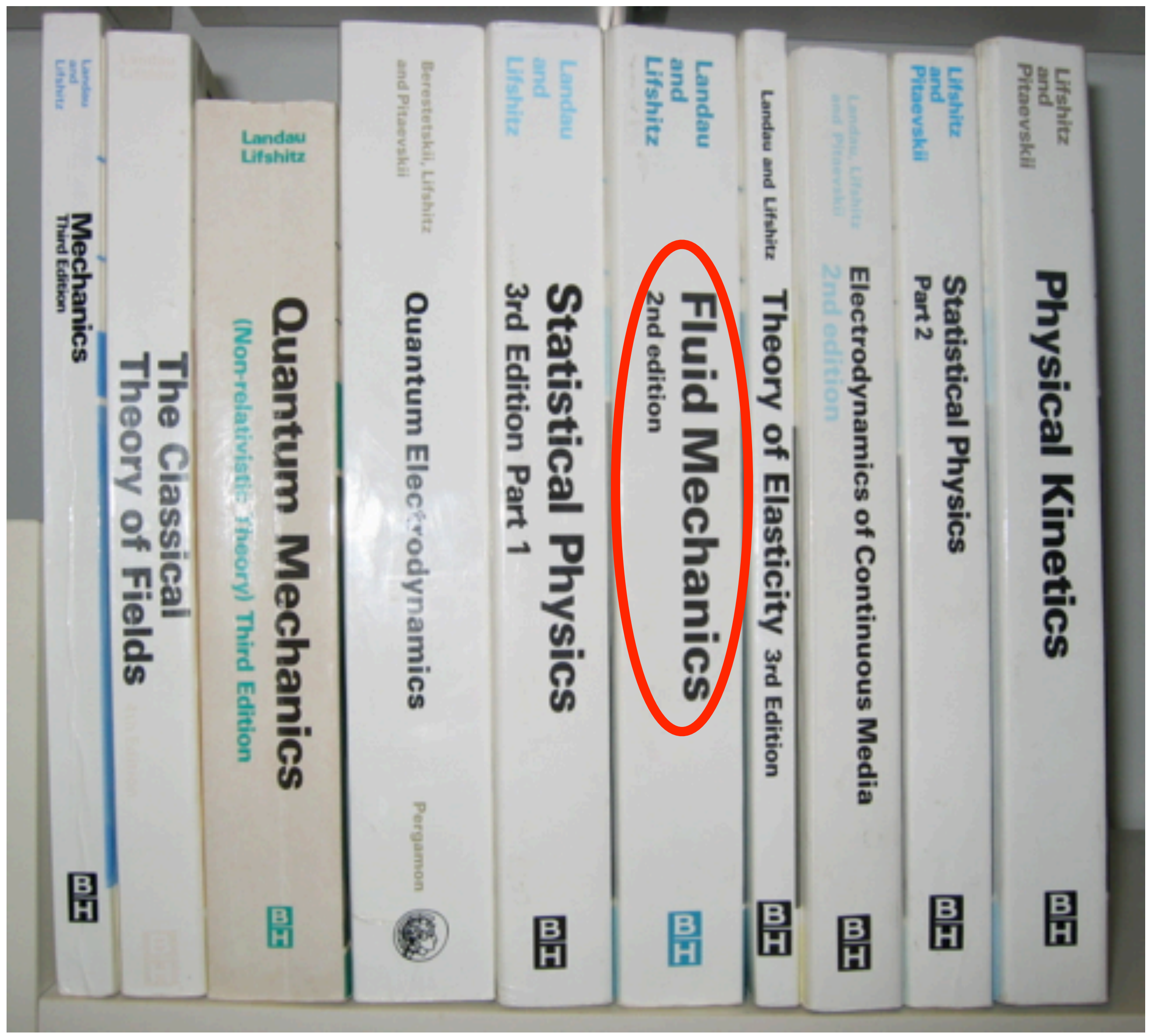

Relativistic hydro: an old subject

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The Thermodynamics of Irreversible Processes

III. Relativistic Theory of the Simple Fluid

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(Received September 26, 1940)

The considerations of the first paper of this series are modified so as to be consistent with the special theory of relativity. It is shown that the inertia of energy does not obviate the necessity for assuming the conservation of matter. *Matter* is to be interpreted as number of molecules, therefore, and not as inertia. Its velocity vector serves to define local proper-time axes, and the energy momentum tensor is resolved into proper-time and -space components. It is shown that the first law of thermodynamics is a scalar equation, and not the fourth component of the energy-momentum principle. Temperature and entropy also prove to be scalars. Simple relativistic generalizations of Fourier's law of heat conduction, and of the laws of viscosity are obtained from the requirements of the second law. The same considerations lead directly to the accepted relativistic form of Ohm's law.

INTRODUCTION

IN the second paper of this series,[1] the theory of ϵ-substitutions was outlined, and it was shown that this device can be used to simplify the derivation of some thermodynamic formulae. However, the author was reluctant to use it in the derivation of any fundamental formulae because the ϵ-substitution depends on the fact rather than inertia. The principles (b) and (c) combine into a single tensor equation, as is well known. This is somewhat disconcerting, for the first law of thermodynamics is a scalar equation; its relation to the energy-momentum principle must be discovered. Moreover, the correct form of the energy-momentum tensor is still a matter of discussion, and some assumption

A low-energy effective theory

Consider a thermal system: $T \neq 0$

Dynamics at large distances $\ell \gg \lambda_{\mathrm{mfp}}$

governed by a simple effective theory:

Hydrodynamics

Degrees of freedom in hydrodynamics

D.o.f. that relax arbitrarily slowly in the long-wavelength limit:

- Conserved densities
- Goldstone modes (superfluids)
- Massless U(1) gauge field (magnetohydrodynamics)

Relativistic hydrodynamics

Conservation laws: $\partial_\mu T^{\mu\nu} = 0$

$\partial_\mu j^\mu = 0$ (if ∃ conserved charge)

Constitutive equations: local thermal equilibrium

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu}$$

$$j^\mu = n u^\mu$$

Total: 5 equations, 5 unknowns

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$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

$$j^\mu = n u^\mu + \nu^\mu$$

Total: 5 equations, 5 unknowns

Dissipative terms

$$\tau^{ij} = -\eta(\partial^i u^j + \partial^j u^i - \frac{2}{3}\delta^{ij}\vec{\nabla}\cdot\vec{u}) - \zeta\delta^{ij}\vec{\nabla}\cdot\vec{u} \qquad \nu^i = -\sigma T \partial^i \left(\frac{\mu}{T}\right)$$

Relativistic hydrodynamics

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$$j^\mu = n u^\mu + \nu^\mu$$

Total: 5 equations, 5 unknowns

Dissipative terms

$$\tau^{ij} = -\underbrace{\eta}_{\text{shear viscosity}}(\partial^i u^j + \partial^j u^i - \frac{2}{3}\delta^{ij}\vec{\nabla}\cdot\vec{u}) - \underbrace{\zeta}_{\text{bulk viscosity}}\delta^{ij}\vec{\nabla}\cdot\vec{u} \qquad \nu^i = -\underbrace{\sigma}_{\substack{\text{conductivity} \\ \text{(diffusion)}}} T\partial^i\left(\frac{\mu}{T}\right)$$

Parity-odd effects?

- QFT: may have *chiral fermions*
  - example: QCD with massless quarks
- Parity invariance does not forbid

$$j^{5\mu} = n^5 u^\mu + \xi(T,\mu)\omega^\mu$$

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$$ vorticity

- The same order in derivatives as dissipative terms (viscosity, diffusion)

cf Hall viscosity in 2+1 D: $\tau_{\mu\nu} = \cdots + \epsilon_{\mu\alpha\beta}u_\alpha u_{\nu\beta} + (\mu \leftrightarrow \nu)$

Landau-Lifshitz frame

- We can also have correction to the stress-energy tensor

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \xi'(u^\mu \omega^\nu + \omega^\mu u^\nu)$$

  - Can be eliminated by redefinition of u^μ

$$u^\mu \to u^\mu - \frac{\xi'}{\epsilon + P}\omega^\mu$$

Only a linear combination $\xi - \frac{n}{\epsilon + P}\xi'$ has physical meaning

Let us set $\xi' = 0$

New effect: chiral separation

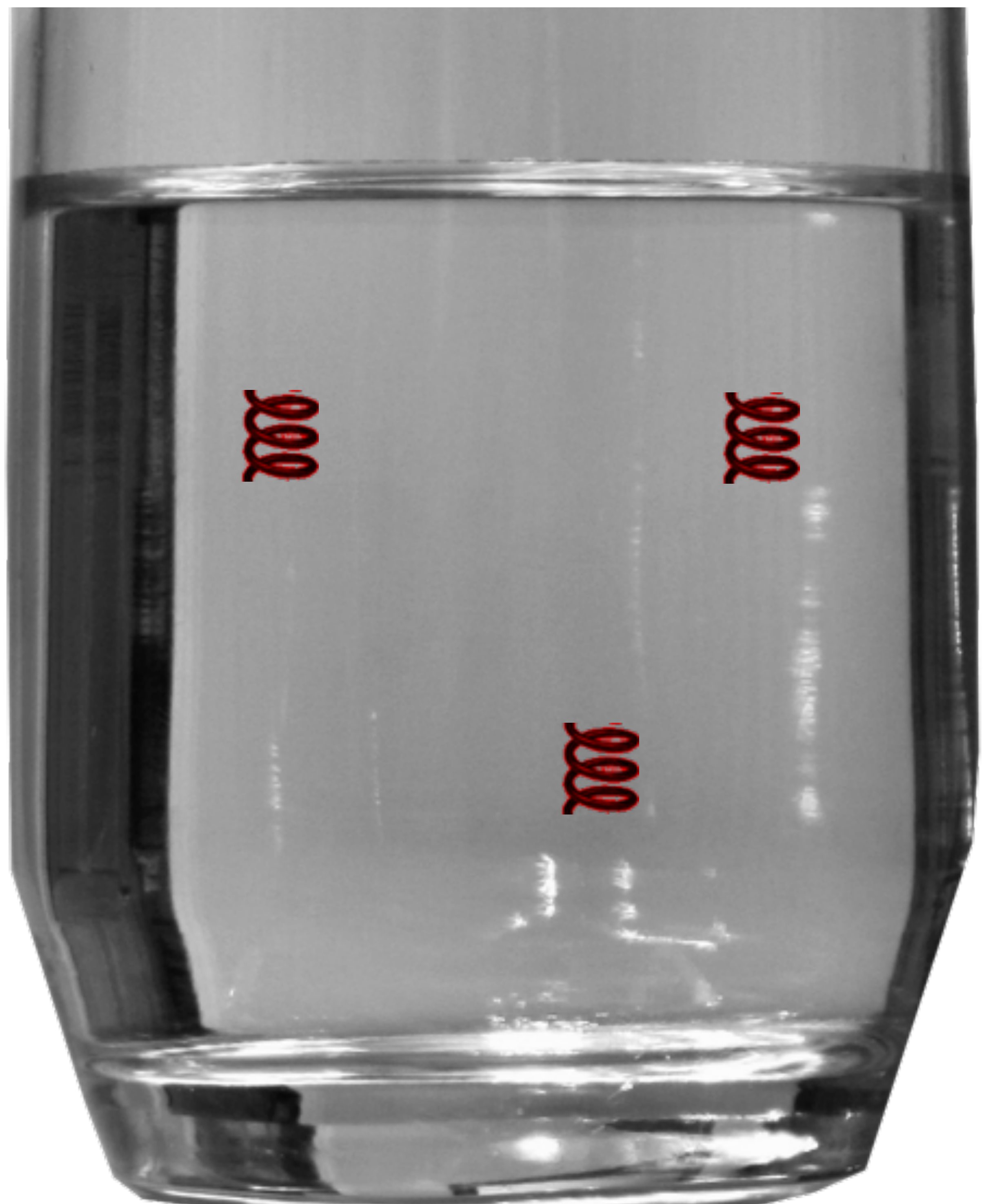
- Rotating piece of quark matter
- Initially only vector charge density $\neq 0$
- Rotation: lead to j^5: chiral charge density develops
- Can be thought of as chiral separation: left- and right-handed quarks move differently in rotation fluid
- Similar effect in nonrelativistic fluids?

Chiral separation by rotation

Chiral separation by rotation

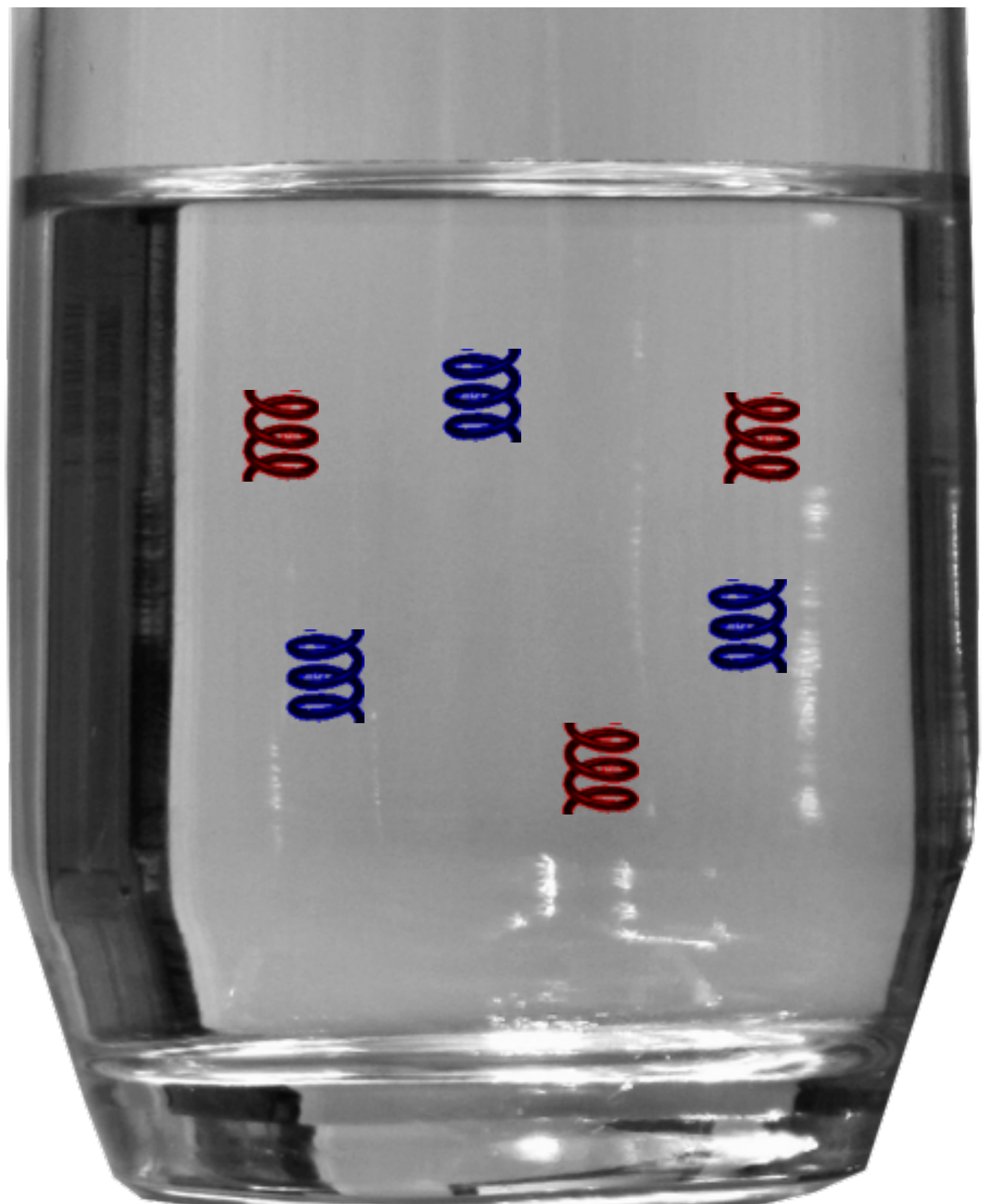
Chiral separation by rotation

Chiral separation by rotation

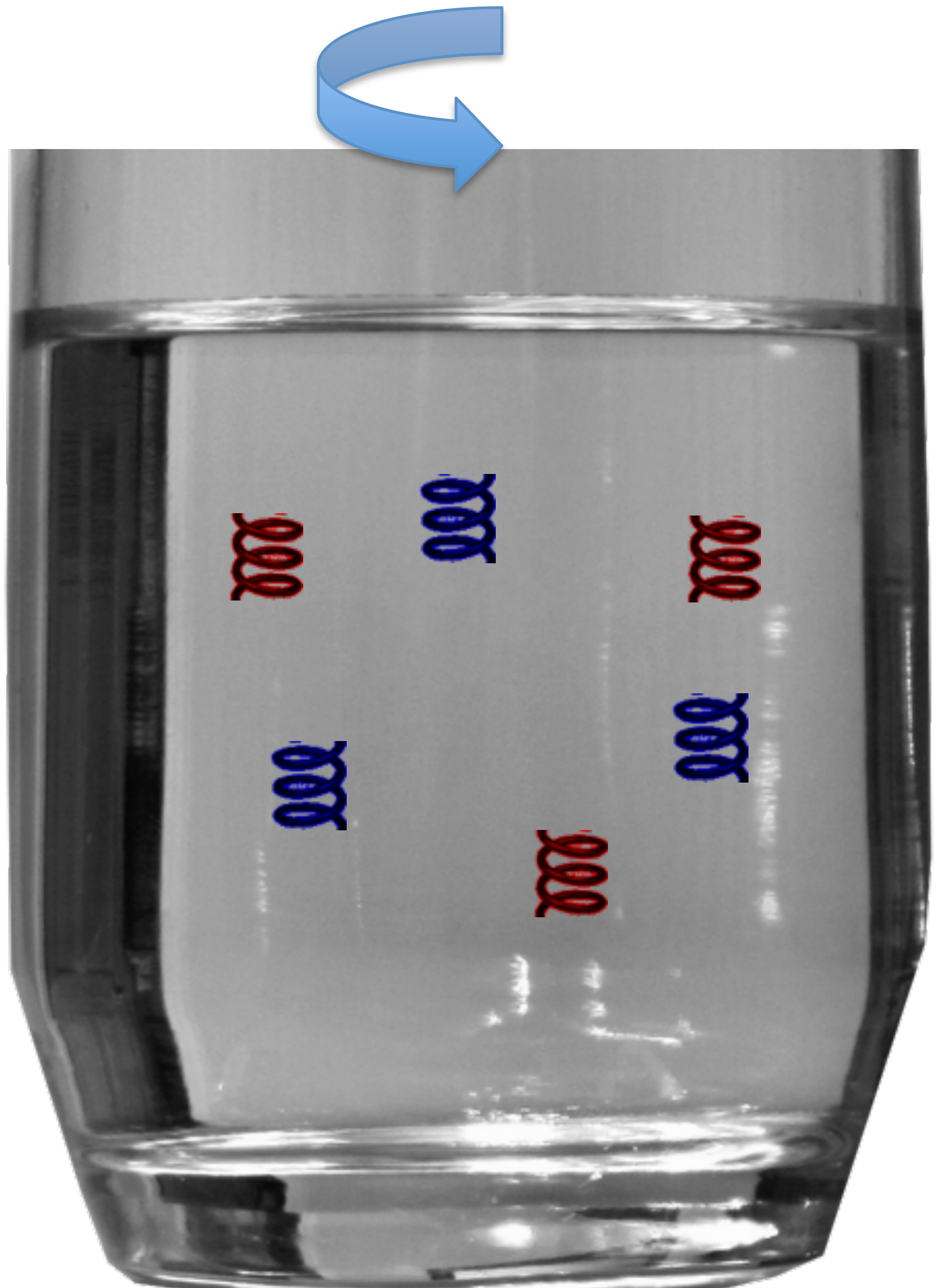


Chiral separation by rotation

Chiral separation by rotation



Chiral separation by rotation



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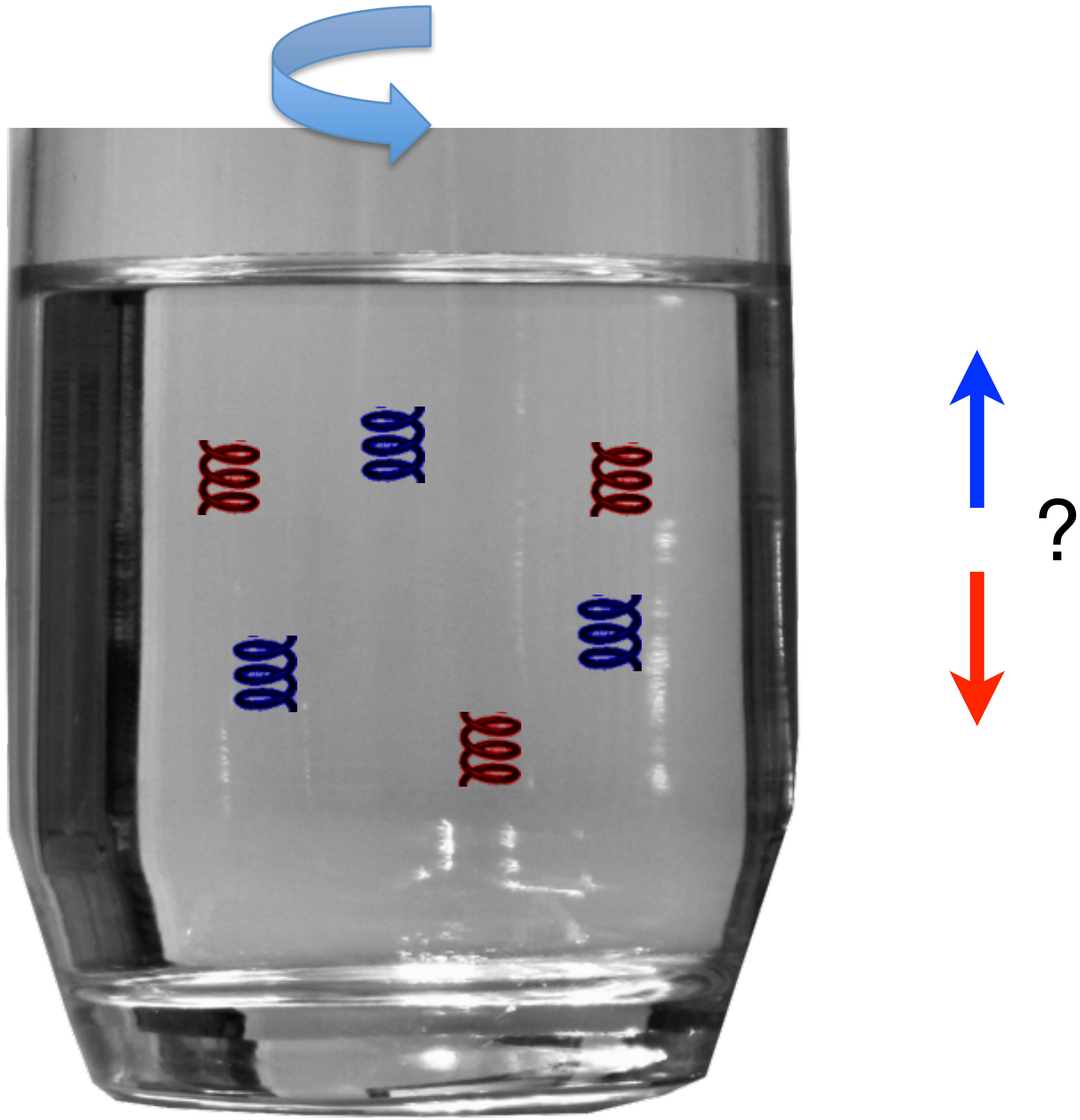


Can chiral separation occur in rigid rotation?

- If a chiral molecule rotates with respect to the liquid, it will moves
- In rigid rotation, molecules rotate with liquid
- $\Rightarrow$ no current in rigid rotation.
- Chiral separation occurs at higher orders in derivative expansion Andreev DTS Spivak

$$j_i^{\rm chiral} \sim (\partial_i v_j + \partial_j v_i)\omega_j + \cdots$$

but NOT $\cancel{\mathbf{j}^{\rm chiral} \sim \omega = \nabla \times \mathbf{v}}$

Relativistic theories are different

- There can be current ~ vorticity
- It is related to triangle anomalies

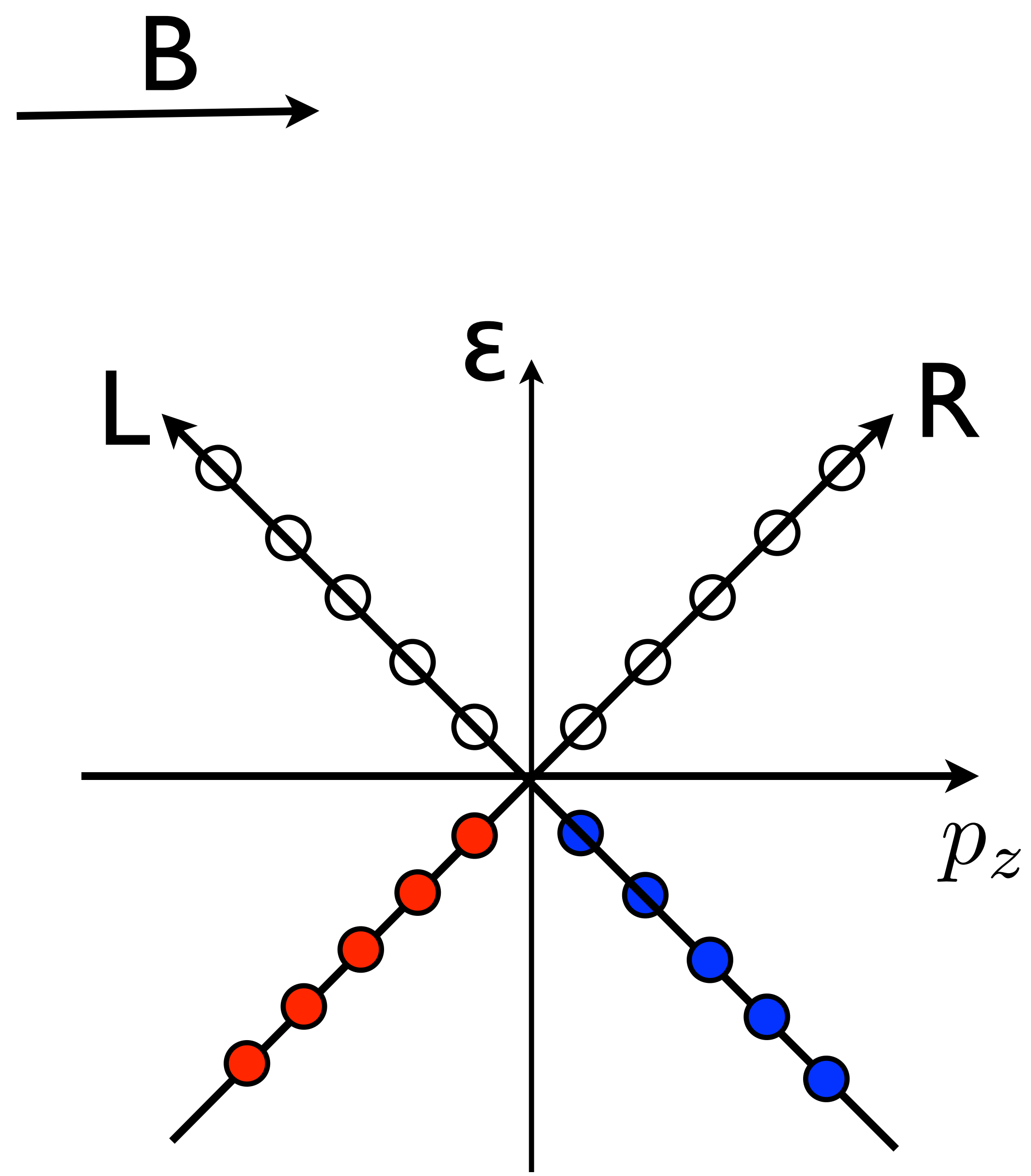
$$\partial_\mu j^{5\mu} = \# E \cdot B$$

  but the effect is there even in the absence of external field

- The kinetic coefficient ξ is determined completely by anomalies and equation of state

Anomalies

Massless fermions: lowest Landau level is chiral



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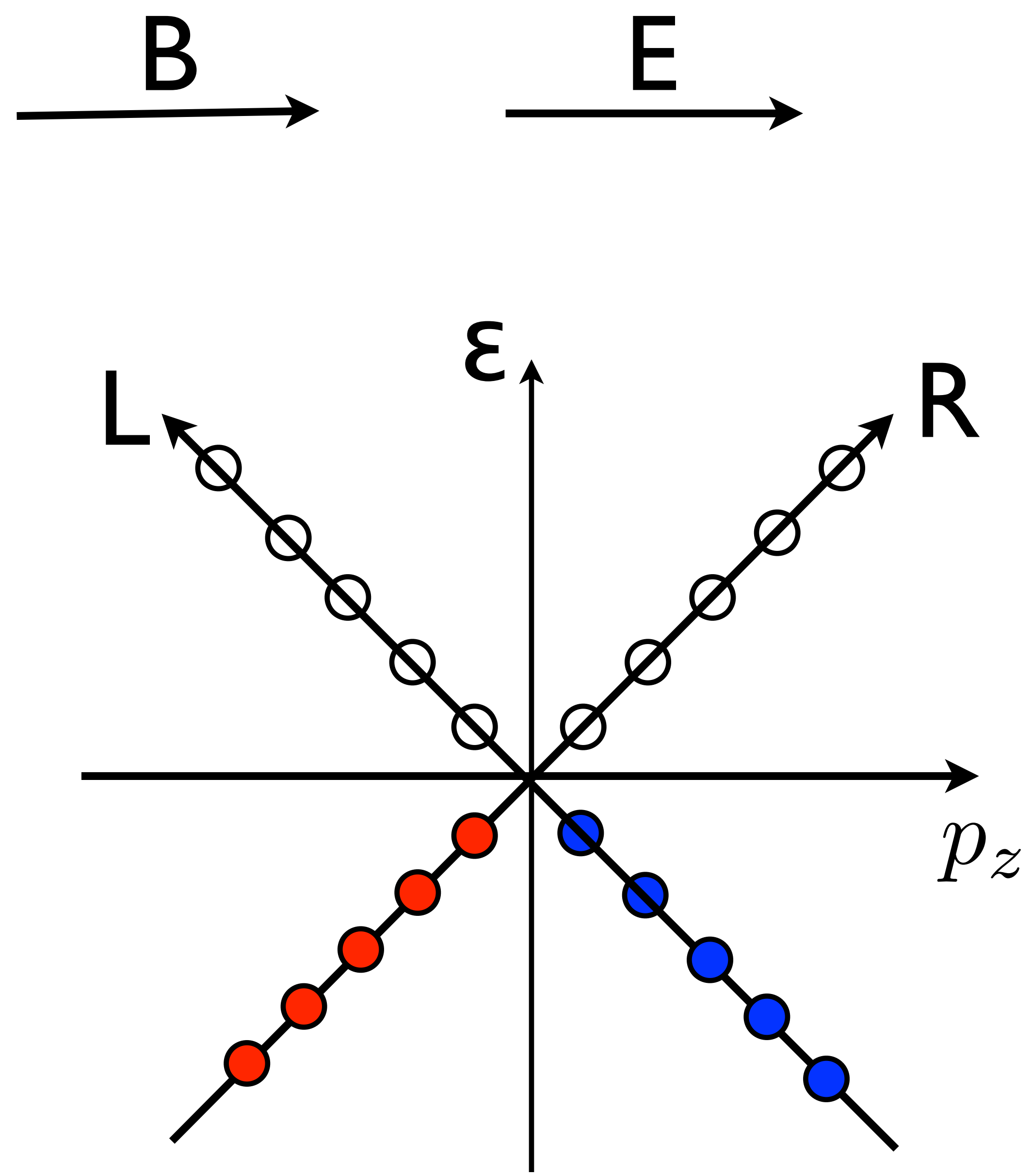

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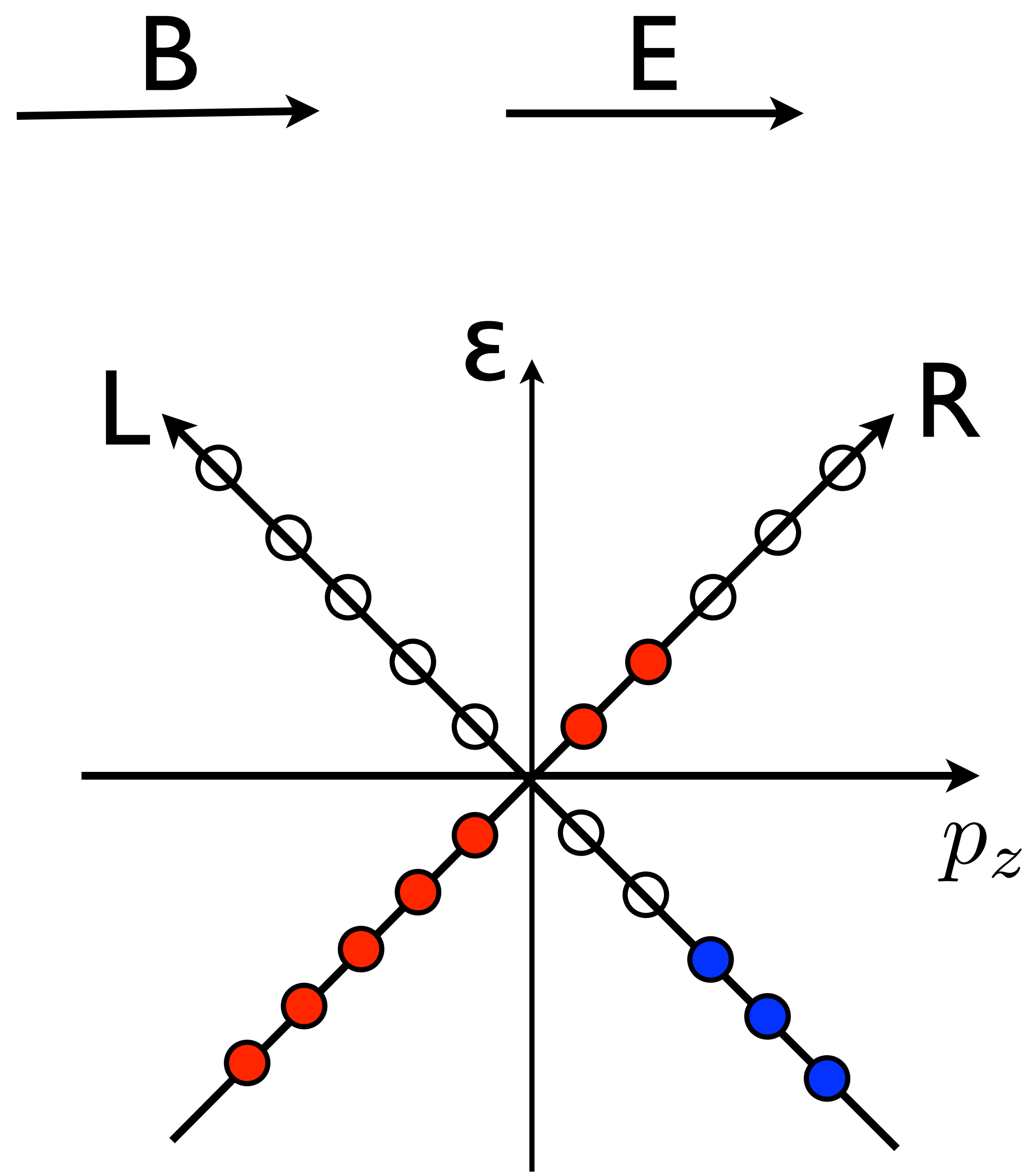

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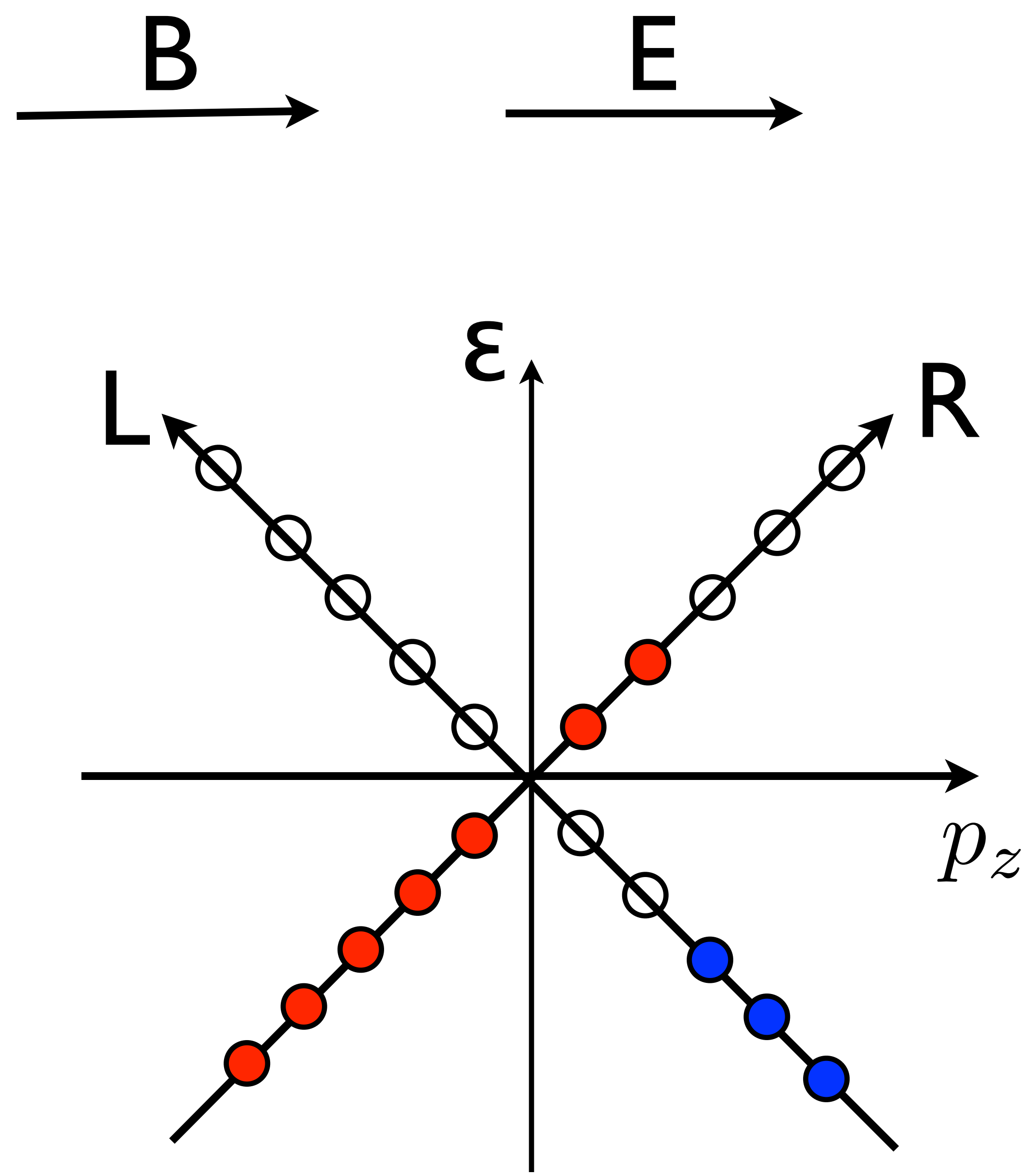

$$\frac{d}{dt}(N_R - N_L) \sim E \cdot B$$

Forbidden by Landau?

- Terms with epsilon tensor do not appear in the standard Landau-Lifshitz treatment of hydrodynamics
- Was it deliberate?

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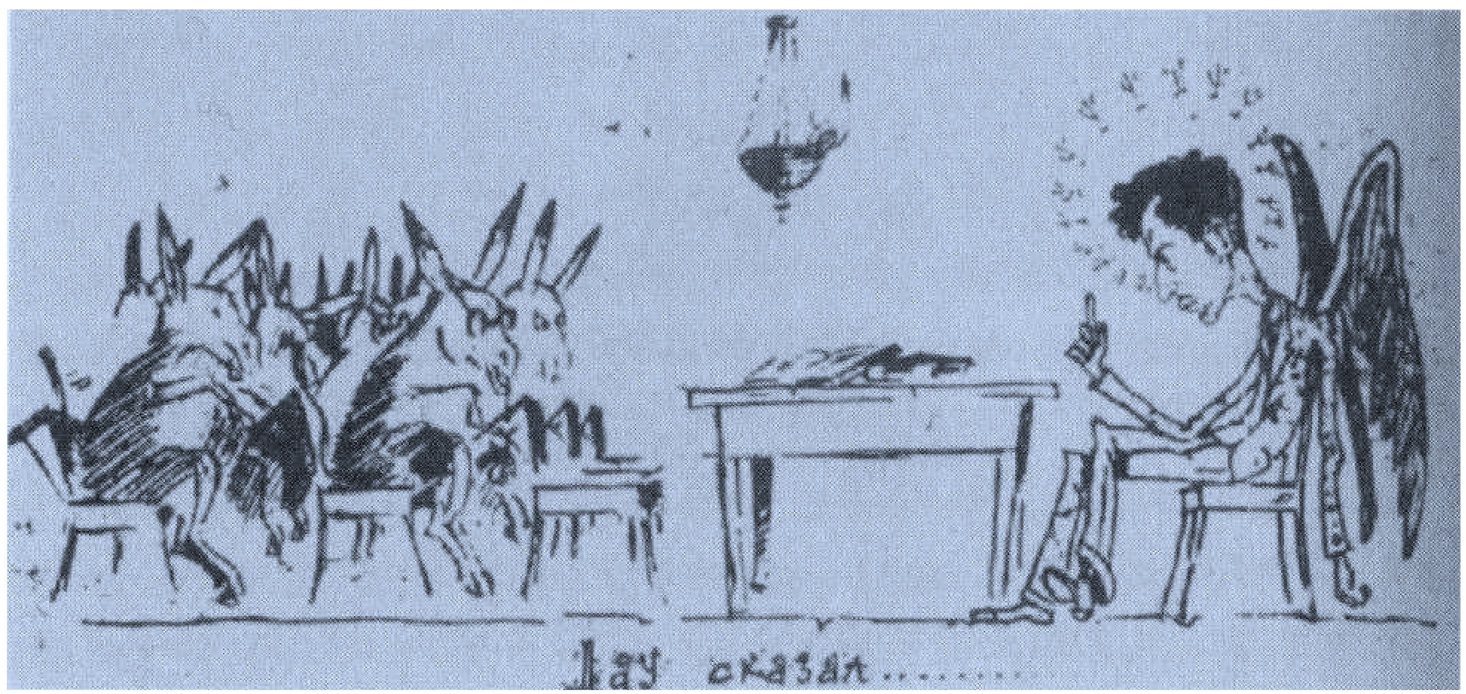

"Landau said..."

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“Landau said...”

Possible reason: 2nd law of thermodynamics

Dissipative terms

Standard textbook manipulations (single U(1) charge)

$$\partial_\mu[(\epsilon + P)u^\mu u^\nu] + \partial^\nu P + \partial_\mu \tau^{\mu\nu} = 0$$

$$\partial_\mu(nu^\mu) + \partial_\mu \nu^\mu = 0$$

Dissipative terms

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$$\partial_\mu[(Ts + \mu n)u^\mu u^\nu] + \partial^\nu P + \partial_\mu \tau^{\mu\nu} = 0$$

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$$-\frac{u_\nu}{T}\times\partial_\mu[(Ts+\mu n)u^\mu u^\nu]+\partial^\nu P+\partial_\mu\tau^{\mu\nu}=0$$

$$-\frac{\mu}{T}\times\partial_\mu(nu^\mu)+\partial_\mu\nu^\mu=0$$

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$$-\frac{\mu}{T}\times\partial_\mu(nu^\mu)+\partial_\mu\nu^\mu=0$$

$$\partial_\mu(su^\mu)\quad=\quad\frac{\mu}{T}\partial_\mu\nu^\mu+\frac{1}{T}\quad u_\nu\,\partial_\mu\tau^{\mu\nu}$$

Dissipative terms

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$$-\frac{u_\nu}{T} \times \partial_\mu[(Ts + \mu n)u^\mu u^\nu] + \partial^\nu P + \partial_\mu \tau^{\mu\nu} = 0$$

$$+$$

$$-\frac{\mu}{T} \times \partial_\mu(nu^\mu) + \partial_\mu \nu^\mu = 0$$

$$\partial_\mu\left(su^\mu - \frac{\mu}{T}\nu^\mu\right) = \quad \frac{\mu}{T}\partial_\mu \nu^\mu + \frac{1}{T} \quad u_\nu \, \partial_\mu \tau^{\mu\nu}$$

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$$\partial_\mu(su^\mu-\frac{\mu}{T}\nu^\mu)=-\partial_\mu\frac{\mu}{T}\quad \nu^\mu-\ \frac{1}{T}\partial_\mu u_\nu\quad \tau^{\mu\nu}$$

Dissipative terms

Standard textbook manipulations (single U(1) charge)

$$-\frac{u_\nu}{T}\times\partial_\mu[(Ts+\mu n)u^\mu u^\nu]+\partial^\nu P+\partial_\mu\tau^{\mu\nu}=0$$

$+$

$$-\frac{\mu}{T}\times\partial_\mu(nu^\mu)+\partial_\mu\nu^\mu=0$$

$$\partial_\mu\Big(\underbrace{su^\mu-\frac{\mu}{T}\nu^\mu}_{\text{entropy current } s^\mu}\Big)=-\partial_\mu\frac{\mu}{T}\quad\nu^\mu-\frac{1}{T}\partial_\mu u_\nu\quad\tau^{\mu\nu}$$

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$$\partial_\mu(su^\mu-\frac{\mu}{T}\nu^\mu)=-\partial_\mu\frac{\mu}{T}\quad\nu^\mu-\frac{1}{T}\partial_\mu u_\nu\quad\tau^{\mu\nu}$$

entropy current s^μ

Positivity of entropy production constrains the dissipation terms: only three kinetic coefficients η, ζ, and σ (right hand side positive-definite)

Is there a place for a new kinetic coefficient?

$$\partial_\mu \left(s u^\mu - \frac{\mu}{T} \nu^\mu \right) = -\frac{1}{T} \tau^{\mu\nu} \partial_\mu u_\nu - \nu^\mu \partial_\mu \left(\frac{\mu}{T} \right)$$

Can we add to the current: $\nu^\mu = \cdots + \xi \omega^\mu$?

Problem: Extra term in current would lead to

$$\partial_\mu s^\mu = \cdots - \xi \omega^\mu \partial_\mu \left(\frac{\mu}{T} \right)$$ not manifestly zero

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This can have either sign, and can overwhelm other terms

cf. Hall viscosity in 2+1 D is manifestly dissipationless

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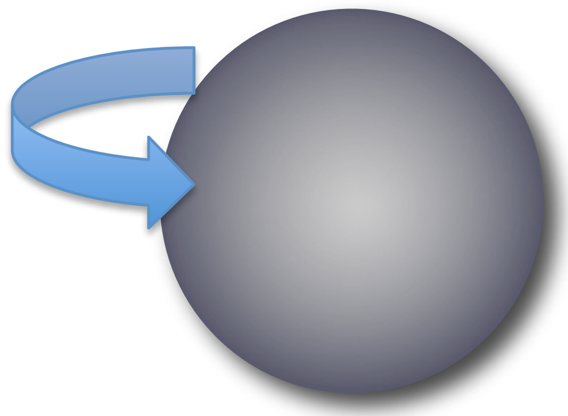
Forbidden by 2nd law of thermodynamics?

Holography

The first indication that standard hydrodynamic equations are not complete comes from considering



rotating 3-sphere of N=4 SYM plasma ↔ rotating BH

If the sphere is large: hydrodynamics should work

no shear flow: corrections ~ 1/R^2

Instead: corrections ~ 1/R   Bhattacharyya, Lahiri, Loganayagam, Minwalla

Holography (II)

Erdmenger et al. arXiv:0809.2488

Banerjee et al. arXiv:0809.2596

considered N=4 super Yang Mills at strong coupling
finite T and μ

should be described by a hydrodynamic theory

discovered that there is a current ~ vorticity

Found the kinetic coefficient $\xi(T,\mu)$

Fluid-gravity correspondence

- Long-distance dynamics of black-brane horizons (in AdS) are described by hydrodynamic equations
  - finite-T field theory ↔ AdS black holes

described by hydrodynamics

- In absence of conserved currents: just Landau-Lifshitz hydrodynamics
- Charged black branes: hydrodynamics with conserved charges
- Anomalies: Chern-Simons term in 5D action of gauge fields

A holographic fluid

$$S = \frac{1}{8\pi G}\int d^5x\,\sqrt{-g}\left(R - 12 - \frac{1}{4}F_{AB}^2 + \frac{4\kappa}{3}\epsilon^{LABCD}A_L F_{AB} F_{CD}\right)$$

encodes anomalies

Black brane solution (Eddington coordinates)

$$ds^2 = 2dvdr - r^2 f(r,m,q)dv^2 + r^2 d\vec{x}^2 \qquad f(m,q,r) = 1 - \frac{m^4}{r^4} + \frac{q^2}{r^6}$$

$$A_0(r) = \#\frac{q}{r^2}$$

Boosted black brane: also a solution

$$ds^2 = -2u_\mu dx^\mu dr + r^2(P_{\mu\nu} - f u_\mu u_\nu)dx^\mu dx^\nu$$

$$A_\mu(r) = -u_\mu \# \frac{q}{r^2}$$

Promoting parameters into variables

$$u_\mu \to u_\mu(x) \qquad m \to m(x) \qquad q \to q(x)$$

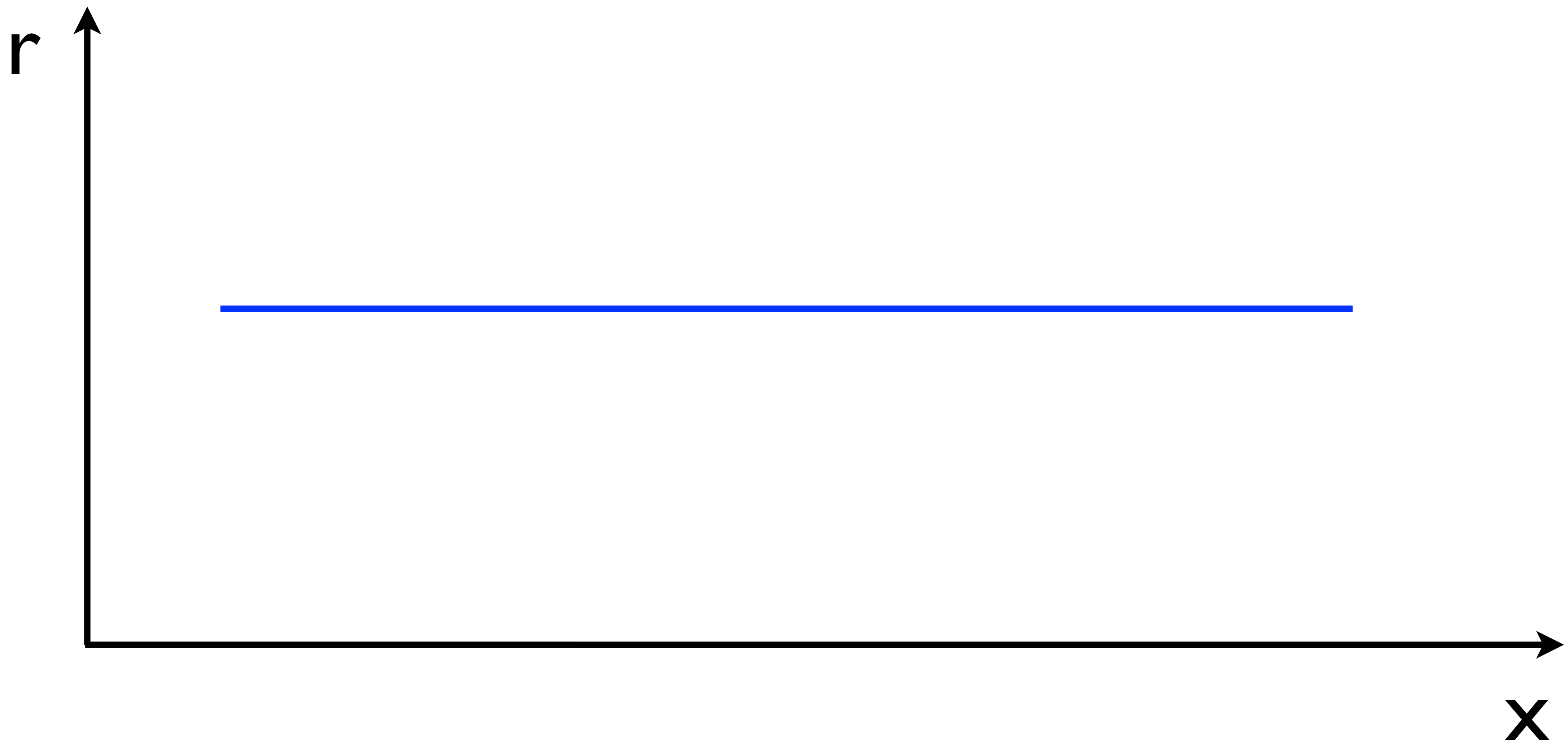
$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(m, q, u) + g^1_{\mu\nu}$$

proportional to ∇m, ∇q, ∇u

Solve for g^1 perturbatively in derivaties

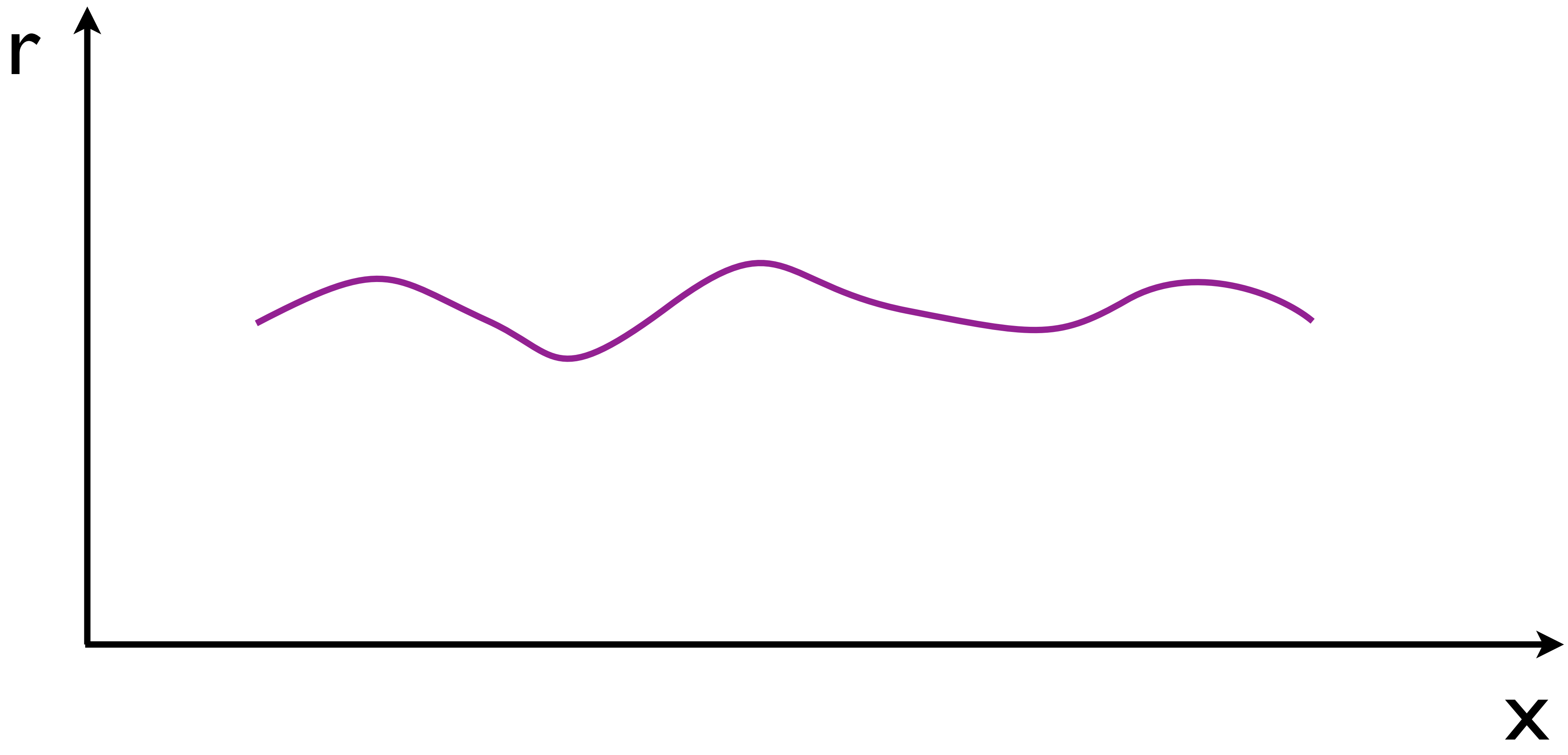
Condition: no singularity outside the horizon

In picture



BH horizon in equilibrium

In picture



BH horizon out of equilibrium

Learning from holography

- Chern-Simons term enters the equation of motion

$$\Box A^\mu \sim \epsilon^{\mu\nu\lambda\alpha\beta} F_{\nu\lambda} F_{\alpha\beta}$$

Learning from holography

- Chern-Simons term enters the equation of motion

$$\Box A^{\underset{\substack{\uparrow\\ i}}{\mu}} \sim \epsilon^{\mu\nu\lambda\alpha\beta} F_{\underset{\substack{\uparrow\\ 0}}{\nu}\underset{\substack{\uparrow\\ r}}{\lambda}} F_{\underset{\substack{\uparrow\\ j}}{\alpha}\underset{\substack{\uparrow\\ k}}{\beta}}$$

Learning from holography

- Chern-Simons term enters the equation of motion

$$\Box A^{\mu} \sim \epsilon^{\mu\nu\lambda\alpha\beta} F_{\nu\lambda} F_{\alpha\beta}$$

i

0 r j k

$A_i \sim u_i$

Learning from holography

- Chern-Simons term enters the equation of motion

$$\Box \underset{\overset{\uparrow}{i}}{A^{\mu}} \sim \epsilon^{\mu\nu\lambda\alpha\beta} \underset{\overset{\uparrow\ \uparrow}{0\ \ r}}{F_{\nu\lambda}} \underset{\overset{\uparrow\ \uparrow}{j\ \ k}}{F_{\alpha\beta}}$$

$$A_i \sim u_i$$

- This lead to correction to the gauge field
    - $\delta A_i \sim \varepsilon_{ijk} \partial_j u_k$
- Current is read out from asymptotics of A near the boundary: $j \sim \omega$

Back to hydrodynamics

- How can the argument based on 2nd law of thermodynamics fail?
  - 2nd law not valid? unlikely...
  - Maybe we were not careful enough?

$$\partial_\mu s^\mu = \cdots - \xi \omega^\mu \partial_\mu \left(\frac{\mu}{T} \right)$$

Can this be a total derivative?

If yes, then all we need to to is to modify s^μ

$$s^\mu \to s^\mu + D(T, \mu) \omega^\mu$$

so our task is to find D so that

$$\partial_\mu [D(T,\mu)\omega^\mu] = \xi(T,\mu)\omega^\mu \partial_\mu \left(\frac{\mu}{T}\right)$$

for all solutions to hydrodynamic equations

Not all ξ(T,μ) are allowed: only a special class

$$\xi = T^2 d' \left(\frac{\mu}{T}\right) - \frac{2nT^3}{\epsilon + P} d\left(\frac{\mu}{T}\right)$$

Instead of a function of 2 variables T, μ: ξ depends on a function of 1 variable: μ/T

Questions

But this raises a number of questions:

- What determines the function $d(\mu/T)$?
- Does anomaly play a role?
  - The holographic example suggests that it does
  - If $d(\mu/T)$ determined by anomalies, then ξ should be in some sense "quantized"
- But how to see anomalies in hydrodymamics?

Turning on external fields

- To see where anomalies enter, we turn on external background U(1) field A_μ
- Now the energy-momentum and charge are not conserved

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

$$\partial_\mu j^\mu = -\frac{C}{8}\epsilon^{\mu\nu\lambda\rho} F^{\mu\nu} F^{\lambda\rho}$$

- Power counting: A~1, F~O(p): right hand side has to be taken into account

Anomalous hydrodynamics

- These equations have to be supplemented by the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu}$$ +viscosities

$$j^\mu = n u^\mu + \xi \omega^\mu + \xi_B B^\mu \qquad B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}$$

+diffusion+Ohmic current

- We demand that there exist an entropy current with positive derivative: $\partial_\mu s_\mu \geq 0$
- The most general entropy current is

$$s^\mu = s u^\mu - \frac{\mu}{T}\nu^\mu + D\omega^\mu + D_B B^\mu$$

Entropy production

- Positivity of entropy production completely fixes all functions ξ, ξ_B, D, D_B

$$\xi = T^2 d'\left(\frac{\mu}{T}\right) - \frac{2nT^3}{\epsilon + P} d\left(\frac{\mu}{T}\right)$$

$$d(x) = \frac{1}{2} C x^2$$

anomaly coeff

$$= C\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + P}\right)$$

$$\xi_B = C\left(\mu - \frac{1}{2}\frac{n\mu^2}{\epsilon + P}\right)$$

$$j^\mu = \cdots + \xi\omega^\mu + \xi_B B^\mu$$

These expressions have been checked for N=4 SYM

A more convenient "frame"

$$j^\mu = nu^\mu + C\mu^2\omega^\mu$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} + \frac{2}{3}C\mu^3(u^\mu\omega^\nu + u^\nu\omega^\mu)$$

can be eliminated by redefinition of u

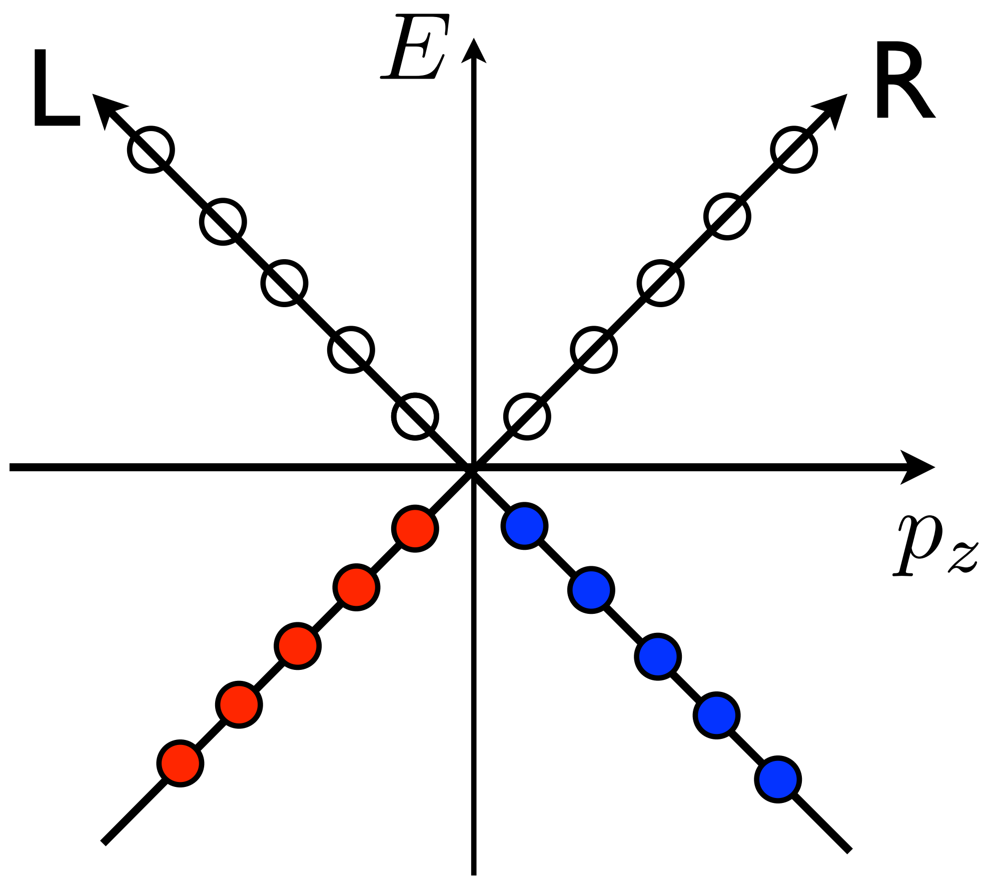
anomalous terms are "quantized"

Current induced by magnetic field

Spectrum of Dirac operator:

$$E^2 = 2nB + p_z^2$$

All states LR degenerate except for n=0



$$j_{\mathrm{L}} \sim -C\mu B$$
$$j_{\mathrm{R}} \sim C\mu B$$

$$j_5 = j_R - j_L \sim C\mu B$$

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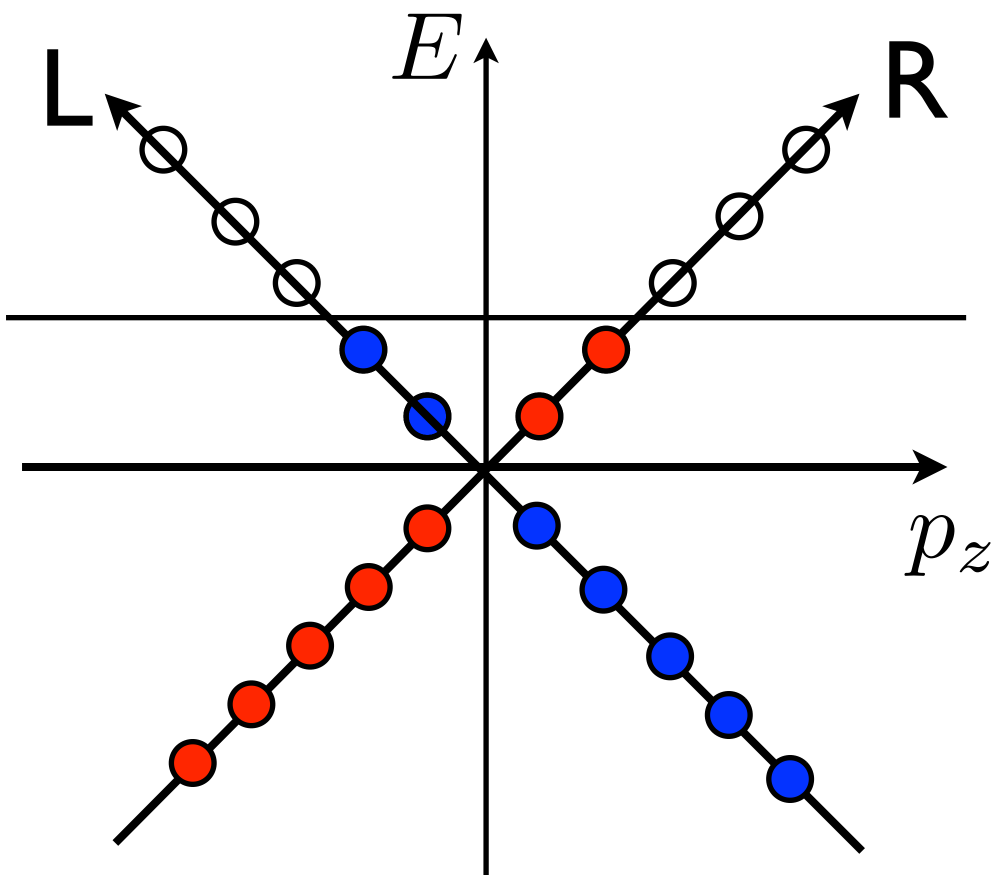


$$j_{\mathrm{L}} \sim -C\mu B$$
$$j_{\mathrm{R}} \sim C\mu B$$

$$j_5 = j_R - j_L \sim C\mu B$$

If there is only right-handed fermions:

$$j^\mu = n u^\mu + C\mu B^\mu$$
$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + \frac{C}{2}\mu^2 (u^\mu B^\nu + u^\nu B^\mu)$$

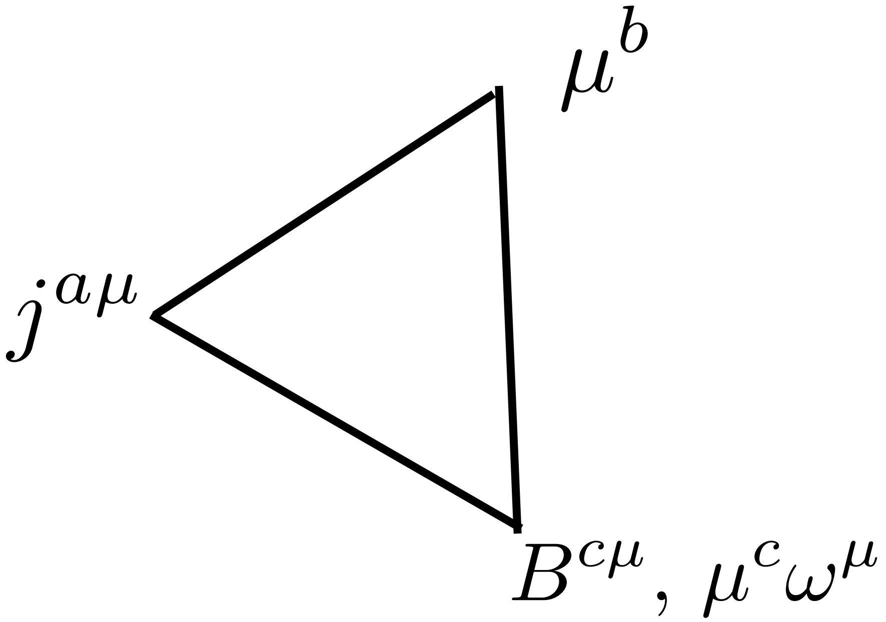
can be made disappear by redefining u^μ

Landau-Lifshitz frame: $u^\mu \to u^\mu - \frac{C}{2}\frac{\mu^2}{\epsilon + P} B^\mu$

$$j^\mu = n u^\mu + \xi_B B^\mu \qquad \xi_B = C\mu - \frac{C}{2}\frac{\mu^2 n}{\epsilon + P}$$

Simple understanding of ξ still lacking...

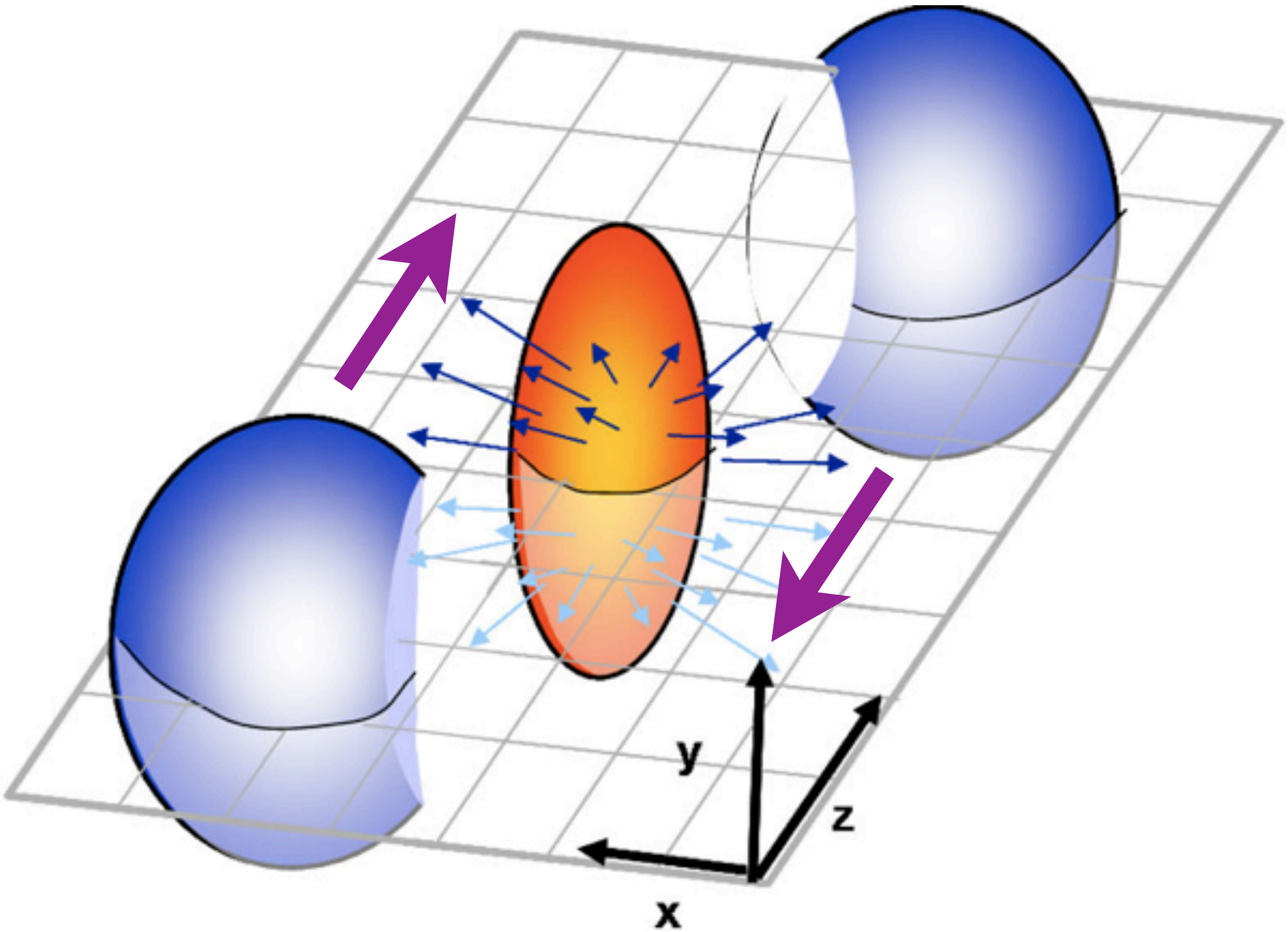
Multiple charges



$$\sim C^{abc}$$

$$j^{a\mu} = \cdots + \# C^{abc}\mu^b\mu^c\omega^\mu + \# C^{abc}\mu^b B^{c\mu}$$

Observable effect on heavy-ion collsions?



Chiral charges accumulate at the poles: what happens when they decay?

A more speculative scenario

- Large axial chemical potential μ_5 for some reason
- Leads to a vector current: charge separation
- π^+ and π^- would have anticorrelation in momenta
- Some experimental signal?
- Attempts to explain the signal by $j \sim \mu_5 B$ Kharzeev et al

From kinetic theory?

- The anomalous hydrodynamics current also exists in weakly coupled theories
- Should be derivable from kinetic theory, for example from Landau's Fermi liquid theories
- which kind of corrections to Landau's Fermi liquid theory?
    - should distinguish left- and right-handed quarks
- Berry's curvature on the Fermi surface?

Conclusions

- A surprising finding: anomalies affect hydrodynamic behavior of relativistic fluids
- First seen in holographic models, but can be found by reconciling anomalies and 2nd law
- Further studies of experimental significance needed
- Anomalies in Landau's Fermi liquid theory?