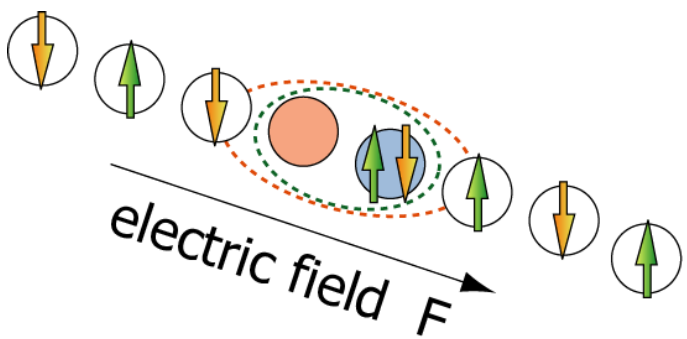
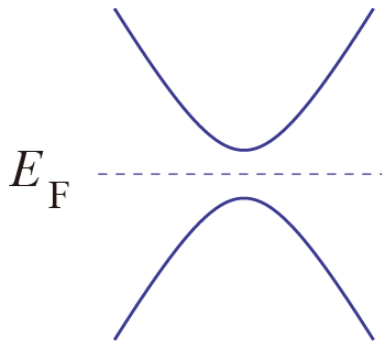
Many-body Schwinger-Landau-Zener Mechanism in Nonequilibrium Strongly Correlated Electron Systems

Takashi Oka, H. Aoki (U-Tokyo)



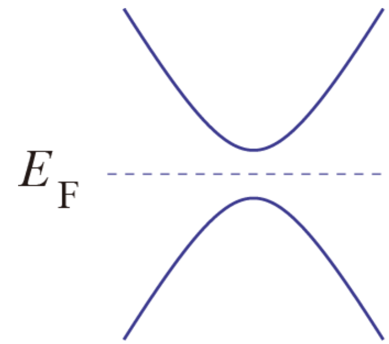
Insulators in Condensed Matter and High Energy

Band Insulator

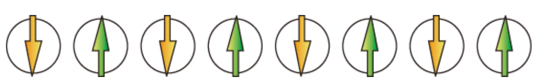


weak correlation

QED vacuum



Mott Insulator



local Coulomb repulsion

strong correlation
non-perturb. g.s.

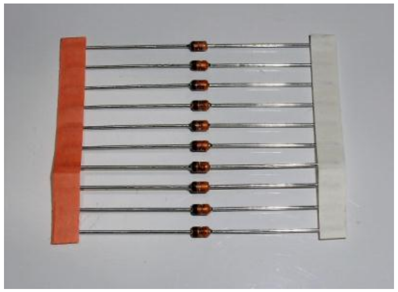
QCD vacuum

chiral condensate

$$\langle \bar{\psi}\psi \rangle \neq 0$$

Strong Field Physics in Condensed Matter and High Energy

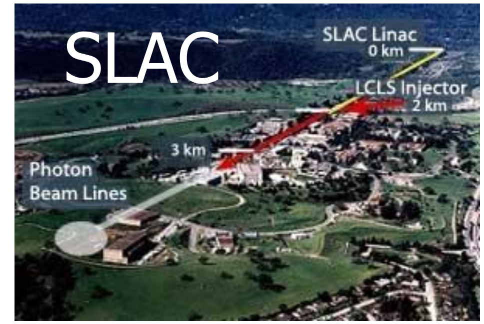
Band Insulator



Landau-Zener mechanism

$$E_{th} = \Delta^2/t_{hop}a \sim 1eV/\mathring{A} \sim 10^8 V/cm$$

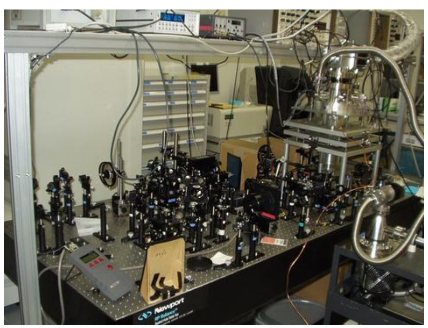
QED vacuum



Schwinger mechanism
(Heisenberg-Euler)

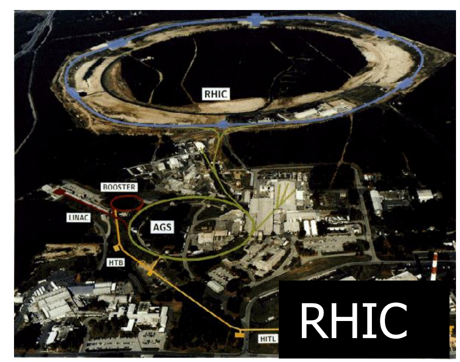
$$E_{th} = m^2/e = 1.3 \times 10^{16} V/cm$$

Mott Insulator



many body Schwinger-
Landau-Zener mechanism

QCD vacuum



Plan of my talk

Effective Lagrangian

$$\mathcal{L}(A_{ext}) = -i \ln \int D[\psi, \bar{\psi}] e^{i \int d^D x L(A_{ext})}$$

$$\Gamma / V = 2 \mathrm{Im} \mathcal{L}$$

Band Insulator

- CM version of the effective Lagrangian
- Dielectric breakdown, induced polarization
- photovoltaic Hall effect

QED vacuum

- Heisenberg-Euler's effective Lagrangian
- Schwinger mechanism

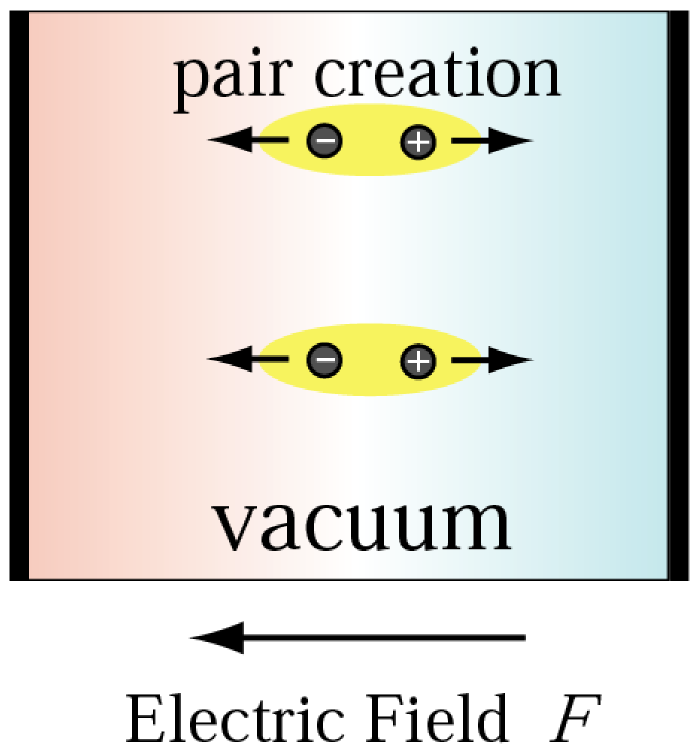
Mott Insulator

single "instanton" apprx. for Γ

- DDP tunneling theory + Hubbard model = non-Hermitian Bethe ansatz analysis

In CM,
Γ is measurable and numerically calculable.

1. Dirac particles in electric fields



Vacuum decay rate (production rate) $= \Gamma$

Effective Lagrangian

Dirac particles coupled to electromagnetic fields

$$L = \bar{\psi}(i\partial\!\!\!/ + ieA\!\!\!/ - m)\psi$$

How does the vacuum behave in background fields?

Effective Lagrangian

$$\mathcal{L}(A_{ext}) = -i \ln \int D[\psi, \bar{\psi}] e^{i \int d^D x L(A_{ext})}$$

$$= -i \ln \mathrm{Det}\left[i\partial\!\!\!/ + ieA\!\!\!/ - m\right]$$

``Consequences of Dirac‘s Theory of Positrons“

W. Heisenberg and H. Euler, **Z. Physik** 98, 714 (1936)

also Weisskopf (1936)

Folgerungen aus der Diracschen Theorie des Positrons.

Von **W. Heisenberg** und **H. Euler** in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

Aus der Diracschen Theorie des Positrons folgt, da jedes elektromagnetische Feld zur Paarerzeugung neigt, eine Abänderung der Maxwellschen Gleichungen des Vakuums. Diese Abänderungen werden für den speziellen Fall berechnet, in dem keine wirklichen Elektronen und Positronen vorhanden sind, und in dem sich das Feld auf Strecken der Compton-Wellenlänge nur wenig ändert. Es ergibt sich für das Feld eine Lagrange-Funktion:

effective Lagrangian

$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2 - \mathfrak{B}^2) + \frac{e^2}{\hbar c}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B})\cdot\frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)+\text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)-\text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}.$$

$\Big(\mathfrak{E}, \mathfrak{B}$ Kraft auf das Elektron.

$|\mathfrak{E}_k| = \frac{m^2c^3}{e\hbar} = \frac{1}{„137“}\frac{e}{(e^2/mc^2)^2} =$ „Kritische Feldstärke“.$\Big)$

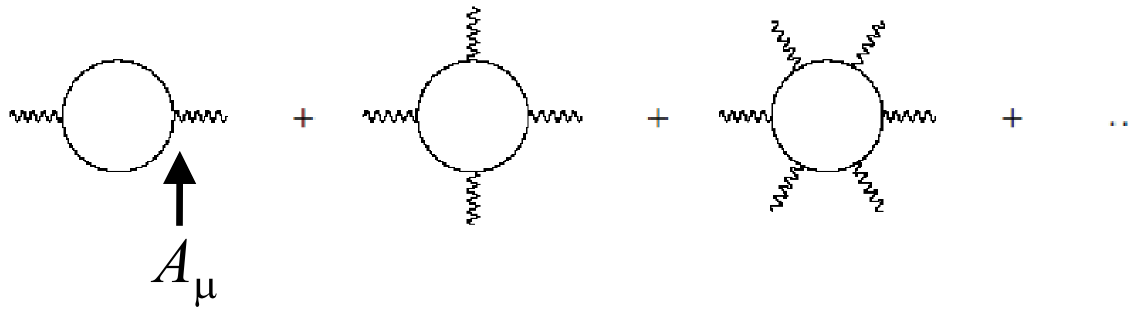
threshold (critical) field

Ihre Entwicklungsglieder für (gegen $|\mathfrak{E}_k|$) kleine Felder beschreiben Prozesse der Streuung von Licht an Licht, deren einfachstes bereits aus einer Störungsrechnung bekannt ist. Für große Felder sind die hier abgeleiteten Feldgleichungen von den Maxwellschen sehr verschieden. Sie werden mit den von Born vorgeschlagenen verglichen.

Polarization, Hall effect, light-light scattering

$$\mathcal{L} = -i \ln \mathrm{Det}\left[i\partial\!\!\!/ + ieA\!\!\!/ - m\right]$$

$=$



second order

$\Delta\varepsilon(\boldsymbol{E}^2 - \boldsymbol{B}^2)$

vacuum polarization

fourth order

$(\boldsymbol{E}^2 - \boldsymbol{B}^2)^2$ light-light scattering,

$(\boldsymbol{E}\cdot\boldsymbol{B})^2$ magneto-electric effect

$\kappa\varepsilon^{\mu\nu\delta}A_\mu\partial_\nu A_\delta$

Chern-Simons term (2+1 dim.)

in CM, this gives quantum (spin) Hall effect in ``topological ins.”

Pair production rate

$$\Gamma/V = 2\mathrm{Im}\mathcal{L}$$
$$\sim \frac{e^2E^2}{4\pi^3}\exp\left(-\pi\frac{m^2}{eE}\right)$$

"Schwinger" mechanism for pair creation

Heisenberg-Euler (1936)
Schwinger Phys. Rev.82, (1951)

Plan of my talk

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- CM version of the effective Lagrangian
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- photovoltaic Hall effect

QED vacuum

- Heisenberg-Euler's effective Lagrangian
- Schwinger mechanism

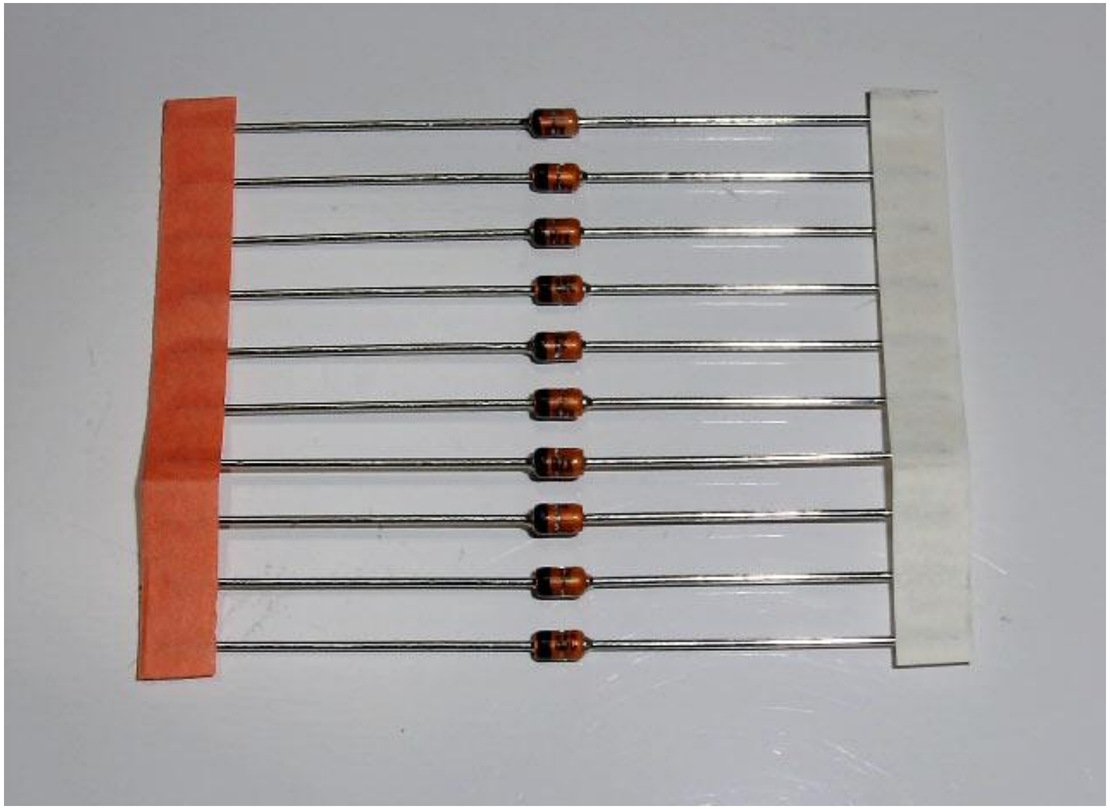
Mott Insulator

single "instanton" apprx. for Γ

- DDP tunneling theory + Hubbard model = non-Hermitian Bethe ansatz analysis

Comment: In CM, Γ is measurable and numerically calculable.

2. Dielectric breakdown in band insulators



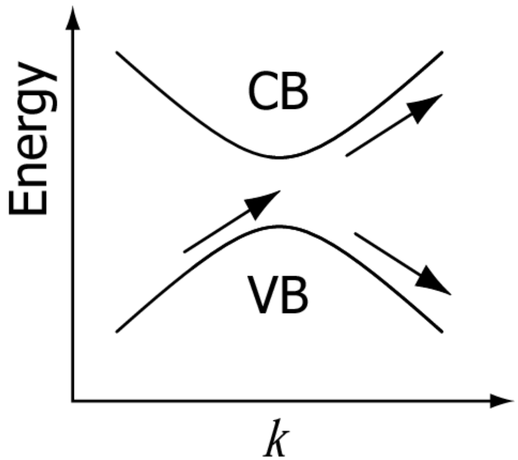
Zener diode

Electric fields in lattice models

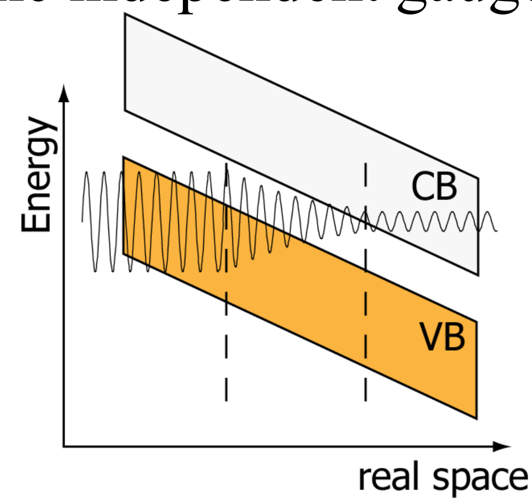
Time dependent gauge

$$H(t) = -\sum_i \left[e^{i\phi(t)} c^\dagger_{i+1} c_i + e^{-i\phi(t)} c^\dagger_i c_{i+1} \right] + \frac{\Delta}{2} \sum_i (-1)^i n_i$$

$\phi(t) = Ft$ time dependent phase



Time independent gauge



Condensed matter version of the effective Lagrangian

Groundstate-to-groundstate amplitude (quantum fidelity)

Time dependent gauge

$$\Xi(t) = \langle 0; \phi(t) | \hat{T} e^{-i \int_0^t H(\phi(s)) ds} | 0; \phi(0) \rangle e^{i \int_0^t E_0(\phi(s)) ds}$$

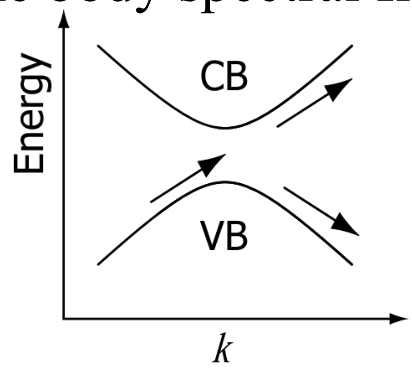
g.s. of H(ϕ)

Time independent gauge (non-adiabatic twist operator)

$$\Xi(t) = \langle 0 | e^{-it(H_0 + F\hat{X})} | 0 \rangle e^{itE_0}$$

$$\Rightarrow \quad \mathcal{L} = \lim_{t\to\infty} \frac{-i}{tL^d} \ln \Xi(t)$$

TO, H. Aoki, Phys. Rev. Lett. **95**, 137601 (2005) one body spectral flow



Time evolution = Application of a 2 × 2Unitary matrix

upper band $c_+^\dagger(\boldsymbol{k}) \to \sqrt{1-p(\boldsymbol{k})}e^{-i\chi(\boldsymbol{k})}c_+^\dagger(\boldsymbol{k}) + \sqrt{p(\boldsymbol{k})}c_-^\dagger(\boldsymbol{k})$,

lower band $c_-^\dagger(\boldsymbol{k}) \to -\sqrt{p(\boldsymbol{k})}c_+^\dagger(\boldsymbol{k}) + \sqrt{1-p(\boldsymbol{k})}e^{i\chi(\boldsymbol{k})}c_-^\dagger(\boldsymbol{k})$.

groundstate-to-groundstate component

$$\Xi(n\Delta t) = \left(\prod_{\boldsymbol{k}} \sqrt{1-p(\boldsymbol{k})}e^{i\chi(\boldsymbol{k})}\right)^n e^{inE_0\Delta t}$$

$\Delta t = 2\pi\hbar/F$

$$\chi(\boldsymbol{k}) = -\theta(\boldsymbol{k}) + \gamma(\boldsymbol{k})$$

dynamical phase non-adiabatic geometric phase

Effective Lagrangian for band insulators

non-adiabatic Berry phase

$$\mathrm{Re}\,\mathcal{L}(F) = -F \int_{\mathrm{BZ}} \frac{d\boldsymbol{k}}{(2\pi)^d} \frac{\gamma(\boldsymbol{k})}{2\pi},$$

induced polarization

tunneling

$$\mathrm{Im}\,\mathcal{L}(F) = -F \int_{\mathrm{BZ}} \frac{d\boldsymbol{k}}{(2\pi)^d} \frac{1}{4\pi} \ln[1 - p(\boldsymbol{k})],$$

TO, H. Aoki, Phys. Rev. Lett. **95**, 137601 (2005)

e.g.) Dirac band

non-adiabatic Berry phase

$$\gamma(\boldsymbol{k}) = \frac{1}{2}\,\mathrm{Im} \int_0^\infty ds\, \frac{e^{-i(\Delta_{\mathrm{band}}(\boldsymbol{k})/2)^2 s}}{s} \left[\cot(vFs) - \frac{1}{vFs}\right]$$

Landau-Zener formula for the tunneling probability

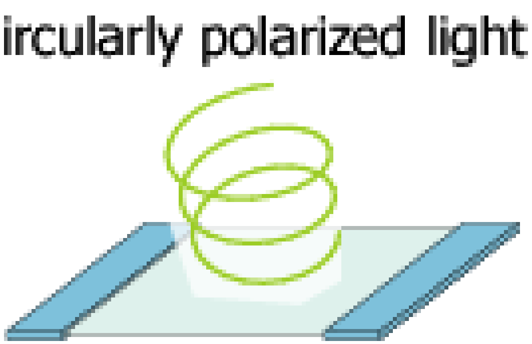
$$p(\boldsymbol{k}) = \exp\left[-\pi \frac{(\Delta_{\mathrm{band}}(\boldsymbol{k})/2)^2}{vF}\right]$$

cf.) Y. Kayanuma, Phys. Rev. B. **47**, 9940 (1993)

→ **Heisenberg-Euler-Schwinger result is recovered.**

Photovoltaic Hall Effect

= Hall effect in circularly polarized light



massless Dirac + circularly polarized light

$$\mathcal{L} = -i \ln \mathrm{Det}\left[i\partial\!\!\!/ + ieA\!\!\!/\right]$$

→ Dynamical topological mass $\kappa = \frac{\sqrt{4A_{ac}^2 + \Omega^2} - \Omega}{2}$

cf) Different from Volkov's solution (1935) since
in CM light breaks "Lorentz invariance"

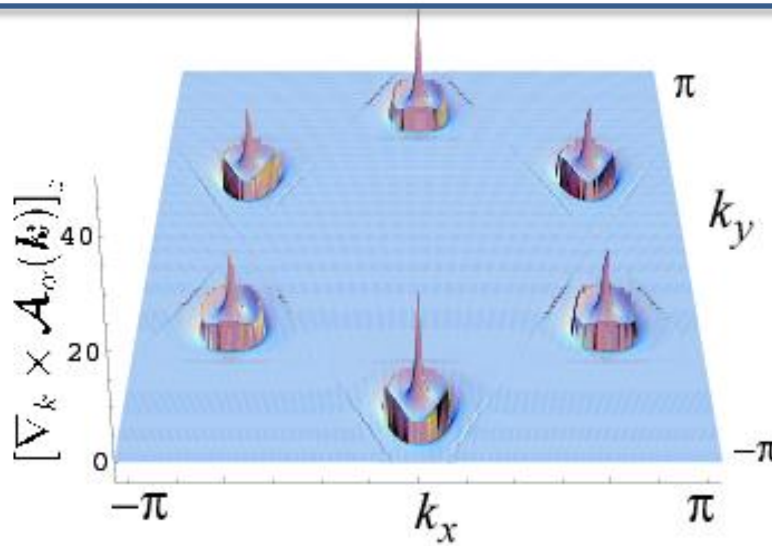
TO and H. Aoki, Phys. Rev. B **79**, 081406 (R) (2009)

photovoltaic Thouless-Kohmoto-Nightingale-Nijs formula
(Chern form induced by light)

$$\sigma_{xy}(\boldsymbol{A}_{\rm ac}) = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_{\alpha} f_\alpha(\boldsymbol{k}) \left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z$$

photovoltaic gauge field $\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k}) \equiv -i\langle\langle \Phi_\alpha(\boldsymbol{k})|\nabla_{\boldsymbol{k}}|\Phi_\alpha(\boldsymbol{k})\rangle\rangle$

Floquet states (time-dependent solution)



Photovoltaic Berry curvature $\left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z$
(graphene; honey comb lattice)

TO and H. Aoki, Phys. Rev. B **79**, 081406 (R) (2009)

Plan of my talk

Band Insulator

- CM version of the effective Lagrangian
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QED vacuum

- Heisenberg-Euler's effective Lagrangian
- Schwinger mechanism

Mott Insulator

single "instanton" apprx. for Γ

- DDP tunneling theory + Hubbard model = non-Hermitian Bethe ansatz analysis

Comment: In CM, Γ is measurable and numerically calculable.

3. Mott insulators: many-body SLZ mechanism

$$H(t) = -\sum_i \left[e^{i\phi(t)} c_{i+1}^\dagger c_i + e^{-i\phi(t)} c_i^\dagger c_{i+1} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

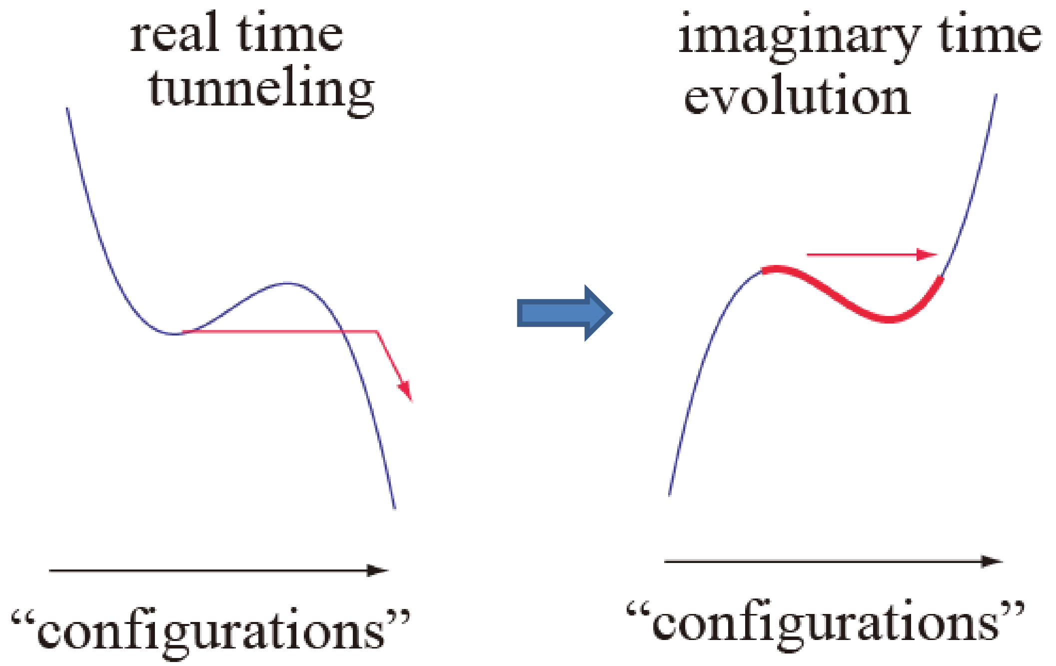
$\phi(t) = Ft$ time dependent phase

single "instanton" approximation

$$\rightarrow \quad \Gamma / L = -\frac{aF}{2\pi} \ln(1-p)$$

How can we obtain p?

Strategy



for 1d Hubbard model

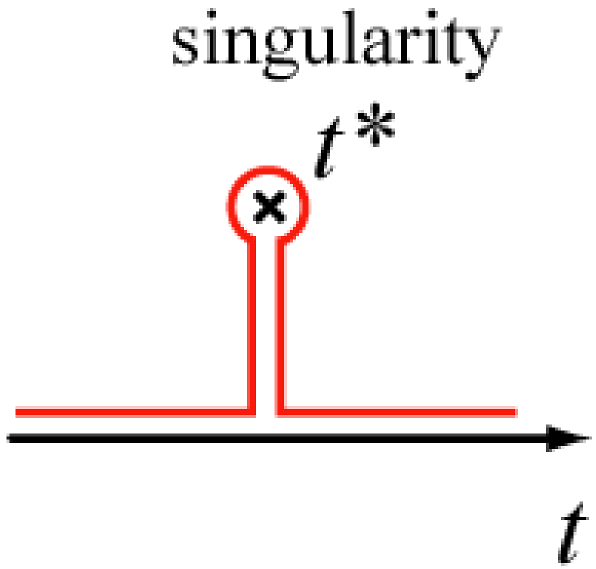
"configuration"
= infinite dimensional space of Bethe ansatz solutions

↕

distribution of "rapidities"

Dykhne-Davis-Pechkas theory of tunneling

Dykhne JETP (1962), Daviis, Pechukas, J.Chem.Phys. (1976)



Tunneling probability

$$p = \exp\left(-2\mathrm{Im}S_{1,2}/\hbar\right)$$

$$S_{1,2} = \int_{t_0}^{t^*} dt'[E_2(\Phi(t')) - E_1(\Phi(t'))]$$

singularity＝Energy crossing in complex time

$$E_2(t^*) = E_1(t^*)$$

Evolution to the other Riemann surface

Dominant contribution comes from the singularity closest to the real time axes.

Imaginary time

$$H(t) = -\sum_i \left[e^{i\phi(t)} c_{i+1}^\dagger c_i + e^{-i\phi(t)} c_i^\dagger c_{i+1} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

⇓

Non-Hermitian Hubbard model

$$H(t) = -\sum_i \left[e^{\Psi} c_{i+1}^\dagger c_i + e^{-\Psi} c_i^\dagger c_{i+1} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

left hopping $\neq$ right hopping

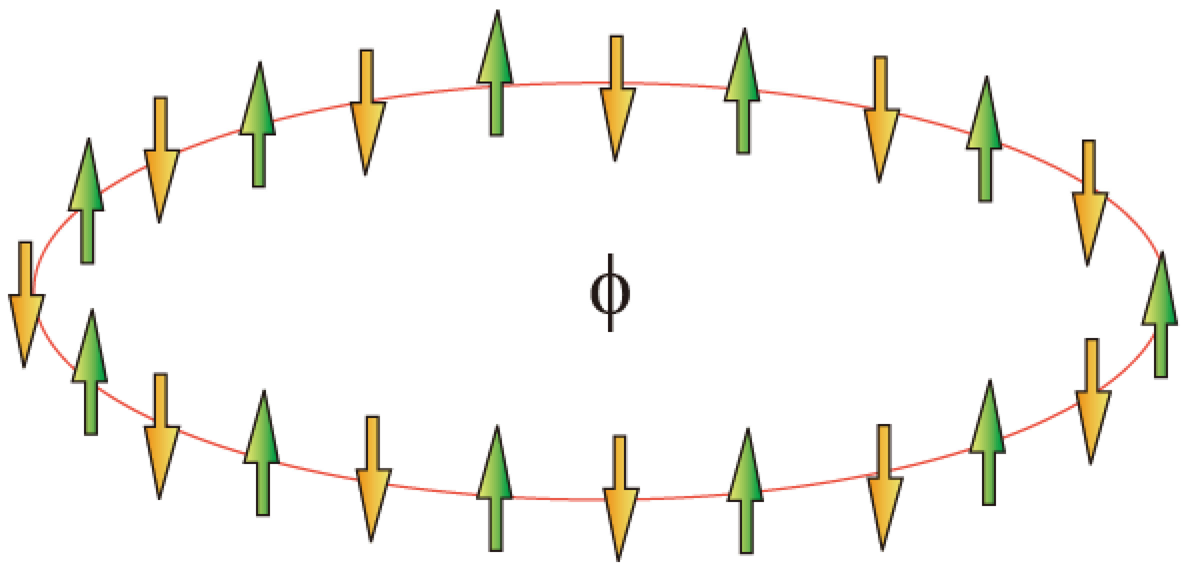
Fukui, Kawakami, Phys. Rev. **B 58**, 160501 (1998)

many-body spectrum

$$H(t) = -\sum_i \left[e^{i\phi(t)} c^\dagger_{i+1} c_i + e^{-i\phi(t)} c^\dagger_i c_{i+1} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

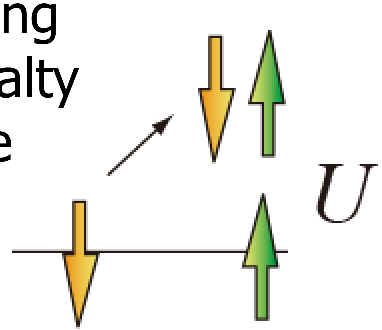
groundstate = Mott insulator

$|0;\phi\rangle =$



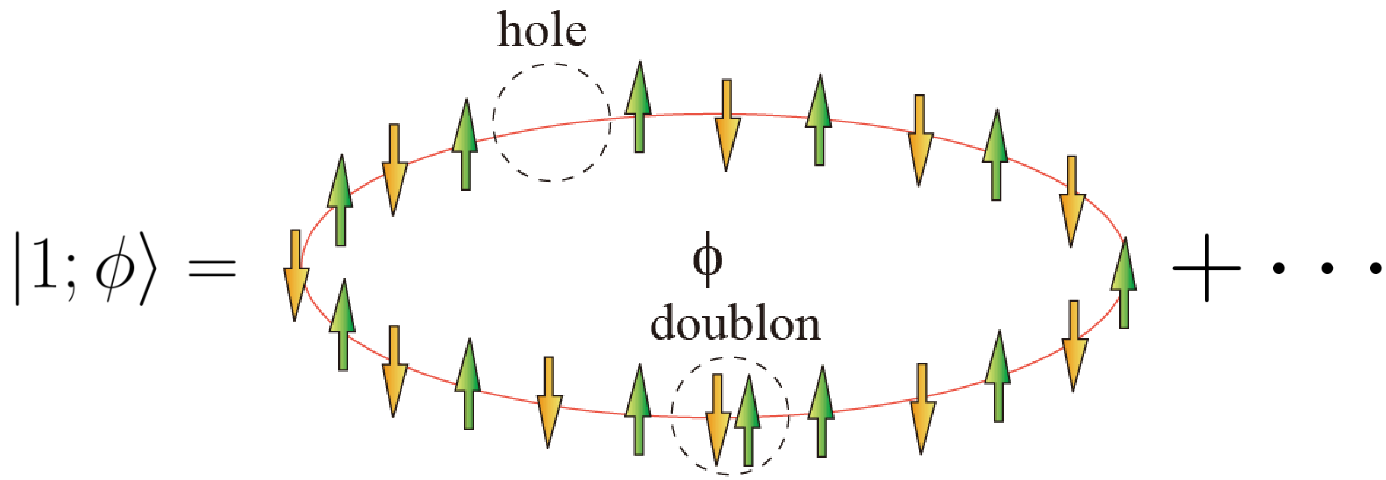
$+ \cdots$

This state is insulating since there is a penalty in energy for charge excitation

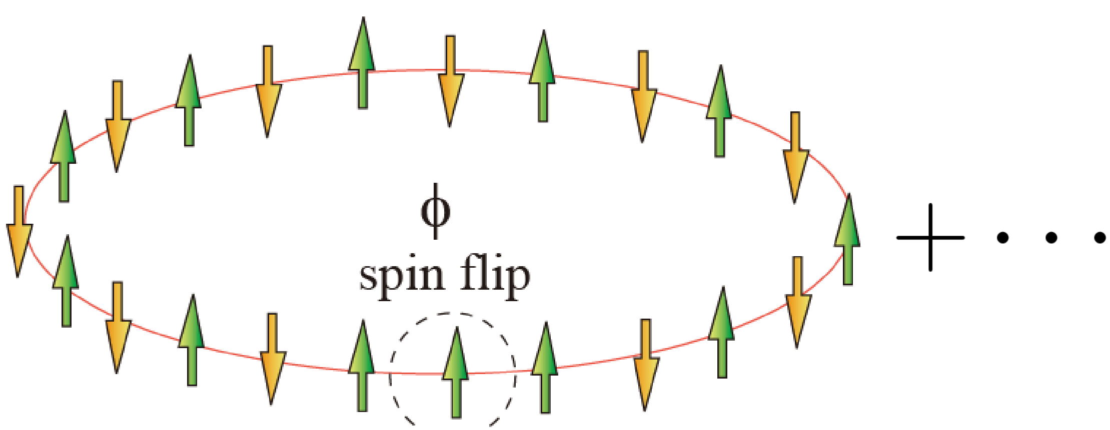


Anti-ferromagnetic order is only quasi-long range. (Mermin-Wagner th.)

1 doublon-hole pair (1 string state)



magnons



we ignore magnons here

many-body wave function

$$|n;\phi\rangle = \frac{1}{\sqrt{N!}} \sum_{x_1,\ldots,x_N=1}^{L} \sum_{s_1,\ldots,s_N=\uparrow\downarrow} \psi(x_1,\ldots,x_L;s_1,\ldots,s_N) c^\dagger_{x_1 s_1} \ldots c^\dagger_{x_N s_N} |0\rangle$$

Bethe ansatz form

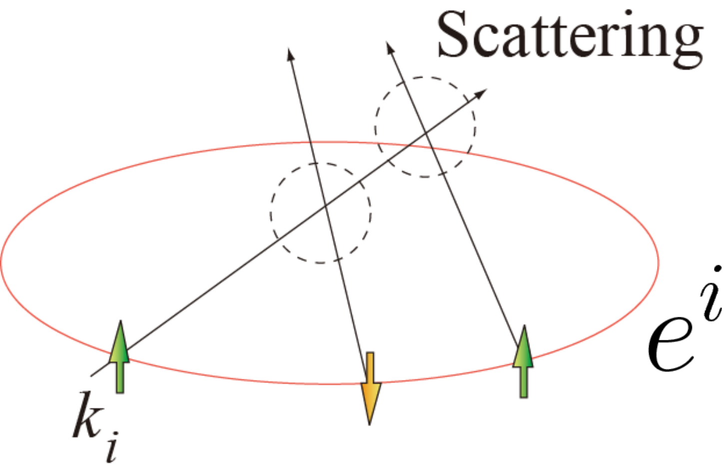
$$\psi(x_1,\ldots,x_L;s_1,\ldots,s_N) = \sum_{P\in S_L} (-1)^{PQ} \varphi_P(s_{Q1}\ldots s_{QL}) e^{i\sum_j k_{Pj} x_{Qj}}$$

yet to be determined $\{k_i\}$ charge rapidities

$\{\lambda_\alpha\}$ spin rapidities

Lieb-Wu equation (integrability condition)

Lieb-Wu, PRL (1968)



$$e^{ik_iL} = \prod_{j \neq i} S(k_j, k_i)$$

scattering matrix
satisfying the Yang-Baxter condition

Lieb-Wu equations with flux ϕ

$$Lk_j = 2\pi I_j + L\phi - \sum_{\alpha=1}^{N_\downarrow} \theta(\sin k_j - \lambda_\alpha),$$

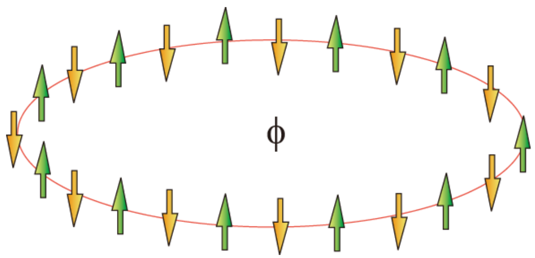
$$\sum_{j=1}^{L} \theta(\sin k_j - \lambda_\alpha) = 2\pi J_\alpha - \sum_{\beta=1}^{N_\downarrow} \theta\left(\frac{\lambda_\alpha - \lambda_\beta}{2}\right),$$

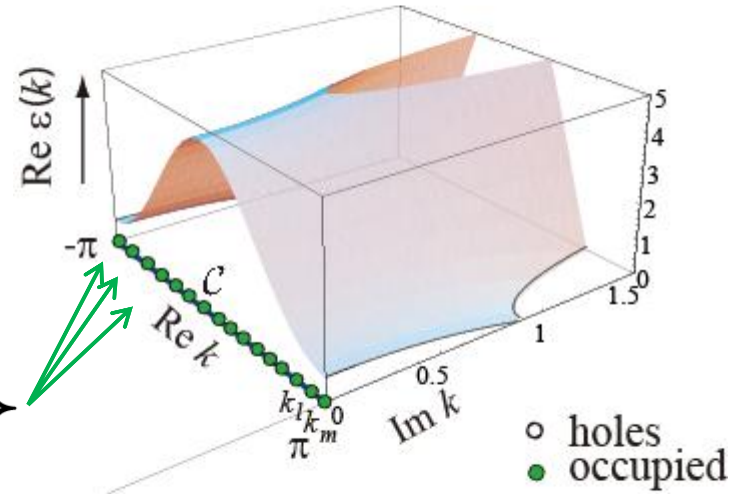
This determines the rapidities $\{k_i\}$ $\{\lambda_\alpha\}$

$$\theta(x) = -2\tan^{-1}(4x/U)$$

Energy of charge rapidity k

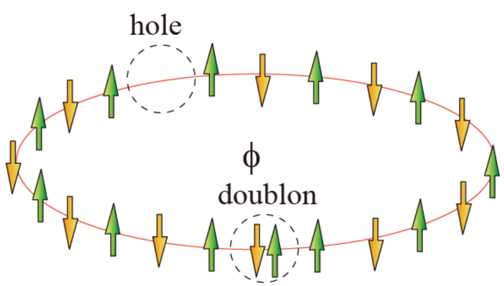
$$\varepsilon(k) = 2u + 2\cos(k) + 2\int_0^\infty \frac{e^{-u\omega}}{\omega \cosh u\omega} J_1(\omega)\cos(\omega \sin k)\, d\omega.$$

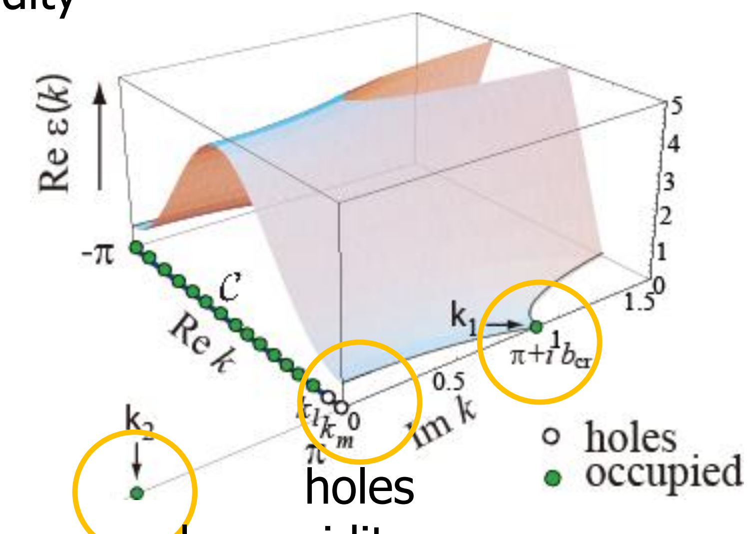
$|0;\phi\rangle =$  $+ \cdots =$



$\{k_i\}$

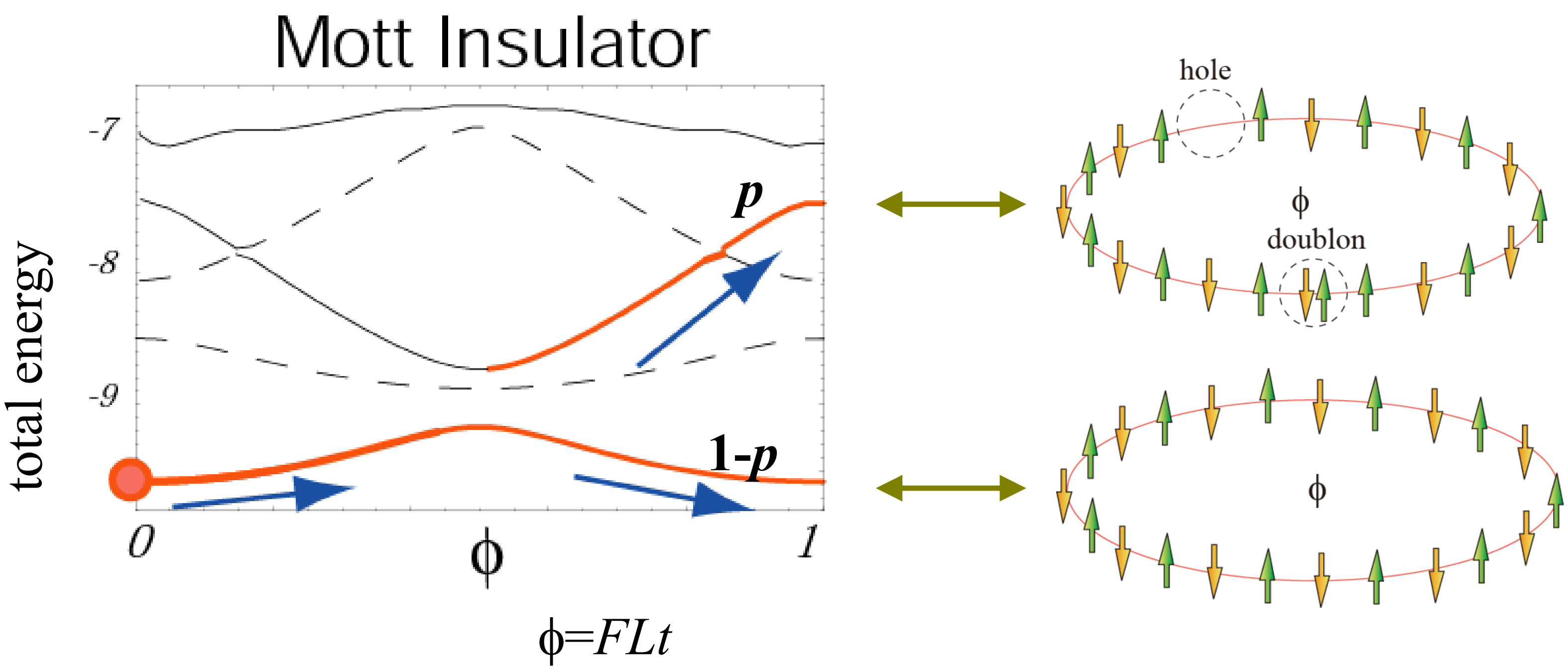
charge rapidity

$|1;\phi\rangle =$  $+ \cdots =$

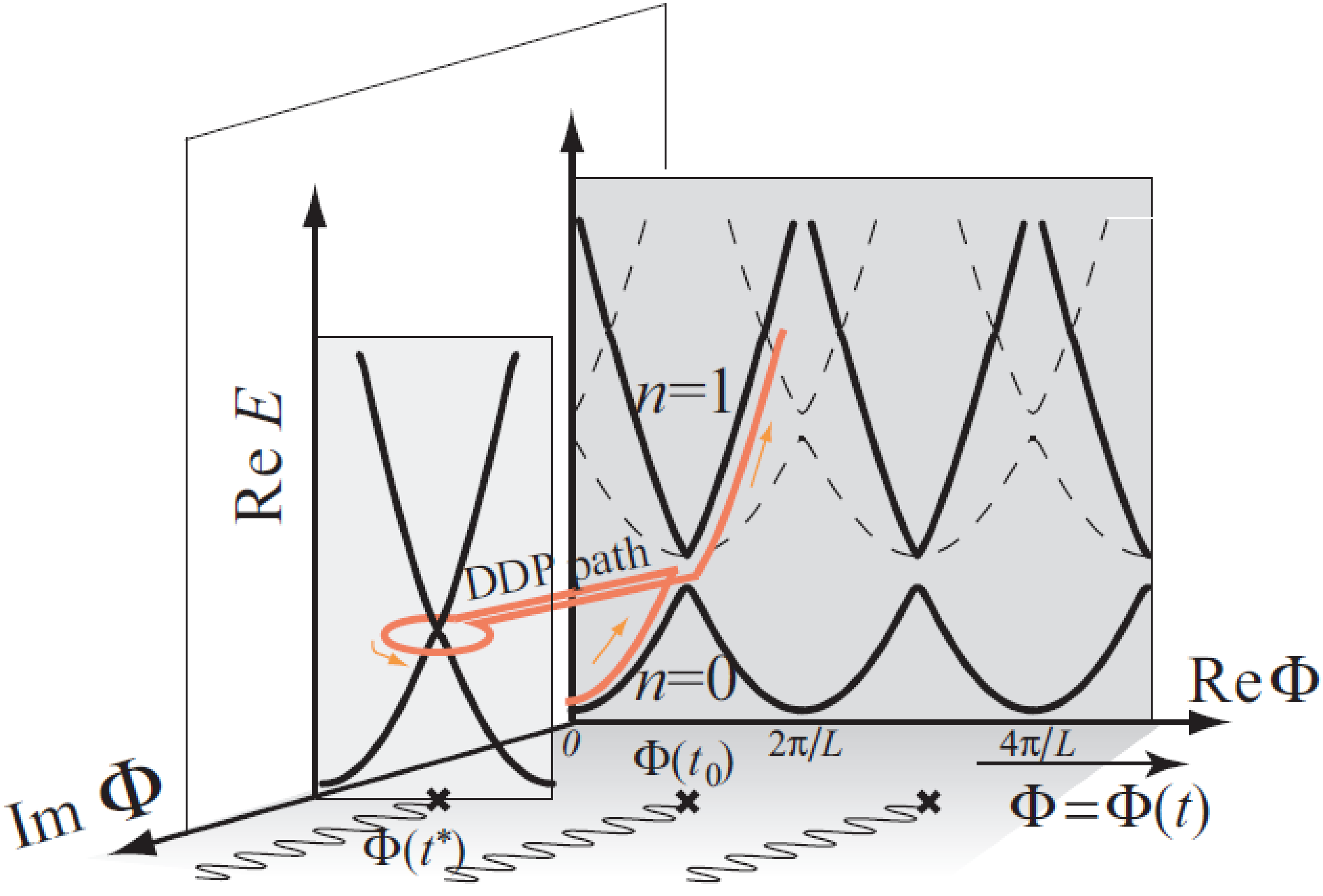


Ovchinikov JETP, 30 (1970), Coll, PRB 9 (1974),
Takahashi, Prog. Theor. Phys. 47 (1972)
Woynarovich, J.Phys.C, (1982)

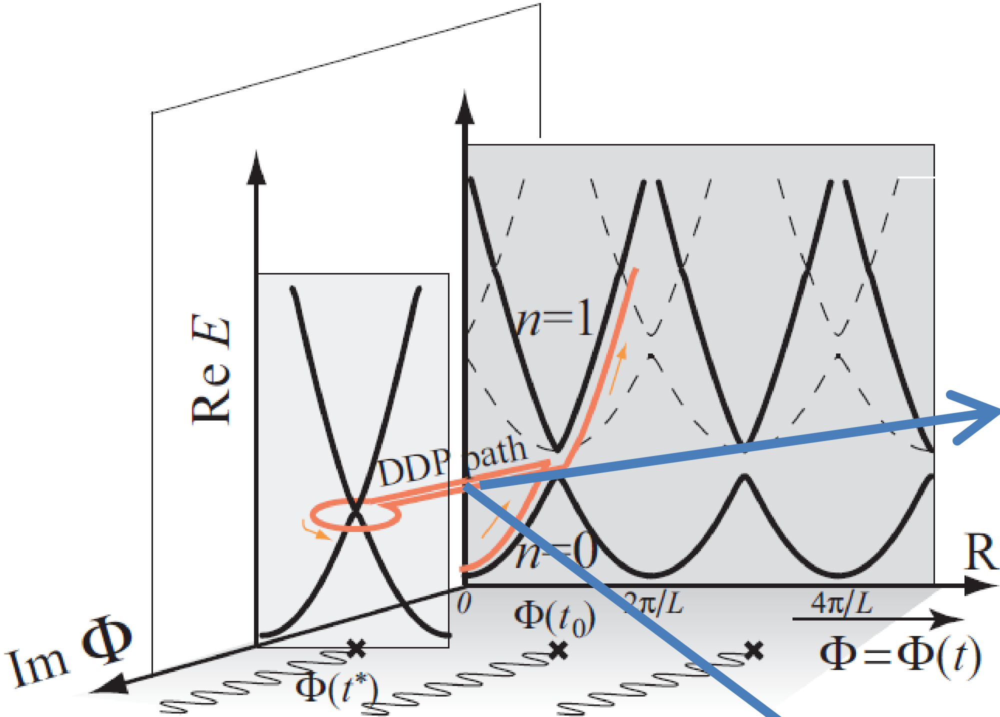
many-body spectral flow

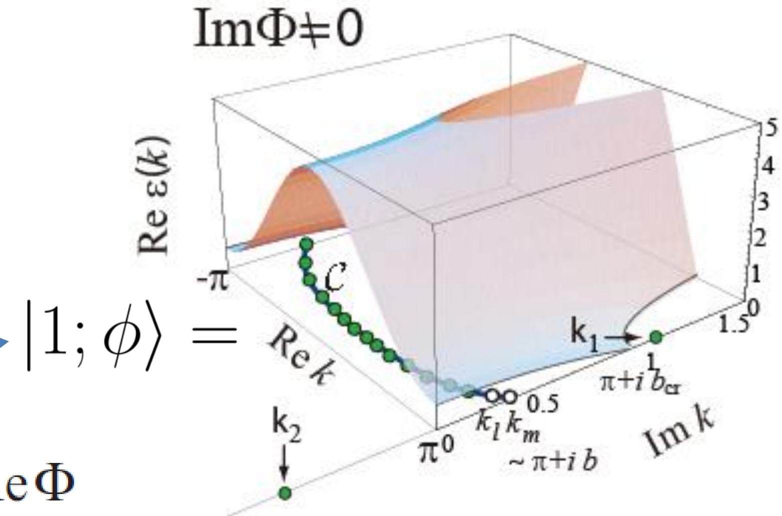


DDP path for the Hubbard model



T. Oka, H. Aoki, Phys. Rev. **B 81**, 033103 (2010)



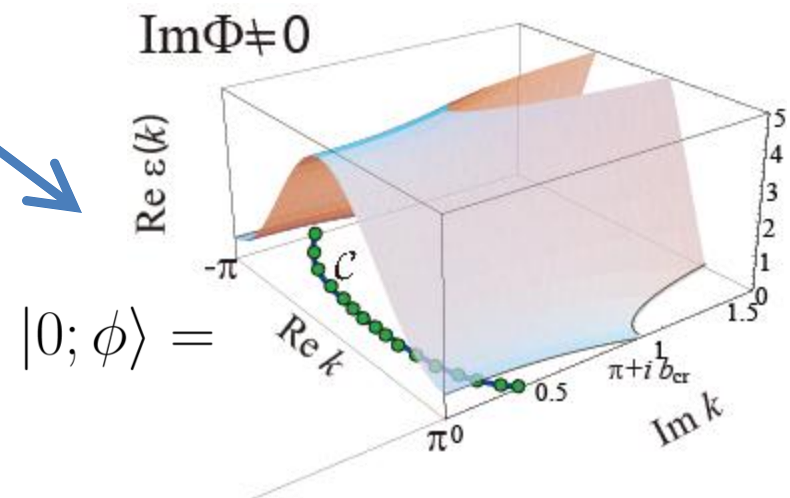


ImΦ≠0

$$Lk_j = 2\pi I_j + L\phi + \sum_{\alpha=1}^{N_\downarrow} \theta(\sin k_j - \lambda_\alpha),$$

$$\sum_{j=1}^{L} \theta(\sin k_j - \lambda_\alpha) = 2\pi J_\alpha - \sum_{\beta=1}^{N_\downarrow} \theta\left(\frac{\lambda_\alpha - \lambda_\beta}{2}\right),$$

all charge rapidities become **complex**



groundstate studied in
Fukui, Kawakami, Phys. Rev. **B 58**, 160501 (1998)

Tunneling probability for the 1d Hubbard model

$$p_{\mathrm{th}}^{\mathrm{DDP}} = \exp\left(-\pi F_{\mathrm{th}}^{\mathrm{DDP}}/F\right)$$

$$\begin{aligned} F_{\mathrm{th}}^{\mathrm{DDP}} &= \frac{2}{\pi}\int_0^{b_{\mathrm{cr}}} (E_1 - E_0)\frac{d\Psi}{db}db \\ = &\ \frac{2}{\pi}\int_0^{\sinh^{-1}u} 4\left[u - \cosh b + \int_{-\infty}^{\infty} d\omega \frac{e^{\omega \sinh b} J_1(\omega)}{\omega(1+e^{2u|\omega|})}\right] \\ &\times \left[1 - \cosh b \int_0^{\infty} d\omega \frac{J_0(\omega)\cosh(\omega \sinh b)}{1+e^{2u\omega}}\right] db. \end{aligned}$$

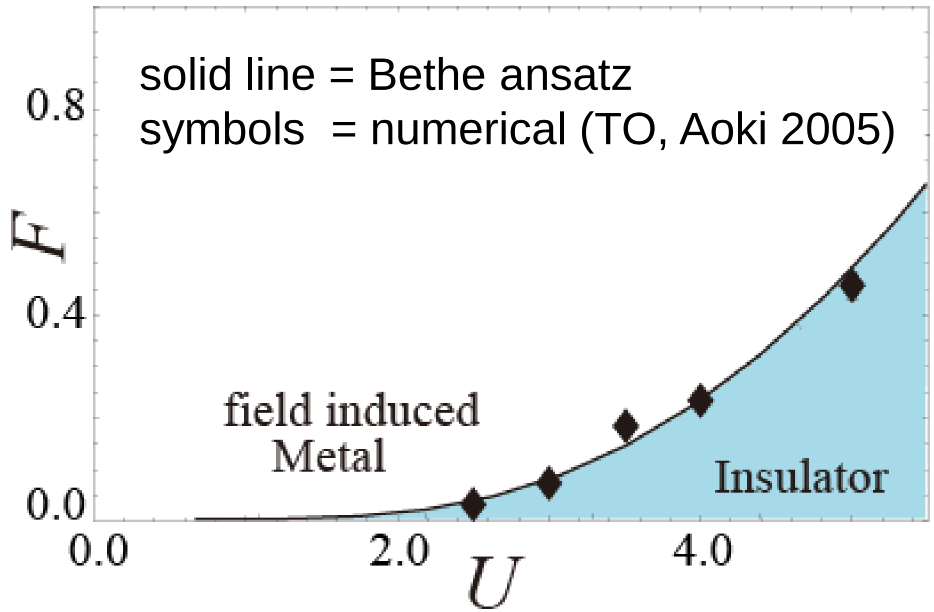
$u=U/4$

cf) In Dirac it was $E_{th} = m^2/e$

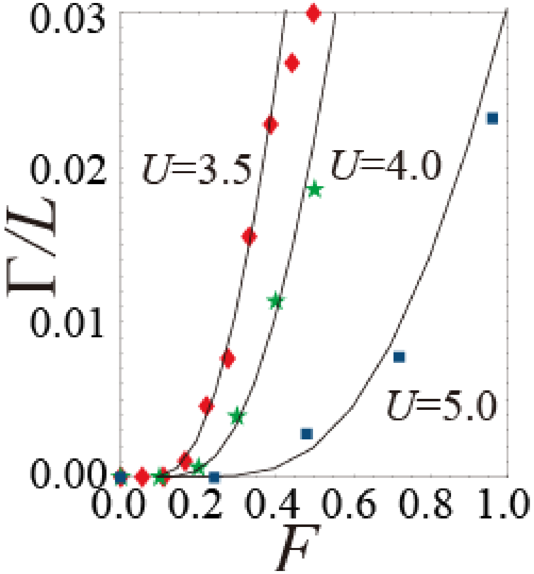
T. Oka, H. Aoki, Phys. Rev. **B 81**, 033103 (2010)

Comparison with numerical calculation (time-dependent density matrix renormalization group)

threshold F_{th}



vacuum decay rate

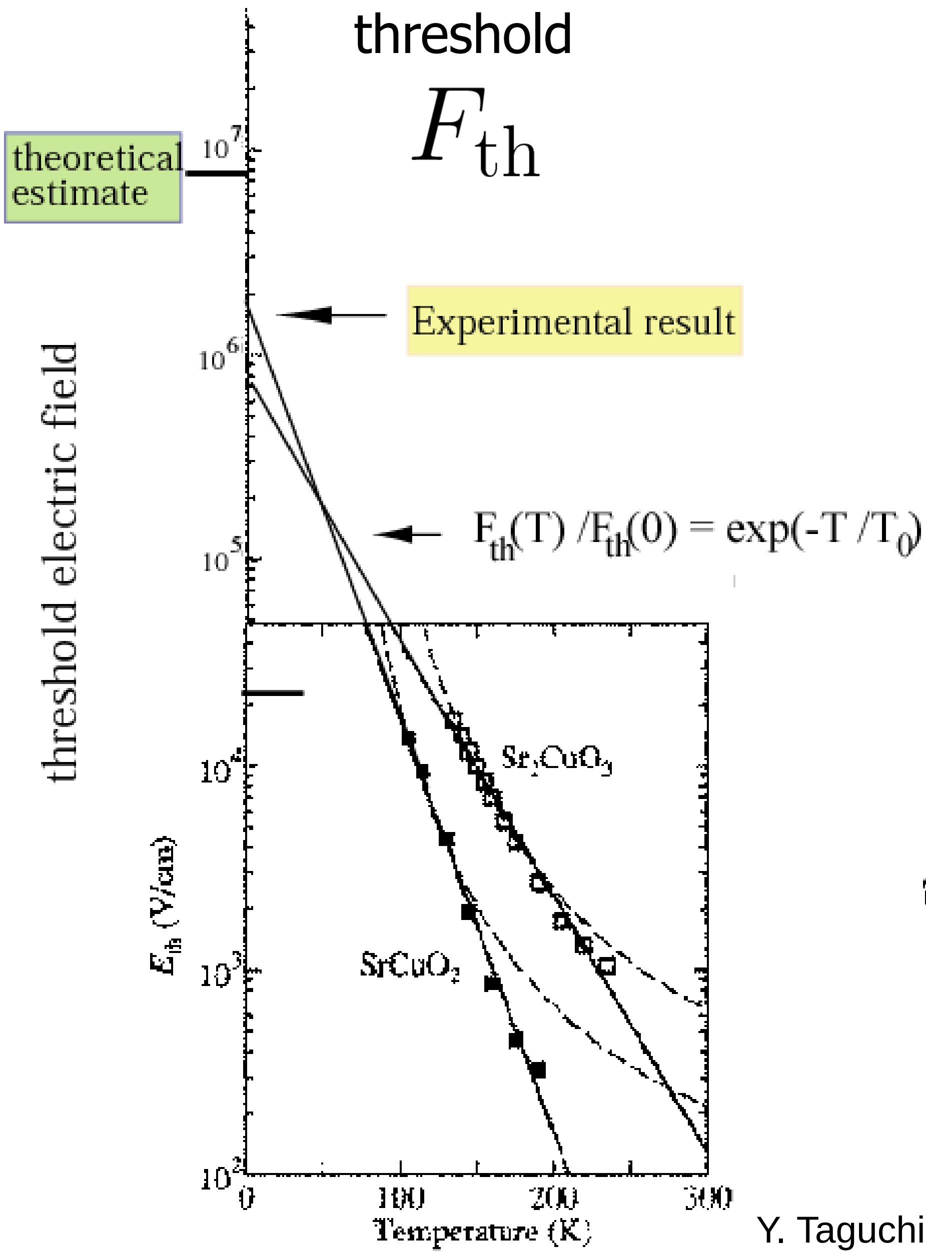


$$\Gamma/L = -\frac{aF}{2\pi}\ln[1-\exp(-\pi F_{\mathrm{th}}^{\mathrm{DDP}}/F)]$$

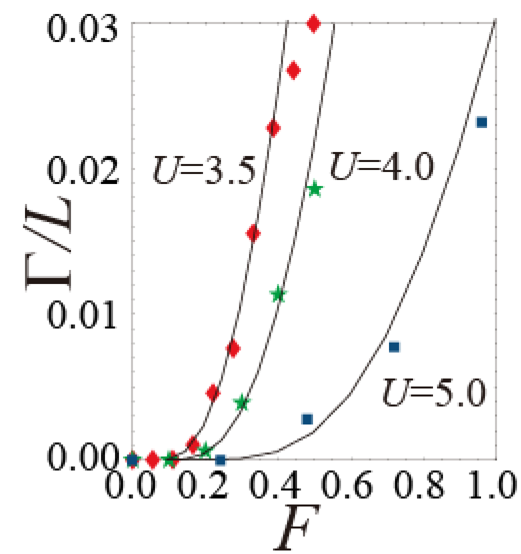
Good agreement!

T. Oka, H. Aoki, Phys. Rev. **B 81**, 033103 (2010)

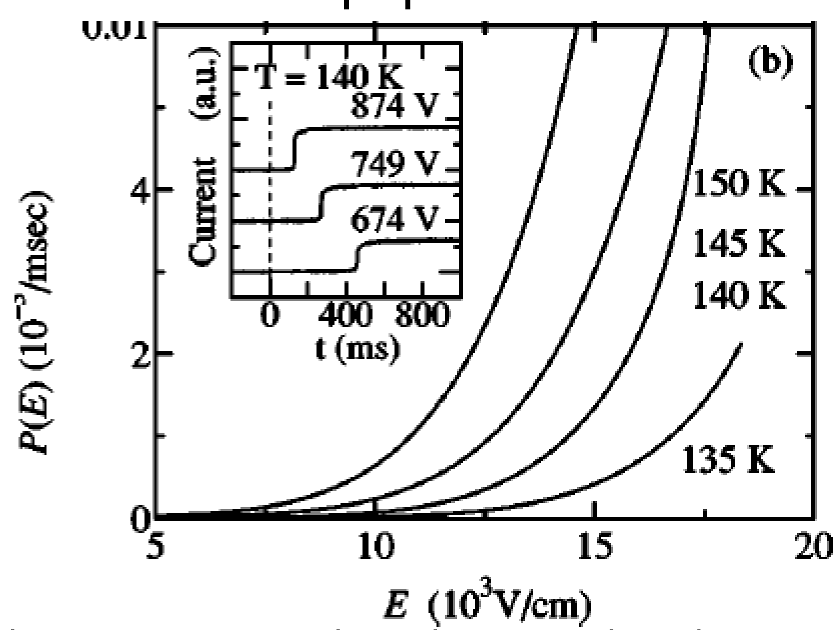
Comparison with experiment



vacuum decay rate



exp. production rate



Y. Taguchi, T. Matsumoto, and Y. Tokura PRB (2000).

Summary

$$\mathcal{L}(A_{ext}) = -i \ln \int D[\psi, \bar{\psi}] e^{i \int d^D x L(A_{ext})}$$

1. Interesting physics, e.g., photovoltaic Hall effect
2. Attempt to calculate in an interacting system, e.g., many-body SLZ mechanism
3. You can do experimental tests! (in CM)

Band Insulator ← QED vacuum

↓

Mott Insulator ↔ QCD vacuum

- lessons from AdS/QCD (non-eqilibrium gravity dual)
  e.g., temperature dependence
- CM realizes QED_{1+1}, QED_{2+1} and cofinement