Critical phenomena in AdS/CFT duality

Makoto Natsuume (KEK)

arXiv: 0809.4074 [hep-th]
arXiv: 0904.1914 [hep-th]

in collaboration w/
Kengo Maeda (Shibaura Institute of Technology) and
Takashi Okamura (Kwansei Gakuin Univ.)

Universality of η/s

According to AdS/CFT,
non-Abelian plasmas at strong coupling: small shear viscosity

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Many people's work
(in particular Son & colleagues)
and
Mas, 0601144
Son - Starinets, 0601157
Saremi, 0601159
Maeda - Natsuume - Okamura, 0602010

The "prediction": very close to the real QGP value at RHIC experiments

With the "success," many people try to apply AdS/CFT to condensed-matter physics as well.

Our aim: Critical phenomena in AdS/CFT duality

Static vs dynamic

condensed-matter basics

In 2nd-order phase transition, the correlation length ξ (& physical quantities) diverge. Two kinds of critical phenomena

- (Static) critical phenomena

  (thermodynamic quantities) ~ $\xi^{\#}$ ← critical exponent

  critical exponent: determined from sym and dimensions "universality"

- (Dynamic) critical phenomena

  (relaxation time) ~ $\xi^{z} \to \infty$ ⇨ critical slowing down

⇨ Study these phenomena from Gravity side
Goal: BHs in AdS/CFT obey the theory of critical phenomena?

Plan

- Gravity system w/ 2nd order phase transition
- Static & dynamic critical phenomena
- Critical phenomena in AdS/CFT
  - "Holographic superconductors" Ginzburg-Landau & Model A
  - R-charged BHs non-GL & Model B

BH w/ 2nd order phase transition?

We consider finite temperature criticality (not quantum criticality)

(Finite temperature) AdS/CFT

Finite temperature gauge theory	⇔	Black hole
at strong coupling		in AdS space
⇓		⇓
thermal		thermal due to the Hawking radiation

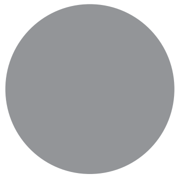
So, above all things we need a BH w/ a 2nd order phase transition.

Is there any?

BH w/ 2nd order phase transition?

- Pure Gravity ↔ $\mathcal{N}$=4 SYM

  Schwarzschild-AdS_5 BH w/ planar horizon
  conformal → no phase transition



- Einstein - Maxwell
  Reissner-Nordström-AdS_5 BH w/ planar horizon
  still no phase transition due to no-hair thm → BH: M, J, Q

  No-hair thm is not entirely true for AdS space though

- Einstein - Maxwell - scalar
  2 systems known

→ "Holographic superconductors"
R-charged BHs

Field theory ingredients

Boundary (4d) ⇔ Bulk (5d)

EM-tensor $T_{\mu\nu}$ ⇔ graviton $h_{\mu\nu}$

U(1) current J_μ ⇔ Maxwell A_μ

scalar op. $\mathcal{O}$ ⇔ scalar ψ

⇒ Field theory w/ $T_{\mu\nu}$, J_μ, $\mathcal{O}$

Bulk fields act as sources of boundary operators.

$$\delta S \sim \int d^4x \; h^{\mu\nu} T_{\mu\nu} + A^\mu J_\mu + \psi \mathcal{O} \cdots$$

Why do we study both?

They belong to different universality classes

- "Holographic superconductors"

  dual to some kind of superconductors

  - (static) univ class: standard Ginzburg-Landau theory
  - (dynamic) univ class: Model A

- R-charged BHs

  dual to $\mathcal{N}$=4 SYM at a finite chemical potential

  - (static) univ class: unconventional
  - (dynamic) univ class: Model B

Static & dynamic critical phenomena

Static critical phenomena

condensed-matter basics

In 2nd order transition, (thermodynamic quantities) ~ $\xi^{\#} \to \infty$

For ferromagnets, 6 exponents (α, β, γ, δ, ν, η)

specific heat: $C \propto |T - T_c|^{-\alpha}$

spontaneous magnetiz: $m \propto |T - T_c|^{\beta}$ $\quad (T < T_c)$

magnetic susceptibility: $\chi = \left.\frac{\partial m}{\partial h}\right|_T \propto |T - T_c|^{-\gamma}$ $\quad (h = 0)$

critical isotherm: $m \propto |h|^{1/\delta}$ $\quad (T = T_c)$

correlation fn: $G(r) \propto e^{-r/\xi}$ $\quad (T \neq T_c)$

$\propto r^{-d+2-\eta}$ $\quad (T = T_c)$

correlation length: $\xi \propto |T - T_c|^{-\nu}$

m: magnetization
h: external magnetic field

d: spatial dimensions

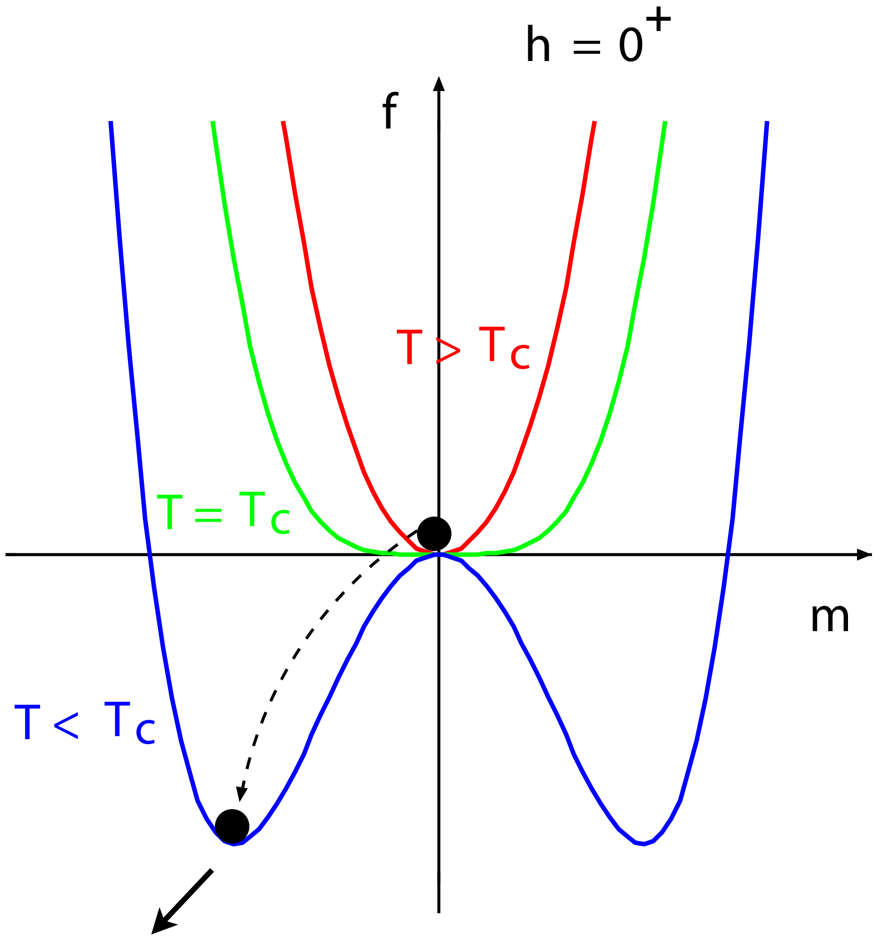
condensed-matter basics

Example: Landau theory

$$f = f_0 + \frac{1}{2} a m^2 + \frac{1}{4!} b m^4 + \cdots - mh$$

$$a = a_0 (T - T_c) + \cdots$$

$$(\alpha, \beta, \gamma, \delta, \nu, \eta) = \left(0, \frac{1}{2}, 1, 3, \frac{1}{2}, 0\right)$$



$$m \propto \sqrt{-a/b} \propto (Tc - T)^{1/2} \quad \Rightarrow \quad \beta = \frac{1}{2}$$

We'll see some BHs have the same values whereas some BHs do not have them.

Dynamic critical phenomena

condensed-matter basics

Near the critical pt,

(relaxation time) $\sim \xi^z \to \infty \Rightarrow$ critical slowing down

The details depend on dynamic univ class

- Additional properties of system, in particular conservation laws affect the dynamic univ class
- Conservation law forces relaxation to proceed more slowly

→ 2 systems w/ the same static univ class may not be the same dynamic univ class

Dynamic universality class

condensed-matter basics

- Macroscopic variables
  - Order parameter
  - Conserved charges
- Classified as (A, B, C, H, F, G, J)

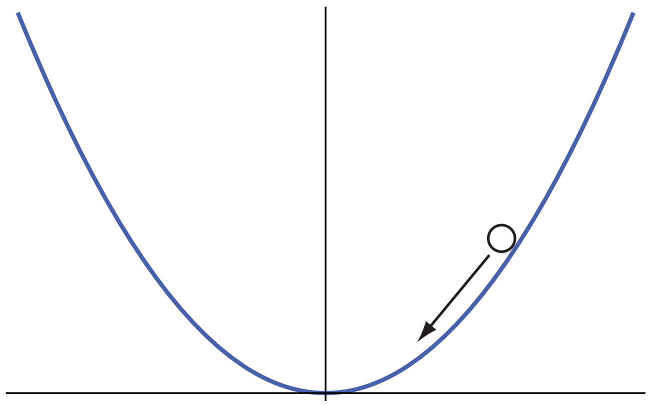
Hohenberg - Halperin (1977)

- Criteria:
  - Order parameter is conserved?
  - Any other conserved charges in the system
- Model A & B ⇨ order parameter only

Critical slowing down: simple example

condensed-matter basics

$$\frac{dm}{dt} = -\Gamma \partial_m f$$
$$= -a\Gamma m$$



where $f = \frac{1}{2} a(T) m^2 + \cdots$

$$m \propto e^{-t/\tau}$$
$$\tau^{-1} = a(T)\Gamma \propto (T - T_c) \to 0$$
$$\propto \xi^{-2}$$

$\Rightarrow$ z = 2 $\quad \tau \propto \xi^z$

Diverging relaxation ⇐ flat potential

In momentum space, $\omega = -ia\Gamma$

Model A & B

condensed-matter basics

- Model A
  m : not a conserved charge $\omega = -ia\Gamma$ $\Rightarrow$ z = 2

- Model B
  m : a conserved charge

$$\omega = -iDq^2 \;\Leftrightarrow\; \partial_t m - D\partial_i^2 m = 0 \quad \text{diffusion eq.}$$

$$\updownarrow$$

$$\omega = -ia\Gamma$$

$$\Gamma \propto q^2, D \propto a(T) \to 0$$ D→0 at critical pt

$$\tau^{-1} \sim aq^2 \sim \xi^{-2}q^2 \sim \frac{(\xi q)^2}{\xi^4}$$ "scaling form" $\Rightarrow$ z = 4

Our job: find AdS counterpart of model A & B

“Holographic superconductors”: GL & Model A

“Holographic superconductors”

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - F_{\mu\nu}^2 - |\nabla\psi - iqA\psi|^2 - m^2|\psi|^2\right]$$

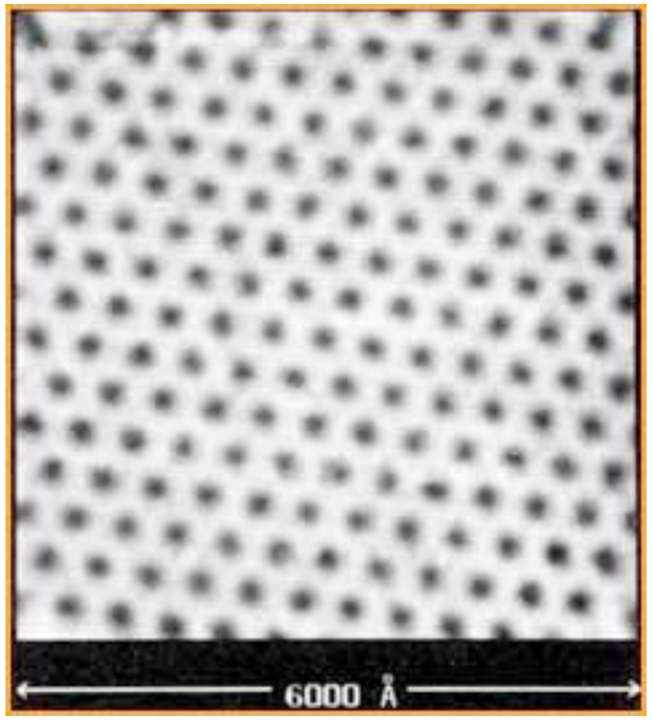
Hartnoll - Herzog - Horowitz, 0803.3295; 0810.1563
Gubser, 0801.2977

- Phase structure
  - T>Tc: Reissner-Nordström-AdS_4 w/ $\psi = 0$
  - T<Tc: Charged BH “w/ hair” $\psi \neq 0$ (no known analytic solution)
    → ψ: order parameter
- Claim: dual to some kind of superconductor
  - charge conductivity diverges
  - some sign of “energy gap”

- extreme type II → magnetic field can penetrate
  → vortex
  Abrikosov lattice (triangular lattice: most favorable)

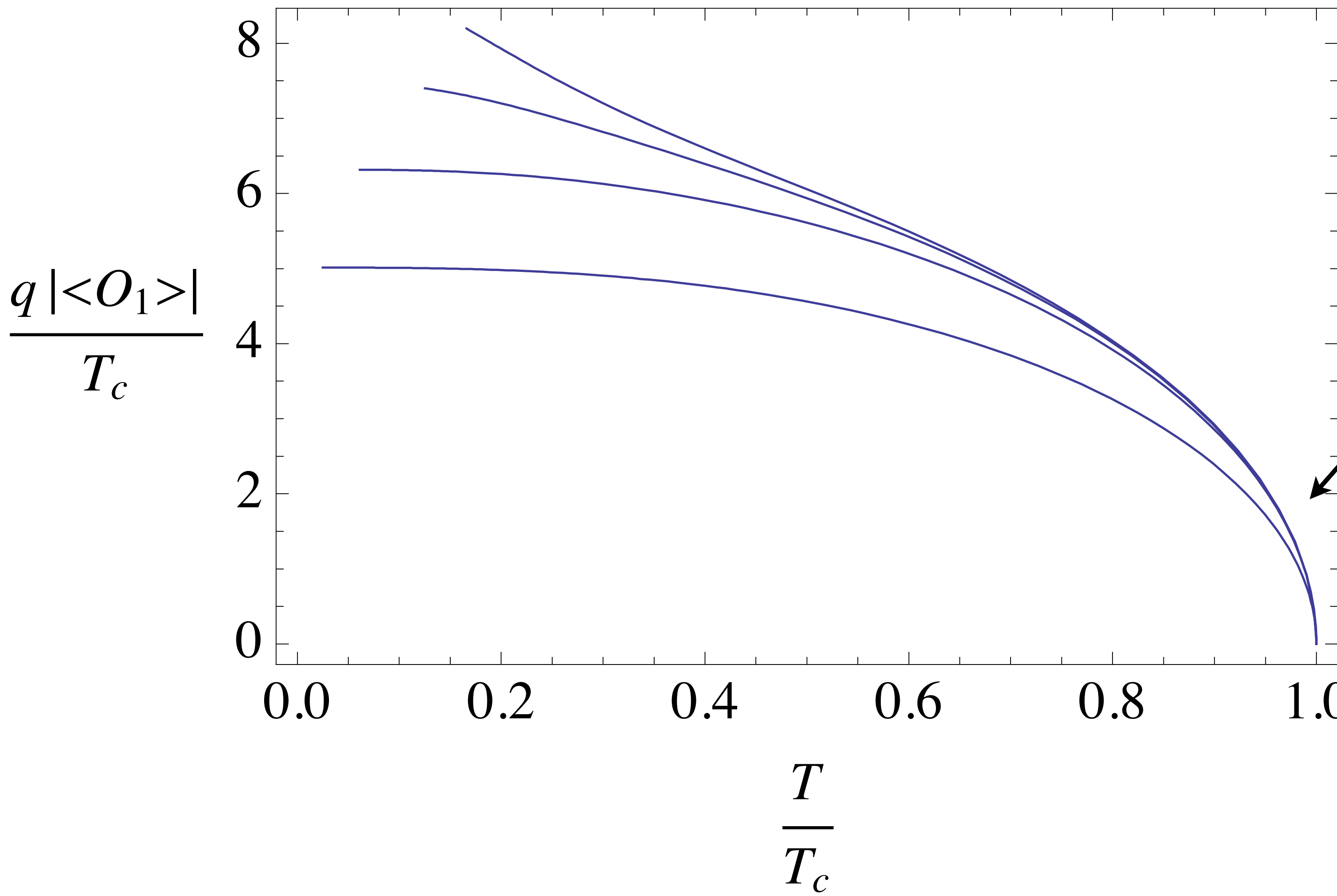
Single vortex: Albash - Johnson, 0906.0519; 0906.1795
Montull - Pomarol - Silva, 0906.2396
vortex lattice: **Maeda - Natsuume - Okamura, 0910.4475**

Hess et al., PRL 62 (1989) 214



Condensate vs T

Adapted from 0810.1563



$\langle \mathcal{O} \rangle \sim (Tc - T)^{1/2}$

typical GL behavior
(β = 1/2)

superconducting phase ←

To study critical phenomena

- Analytic solution: T<Tc unknown

  T>Tc known (RN-AdS)

  - Approach from high temperature
  - Bulk perturb. eq.: the scalar ψ decouples from the rest
    → Enough to solve ψ EOM
  - Linear perturb.

- Analyze both analytically & numerically

  (in "probe" approx)

Exponents

Maeda - Natsuume - Okamura, 0904.1914

- Static exponents

$$(\alpha,\beta,\gamma,\delta,\nu,\eta)=\left(0,\frac{1}{2},1,3,\frac{1}{2},0\right)$$

➡ standard values

exponents we compute

Previously known results:
Herzog - Kovtun - Son, 0809.4870
Maeda - Okamura, 0809.3079
Horowitz - Roberts, 0810.1077

- Dynamic exponent
  - Order parameter: scalar condensate $\mathcal{O}$ → Model A
  - We indeed show z = 2
- Exponents: independent of d → typical mean-field results (Large-N: fluctuations suppressed)

R-charged BHs: non-GL & Model B

R-charged BH

Dual to $\mathcal{N}$=4 SYM at a finite chemical potential

Action: D=5 gauged SUGRA coupled w/ a scalar H

$$\mathcal{L}_5 = \sqrt{-g}\left[R - \frac{L^2}{8}H^{4/3}F_{\mu\nu}^2 - \frac{1}{3}\frac{(\nabla H)^2}{H^2} + \frac{4}{L^2}(H^{2/3} + 2H^{-1/3})\right]$$

Solution:

$$ds_5^2 = -H^{-2/3}fdt^2 + H^{1/3}\left(\frac{dr^2}{f} + r^2 d\vec{x}^2\right)$$

$$f = \frac{r^2}{L^2}\left\{1 + \kappa\left(\frac{r_+}{r}\right)^2 - (1+\kappa)\left(\frac{r_+}{r}\right)^4\right\}$$

$$H = 1 + \kappa\left(\frac{r_+}{r}\right)^2$$

$$A_0 \neq 0$$

$r = r_+$: outer horizon
L: AdS radius

Static properties

- Analytic solution: known
- Thermodynamic quantities: computable
  - f'': R-charge susceptibility $\to \infty$ at $\kappa=2$
  - f': regular there (e.g. s & ρ) $\to$ 2nd order transition
- Static exponents: computable

Cai - Soh (98)
Cvetic - Gubser (99)

  - 2 exponents related to correlation fn: hard to compute

$$(\alpha,\beta,\gamma,\delta,\nu,\eta)=\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},2,?,?\right)$$ ⇨ nonstandard values

cf. GL: $(\alpha,\beta,\gamma,\delta,\nu,\eta)=\left(0,\frac{1}{2},1,3,\frac{1}{2},0\right)$

Not described by conventional Ginzburg-Landau Hamiltonian

Dynamic properties

Maeda - Natsuume - Okamura, 0809.4074

- Order parameter: ρ → Model B

  Expect D→0 at critical pt

- D: computable by Kubo formula from $\langle J_x J_x \rangle$ ⇒ Bulk Ax
- Standard AdS/CFT technique exists to compute it
- Result

$$D = \frac{1}{2\pi T} \frac{(1+\kappa/2)^3}{1+\kappa} \frac{2-\kappa}{2+5\kappa-\kappa^2} \to 0 \qquad \text{as } \kappa \to 2$$

- z? Hard to compute ξ $\qquad \tau_0 \propto \xi^z$

Summary

- Some BHs do obey the theory of critical phenomena
  - We're talking of critical phenomena of BHs
    In presence of gravity, usual notion of statistical mechanics does not hold in general e.g. thermal equilibrium: not stable
  - But AdS/CFT: AdS BHs equivalent to usual statistical systems
- "Holographic superconductors":
  static universality class: standard Ginzburg-Landau theory
  dynamic universality class: Model A
- The R-charge BH:
  static universality class: unconventional
  dynamic universality class: Model B

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Other related works

- Static/dynamic critical phenomena in AdS/CFT
  Jain - Mukherji - Mukhopadhyay 0906.5134
  Buchel - Pagnutti, 0912.3212

- "Holographic superconductors" under magnetic field
  Nakano - Wen, 0804.3180
  Albash - Johnson, 0804.3466
  Keranen - Keski-Vakkuri - Nowling - Yogendran, 0911.1866, 0912.4280

- Yarom's talk, Posters (J. Chen, Garcia-Garcia, Hong, Maeda)...