IPMU Focus Week, Feb.8-12, 2010
Condensed Matter Physics Meets High Energy Physics

Entanglement Entropy and Topological Insulators from String Theory

Tadashi Takayanagi (IPMU)

Mainly based on the works with Shinsei Ryu (Berkeley)

Contents

Mainly Based on

D-branes and Topological Insulators : S. Ryu and TT: arXiv:1001.0763

See also Fujita-Li-Ryu-TT: JHEP 0906:066 [arXiv:0901.0924]

Entanglement Entropy from AdS/CFT:

S. Ryu and TT, PRL 96(2006)181602 [hep-th/0603001]

:

Recent Review: T. Nishioka, S. Ryu and TT,

J.Phys.A42:504008,2009 [arXiv:0905.0932]

① Introduction

As explained in previous excellent talks, recently there are many interesting applications of string theory to CMP mainly via AdS/CFT.

For example, AdS/CFT (or Holography) can describe

- Superfluid and Superconductivity

  [Hartnoll, Minwalla, Nakamura, Natsuume, Yarom's talk]
- Fermi Liquids/Non-Fermi Liquids [Hartnoll, Liu's talk]
- Non-relativistic Fixed Points [Kachru, Kiritsis, Nakayama, Son's talk]

  :

  :

(i) How about topological phases ?

[Aoki, Fujimoto, Hatsugai, Kitaev, Nomura, Read, Sato, Wen, Zhang's talk]

→ Let us start with topological insulators (quantum Hall effects etc.).

Moreover, AdS/CFT can compute

- Free energy (→ Phase diagrams)
- Specific heat
- Conductivity
- Viscosity
- Various other transportation coefficients

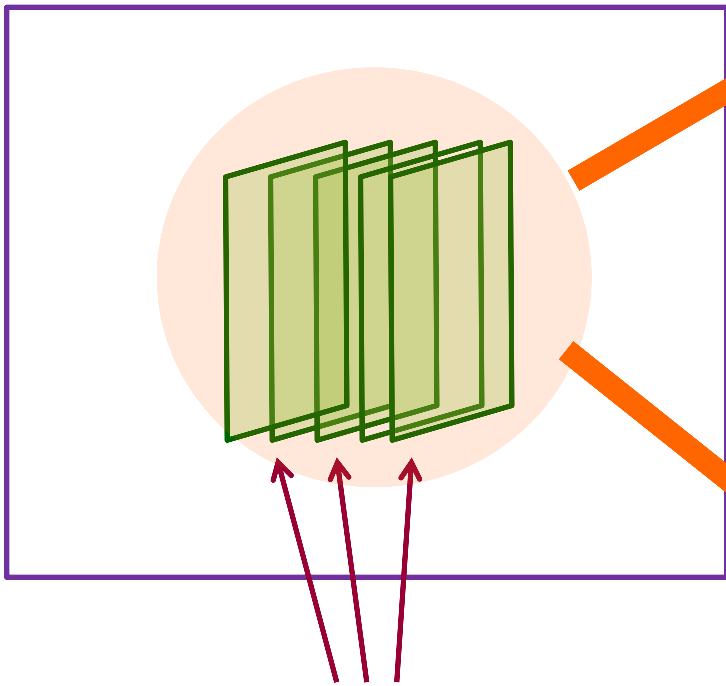
:

:

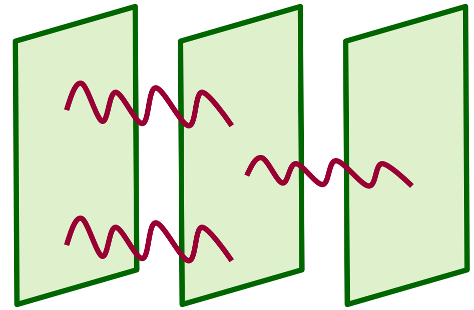
(ii) As we will explain, we can also calculate the entanglement entropy, which plays a role of order parameter in quantum phase transitions.

<u>The basic idea of AdS/CFT</u> ex. AdS5/CFT4

10 dimensional type IIB string theory
with N **D3-branes**



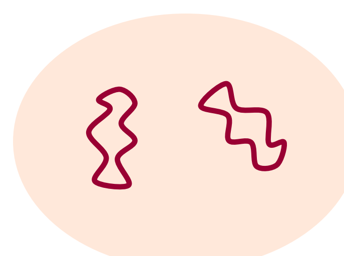
N D3-branes
= (3+1) dimensional sheets



Open Strings between D-branes
→ SU(N) gauge theories

Equivalent !

Type IIB closed string on AdS5 × S5
→ Gravity on AdS5 spacetime



AdS/CFT
(Holography)

$$\boxed{\text{Gravity on } M = \text{Large N Gauge Theory on } \partial M}$$

In this way, the existence of D-branes in string theory is the origin of the AdS/CFT correspondence.

Thus, the first step to find a gravity dual of a given quantum many-body system is to understand its string theory description using D-branes.

② D-branes and Topological Insulators [Ryu-TT 10’]

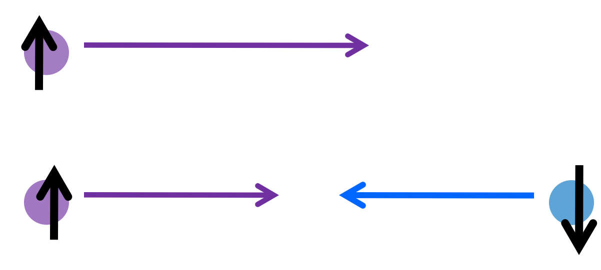
Recently, insulators (or superconductors) with non-trivial topological properties have been discussed and realized in real materials (BiSb, BiTe, BiSe, HgTe etc.). [e.g. Zhang’s talk]

They are called
Topological Insulators/superconductors.

Boundary: gapless
(massless fermions)
Edge State

Basic examples: Quantum Hall Effect (Z)

Quantum Spin Hall Effect (Z_2)



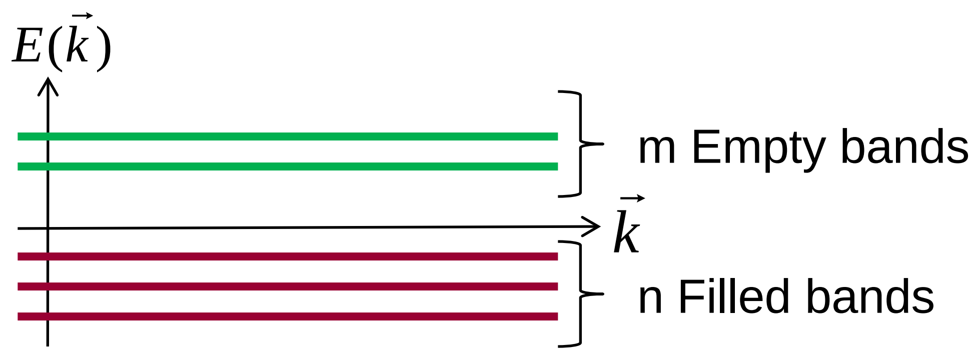
The topological insulators have recently been classified by studying the topological property of the map [Schnyder-Ryu-Furusaki-Ludwig 08', Kitaev's talk]

k-space (Brillouin zone) → Hamiltonian with Randomness

S^d (or T^d)

↕

10 class of symmetric spaces



$$\text{e.g.}\quad \text{Class A}: \ \text{Grasmannian}\ \frac{U(n+m)}{U(n)\times U(m)}$$

Mathematically, the classification of such maps is equivalent to the K-theory, which classifies the difference of two vector bundles.

[Kitaev 09']

classification of topological insulators/superconductors

spatial dimensions

presence/absence
of topological band structure

AZ\d	0	1	2	3	4	5	6	7	8	9
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	...
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	...
AI	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	...
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	...
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	...
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	...
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	...
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	...
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	...
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	...

Type II String (rows A, AIII)

Type I String (rows AI–CI)

10=2+8 symmetry classes of quadratic fermionic Hamiltonians (Altland-Zirnbauer)

$\mathbb{Z}$ integer classification

$\mathbb{Z}_2$ Z2 classification

0 no top. ins./SC

10 = 2+8 = # of classes of topological insulators
= The critical dimension of superstring theory

classification of topological insulators/superconductors

IQHE

p+ip wave SC

polyacetylene

3He B

TMTSF

AZ\\d	0	1	2	3	4	5	6	7	8	9
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\cdots$
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\cdots$
AI	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\cdots$
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\cdots$
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\cdots$
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\cdots$
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$\cdots$
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$

Z2 topological insulator

QSHE

d+id wave SC

On the other hand, D-branes in string theory have also been classified by K-theory based on the tachyon condensation [Sen, Witten 98'].

	D(-1)	D0	D1	D2	D3	D4	D5	D6	D7	D8	D9
Type IIB	Z	0	Z	0	Z	0	Z	0	Z	0	Z
$O9^-$(Type I)	Z_2	Z_2	Z	0	0	0	Z	0	Z_2	Z_2	Z
$O9^+$	0	0	Z	0	Z_2	Z_2	Z	0	0	0	Z

D(-1)-$\overline{\text{D(-1)}}$ **Non-BPS D0-brane** D-brane charges

The ten fold classification of topological insulators

⟺ The classification of Hamiltonian of massive fermions with certain symmetries imposed [Bernard-LeClair 02']

(i) PHS (=C) ⇔ Orientation Ω projection (Sp and SO)

(ii) SLS (sublattice) ⇔ Parity (inversion) I

(iii) TRS (=T) ⇔ Orientifold action Ω･I

Indeed, we can realize the topological insulators in string theory by considering Dp-Dq systems (+ Orientifold) !

(i) Dp-Dq system in Type II

$\rightarrow$ Class A and AIII (Complex K-group K(X))

d	brane	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	Charge	Fermion	CS-term
Dp	D5	*	*	*	*	*	*							
0	D3	*						*	*	*		$\mathbf{Z}$	2 Real	$\int A$
2	D5	*	*	*				*	*	*		$\mathbf{Z}$	2 Maj	$\int AF$
4	D7	*	*	*	*	*		*	*	*		$\mathbf{Z}$	1 Dirac	$\int AFF$

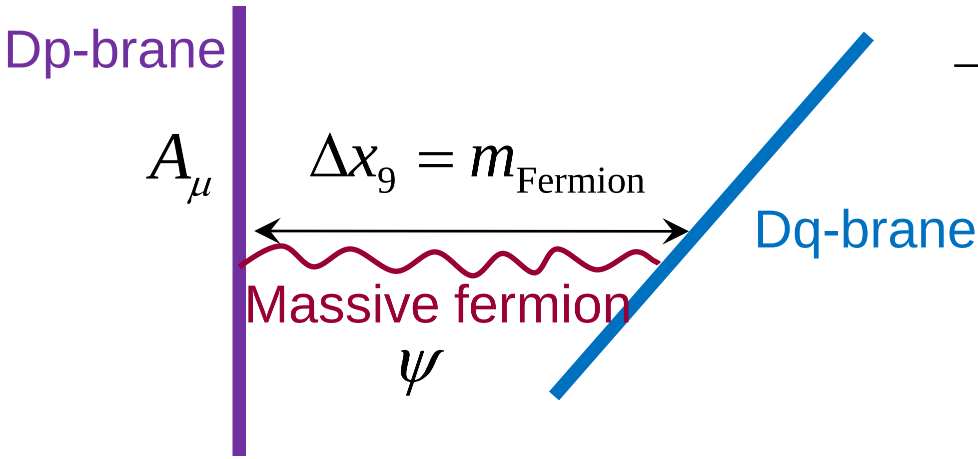
Table 1: Class A Topological Insulators from Intersecting D-branes $\boxed{\texttt{aintd}}$

d	brane	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	Charge	Fermion
Dp	D4	*		*	*	*	*						
1	D4	*		*				*	*	*		$\mathbf{Z}$	2 Maj
3	D6	*		*	*	*		*	*	*		$\mathbf{Z}$	2 Maj

Table 2: Class AIII Topological Insulators from Intersecting D-branes $\boxed{\texttt{aintdd}}$

Ex. Class A d=2 (IQHE) [See also Davis-Kraus-Shah 08', Fujita-Li-Ryu-TT 09']

p=q=5



$$L = \overline{\psi}[\gamma^\mu (i\partial_\mu - A_\mu) - m]\psi + \ldots,$$

$$\rightarrow \quad \frac{m}{4\pi |m|} \int A \wedge dA = \int F \wedge F \wedge C_{RR}^{(2)}$$

Hall Conductivity:

$$\sigma_{xy} = \frac{m}{4\pi |m|} = \pm \frac{e^2}{2h}$$

(ii) Dp-Dq-O9,O8 systems

→ The other eight classes (Real K-group KO(X))

d	brane	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	$O9^-$	$O9^+$
Dp	D5	*	*	*	*	*	*					C	D
0	D3	*						*	*	*		0	$\mathbf{Z}_2$ (2 Real)
1	D4	*	*					*	*	*		0	$\mathbf{Z}_2$ (1 Maj)
2	D5	*	*	*				*	*	*		$\mathbf{Z}$ (4 Maj)	$\mathbf{Z}$ (1 Maj)
3	D6	*	*	*	*			*	*	*		0	0
4	D7	*	*	*	*	*		*	*	*		$\mathbf{Z}_2$ (2 Dirac)	0

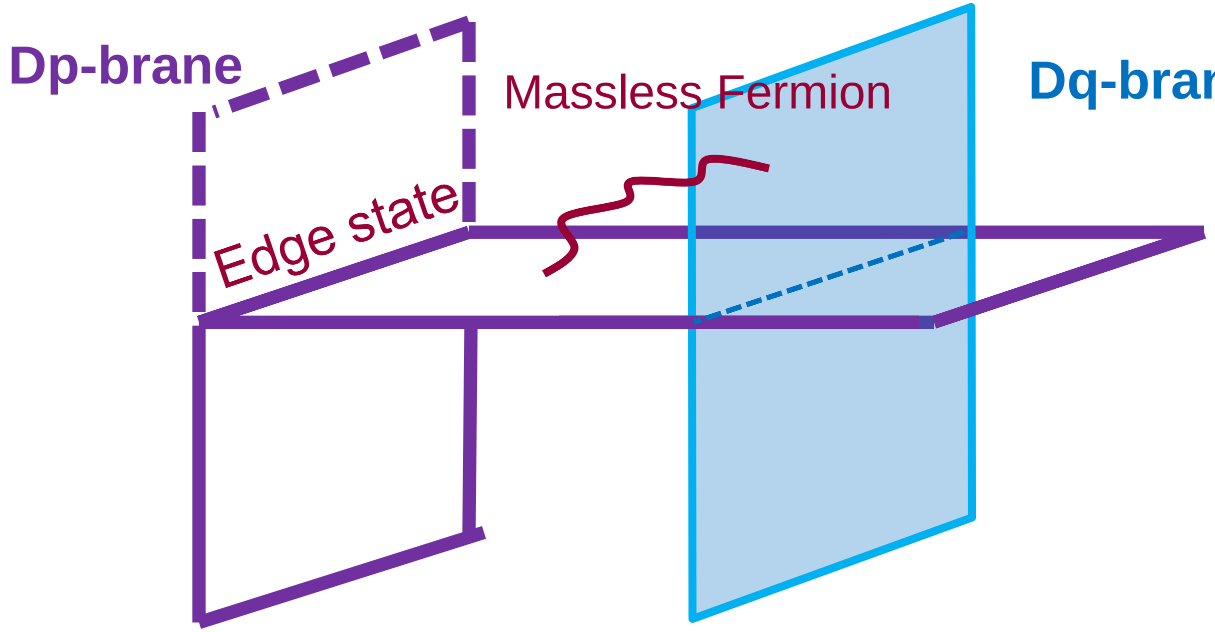
Table 1: Class C and D topological Insulators from Intersecting D-branes

d	brane	x^0	x^1	x^2	x^3	x^4	$x^5(O8)$	x^6	x^7	x^8	x^9	$O8^-$	$O8^+$
Dp	D4	*	*	*	*	*						AII	AI
0	D4	*						*	*	*	*	$\mathbf{Z}$ (4 real)	$\mathbf{Z}$ (1 real)
1	D5	*	*					*	*	*	*	0	0
2	D6	*	*	*				*	*	*	*	$\mathbf{Z}_2$ (4 Maj)	0
3	D7	*	*	*	*			*	*	*	*	$\mathbf{Z}_2$ (2 Maj)	0
4	D8	*	*	*	*	*		*	*	*	*	$\mathbf{Z}$ (1 Dirac)	$\mathbf{Z}$ (1 Dirac)

Table 2: Class AI and AII topological Insulators from Intersecting D-branes . The O8-plane extends except x^5.

In summary, we can realize all ten classes of d dim. topological insulators with d=0,1,2,3,4 from the D-brane systems.

It is also easy to find the boundary gapless modes (edge states) as follows:



Two comments:

(i) Interestingly, they exhaust all possible Dp-Dq brane systems without tachyons in string theory (with some minor exceptions).

(ii) The existence of Z_2 Top. insulators in real world requires stable non-BPS D-branes in string theory.

[Bergman-Gaberdiel 98', Sen 98']

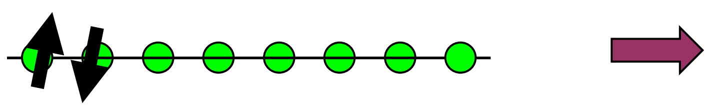
③ Entanglement Entropy from AdS/CFT

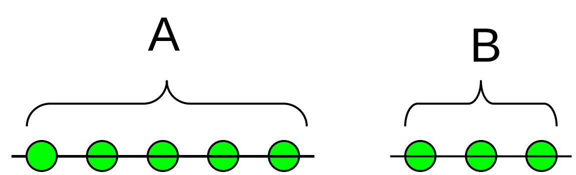
(3-1) What is entanglement entropy ?

Divide a given quantum system into two parts A and B.
Then the total Hilbert space becomes factorized

$$H_{tot} = H_A \otimes H_B \quad .$$

Example: Spin Chain





We define the reduced density matrix ρ_A for A by

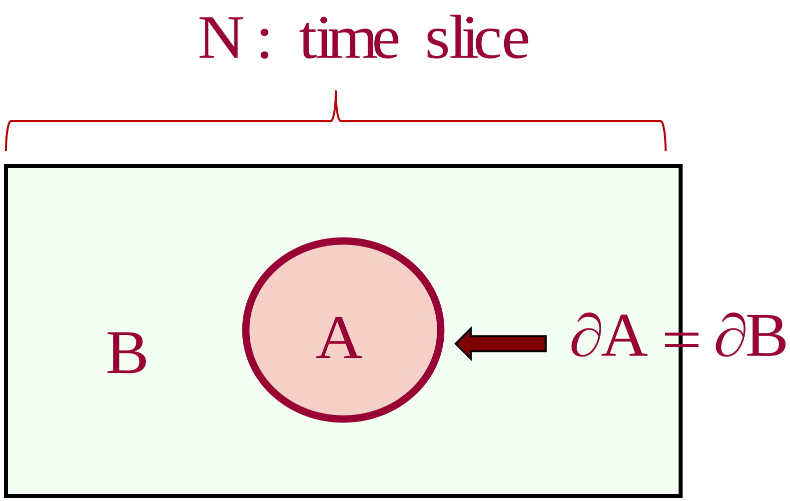
$$\rho_A = \mathrm{Tr}_B \rho_{tot} \quad ,$$

taking trace over the Hilbert space of B .

Now the entanglement entropy S_A is defined by the von-Neumann entropy

$$S_A = -\mathrm{Tr}_A\, \rho_A \log \rho_A \quad .$$

In QFTs, it is defined geometrically (called geometric entropy).



The entanglement entropy (EE) measures how A and B are entangled quantum mechanically.

(1) EE is the entropy for an observer who is only accessible to the subsystem A and not to B.

(2) EE is a sort of a `non-local version of correlation functions', which captures some topological information. (cf. Wilson loops)

(3) EE is proportional to the degrees of freedom.
It is non-vanishing even at zero temperature.

Various Applications

- Quantum Information and Quantum Computing

EE = the amount of quantum information

[see e.g. Nielsen-Chuang's text book 00']

- Condensed matter physics

EE = how much difficult to perform a computer simulation (DMRG)

[Gaite 03',…]

$\Rightarrow$ This gets divergent at phase transition point !

$\Rightarrow$ A new quantum order parameter !

- Quantum Gravity and String Theory

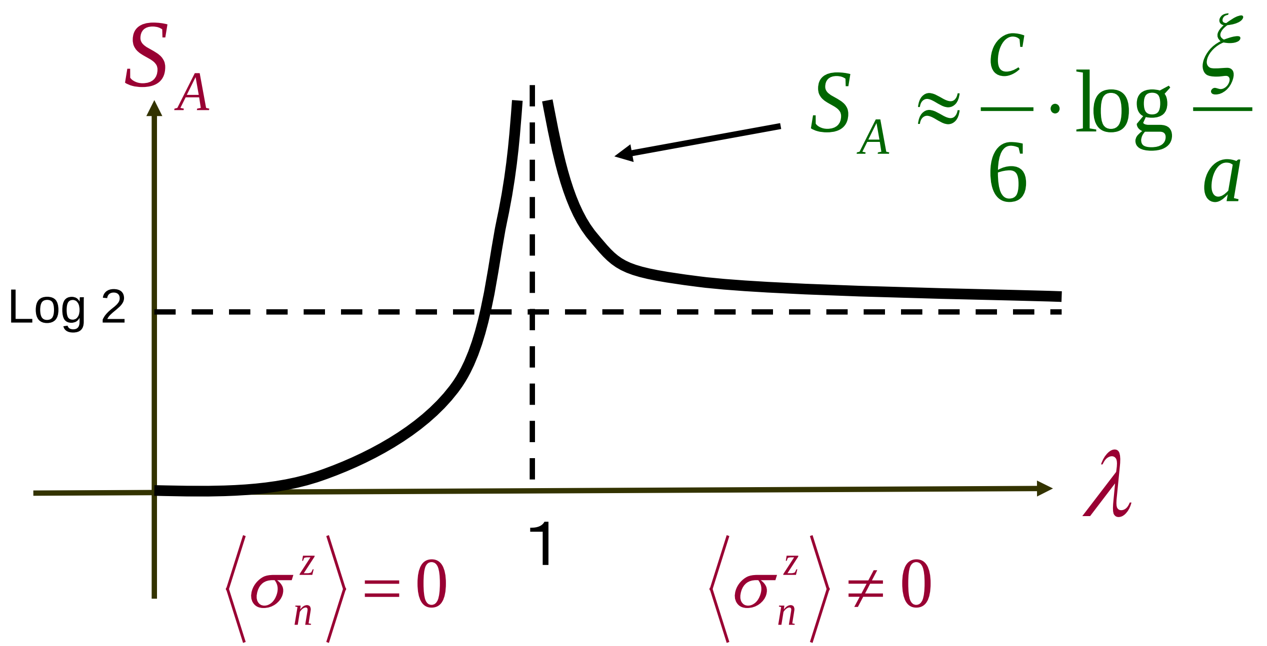
EE $\Leftrightarrow$ Black hole entropy ? (A partial answer will be given in this talk.)

A Basic Example

[02' Vidal et.al.]

A phase transition occurs in the quantum Ising model at zero temperature when we change the magnetic field.

$$H = -\sum_n \sigma_n^x - \lambda \sum_n \sigma_n^z \sigma_{n+1}^z$$



Basic property: Area law

The EE in d+1 dim. QFTs includes UV divergences.

Its leading term is proportional to the area of the (d-1) dim. boundary $\partial \mathrm{A}$

$$S_A \sim \frac{\mathrm{Area}(\partial \mathrm{A})}{a^{d-1}} + (\text{subleading terms}),$$

[Bombelli-Koul-Lee-Sorkin 86', Srednicki 93']

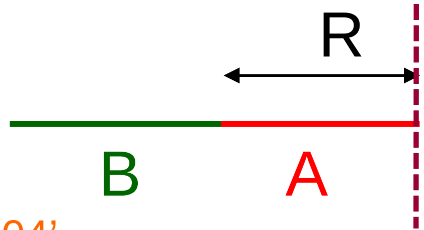
where a is a UV cutoff (i.e. lattice spacing).

Very similar to the Bekenstein-Hawking formula of black hole entropy

$$S_{BH} = \frac{\mathrm{Area}(\text{horizon})}{4G_N}.$$

Ex. 2D CFT

The entanglement entropy in 2D CFT (with a boundary) looks like



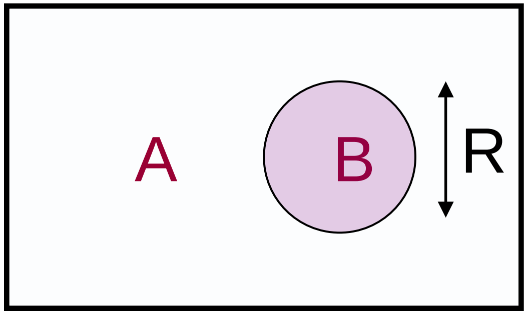
[Holzhey-Larsen-Wilczek 94' ; Calabrese-Cardy 04',
Recent review: Calabrese-Cardy arXiv:0905.4013]

$$S_A = \frac{c}{3}\log\left(\frac{R}{a}\right) + \log g ,$$

where c is the central charge and g is the boundary entropy introduced by Affleck-Ludwig.

[Holographic Calculation: Azeyanagi-Karch-Thompson-TT 07']

Ex. Entanglement entropy in 3D QFT



The general structure:

$$S_A = \gamma \cdot \frac{R}{a} - \log D,$$

UV divergence
(Area law)
a=cut off length

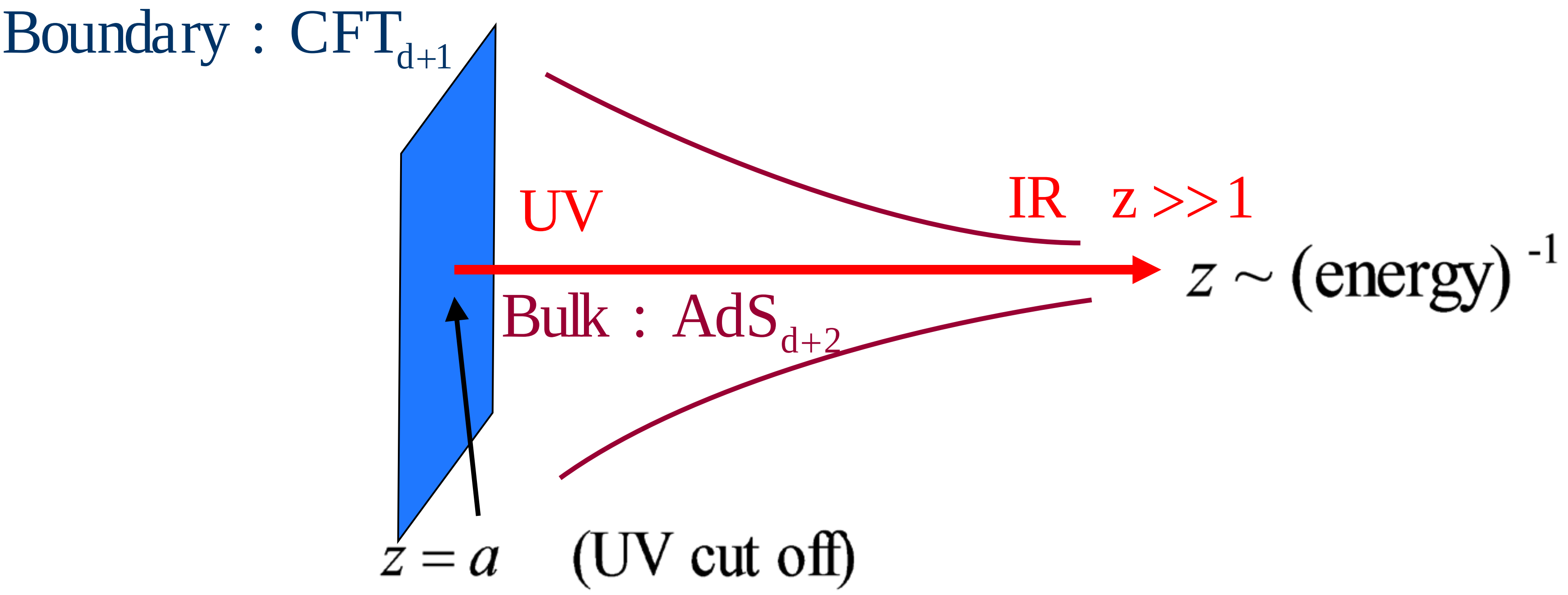
Universal finite constant
D: total quantum dimension
In a massive theory, this becomes topological i.e. does not change under small deformation of B.

Topological Entanglement Entropy
[Kitaev-Preskill, Levin-Wen 05']

(3-2) Entanglement Entropy from AdS/CFT

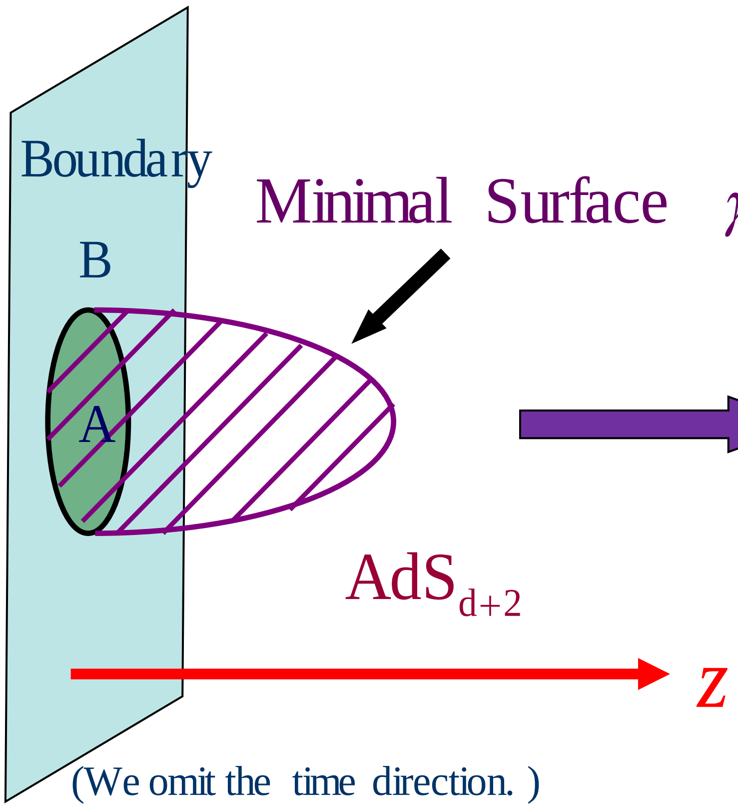
Consider AdS/CFT correspondence in Poincare coordinate:

$$ds^2 = R^2 \frac{dz^2 - dx_0^2 + \Sigma_{i=1}^{d-1} dx_i^2}{z^2}.$$



<u>Holographic Entanglement Entropy</u> [Ryu-TT 06']

The holographic formula of entanglement entropy S_A is given by the area of minimal surface γ_A whose boundary coincides with ∂A .

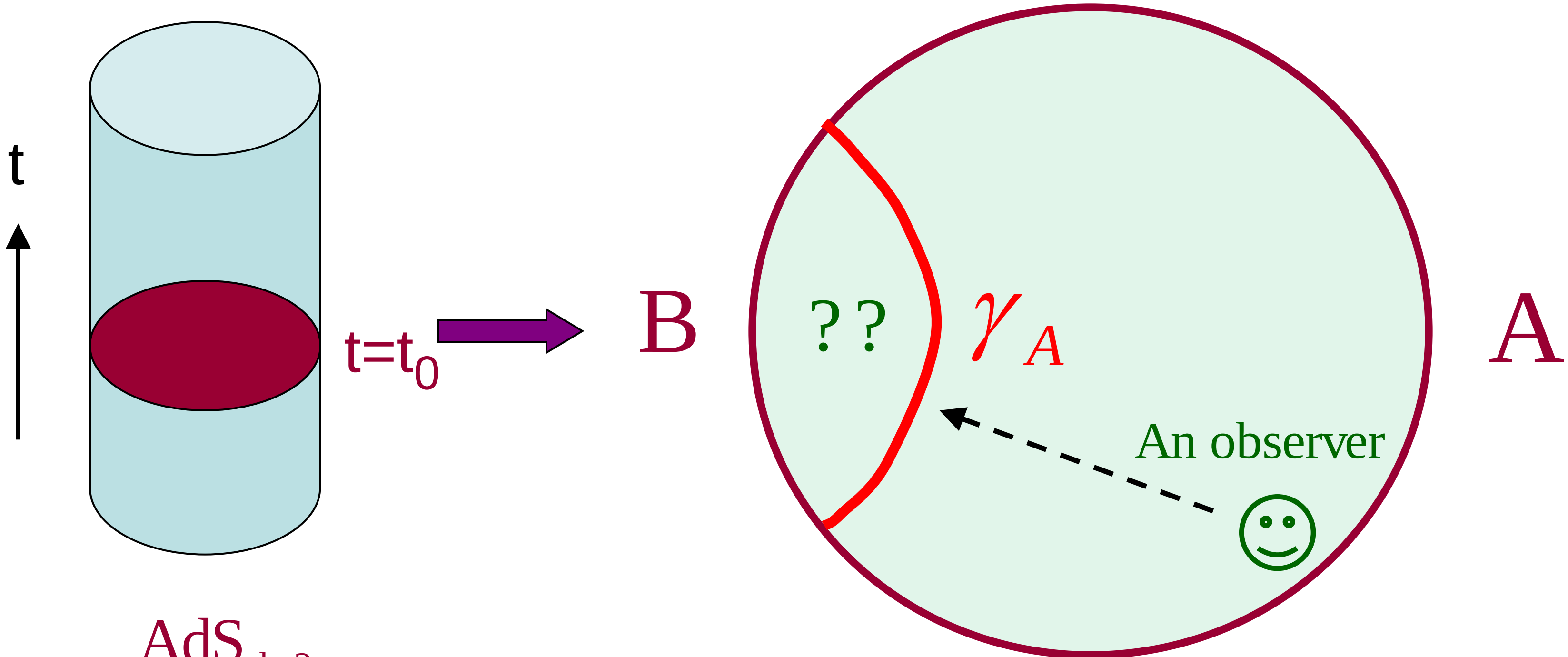


$$\boxed{S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N}}$$

(`Bekenstein-Hawking formula' when γ_A is the horizon.)

Heuristic Interpretation

Here we employ the global coordinate of AdS space and take its time slice at $t=t_0$.



AdS_{d+2}

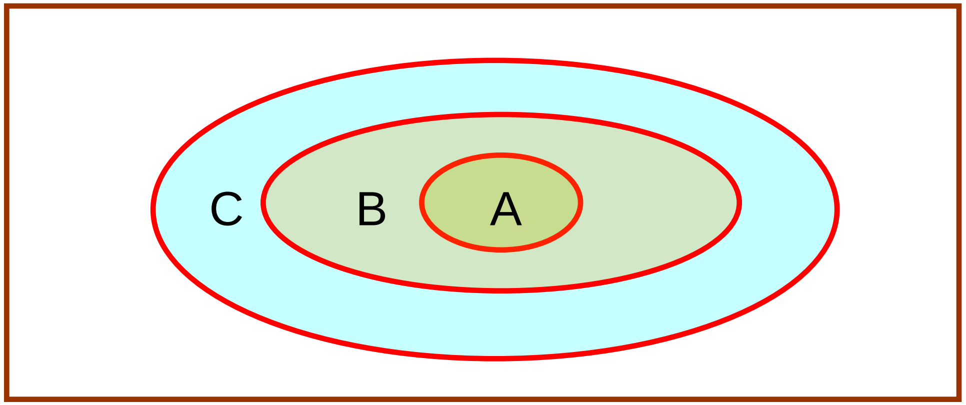
in global Coordinate

Minimal area surface gives the most strict entropy bound (Bousso bound is saturated).

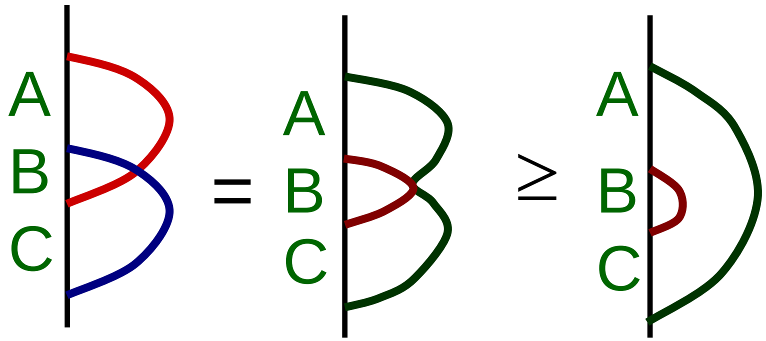
Holographic Proof of Strong Subadditivity [06' Hirata-TT, 07' Headrick-TT]

The strong subadditivity is known as the most important inequality satisfied by EE. [Lieb-Ruskai 73']

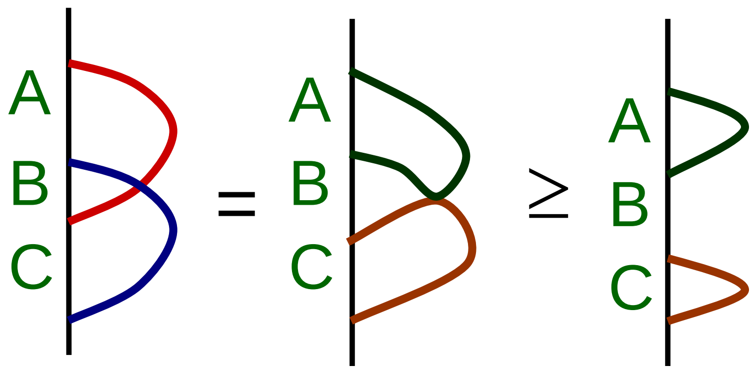
$$S_{A+B} + S_{B+C} \geq S_{A+B+C} + S_{B} \quad ,$$
$$S_{A+B} + S_{B+C} \geq S_{A} + S_{C} \quad .$$



The holographic proof of this inequality is very quick and intuitive !!



$$\Rightarrow S_{A+B} + S_{B+C} \geq S_{A+B+C} + S_B$$



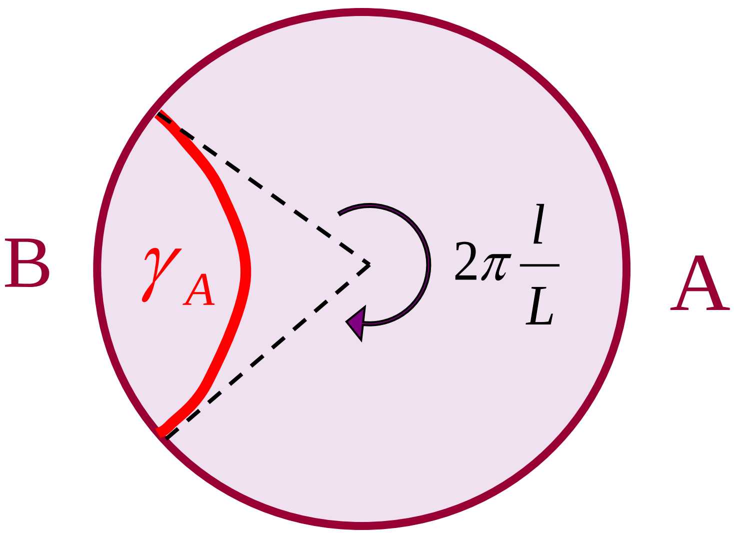
$$\Rightarrow S_{A+B} + S_{B+C} \geq S_A + S_C$$

(3-3) EE from AdS3/CFT2

Consider AdS3 in the global coordinate

$$ds^2 = R^2(-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\theta^2).$$

In this case, the minimal surface is a geodesic line which starts at $\theta = 0, \quad \rho = \rho_0$ and ends at $\theta = 2\pi l / L, \quad \rho = \rho_0$ ($\rho = \rho_0 \to \infty : \quad \text{UV cut off}$) .



The length of γ_A , which is denoted by $|\gamma_A|$, is found as

$$\cosh\frac{|\gamma_A|}{R} = 1 + 2\sinh^2\rho_0 \sin^2\frac{\pi l}{L}.$$

Thus we obtain the prediction of the entanglement entropy

$$S_A = \frac{|\gamma_A|}{4G_N^{(3)}} = \frac{c}{3}\log\left(e^{\rho_0} \sin\frac{\pi l}{L} \right),$$

where we have employed the celebrated relation

$$c = \frac{3R}{2G_N^{(3)}}.$$

[Brown-Henneaux 86’]

Furthermore, the UV cutoff a is related to ρ_0 via $e^{\rho_0} \sim \frac{L}{a}.$

In this way we reproduced the known formula

[94’ Holzhey-Larsen-Wilczek, 04’ Calabrese-Cardy]

$$S_A = \frac{c}{3}\log\left(\frac{L}{a}\sin\frac{\pi l}{L} \right).$$

(3-4) Finite temperature case

We assume the length of the total system is infinite.

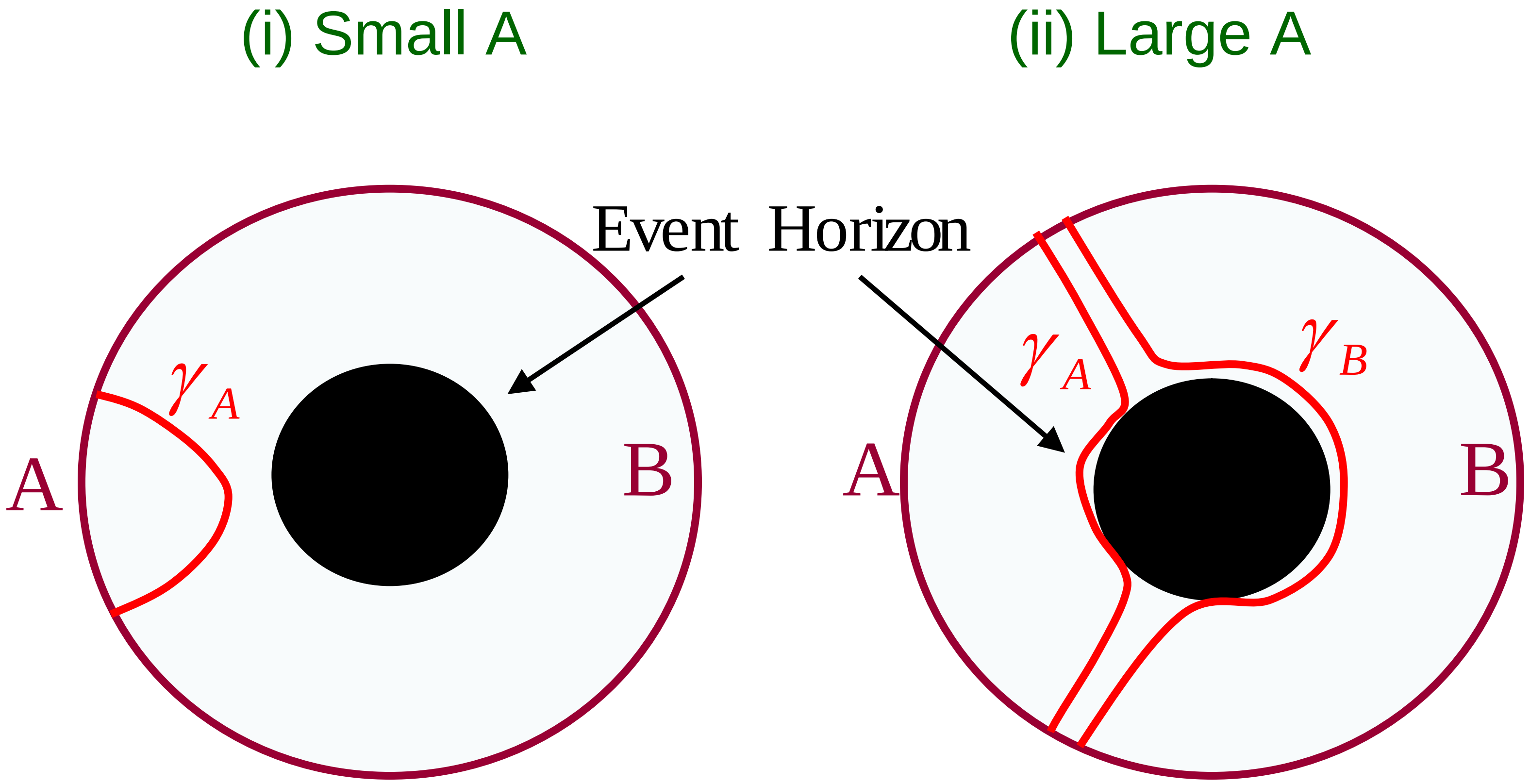
In this case, the dual gravity background is the BTZ black hole and the geodesic distance is given by

$$\cosh\frac{|\gamma_A|}{R} = 1 + 2\cosh^2\rho_0 \sinh^2\frac{\pi l}{\beta}.$$

This again reproduces the known formula at finite T.

$$S_A = \frac{c}{3}\log\left(\frac{\beta}{a}\sinh\left(\frac{\pi l}{\beta}\right)\right) .$$

Geometric Interpretation



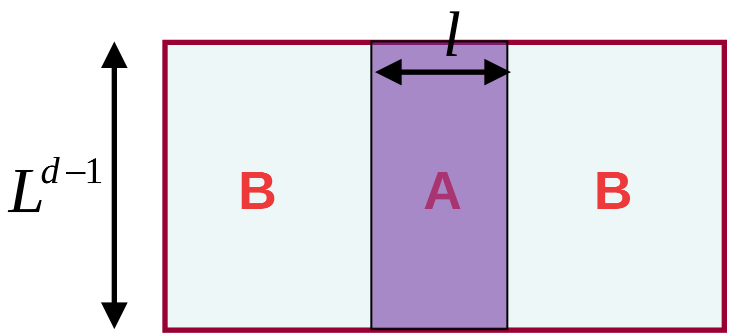
$S_A \neq S_B$ *if* ρ_{tot} is not pure.

(3-5) EE in Higher Dimension

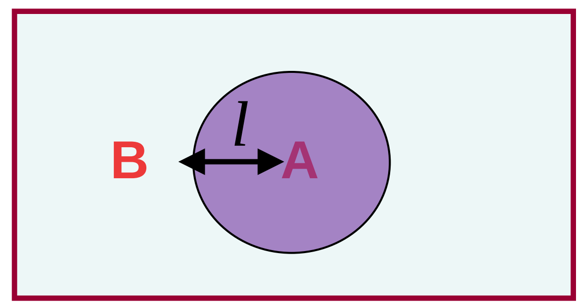
Since it is very complicated to compute EE in higher dim., our AdS/CFT provides a powerful analytical method for this purpose.

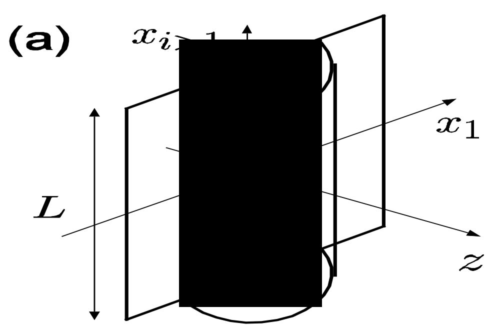
Two examples of the subsystem A:

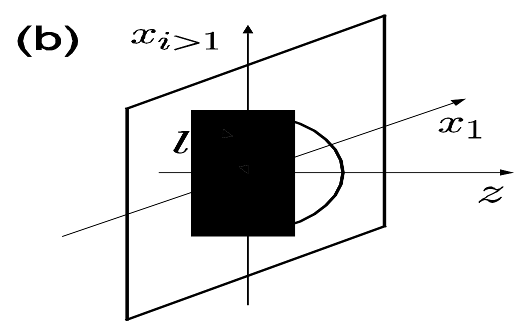
(a) Infinite strip



(b) Circular disk







Entanglement Entropy for (a) Infinite Strip from AdS

$$S_A = \frac{R^d}{2(d-1)G_N^{(d+2)}}\left[\underbrace{\left(\frac{L}{a}\right)^{d-1}} - C\cdot\underbrace{\left(\frac{L}{l}\right)^{d-1}}\right],$$

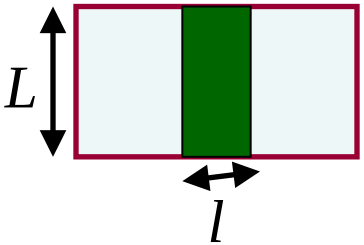
where $\quad C = 2^{d-1}\pi^{d/2}\left(\Gamma\left(\frac{d+1}{2d}\right)\Big/\Gamma\left(\frac{1}{2d}\right)\right)^{d}.$

Area law divergence

This term is finite and does not depend on the UV cutoff.

Basic Example of AdS5/CFT4

$$\mathrm{AdS}_5 \times \mathrm{S}^5 \Leftrightarrow N = 4\ \mathrm{SU(N)\ SYM}$$



$$CFT: \quad S_A^{freeCFT} = K \cdot \frac{N^2 L^2}{a^2} - 0.087 \cdot \frac{N^2 L^2}{l^2}.$$

$$Gravity: \quad S_A^{AdS} = K' \cdot \frac{N^2 L^2}{a^2} - 0.051 \cdot \frac{N^2 L^2}{l^2}.$$

The order one deviation is expected since the AdS result corresponds to the strongly coupled Yang-Mills.

[cf. 4/3 in thermal entropy, Gubser-Klebanov-Peet 96']

Entanglement Entropy for (b) Circular Disk from AdS

$$S_A = \frac{\pi^{d/2} R^d}{2G_N^{(d+2)}\Gamma(d/2)}\left[\underline{p_1\left(\frac{l}{a}\right)^{d-1}} + p_3\left(\frac{l}{a}\right)^{d-3} + \cdots\right.$$

$$\left. \cdots + \begin{cases} p_{d-1}\left(\frac{l}{a}\right) + \underline{p_d} & (\text{if } d = \text{even}) \\ p_{d-2}\left(\frac{l}{a}\right)^2 + \underline{q\log\left(\frac{l}{a}\right)} & (\text{if } d = \text{odd}) \end{cases}\right],$$

Area law divergence

where $p_1 = (d-1)^{-1}, p_3 = -(d-2)/[2(d-3)], \ldots.$

$\ldots.. \quad q = (-1)^{(d-1)/2}(d-2)!!/(d-1)!! \quad .$

A universal quantity which characterizes odd dimensional CFT

Conformal Anomaly (~central charge)

Log term in 4d CFT

Our method based on AdS/CFT predicts

$$S_A = k \cdot \frac{l^2}{a^2} - Q \cdot \log \frac{l}{a} \ .$$

[Ryu-TT 06', Casini-Huerta 09']

The coefficient Q is given by $Q = 4a$ in terms of central charge **a**.

Ex. Free scalar field in 4d

$$a = \frac{1}{360}, \quad Q = \frac{1}{90} \quad \rightarrow$$ Recently confirmed to 0.2% accuracy

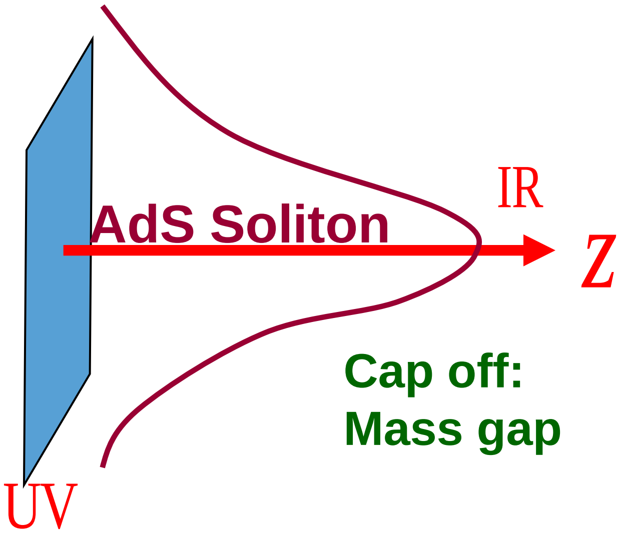
[Lohmayer-Neuberger-Schwimmer-Theisen 09'
cf. Schwimmer-Theisen 08']

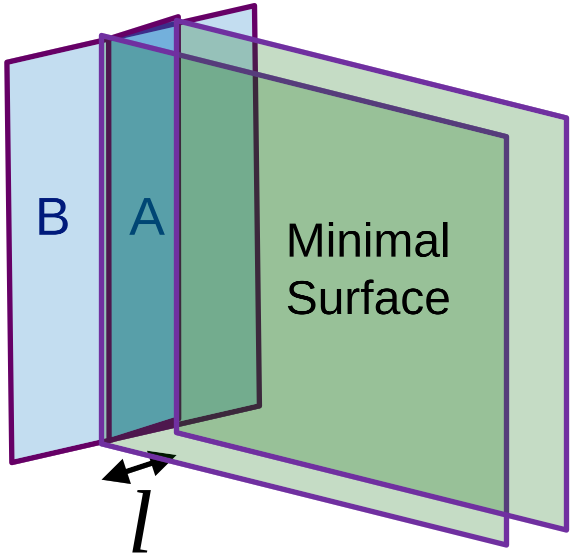
(cf. N=4 large N SYM: $Q = N^2$)

(3-6) EE as Order Parameter of Confining Gauge Theories

As a final example, we will show that EE will play a role of order parameter of confinement/deconfinement phase transition.

One of the simplest gravity dual of the confining gauge theory is known as the AdS soliton. In particular, consider the AdS5 soliton dual to the (2+1) dim. pure SU(N) gauge theory (~ insulator in cond-mat) .



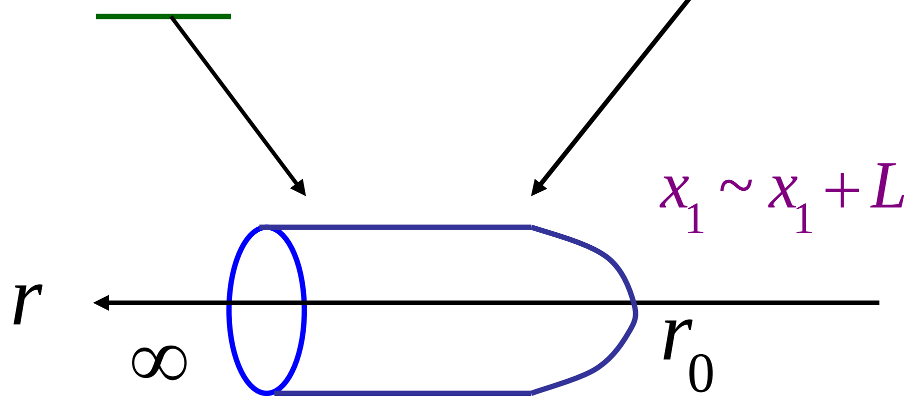


The metric of AdS soliton is given by the double Wick rotation of the AdS black hole solution.

$$ds^2_{\text{AdS BH}} = \frac{R^2 dr^2}{r^2 f(r)} + \frac{r^2}{R^2}(-f(r)dt^2 + dx_1^2 + dx_2^2 + dx_3^2),$$

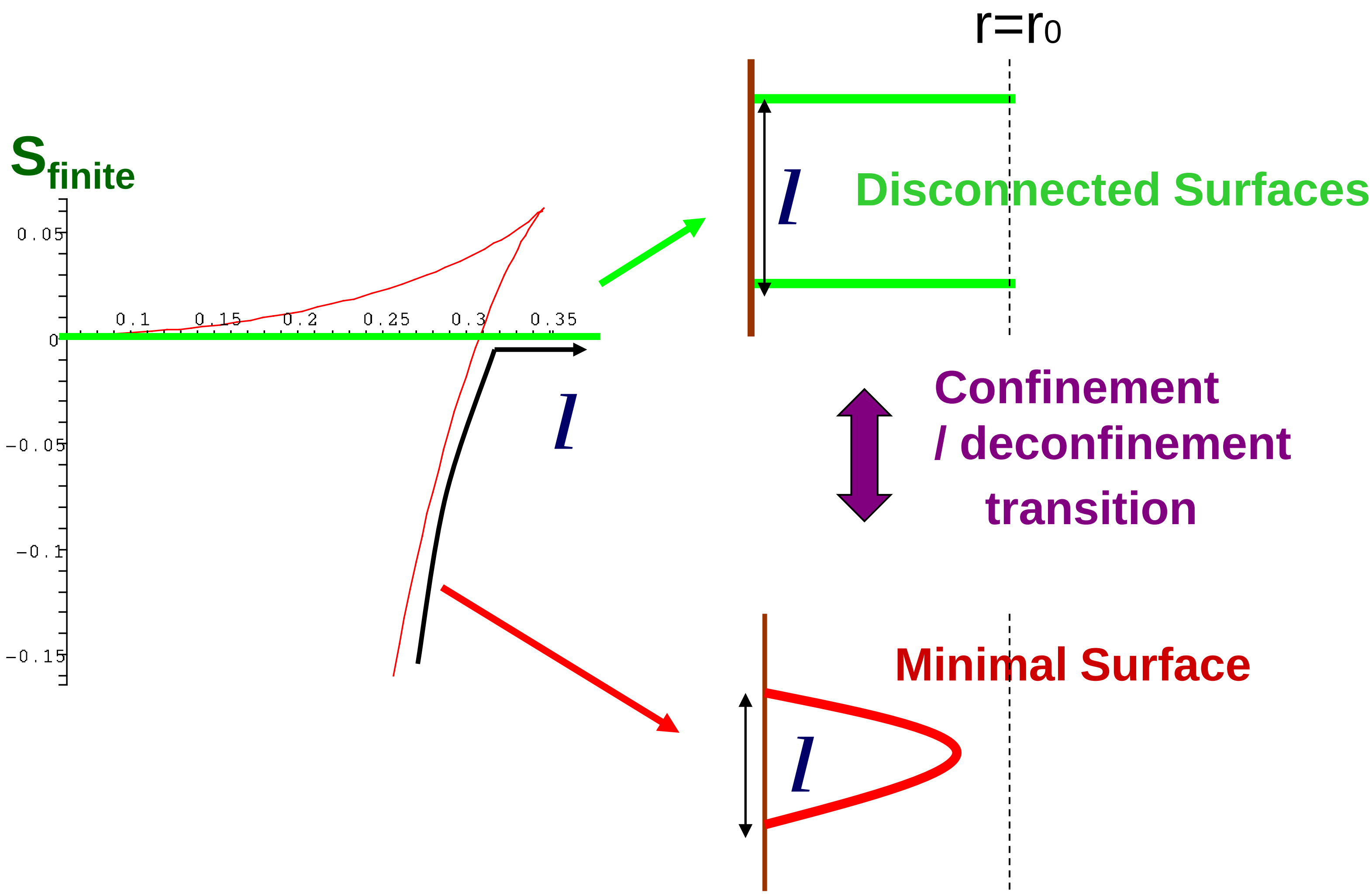
$$f(r) \equiv 1 - \frac{r_0^4}{r^4},$$

$$ds^2_{\text{AdS Soliton}} = \frac{R^2 dr^2}{r^2 f(r)} + \frac{r^2}{R^2}(-dt^2 + f(r)dx_1^2 + dx_2^2 + dx_3^2),$$



In the holographic calculation, two different surfaces compete and this leads to the phase transition.

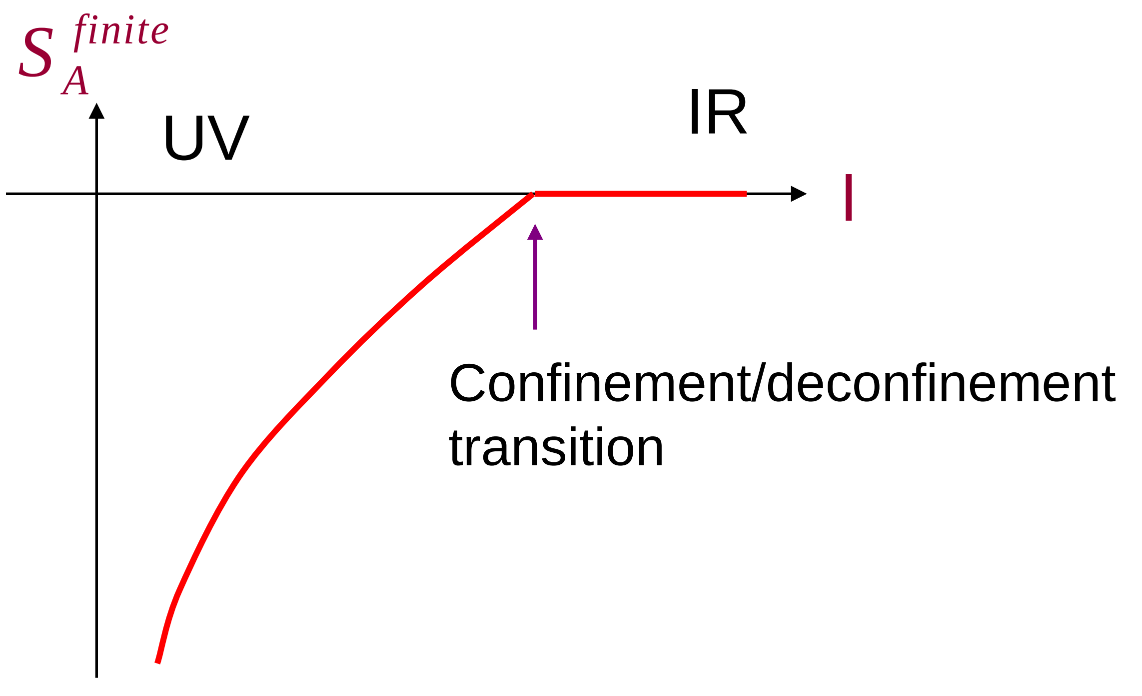
[Nishioka-TT 06’, Klebanov-Kutasov-Murugan 07’]



In summary, we find the following behavior

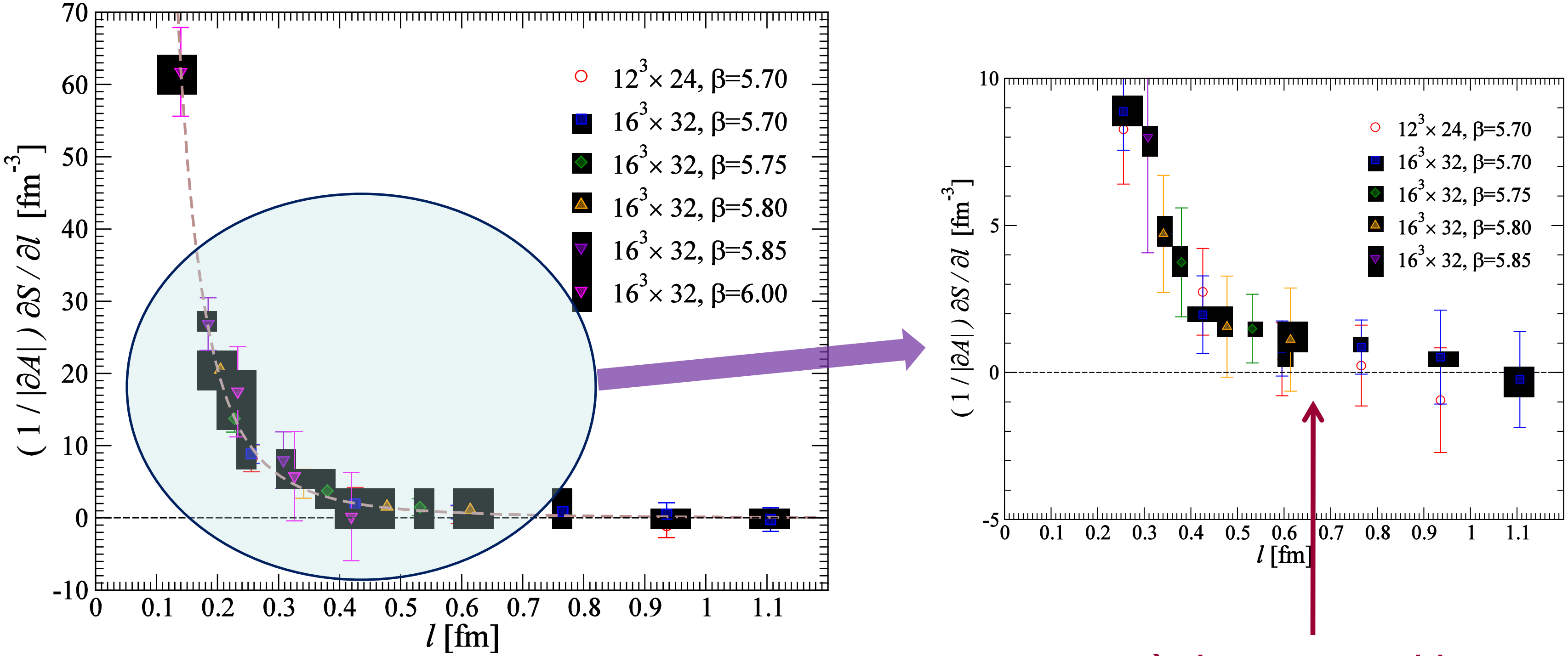
$$S_A^{finite} \approx -N^2 \cdot \frac{L^2}{l^2} \qquad (l \to 0:\ \text{Asymptotic Free}),$$

$$S_A^{finite} \approx 0 \cdot N^2 + O(1) \qquad (l \to \infty : \text{Confined})$$



Recent Lattice Results for 4D Pure SU(3) YM

[Nakagawa-Nakamura-Motoki-Zakharov 09']



`Phase Transition'

④ Conclusions

- We constructed brane systems which realizes top. insulators.

The K-theory classification of D-branes agrees that of top. insulators without interactions.

⇒ Can we take interactions into account ?

⇔ Open string tachyon condensation on edges states ?

How about the randomness ? (Ghost D-branes?)

- We explained how we can calculate entanglement entropy via

AdS/CFT, which will be a powerful tool especially in 2d or 3d systems. We also find that it is a nice order parameter of phase transition.

von-Neumann entropy = Area of minimal area surface in AdS

⇒ Can we compute the topological entanglement entropy ?

⇔Topological terms in the bulk supergravity ?