IPMU Focus Week
Condensed Matter meets High Energy Physics

Gauge & gravitational anomalies in graphene-related systems

National Institute for Materials Science

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and topological field theories

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Acknowledgments (collaborators on related work)
K. Totsuka (YITP, Kyoto)
J. Inoue (NIMS)

Purpose of this study

Admire how massive 2+1d Dirac fermions with graphene-like chiral structure gives rise to condensed-matter-realizations of various topological terms:

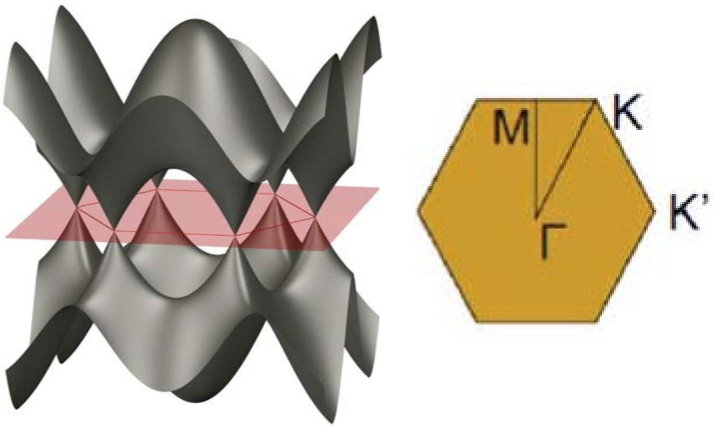
1st part (theory of competing orders):

- ***WZ-terms***
- ***Θ-terms***

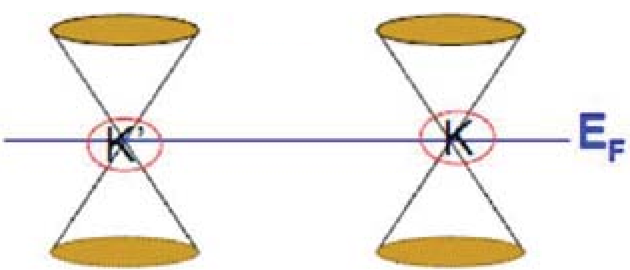
2nd part (quantized linear responses)

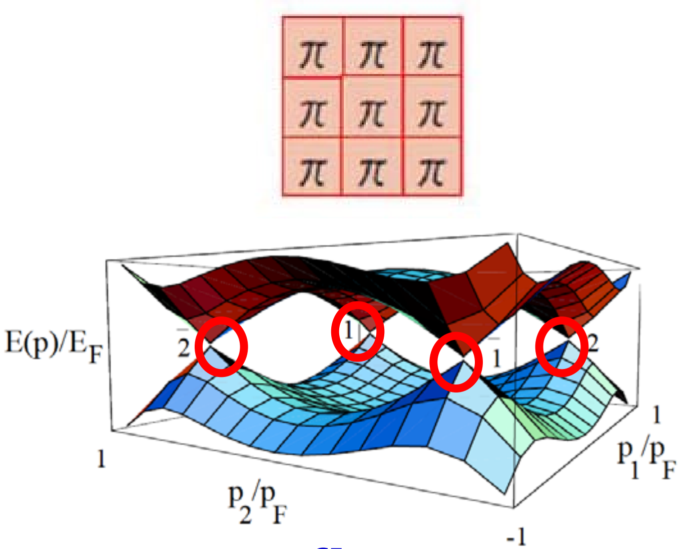
- ***Gauge/gravitational Chern-Simons terms***
- ***Gauge/gravitational BF terms***

The chiral structure:
stems from having two dirac nodes ('valleys')



graphene





π-flux state (square-lattice)

2 dirac fermions ⇒ combine into single 4-component Dirac fermion

gamma-matrices in a reducible representation

⇒ 5 generators of Clifford algebra:

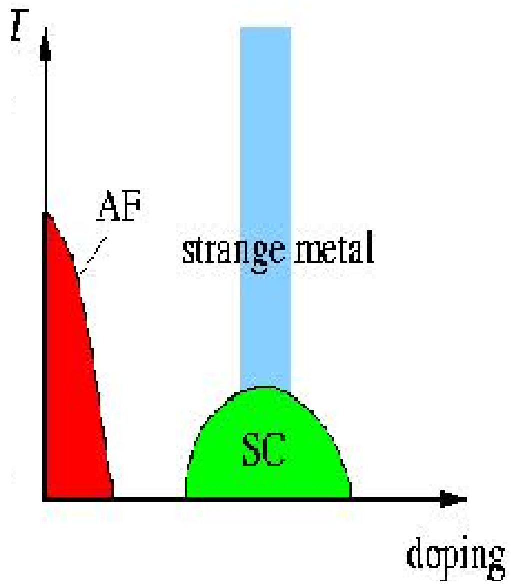
$\gamma_0, \gamma_1, \gamma_2$ Space-time components

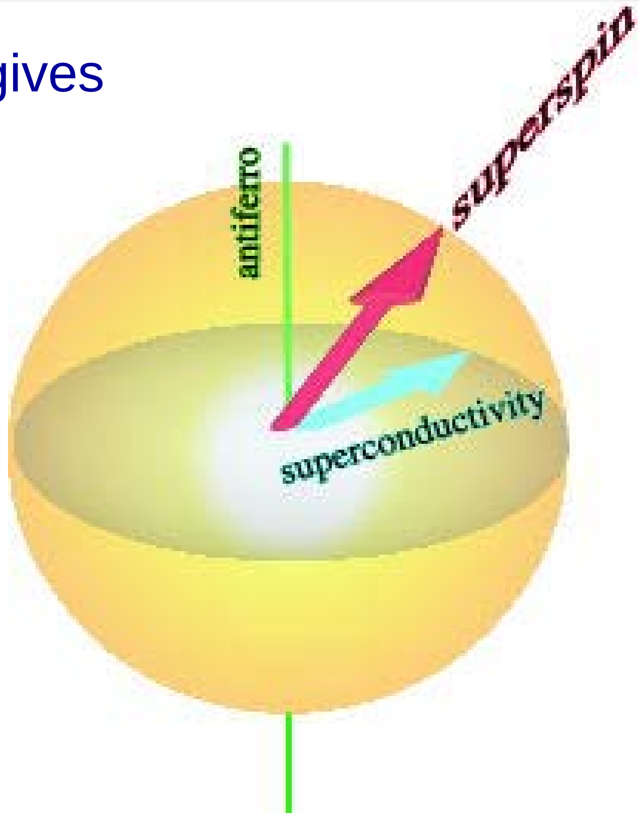
$\gamma_3, \gamma_5,$ 2 generators of chiral transformations

Part 1: Effective theories of competing orders with topological terms

Study in this direction: *stems in part from high T_c superconductor physics*

Proximity to antiferromagnetic phase gives rise to many peculiar features.

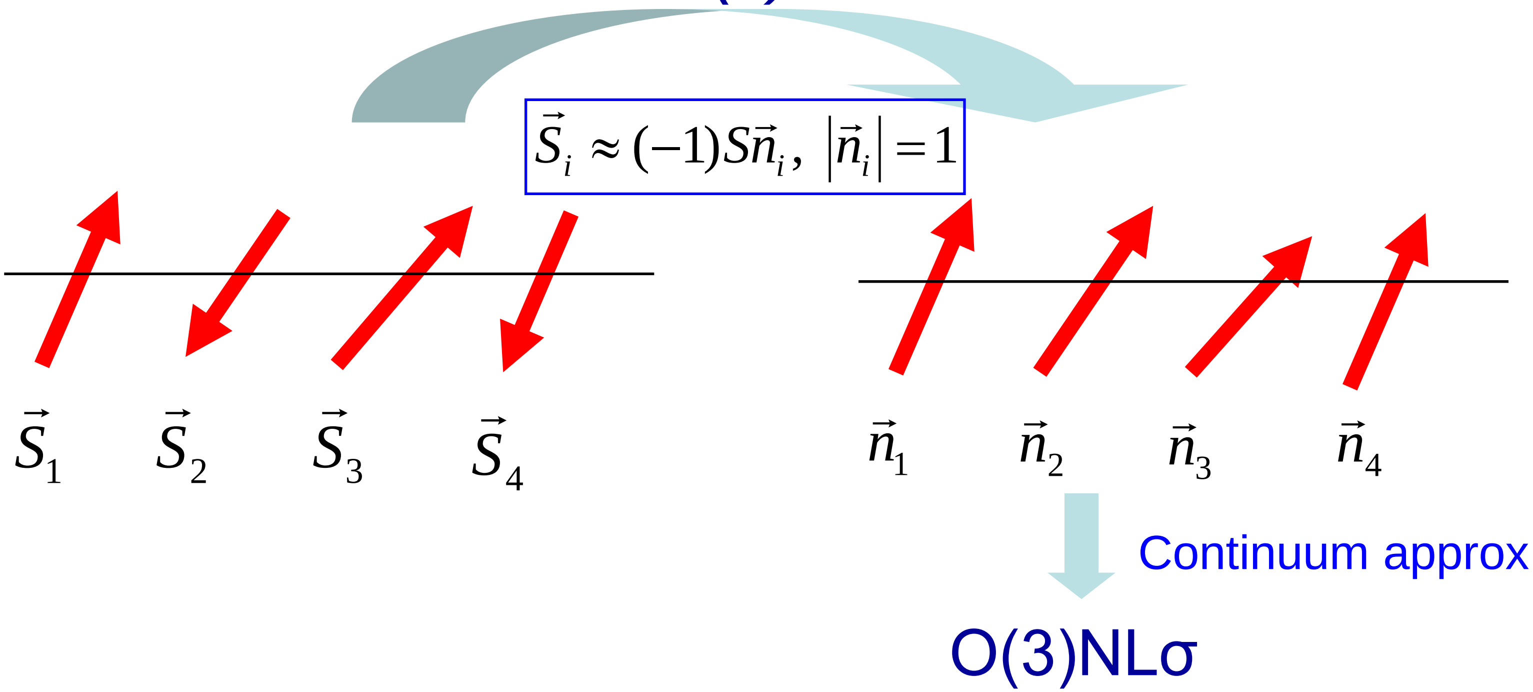




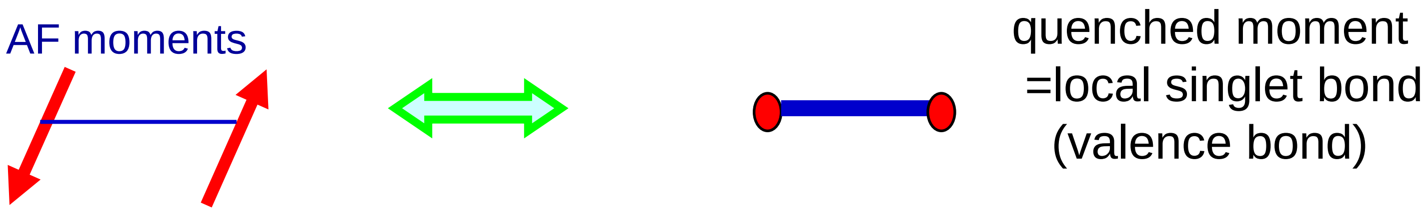
Attempt at a 'unification' of superconducting order (U(1)∼O(2)) and antiferromagnetic order (O(3)) ⇒O(5) theory (S.-C. Zhang)

Here we discuss a similar 'unification' in quantum magnets give rise to effective field theories with new topological terms.

Spin systems with antiferromagnetic (AF) interactions:
often described as an *O(3) nonlinear σ model*

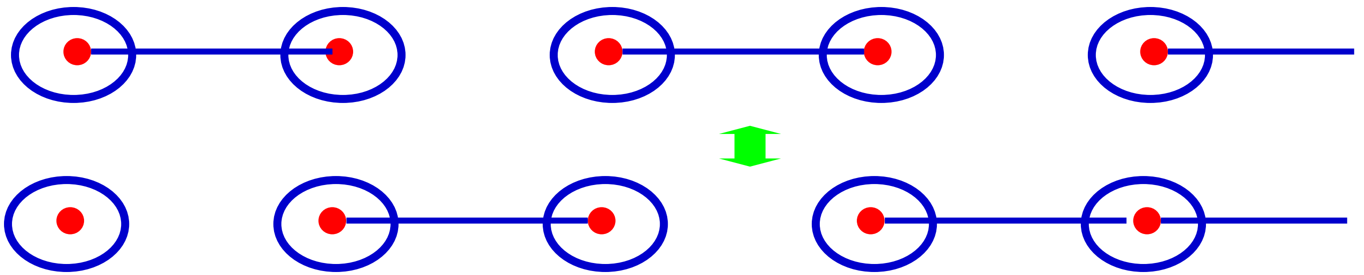


One often tries to stick to this language even when the system is in a spin singlet state.



e.g. combination of local singlet bonds:
valence bond solids (VBS).

These are naturally described in terms of a density wave like order parameter.



$\phi_{VBS} = 1$

$\phi_{VBS} = -1$

Unification?

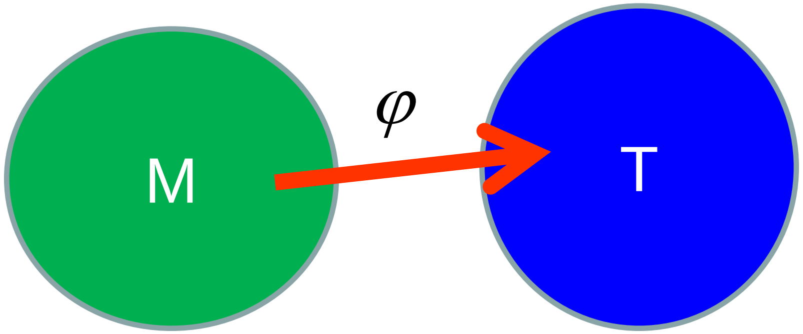
AF: 3 components
VBS: d components (d: spatial dimensionality)
$\Rightarrow$ O(d+3) NLσ model ?

For what cases is such a picture plausible in quantum magnets?
Can topological terms enter the theory and alter the physics ?

Brief aside: Generalities of topological terms of sigma models (especially on S^N)

field $\varphi : M(\sim S^M) \to T(\sim S^N)$

base **target**



■ **When** $\dim M = \dim T$

Θ-terms can (in principle) arise:

$$\boxed{S_\theta = i\theta \int_M \varphi^* \omega_T}$$

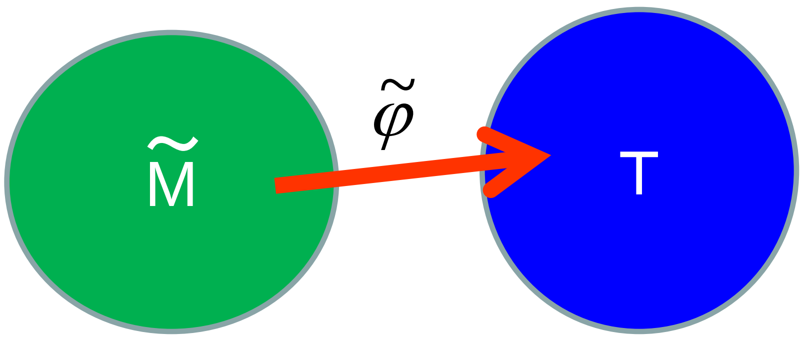
ω_T : **volume –form on T**

φ^* : **pull-back onto M**

θ : **unquantized**

■ **When** $\dim M + 1 = \dim T$

WZ-terms can be constructed via extension of the map φ:



$$\tilde{\varphi}(t) : M \times [0,1] \to T, \ t \in [0,1],$$

$$\tilde{\varphi}(0) \equiv \varphi_0 \in T, \tilde{\varphi}(0) = \varphi$$

$$\boxed{S_{WZ} = ik \int_{M \times [0,1]} \tilde{\varphi}^* \omega_T}$$

k : **level (quantized)**

well-known predecessor : 1+1d theory of competing orders in AFs

■ *level-1 SU(2) WZW model* (S=1/2 Heisenberg AF)

Can be rewritten as a model for ***4-component rotors*** i.e. O(4)

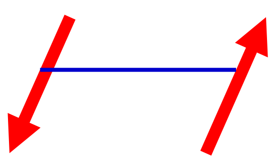
$$g = i\vec{\phi}\cdot\vec{\sigma} + \phi_4 \in SU(2), \quad \vec{\phi} = (\phi_1, \phi_2, \phi_3)$$

$$(\phi_1)^2 + (\phi_2)^2 + (\phi_3)^2 + (\phi_4)^2 = 1$$

Physically, nonabelian bosonization shows:

ϕ_1, ϕ_2, ϕ_3 : AF($\propto \boldsymbol{n}$)

ϕ_4 : VBS



$\Longleftrightarrow$

quenched moment =local singlet (VBS)

AF moment

∴ 1+1d critical theory of competing AF-VBS orders

des Cloizeaux-Pearson mode

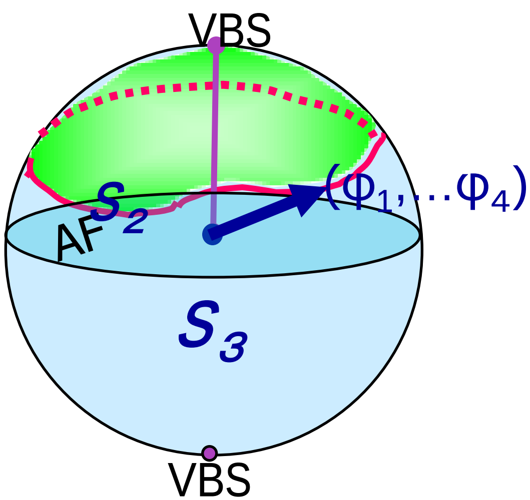
The WZW model written as O(4) model with WZ term

(Note: $\dim M + 1 = \dim T$ ***)***

$$S_{WZW}[\phi_\alpha] = \frac{1}{8\pi}\int d\tau dx\left[\left(\partial_\tau \phi_\alpha\right)^2 + \left(\partial_x \phi_\alpha\right)^2\right] + i\Gamma[\phi_\alpha]$$

$$\Gamma[\phi_\alpha] = \frac{\varepsilon^{abcd}}{\pi}\int_0^1 ds \int d\tau dx\, \phi_a \partial_t \phi_b \partial_\tau \phi_c \partial_x \phi_d = \int_{S_2\times[0,1]} \tilde{\phi}^* \omega_{S_3}$$

$$\phi_\alpha \ (\alpha = 1,2,3,4)$$



Berry phase ('solid angle') swept out by imaginary-time evolution of unit 4-vector on hypersphere S_3.

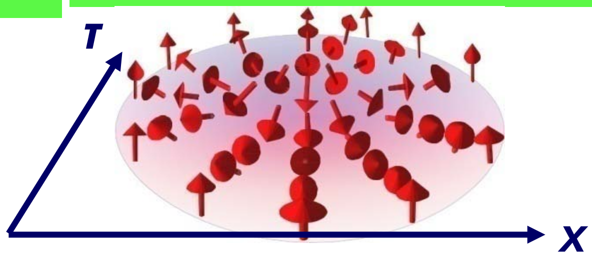
■ *WZW model* ⇒ NLσmodel + θterm

$$\vec{\phi} = (\phi_1, \phi_2, \phi_3) = \left|\vec{\phi}\right| \mathbf{n} : \textit{AF} \qquad \phi_4 = (1 - \left|\vec{\phi}\right|^2)^{\frac{1}{2}} : \textit{VBS}$$

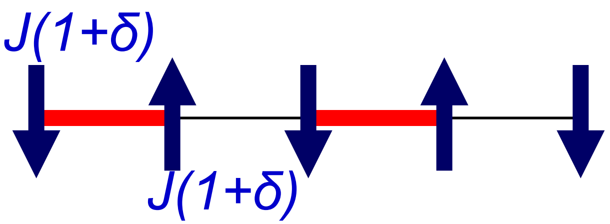
fix $\left|\vec{\phi}\right|$ ⇒residual o.p. is unit Neel vector ***n*** *i.e.* O(3) NLσmodel

Satisfies **rule for θ-terms: dim. (space-time) = effective dim. of o.p.**

$$S_{NL\sigma}[\mathbf{n}] = \frac{1}{g}\int d\tau dx \left[(\partial_\tau \mathbf{n})^2 + (\partial_x \mathbf{n})^2\right] + i\theta Q_{\tau x} \qquad Q_{\tau x} = \frac{1}{4\pi}\int d\tau dx\, \vec{n}\cdot\partial_\tau \vec{n}\times\partial_x \vec{n}$$



How to ' fix $\left|\vec{\phi}\right|$ ' : Introduce weak/strong bonds

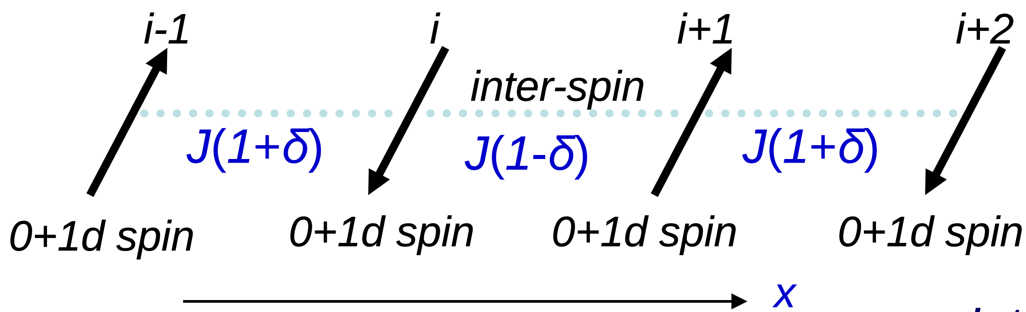


Explicit mapping shows that **θ=2πS(1-δ)**

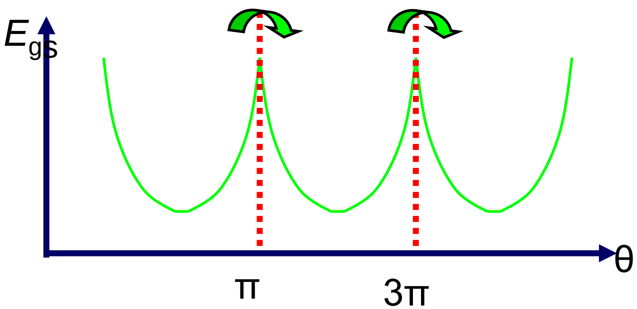
Note:

1.Emergence of a θvacua in spin chains

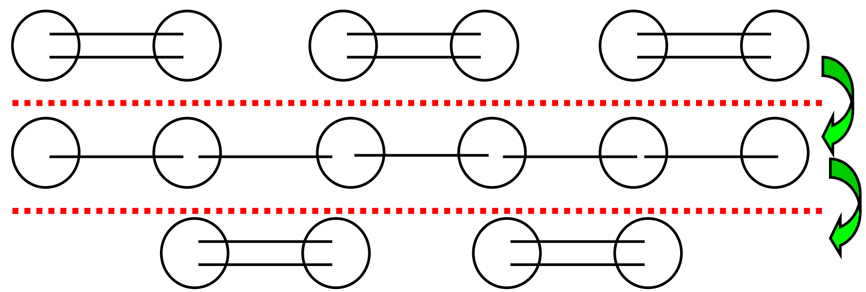
2.Quantum critical point at θ=π (mod 2π)

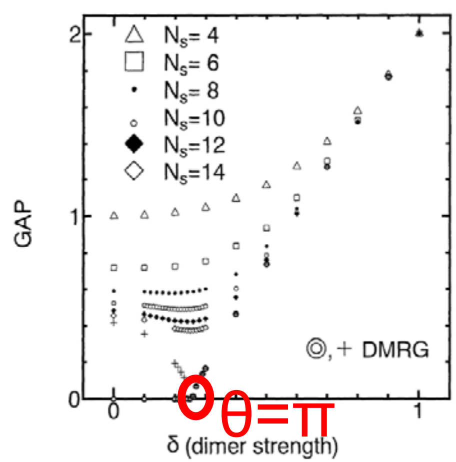


1+1d O(3) NLσ
$\theta=2\pi S(1-\delta)$



Inter-VBS transitions

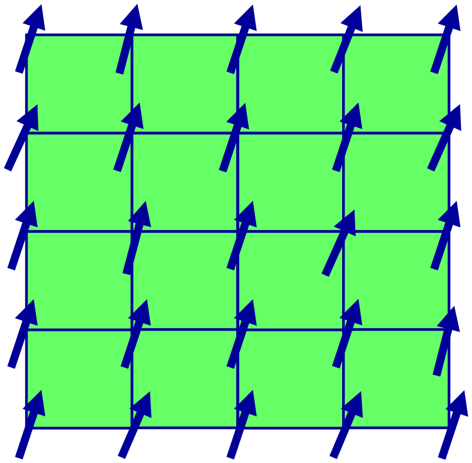




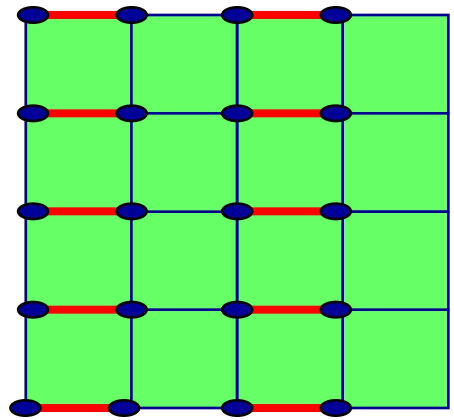
Y. Kato and A.T.   JPSJ 1994

Theory of competing orders in 2+1d AFs

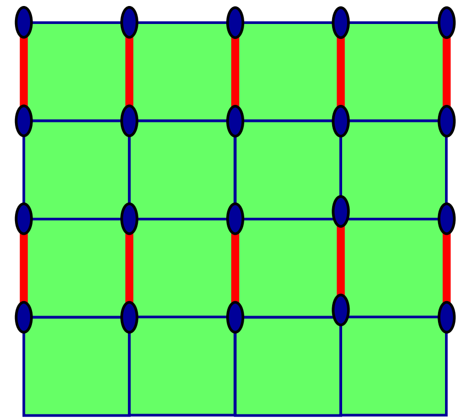
Naively:



Competing orders



&



AF

ϕ_1, ϕ_2, ϕ_3

VBS (valence bond solid)

ϕ_4 ϕ_5

(Note that again $\dim M + 1 = \dim T$ $\rightarrow$ ***WZ term?)***

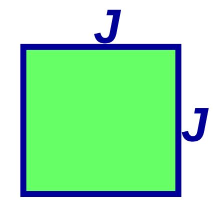
Can such an O(5) theory actually arise ?

⇒yes! It can arise through a quantum anomaly.

A.T. and X.Hu PRL 95 036402 (2005)

2D AF Heisenberg Hamiltonian on Square Lattice

$$H = J\sum_{NN} \vec{S}_i \cdot \vec{S}_j \qquad J > 0$$



Point of departure:

π-flux ansatz state of 2d Heisenberg AF (Marston-Affleck; Kotliar)

Mean Field Theory (decoupling of 4-fermion term)

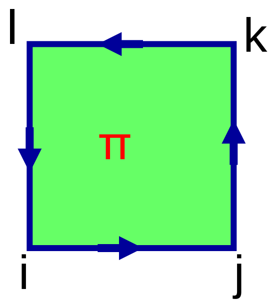
$$\vec{S}_i = c_{i\alpha}{}^{\dagger} \frac{(\vec{\sigma})_{\alpha\beta}}{2} c_{i\beta}, \qquad \chi_{ij} \equiv < c_{i\sigma}{}^{\dagger} c_{i\sigma} >$$

$$H_{\mathrm{MFT}} = J\sum_{NN} \chi_{ij} c_j{}^{+} c_i$$

$$H_{\mathrm{MFT}} = J \sum_{NN} \chi_{ij} c_{j\sigma}^{\dagger} c_{i\sigma} + h.c.$$

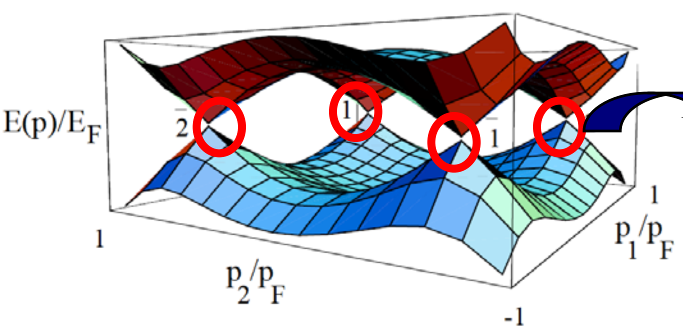
The **π-flux state** (π-flux (Aharanov-Bohm flux) piercing each plaquette) is a good mean field solution.

(Kotliar, Affleck-Marston '88)



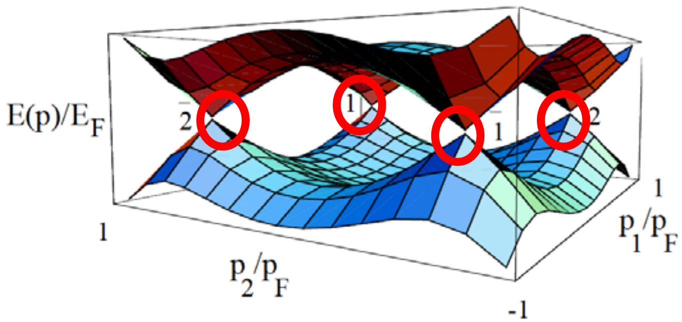
$$\chi_{ij}\chi_{jk}\chi_{kl}\chi_{li} = e^{i\pi} = -1$$

$$\Rightarrow E(\mathbf{k}) = \pm J\sqrt{\cos^2 k_x + \cos^2 k_y}$$

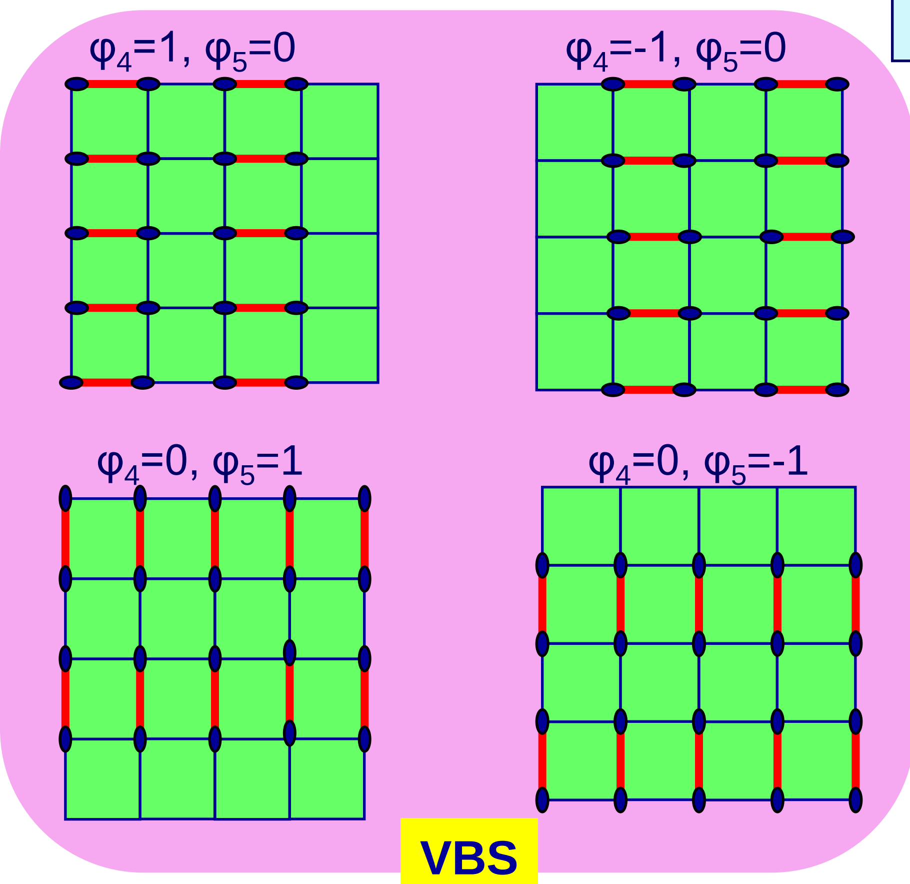


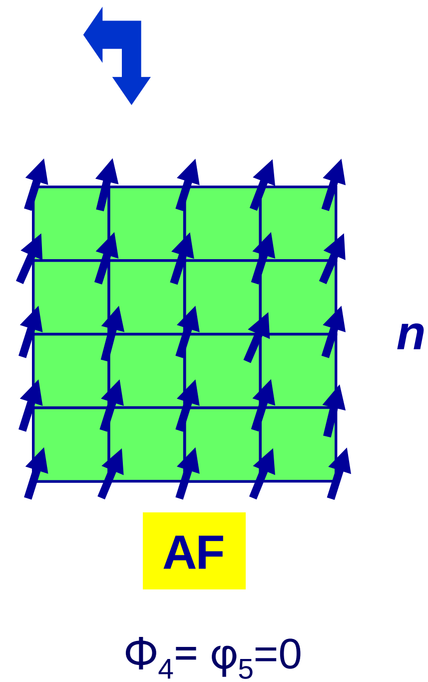
Light-cone like dispersions (**Dirac fermions**) at
K=(π/2,π/2)
K' =(π/2,-π/2)

Note similarity to graphenes (tightbinding model on honeycomb lattice) .



Findings: low energy Dirac fermions have large ***chiral symmetry*** which rotates VBS into AF states and vice versa.





Mass terms (potential energy terms)

$im\bar{\psi}Q\psi,\ Q=\mathbf{n}\cdot\boldsymbol{\sigma},\ |\mathbf{n}|=1$ $\qquad$ $\mathbf{n}$: Neel unit vector

$\sim m\sum_{x,y}(-1)^{x+y}\mathbf{n}(\mathbf{r})\cdot\boldsymbol{\sigma}$ $\qquad$ $\rightarrow$ AF order

Improves variational energy
Gros Phys. Rev. B 38 931('88)

$m\bar{\psi}\gamma_3\psi \quad \sim\sum_{\mathbf{r},\sigma}(-1)^x\delta t\ \ c_{\mathbf{r}\sigma}{}^{\dagger}c_{\mathbf{r}+\hat{x}\sigma}+\text{h.c.}$

$\rightarrow$bond alternation (VBS order on horizontal bonds)

$m\bar{\psi}\gamma_5\psi \quad \sim\sum_{\mathbf{r},\sigma}(-1)^y\delta t\ \ c_{\mathbf{r}\sigma}{}^{\dagger}c_{\mathbf{r}+\hat{y}\sigma}+\text{h.c.}$

$\rightarrow$bond alternation (VBS order on vertical bonds)

All three of the above belong to the family of chirally rotated mass term

$$im\bar{\psi}Qe^{i(\alpha\gamma_3+\beta\gamma_5)\otimes Q}\psi,\ \alpha,\beta\in\mathbf{R}$$

i.e. ***transform into each other*** via the aforementioned chiral symmetry.

Natural framework for studying AF-VBS competition!

Note: $\bar{\psi}\gamma_3\gamma_5\psi$ $\quad$ related to diagonal hopping, T-violating (chiral spin liquid)

■Chiral Dirac fermion from AF-VBS(dimer) competition

$$S=\int d\tau d^2x\, i\overline{\psi}(\partial\!\!\!/+mV)\psi,$$

$$V\equiv \underbrace{-i\phi_4 1\otimes\gamma_5}_{\text{VBS (//x)}}\underbrace{-i\phi_5 1\otimes\gamma_3}_{\text{VBS (//y)}_5}\underbrace{+\vec{\phi}_{\mathrm{AF}}\cdot\boldsymbol{\sigma}\otimes 1}_{\text{AF}}$$

$$\vec{\phi}_{\mathrm{AF}}\equiv(\phi_1,\phi_2,\phi_3)//\mathbf{n},\ \sum_{\mathrm{i}=1}^{5}(\phi_{\mathrm{i}})^2=1$$

Similar chiral structure in context of QED_3 theory of cuprates derives a rich variety of electronic states e.g. stripes. (Herbut et al, Tesanovic *et al* '02)

Here it leads to a ***nontrivial Berry phase term***.

$$Z=\int D\psi D\overline{\psi}e^{-\int d^3x\overline{\psi}D\psi}=\det D\equiv e^{-S_{eff}[\mathbf{v}]}$$

$$S_{eff}[\mathbf{v}]=-\ln\det D=-Tr\ln D$$

method due to: Abanov-Wiegmann '00

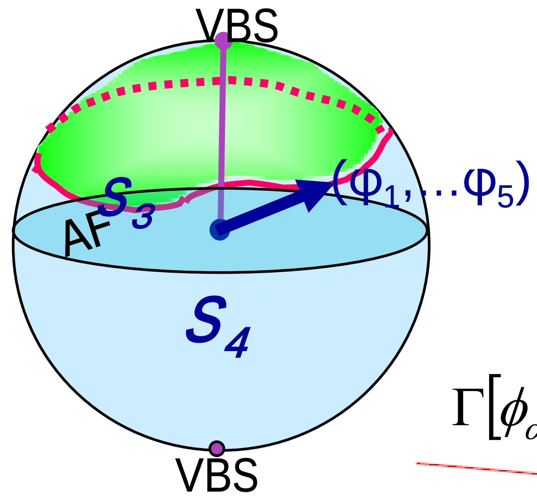
$$\delta S_{eff}=-Tr[D^{-1}\delta D]=-Tr[\frac{1}{-\partial^2+m^2-m\partial\!\!\!/V}D^{-1}im\delta V]$$

$$\Rightarrow \delta S_{eff}^{\mathrm{Im}}=-tr\int\frac{d^3p}{(2\pi)^3}\frac{m^5}{(p^2+m^2)^4}\int d^3x[(\partial V)^3V^{+}\delta V]$$

effective O(5) theory for competing orders *(after integrating out fermions)*

$$S_{eff}[\phi_\alpha] = \frac{1}{g}\int d\tau d^2x \left(\partial_\mu \phi_\alpha\right)^2 + i\Gamma[\phi_\alpha]$$

$$\Gamma[\phi_\alpha] = \frac{3\varepsilon^{abcde}}{4\pi}\int_0^1 ds \int d\tau d^2x\, \phi_a \partial_s \phi_b \partial_\tau \phi_c \partial_x \phi_d \partial_y \phi_e = \int_{S_3\times[0,1]} \tilde{\phi}^* \omega_{S_4}$$

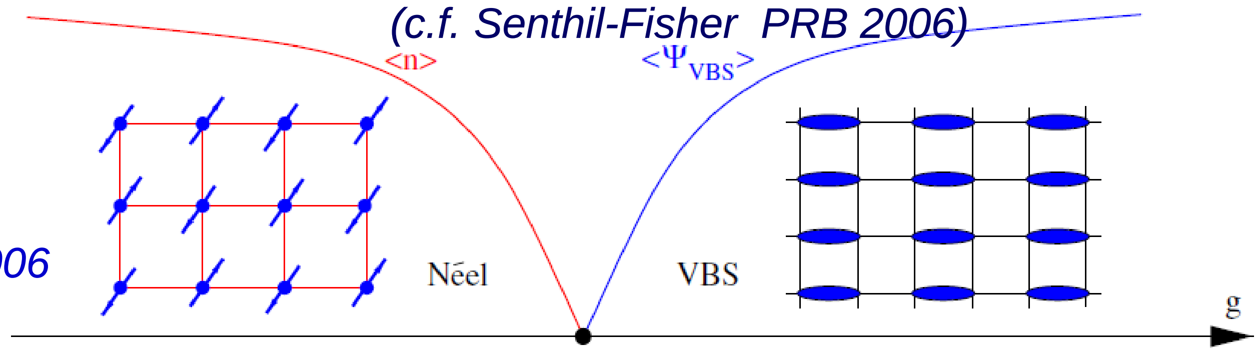


Berry phase ('solid angle') swept out by 5 component superspin on hypersphere S_4.

- ***2+1d extension of WZW (critical theory?)***
- ***Reproduces Haldane-Read-Sachdev Berry phases***

$\Gamma[\phi_\alpha]$: Key to *LGW description of* '*deconfined criticality*'?

(c.f. Senthil-Fisher PRB 2006)



Alet et al, Physica A 2006

emergent U(1) symmetry corresponding to Skyrmion number conservation

An emergent AF-VBS duality is present in the O(5) theory
(c.f. Xu-Sachdev, 2008 PRL for claim to the contrary)

$\{\gamma_3, \gamma_5, \gamma_{35} \equiv \gamma_3\gamma_5\}$ ***forms SU(2) subalgebra***
$\rightarrow \gamma_{35}$ ***generates rotation of VBS OP***

Couple fermions to following gauge field: $i\partial\!\!\!/ \rightarrow i\partial\!\!\!/ + \tilde{a\!\!\!/}\gamma_{35}$

(c.f. in graphene, corresponds to ripple gauge field)

$\rightarrow$ ***a nontrivial Goldstone-Wilczek current arises***

Consider case where a U(1) symmetry arises in the VBS-sector.

Noether current : $$j_\mu^{Noether} = \left\langle \frac{\delta S_F\,(i\partial \to i\partial + \tilde{a}\gamma_{35})}{\delta \tilde{a}_\mu} \right\rangle_{\vec{N}(x)} \Big|_{\tilde{a}_\mu = 0} = \frac{1}{4\pi}\varepsilon_{\mu\nu\lambda}\mathbf{n}\cdot\partial_\nu \mathbf{n}\times\partial_\lambda \mathbf{n}$$

Emergent AF-VBS duality from underlying O(5) theory

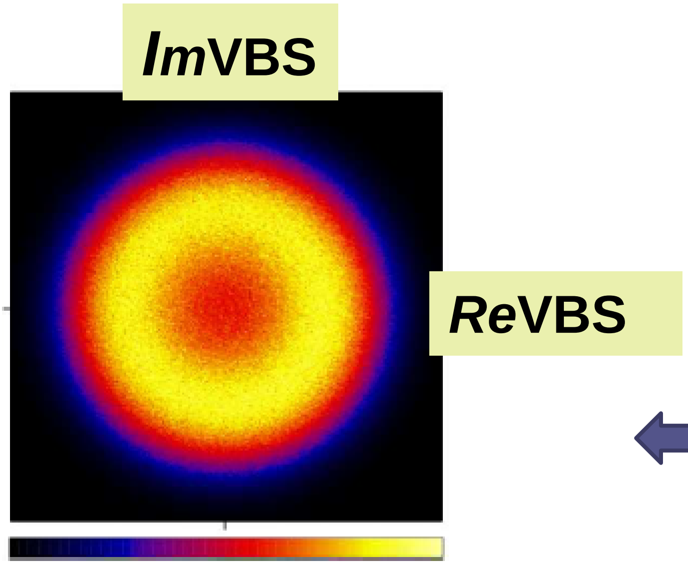
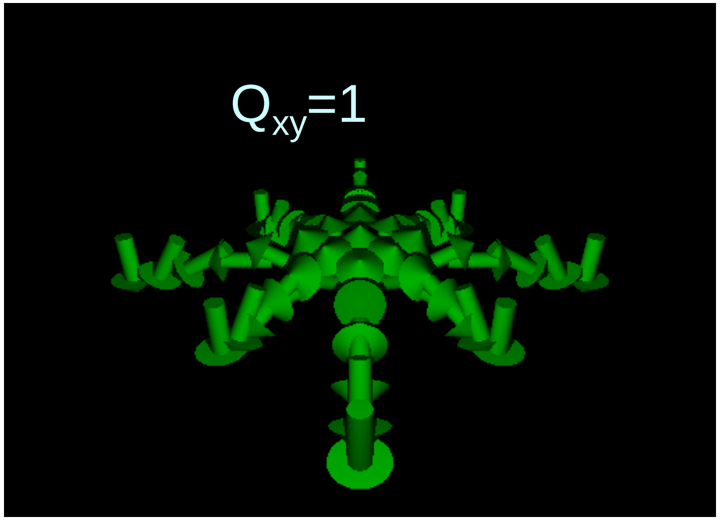


FIG. 5: (Color online) Histogram of the dimer order parameter for an $L = 32$ system at $J/Q = 0$. The ring shape demonstrates an emergent $U(1)$ symmetry, i.e., irrelevance of the Z_4 anisotropy of the VBS order parameter.

A. Sandvik 2007

***Conserved skyrmion number in AF sector** (hallmark of DC)*



This can be continued on to arbitrary dimensions

Dimensionality	unit cell	components in spinor	gamma matrices
d=1 (1+1D)	2^1 sites	2	$\gamma_0\ \gamma_1\ \gamma_5$
d=2 (2+1D)	2^2 sites	4	$\gamma_0\ \gamma_1\ \gamma_2\ \gamma_3\ \gamma_5$
d=3 (3+1D)	2^3 sites	8	$\gamma_0\ \gamma_1\ \gamma_2\ \gamma_3\ \gamma_5\ \gamma_6\ \gamma_7$

Resulting Berry phase term:
Wess-Zumino term for O(d+3) dimensions

$$S_{BP}^{1+1D} = \frac{-i2\pi}{Area(S^3)} \int_0^1 dt \int d\tau dx \varepsilon^{\mathrm{abcd}} \mathrm{v_a} \partial_t \mathrm{v_b} \partial_\tau \mathrm{v_c} \partial_x \mathrm{v_d}$$

$$S_{BP}^{2+1D} = \frac{-i2\pi}{Area(S^4)} \int_0^1 dt \int d\tau d^2 x \varepsilon^{\mathrm{abcde}} \mathrm{v_a} \partial_t \mathrm{v_b} \partial_\tau \mathrm{v_c} \partial_x \mathrm{v_d} \partial_y \mathrm{v_e}$$

$$S_{BP}^{3+1D} = \frac{-i2\pi}{Area(S^5)} \int_0^1 dt \int d\tau d^3 x \varepsilon^{\mathrm{abcdef}} \mathrm{v_a} \partial_t \mathrm{v_b} \partial_\tau \mathrm{v_c} \partial_x \mathrm{v_d} \partial_y \mathrm{v_e} \partial_z \mathrm{v_f}$$

O(5) NLσ+WZ term ⇒ O(4) NLσmodel + θ-term

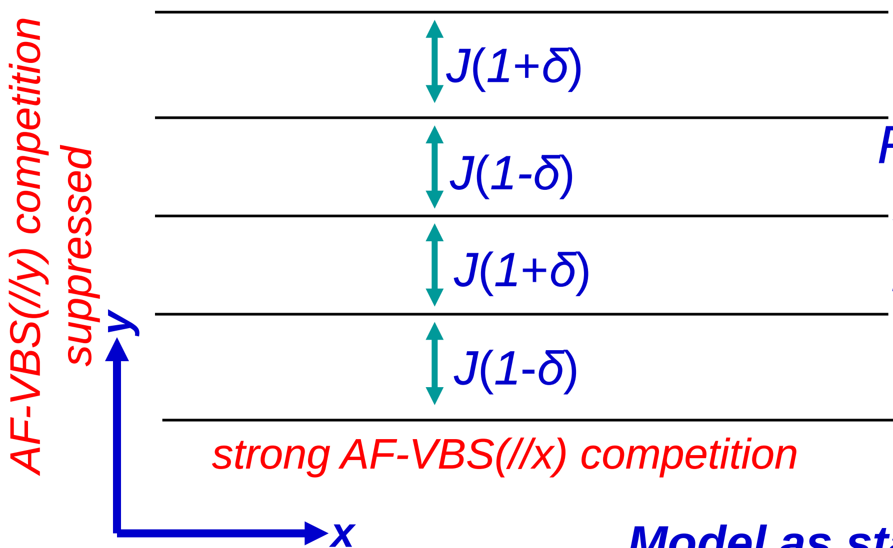
Fix $(\phi_1)^2 + (\phi_2)^2 + (\phi_3)^2 + (\phi_4)^2 \equiv A^2,\ |\phi_5| = \sqrt{1 - A^2}$

Satisfies **rule for θ-terms: dim. (space-time) = effective dim. of o.p.**

$$S_{reduced} = \int d\tau d^2x \frac{1}{g}(\partial_\mu \phi_\alpha)^2 + i\theta\, Q_{\tau xy} \quad (\alpha = 1,\dots,4)$$

$$Q_{\tau xy} = \frac{1}{2\pi^2}\int d\tau dx dy\, \varepsilon^{abcd}\, \phi_a \partial_\tau \phi_b \partial_x \phi_c \partial_y \phi_d$$

How to fix *A*

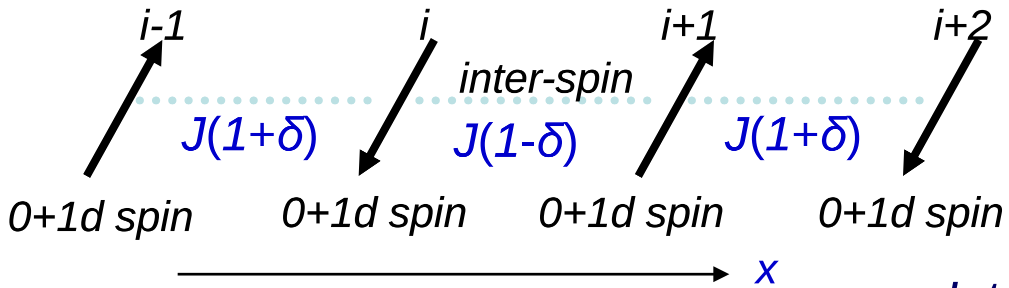


Reduce effective # of components 5 → 4 by externally imposing bond alternation in one direction

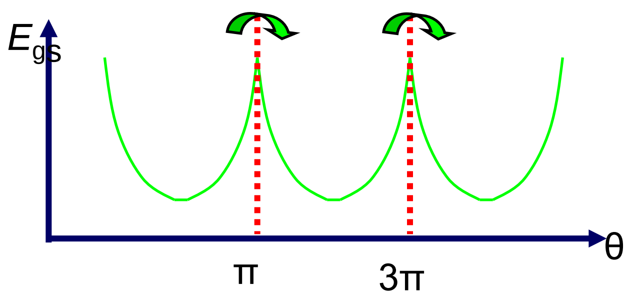
Model as stack of level-2S WZW models

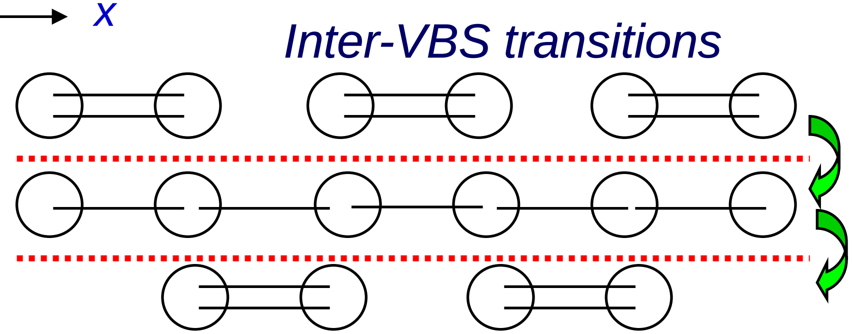
Route towards an effective theory

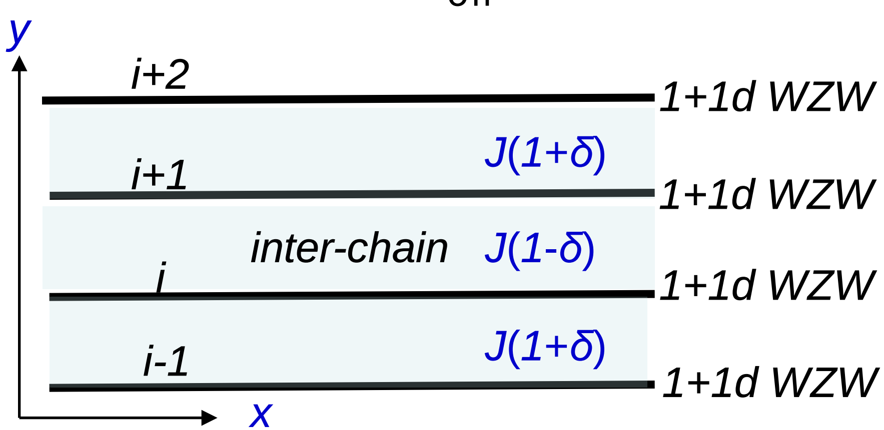
Parallels Haldane-Affleck mapping of spin chains into a NLσmodel with a θterm.



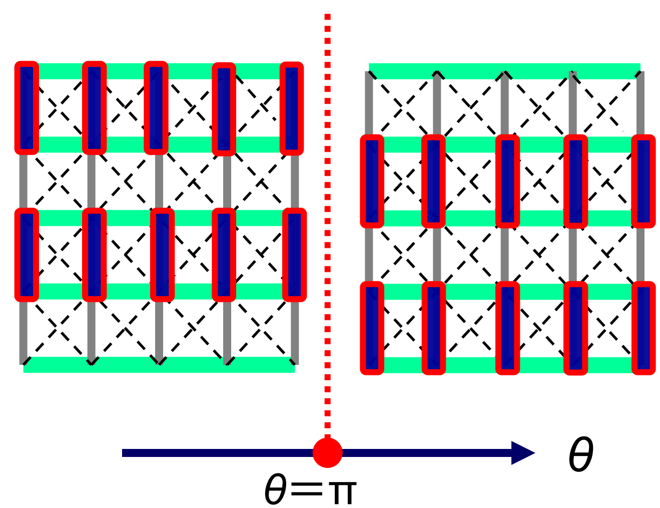
1+1d O(3) NLσ
$\theta=2\pi S(1-\delta)$







2+1d O(4) NLσ
$\theta=2\pi S(1-\delta)$



Summary

New topological terms for effective models in any dimensions with competing AF-VBS orders

1+1d		***2+1d***
WZW model	*analogy* ⟷	WZW-like model (deconfined criticality?)
↓ *symmetry reduction*		↓ *symmetry reduction*
tunable θ-term	*analogy* ⟷	tunable θterm (quantum phase transitions?)

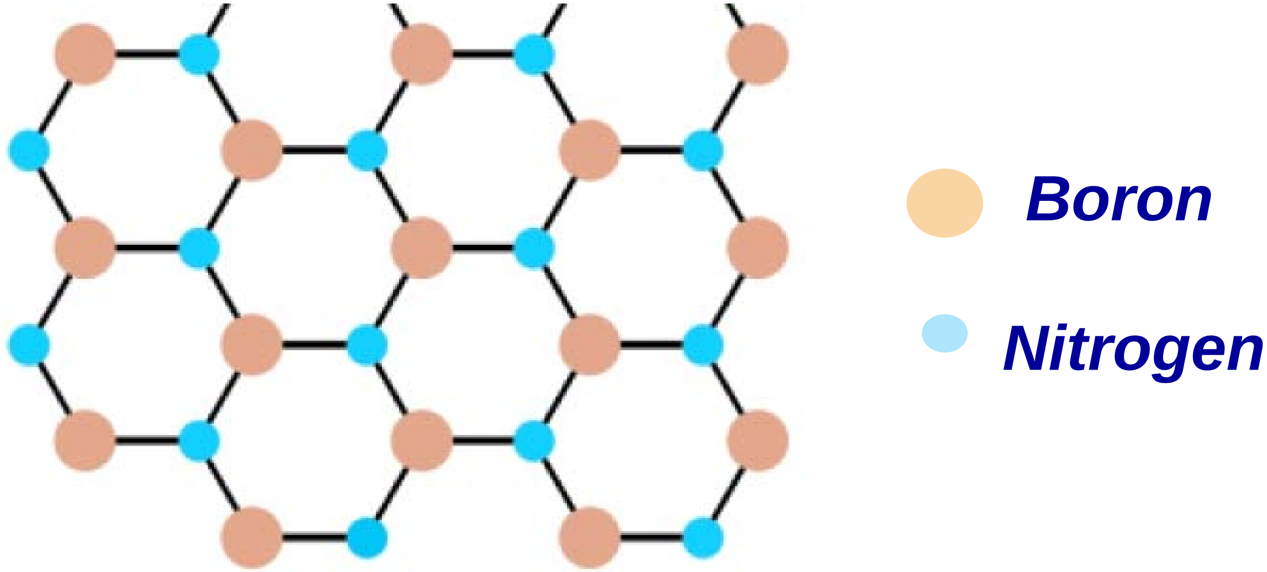
Work in this direction is now pursued in context of topological insulators (Grover –Senthil 2007; Gaemi et al 2009; Ryu et al, 2009)

Quantized linear responses in Graphene-Derived Insulators

Various ways to open up energy gap in graphene…

Various ways to open up energy gap in graphene…

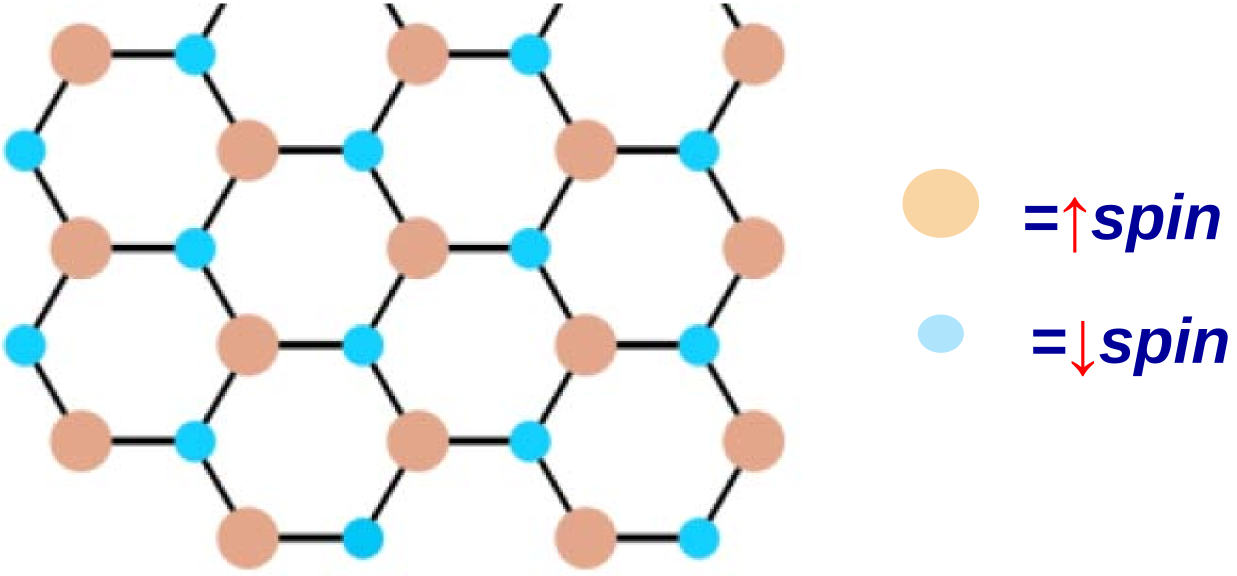
e.g. by imposing a "staggered potential"



nontopological insulator

Various ways to open up energy gap in graphene…

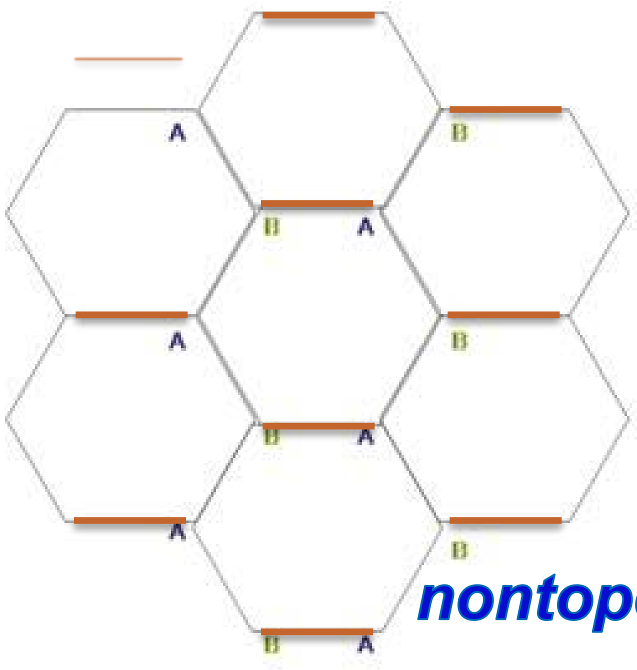
e.g. by imposing an "SDW potential"



nontopological insulator

Various ways to open up energy gap in graphene...

e.g. by imposing a "Kekule pattern"



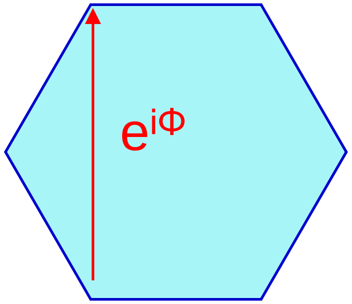
"strong" bonds

nontopological insulator

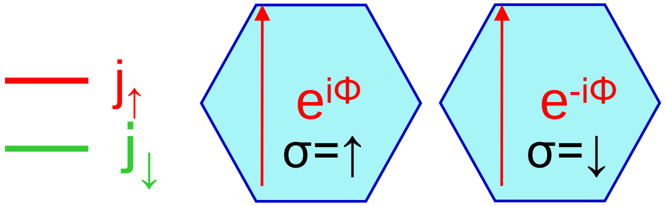
Various ways to open up energy gap in graphene…

NNN hopping with complex phase factors

“*Haldane model*”



spin-dependent NNN hopping w. complex phase factors



“*Kane-Mele model*”

topological insulators

We seek below quantal response of topological origin to gauge fields (charge/spin/valley) an to gravitational coupling (curved space-time).

Chiral structure of graphene-related insulators

$$L = \overline{\psi}\, D\psi; \;\; D = i\partial\!\!\!/ + a\!\!\!/ + \hat{M}$$

4×4 matrices

$\gamma_0, \gamma_1, \gamma_2$: **space-time** γ_3, γ_5 : **chiral (internal) symmetry** $\dagger$

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \qquad \gamma_0\gamma_1\gamma_2\gamma_3\gamma_5 = \mathbf{1}$$

$\dagger$
$$\begin{cases} \psi \to e^{i(\alpha\gamma_3 + \beta\gamma_5)}\psi \\ \overline{\psi} \to \overline{\psi} e^{i(\alpha\gamma_3 + \beta\gamma_5)} \end{cases} \quad \alpha, \beta \in \mathbf{Z}$$

: **symmetry at** $\hat{M} = 0$

chiral transformation

Chiral structure of graphene-related insulators

$$L = \overline{\psi}\, D\psi;\;\; D = i\partial\!\!\!/ + a\!\!\!/ + \hat{M}$$

$\gamma_0, \gamma_1, \gamma_2$: ***space-time*** γ_3, γ_5 : ***chiral (internal) symmetry*** †

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†

$$\begin{cases} \psi \to e^{i(\alpha\gamma_3 + \beta\gamma_5)\otimes\sigma_3}\psi \\ \overline{\psi} \to \overline{\psi} e^{i(\alpha\gamma_3 + \beta\gamma_5)\otimes\sigma_3} \end{cases}$$

$\alpha, \beta \in \mathbf{Z}$: ***symmetry at*** $\hat{M} = 0$

spin-dependent chiral transformation

A.T. PRL 2002;2005

Zoology of insulators

charge

mass term $\hat{M}$	type of perturbation	type of insulator
$m\mathbf{1}$	"staggered" (BN-sheet)	conventional
$im\gamma_3, im\gamma_5$	"Kekule' "(bond alternation)	conventional
$m\gamma_{35}\ (\gamma_{35} \equiv \gamma_3\gamma_5)$	"Haldane" (NNN hopping)	topological (IQHE)

spin

mass term $\hat{M}$	type of perturbation	type of insulator
$m\mathbf{1} \otimes \sigma_3$	SDW	conventional
$m\gamma_{35} \otimes \sigma_3$	"Kane-Mele" (spin-dependent NNN hopping)	topological (SQHE)

charge

No current-flow (insulators)

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Response to gauge fields

Response to gauge fields

spin sector

$$i\partial\!\!\!/ \rightarrow i\partial\!\!\!/ + a\!\!\!/^{c} + \sigma_3 \otimes a\!\!\!/^{s}$$

$$Z[a^c, a^s] = \int D\psi D\overline{\psi} e^{-\int \overline{\psi} D \psi} = \det D \equiv e^{-S_{eff}\left[a^c, a^s\right]}$$

$$j^s_\mu = \frac{\delta S_{eff}\left[a^c, a^s\right]}{\delta a^s_\mu} = \mathrm{tr}[\frac{1}{D^\dagger D} \cdot D^\dagger \cdot \frac{\delta D}{\delta a^s_\mu}]$$

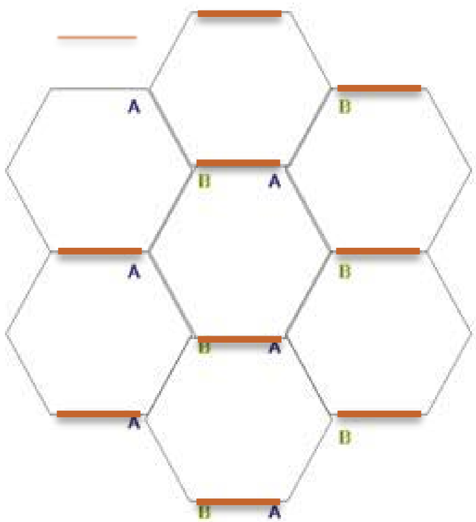
$$= \mathrm{tr}[\frac{1}{D^\dagger D} \cdot \hat{M} \cdot \gamma_\mu \otimes \sigma_3]\Big|_{\partial a\!\!\!/^{c} \neq 0, \partial a\!\!\!/^{s} = 0}$$

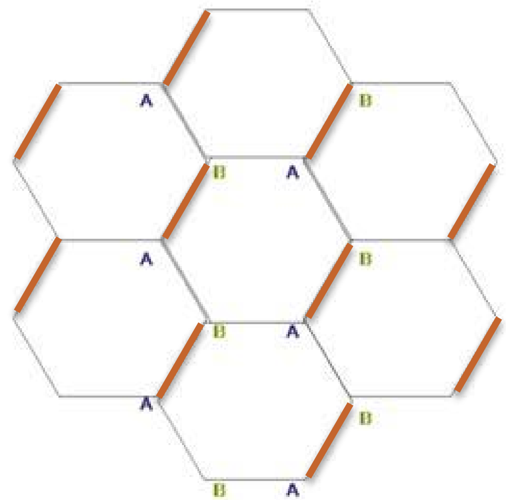
$$\gamma_0\gamma_1\gamma_2\gamma_3\gamma_5 = \mathbf{1}$$

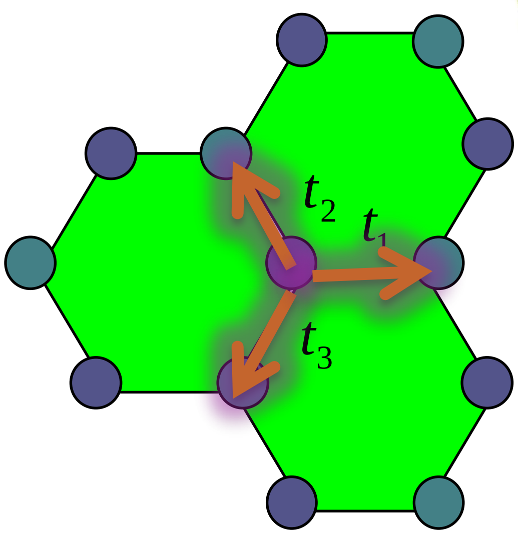
$$\propto \mathrm{tr}[i\partial a\!\!\!/^{c} \cdot \underline{\hat{M}} \cdot \gamma_\mu \otimes \sigma_3]$$

γ_3: bond alternation (horizontal component)

γ_5: bond alternation (vertical component)







t_1 hopping term	$m\bar{\Psi}(i\gamma_3)\Psi$
t_2 hopping term	$m\bar{\Psi}(i\cos(\frac{2}{3}\pi)\gamma_3 + i\sin(\frac{2}{3}\pi)\gamma_5)\Psi$
t_3 hopping term	$m\bar{\Psi}(i\cos(\frac{4}{3}\pi)\gamma_3 + i\sin(\frac{4}{3}\pi)\gamma_5)\Psi$

No responses (insulators)

charge

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$m\gamma_{35} \otimes \sigma_3$	"Kane-Mele" (spin-dependent NNN hopping)	topological (SQHE)

Zoology of insulators

Finite response !

charge

mass term $\hat{M}$	type of perturbation	type of insulator
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Responses of topological insulators

charge

$m\bar{\psi}\gamma_{35}\psi$

Haldane model

Effective theory of EM-response: Chern-Simons action

$$S_{\mathrm{eff}}[a_\mu^c] = \frac{i}{2\pi}\int d^3x\ \varepsilon_{\mu\nu\lambda} a_\mu^c \partial_\nu a_\lambda^c$$

spin

$m\bar{\psi}\gamma_{35}\otimes\sigma_3\psi$

Kane-Mele model

Effective theory of EM-response: BF theory (mixed Chern-Simons)

$$S_{\mathrm{eff}}[a_\mu^c, a_\mu^s] = \frac{i}{2\pi}\int d^3x\ \varepsilon_{\mu\nu\lambda} a_\mu^s \partial_\nu a_\lambda^c$$

Grover-Senthil PRL 2008

There is room for topological responses (cross-effects) in conventional insulators

Generalize response by coupling to ripple gauge field

$$i\partial\!\!\!/ \rightarrow i\partial\!\!\!/ + a\!\!\!/^{c} + \sigma_3 \otimes a\!\!\!/^{s} + \underline{i\gamma_{35}\tilde{a\!\!\!/}}$$

$$\varepsilon_{\mu\nu\lambda}\partial_\nu \tilde{a}_\lambda$$

pseudo-EM field (curvature)

New topological (quantized) responses

curvature-induced charge current

$$j_\mu^c = \left.\frac{\delta S}{\delta a_\mu^c}\right|_{\varepsilon_{\mu\nu\lambda}\partial_\nu \tilde{a}_\lambda \neq 0}$$

staggered insulator

$$S_{eff}[a_\mu^c, \tilde{a}_\mu] = \frac{i}{2\pi}\int d^3x \ \varepsilon_{\mu\nu\lambda} a_\mu^c \partial_\nu \tilde{a}_\lambda$$

curvature-induced spin current

$$j_\mu^s = \left.\frac{\delta S}{\delta a_\mu^s}\right|_{\varepsilon_{\mu\nu\lambda}\partial_\nu \tilde{a}_\lambda \neq 0}$$

SDW insulator

$$S_{eff}[a_\mu^s, \tilde{a}_\mu] = \frac{i}{2\pi}\int d^3x \ \varepsilon_{\mu\nu\lambda} a_\mu^s \partial_\nu \tilde{a}_\lambda$$

EM-field induced "valley-current" (valley Hall effect)

$$j_\mu^{\mathrm{v}} \equiv \left.\frac{\delta S}{\delta \tilde{a}_\mu}\right|_{\varepsilon_{\mu\nu\lambda}\partial_\nu a_\lambda^c \neq 0}$$

staggered insulator

Q. Niu group, 2008 PRB

$$S_{eff}[a_\mu^c, \tilde{a}_\mu] = \frac{i}{2\pi}\int d^3x \ \varepsilon_{\mu\nu\lambda} \tilde{a}_\mu \partial_\nu a_\lambda^c$$

Coupling to gravity-- motivations

(1) In the presence of defects…

$$i\partial \rightarrow i\partial + i\gamma_{35}\tilde{\not{a}}$$

ripple gauge field :
accounts for disturbed
sublattice structure

$\gamma_3, \gamma_5, \gamma_{35}$: ***SU(2) algebra***

Full account requires consideration of
metric fluctuation & 'space-time torsion'

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \rightarrow \{\gamma_\mu(x), \gamma_\nu(x)\} = 2g_{\mu\nu}(x)$$

(2) Thermal responses (Luttinger 1962)
$\rightarrow$ ***"gravitational Hall effect" (Callan-Harvey)***

Vielbein formalism

$$\gamma_\mu(x) = \gamma^a e_\mu^a(x) \quad (a = \tau, x, y)$$

γ^a ***local Euclidean ('falling elevator') frame***

$$\{\gamma^a, \gamma^b\} = 2\delta_{ab}$$

e_μ^a ***dreibein*** $\qquad e_\mu^a e_\nu^a = g_{\mu\nu}$

$\psi \to \exp[i\sigma_{ab}\phi_{ab}(x)]\psi$ ***local Euclidean (Lorentz) transf.***

$\sigma_{ab} = \frac{1}{2i}[\gamma^a, \gamma^b]$ ***generator of rotation in ab-plane***

insulator with metric fluctuation

$$L = \sqrt{g(x)}\,\overline{\psi}[\gamma_\mu(\partial_\mu + \boxed{i\omega_\mu^{ab}(x)\sigma_{ab}} + \boxed{i\tilde{\omega}_\mu^{ab}\sigma_{ab}\otimes\gamma_{35}}) + \hat{M}]\psi$$

$$\omega_\mu^{ab} = e_\nu^a(\partial_\mu + \Gamma_{\mu\lambda}^\nu)e_\lambda^b$$

$$R = d\omega + \omega^2$$

spin connection

valley-contrasting spin connection

Effective response action for metric fluctuation

$\omega \to \omega + \delta\omega$ ***(w.r.t. flat metric)***

$\hat{M} = m\gamma_{35}$ ***(Haldane)***

$\hat{M} = m$ ***(staggered)***

Implications

Response w.r.t. $\omega \to \omega + \delta\omega$:

$$T_{\mu\nu} = \frac{\delta S_{eff}}{\delta g_{\mu\nu}}$$

essentially probes behavior of *energy flux*

Coefficient of CS theory gives information on Leduc-Righi thermal conductivity κ_{xy} . (~central charge of CFT for chiral edge state)

Gravitational Hall effect Callan-Harvey 1985

$$\delta S_{CS} \overset{\omega_\mu \to \omega_\mu + \partial_\mu \eta}{=} \frac{1}{64\pi} \int d^3x\, \Theta(x - x_{\mathrm{dom.\,wall}})\, \varepsilon_{\mu\nu\lambda}\, \partial_\mu \mathrm{tr}(\eta\, \partial_\nu \omega_\lambda)$$

$$\to \delta S_{edge} \sim -\frac{1}{64\pi} \int d^2x\, \varepsilon_{\alpha\beta}\, \mathrm{tr}(\eta\, \partial_\alpha \omega_\beta)$$

trace anomaly at edge

Response cancels in Kane-Mele due to counter-propagating nature of edge states. (c.f. Kane-Fisher)

Message of 2nd part

Topological responses can arise in 'nontopological insulators' depending on probe:

→ hints at directions to extend notion of topological order?

May serve as a useful playground to develop methods to analyze quantized thermal responses.

(→may have bearings for frustrated magnets.)