A Simple Harmonic Universe

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Outline

1. Motivation

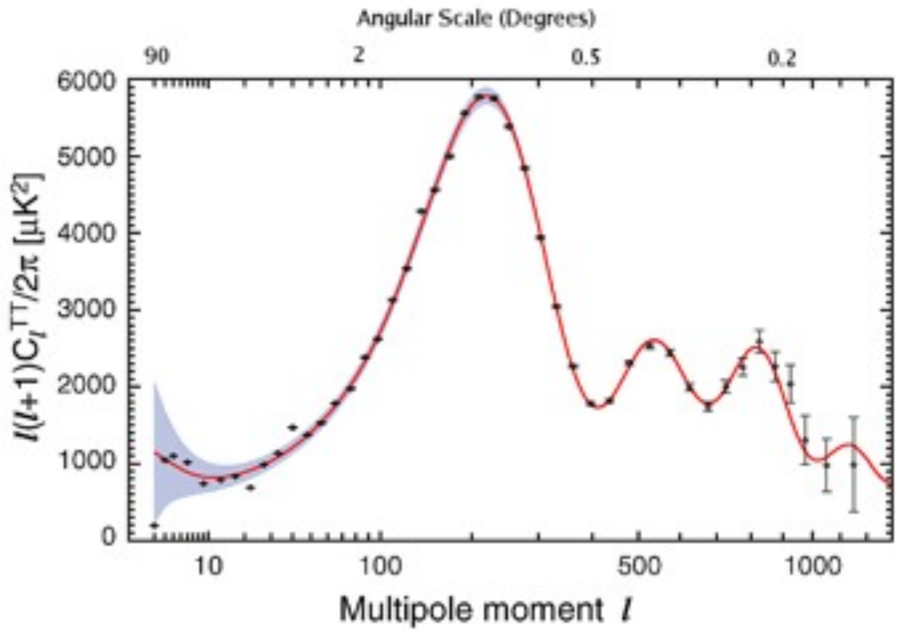
2. An Oscillating Universe Model

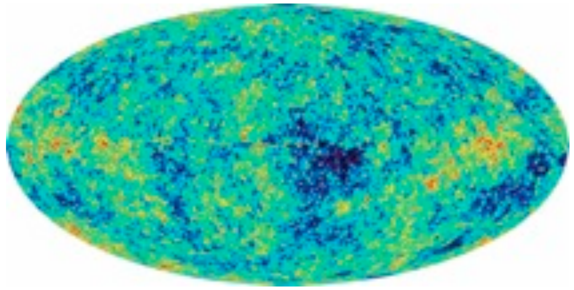
3. Classical and Quantum Stability

4. Conclusions and Future Questions

Motivation

Concordance Cosmology



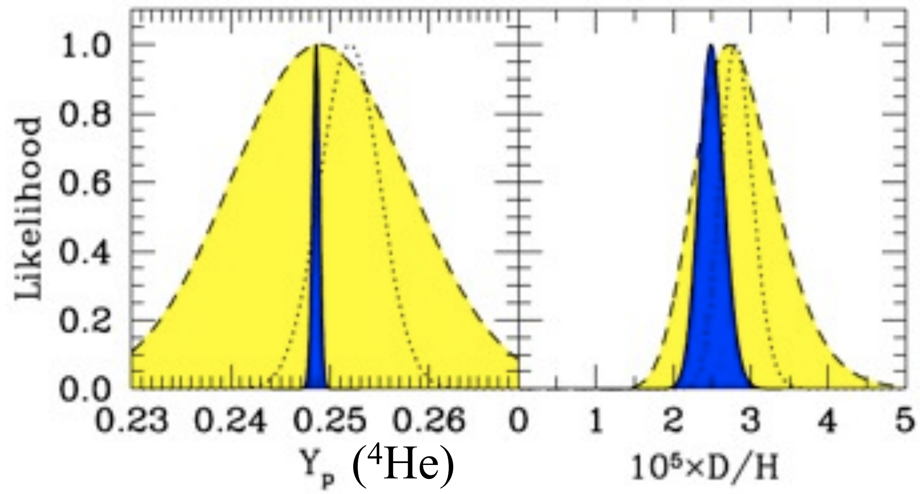


ΛCDM Cosmology and Inflation work extremely well

All observations explained with *high precision*

prediction

observation



We really understand the last ~14 billion years well!

Before Inflation?

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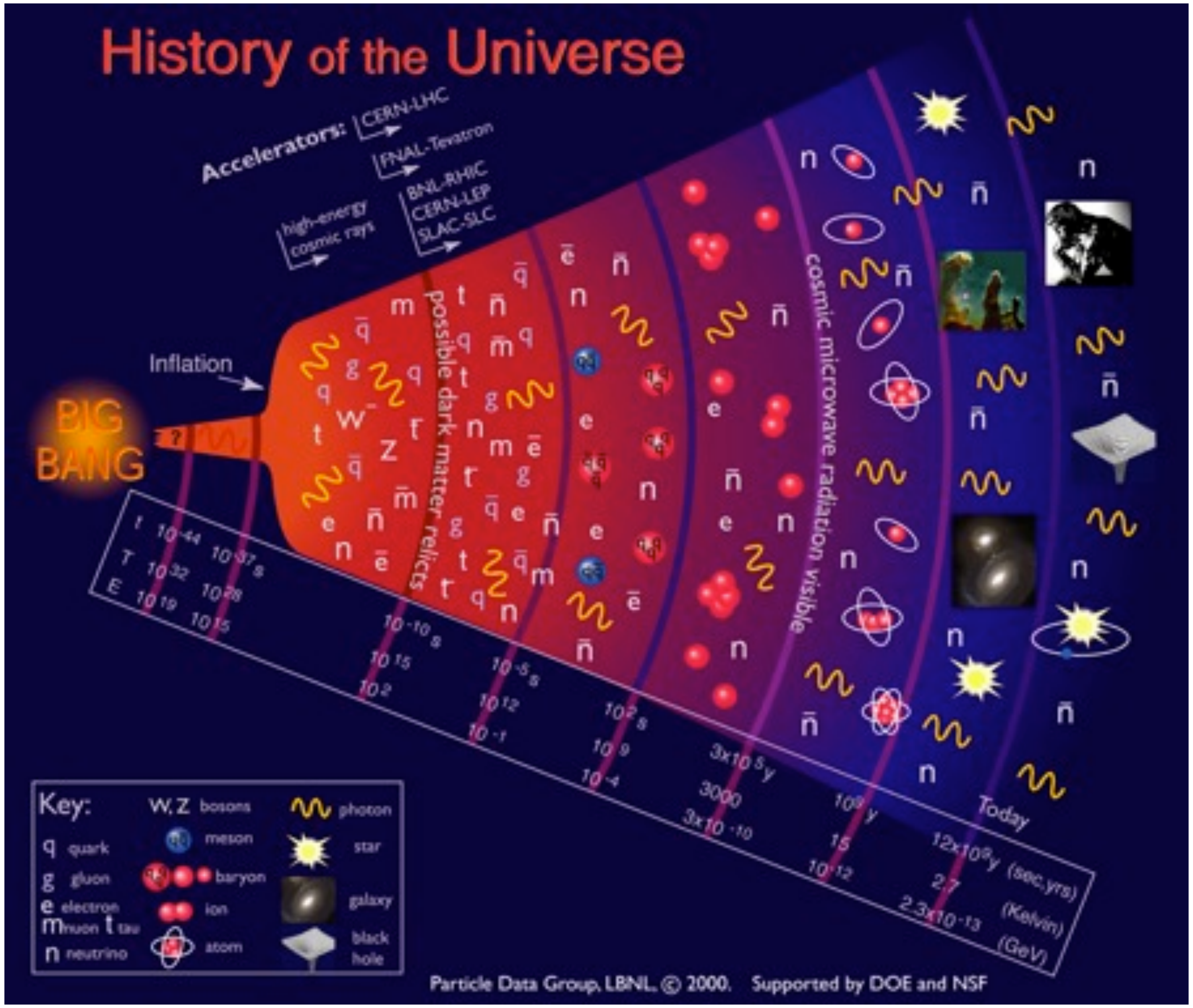

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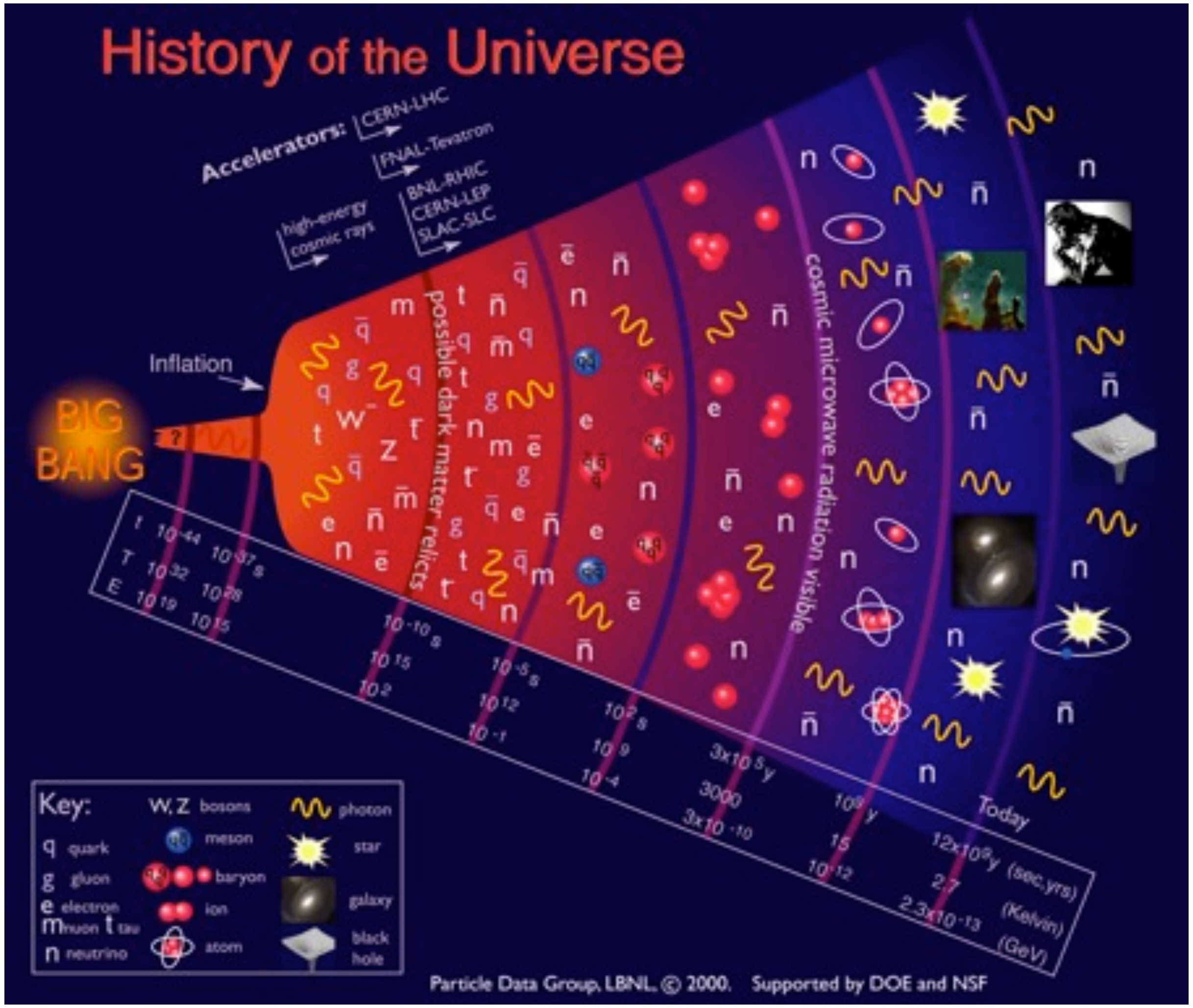

we have no direct evidence of
$T \gtrsim 100$ MeV (BBN)

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even if inflation is near GUT scale
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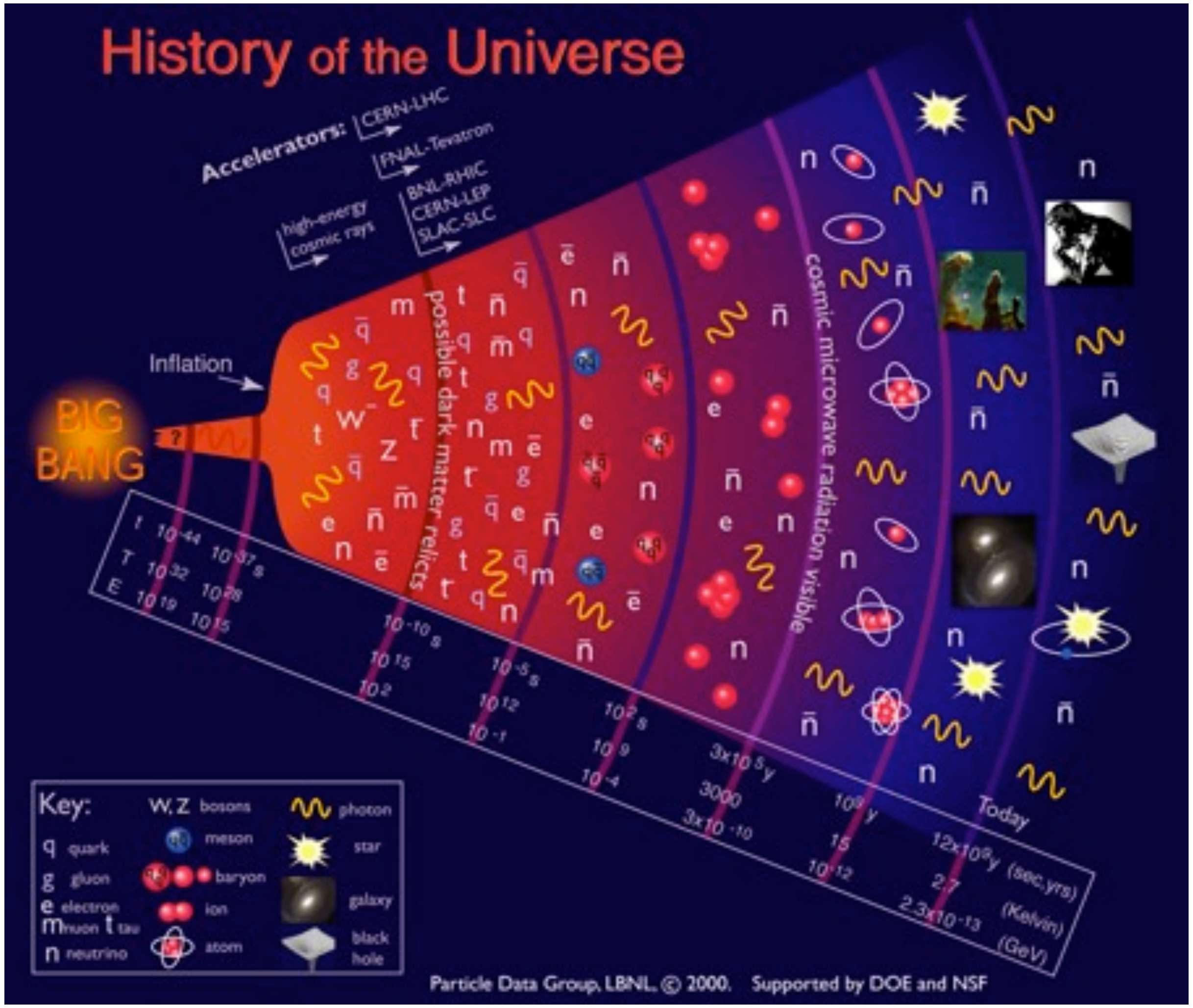

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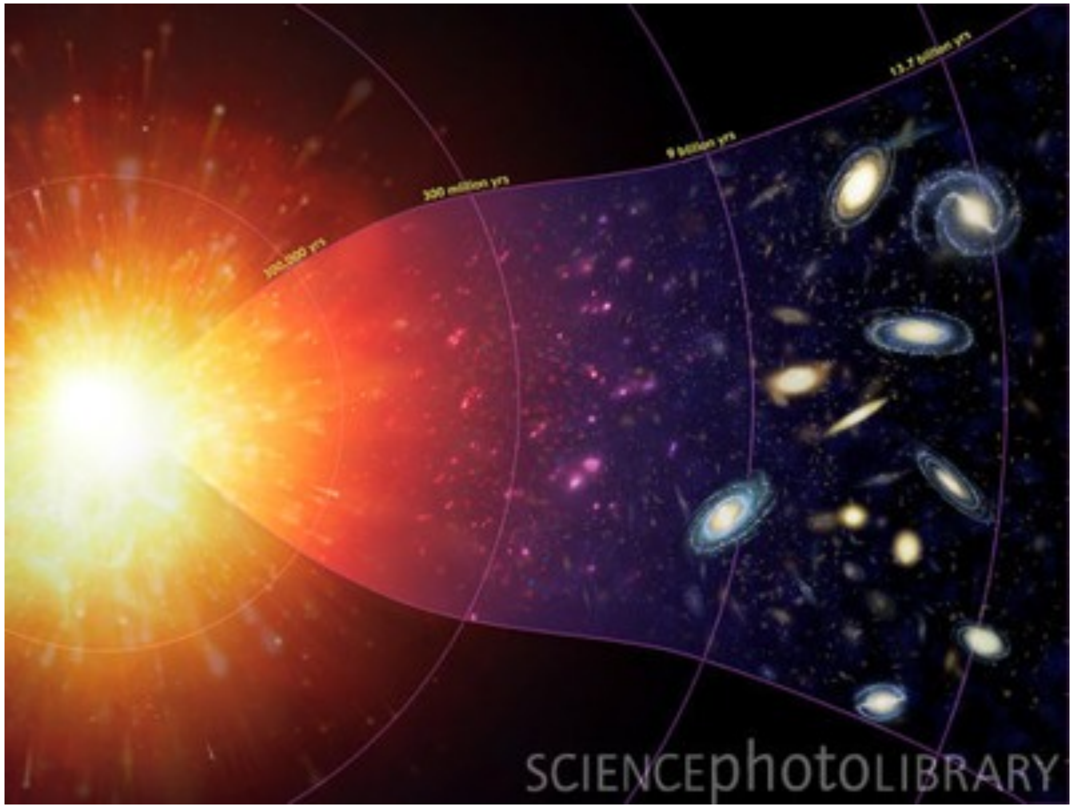
Could there in fact have been no Big Bang?

No Big Bang?



several problems, especially the Cosmological Constant,
lead to speculations about the period before inflation

But even in an eternal inflation scenario there is an initial singularity

Borde, Guth, & Vilenkin (2001)

In general the singularity theorems of Penrose and Hawking would seem to rule this out

Singularity Theorems

these theorems assume an energy condition, e.g.: $T_{\mu\nu}v^\mu v^\nu \geq 0$ for a class of vectors v^μ

and show that spacetime must be geodesically incomplete ("singular")

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As an example, consider the FRW universes:

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2(\theta) d\phi^2) \right)$$

for k = 0 or -1 need only assume the Null Energy Condition (NEC)

that v^μ is a future-pointing null vector field

in FRW this is just the statement that $\rho + p \geq 0$

or for $p = w\rho$ this is just $\rho\,(1+w) \geq 0$ or $w \geq -1$ (for $\rho > 0$)

this is reasonable, in agreement with everything known in our world, and generally allows avoidance of microphysical problems such as ghosts

Singularity Theorems

However for k = +1 need to assume the Strong Energy Condition (SEC)

if v^μ is a future-pointing timelike vector field $$\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)v^\mu v^\nu \geq 0$$

in FRW this requires $\rho + 3p \geq 0$ (and $\rho + p \geq 0$)

This is violated by a cosmological constant (dark energy, inflation...)!

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We will make an oscillating ("bouncing") cosmology
this has attracted interest even if not eternal (e.g. to replace inflation)

Tolman (1931) Lemaitre (1933) "Phoenix Universe"

Creminelli, Luty, Nicolis, & Senatore (2006) NEC violating

Gasperini & Veneziano (2002) Khoury, Ovrut, Steinhardt, & Turok (2001)

relies on as yet unknown high energy theory

and many more...

An Oscillating Universe Model

Conditions For Oscillation

FRW metric: $$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2(\theta) d\phi^2) \right)$$

FRW equations: $$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi}{3} G \left(\rho + 3p\right)$$

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$\ddot{a} > 0$ at a_-

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$\dot{a} = 0$ $\Rightarrow$ need a negative CC at a_+ so that $\rho = 0$

We need a negative CC and positive curvature and a "matter" component

Minimal Oscillatory Model

We need a negative CC and positive curvature and a “matter” component

take $\rho = \Lambda + \rho_0 \, a^{-3(1+w)}$ and $k = +1$

can show that this creates an oscillating universe if and only if $-1 < w < -\frac{1}{3}$

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FRW equations:

$$\frac{\ddot{a}}{a} + \left(\frac{1+3w}{2}\right)\frac{\dot{a}^2}{a^2} = 4\pi\,(1+w)\,G\Lambda - \left(\frac{1+3w}{2}\right)\frac{k}{a^2}$$

conditions for oscillation:

$$\ddot{a} > 0 \quad \text{at } a_- \quad \Leftrightarrow w < -\frac{1}{3}$$

$$\ddot{a} < 0 \quad \text{at } a_+ \quad \Leftrightarrow w > -1$$

This model will continually oscillate between two fixed scale factors, $a_- \leftrightarrow a_+$ though not analytically solvable

A Simple Harmonic Universe

In the special case of $w = -\frac{2}{3}$ it is analytically solvable

$\frac{\ddot{a}}{a} \sim \Lambda + \frac{\rho_0}{a}$ is just a (constrained) simple harmonic oscillator

the solution is $$a = \frac{\rho_0}{2|\Lambda|} + a_0 \cos(\omega t + \psi)$$

where $\omega \equiv \sqrt{\frac{8\pi}{3} G|\Lambda|}$ and $a_0 \equiv \frac{1}{2|\Lambda|}\sqrt{\frac{3\Lambda}{2\pi G} + \rho_0^2}.$

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unlike Einstein Static, this is not tuned, occurs in an open set of parameter space

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in conformal time: $d\eta^2 = dt^2/a(t)^2$ $$a(\eta) = \frac{1}{\omega}\frac{\sqrt{\gamma}}{1 - \sqrt{1-\gamma}\cos(\eta)}$$

where $\gamma \equiv \frac{3|\Lambda|}{2\pi G \rho_0^2}$ which is $\approx 4\frac{a_-}{a_+}$ for small γ

Classical and Quantum Stability

Stability Equations

the gauge invariant description of tensor perturbations is

$$ds^2 = a(\eta)^2 \left[-d\eta^2 + (\delta_{ij} - h_{ij})\, dx^i dx^j\right]$$

the equation of motion for the tensor perturbations is then:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2_{S^3} h_{ij} = 0 \qquad \text{where} \quad \mathcal{H} = \frac{a'}{a}$$

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also consider general anisotropic perturbations $ds^2 = -dt^2 + \sum_{i=1}^{3} a_i^2(t)\,\sigma_i^2$

σ_i are the Maurer-Cartan forms on S^3 $\qquad$ $\beta_\pm$ are functions of the a_i

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All these equations of motion are equivalent

Scalar Perturbations

the general description of scalar perturbations is

$$ds^2 = a(\eta)^2 \left[-(1+2\Phi(\eta,x))d\eta^2 + (1-2\Psi(\eta,x))d\Omega_3^2\right]$$

for perfect fluids: $\Phi = \Psi \qquad \delta p = c_s^2 \delta\rho$

the equation of motion is:

$$\Psi'' + 3\mathcal{H}(1+c_s^2)\Psi' + \left[2\mathcal{H}' + (1+3c_s^2)(\mathcal{H}^2 - k)\right]\Psi - c_s^2 \nabla_{S^3}^2 \Psi = 0$$

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$c_s^2 < 0 \;\Rightarrow$ drastic high-momentum instability

don't take a perfect fluid, use a "solid" with shear resistance, see e.g. Bucher & Spergel (1998)

e.g. a frustrated network of domain walls gives $w = -\frac{2}{3}$ but $c_s^2 > 0$

solid could have microscopic dynamics leading to entropy production

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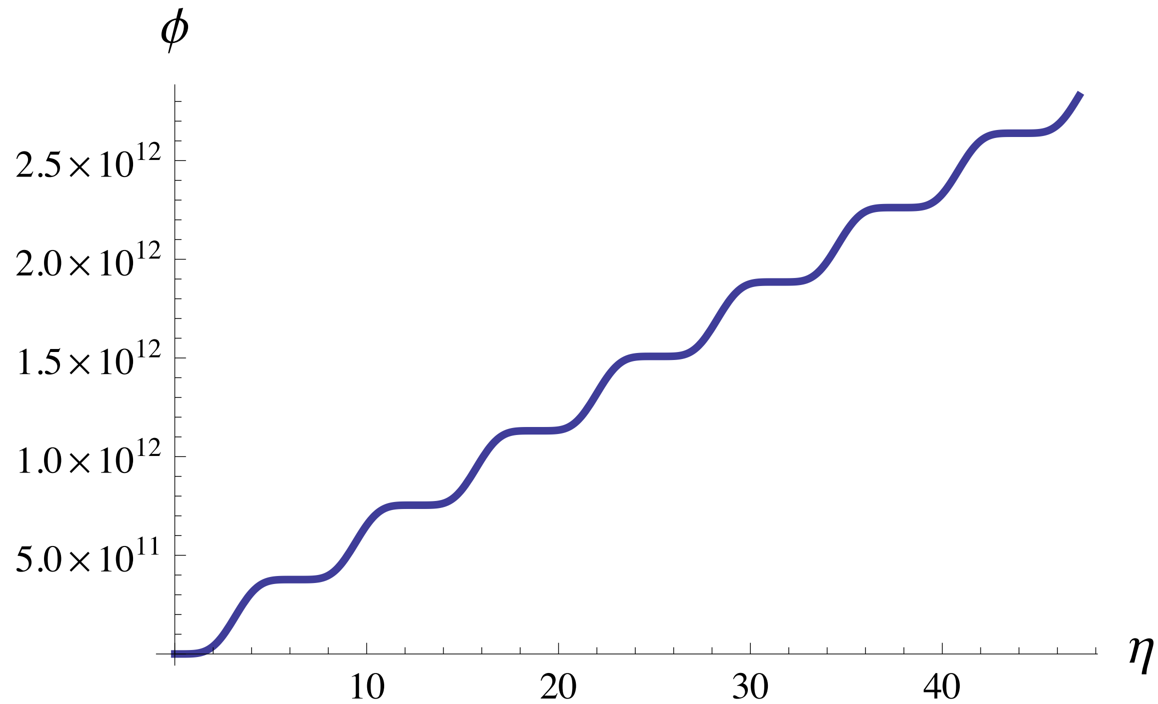
the scalar perturbation equation is qualitatively similar to the other perturbation equations in behavior, though not quantitatively the same

Homogeneous Perturbations

homogeneous mode: $l = 0$



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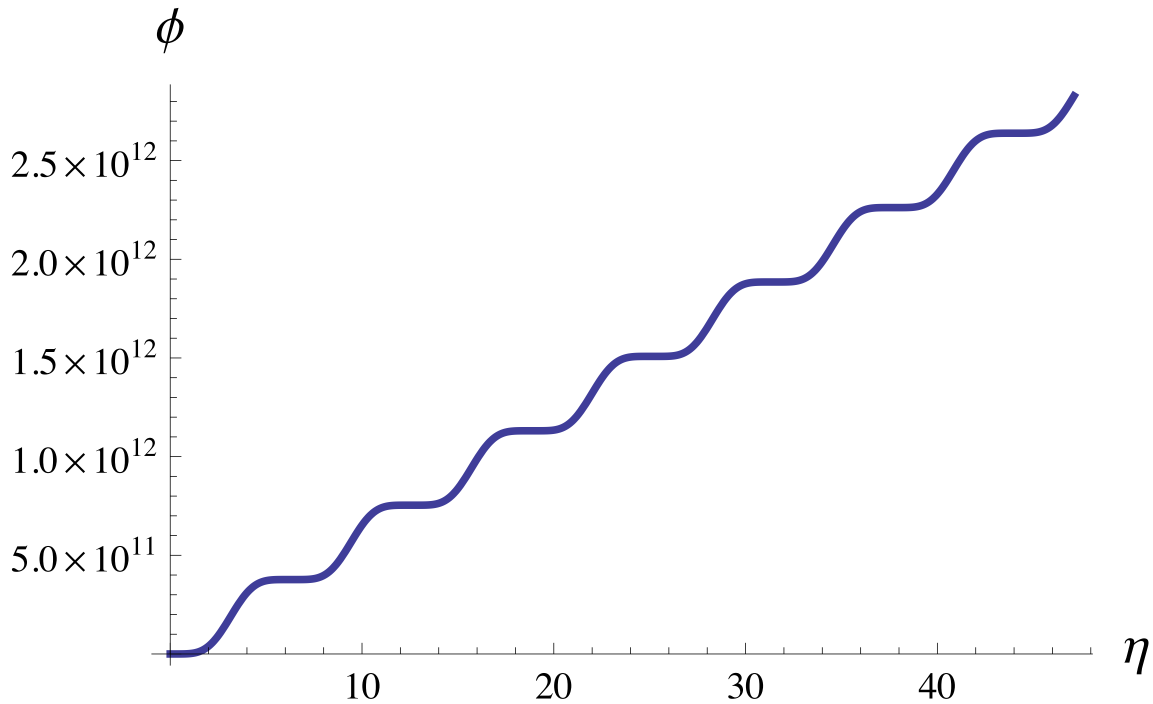

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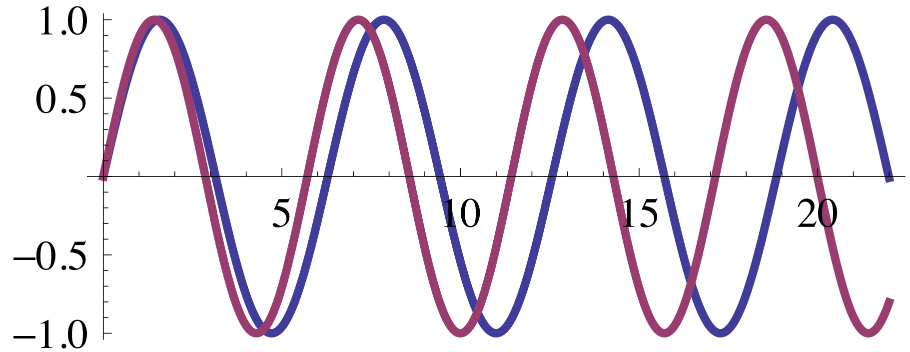
a homogeneous perturbation just moves us to a different one of our family of solutions, since we have an open set (unlike Einstein Static) it is stable against linear perturbations

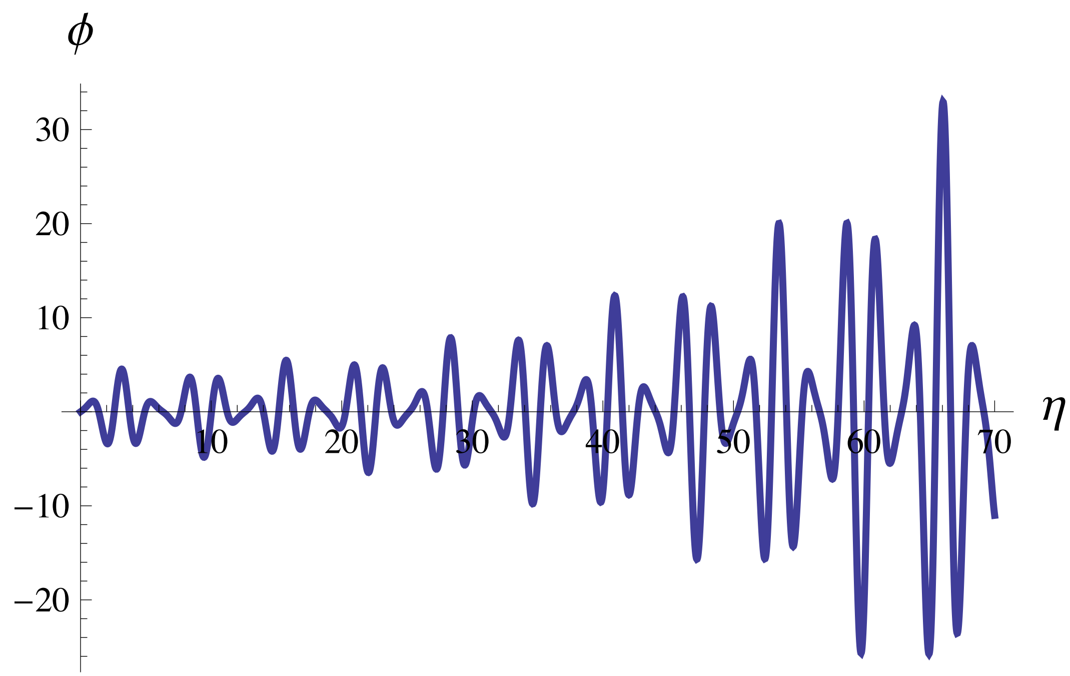
two cosines of slightly different periods differ linearly:



Intermediate Wavelengths

intermediate wavelength modes:

$$2 \leq l \lesssim \frac{1}{\sqrt{\gamma}}$$



differentiate two cases:

1) $\gamma \sim \mathcal{O}(1)$ = O(1) oscillations

all these modes are stable

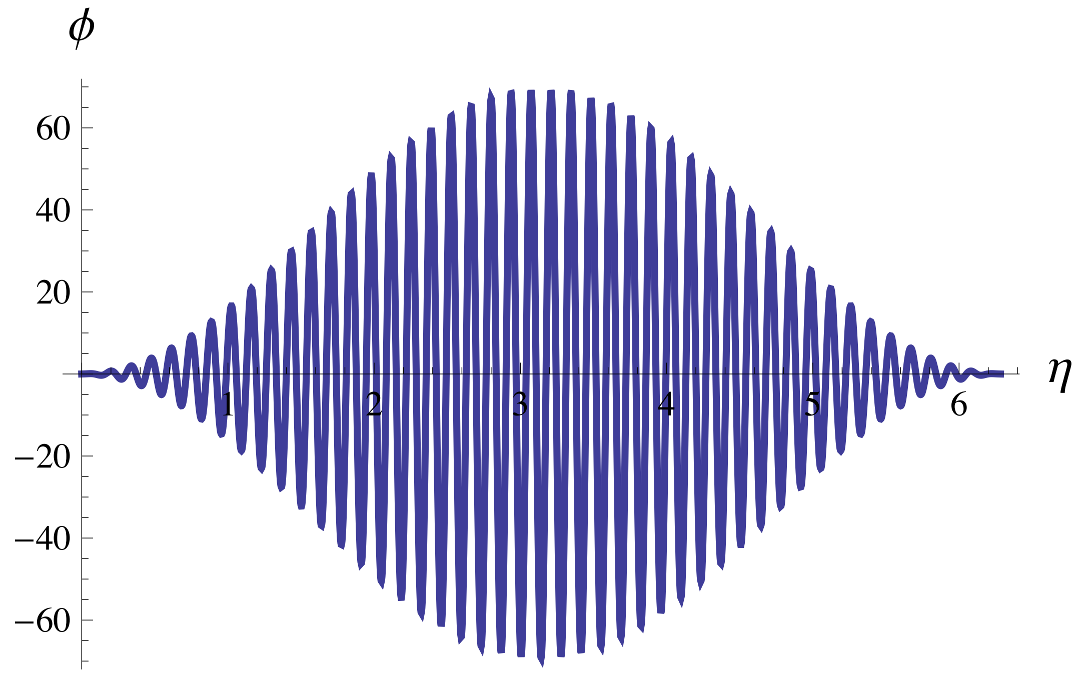
2) $\gamma \ll 1$ = large oscillations

some of these modes are unstable, grow exponentially
the instability starts at the smallest l-modes

Short Wavelengths

short wavelength modes:

$$l \gg \frac{1}{\sqrt{\gamma}}$$



these modes are much faster than frequency of universe's oscillation,
much smaller wavelength than size of universe,
should behave like normal Minkowski space modes

These modes are stable for both small and large oscillations in the universe

$\gamma \sim \mathcal{O}(1)$ and $\gamma \ll 1$

The l=1 Mode

Can show that the scalar $l = 1$ mode is always unstable

in our simple model it is not physical, just pure gauge, however in multi-fluid models a relative velocity between the fluids is physical

likely to be killed by non-gravitational damping which we have not included, e.g. free streaming rate is larger than growth rate of mode for $\gamma \sim O(1)$

Even when not, can project out by orbifolding the S^3

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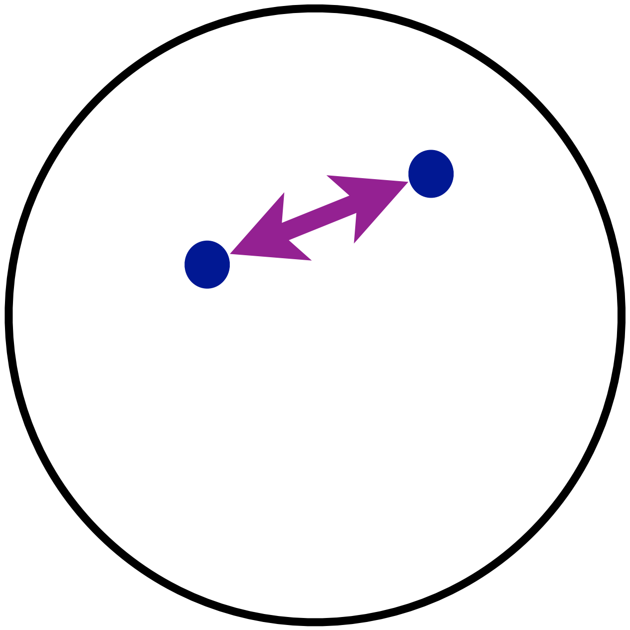
Summary of entire stability analysis:

$\gamma > \gamma_c \sim \mathcal{O}(1)$ all modes can be made stable

$\gamma < \gamma_c$ there are unavoidable instabilities in certain low-l modes

Gravitational Radiation - An Example

Why don't inhomogeneous perturbations radiate gravitational waves?



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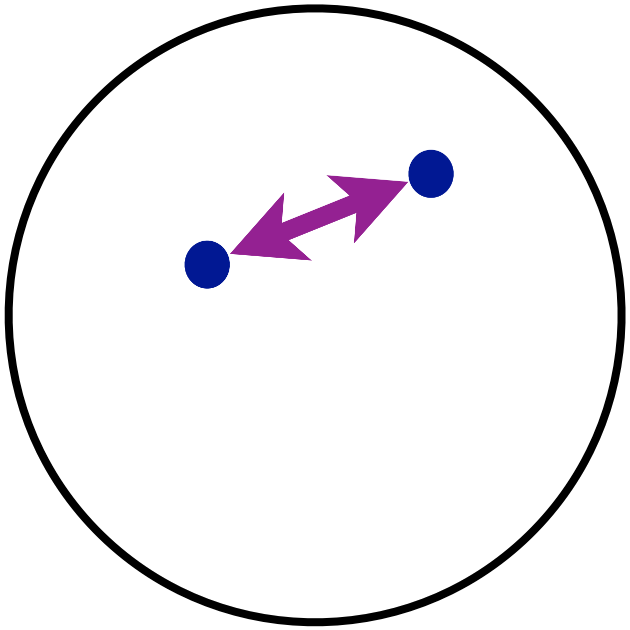

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$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2_{S^3} h_{ij} = 0 \ + \sim \sin(\omega t)$$

of course, do radiate at higher order

mode spectrum is $\nabla^2_{S^3} \propto -l(l+2)$

this just drives modes of h_{ij} off-resonance,
they excite but don't runaway because they're gapped.

without a resonance, non-linear effects remain small

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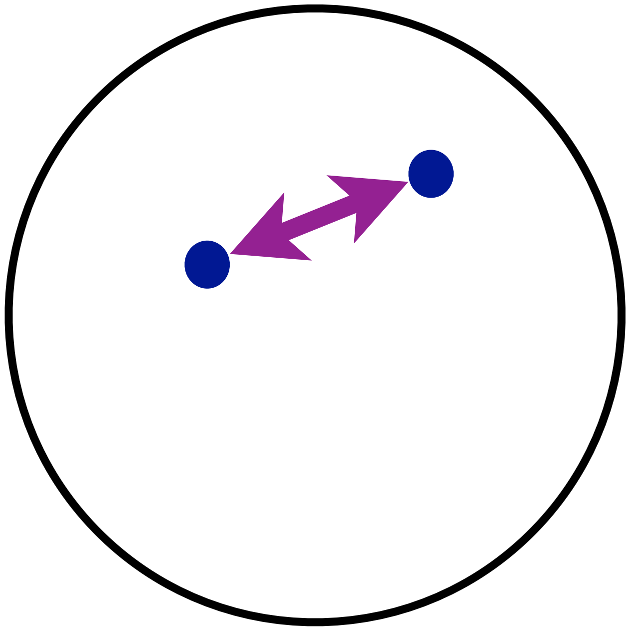

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size of universe is $a \sim \frac{\rho_0}{2|\Lambda|}$ frequency of oscillation is $\omega = \sqrt{\frac{8\pi}{3}G|\Lambda|}$

for $\gamma \sim O(1)$ the size of the universe is slightly larger than its period

never in the far-field regime compared to wavelength of gravitational waves,
don't have radiation modes at frequency ω available $\Rightarrow$ no resonant excitation

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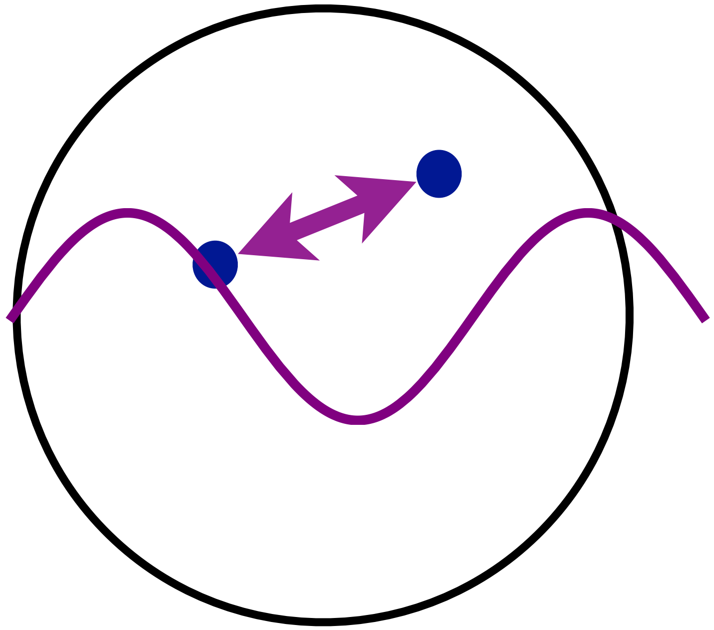

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size of universe is $a \sim \frac{\rho_0}{2|\Lambda|}$ frequency of oscillation is $\omega = \sqrt{\frac{8\pi}{3} G|\Lambda|}$

for $\gamma \sim O(1)$ the size of the universe is slightly larger than its period

never in the far-field regime compared to wavelength of gravitational waves,
don't have radiation modes at frequency ω available $\Rightarrow$ no resonant excitation

Quantum Particle Production

for $\gamma \ll 1$ always have unstable modes

growing modes are roughly $\phi_l(N) \sim \phi_0 \exp\left(c\sqrt{1-\frac{l^2}{l_c^2}} \times N\right)$ ← number of bounces

can classically tune initial amplitude of these modes to allow arbitrarily many bounces

however, quantum mechanically the growing modes mean particle production

have $\langle \phi_0^2 \rangle > 0 \Rightarrow$ mode exceeds background energy density at $N_c \sim \log\left(\frac{M_P}{\phi_0}\right)$

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canonically normalizing: $\chi \equiv a(\eta)\phi$

and quantizing: $[\chi(\theta), \partial_\eta\chi(\theta')] = i\delta^{(3)}(\theta-\theta')$

→ $a^2\phi_0^2 \sim 1$

choosing the background associated to the maximal size of the universe: $a_+ = \frac{2}{\omega\sqrt{\gamma}}$

gives a bound: $N_c \sim \log\left(\frac{M_P}{\omega\sqrt{\gamma}}\right)$ can be made parametrically long

$\gamma \sim \mathcal{O}(1)$ is stable quantum mechanically

Conclusions and Future Questions

Summary

1. Constructed a minimal model of an oscillating universe with positive curvature, negative CC, and a $-1 < w < -1/3$ matter
   - avoids singularity theorems with positive curvature and SEC violation
   - does not violate the NEC
   - bounces under full control in low energy effective theory
   - makes a “static” (on average) universe, very different from Einstein Static
2. Analyzed stability at the linear level with gravitational backreaction
   - for $\gamma \sim O(1)$ it appears all perturbations can be made stable
   - for $\gamma \ll 1$ have unavoidable instabilities in low-l modes
   - evidence (not proof) of stability at non-linear level

Future Questions

1. Do microscopic dynamics of our solid produce entropy?
    - would lead to singularity even in our seemingly eternal models
2. What does this actually have to do with our universe?
3. Can we tunnel out of this eternal phase to a realistic inflationary cosmology, thus removing the initial singularity in our universe?
4. Can our observed universe fit into the expansion phase of one oscillation?
    - not necessarily eternal, but perhaps has many recurrences
    - perhaps non-gravitational damping (e.g. free streaming) ameliorates the instabilities in such a large oscillation model?
    - requires “Higgsing” light SM modes above a high energy scale so curvature can dominate
5. Can we prove a general quantum singularity theorem?