Combinatorial rigidity in curve complexes and mapping class groups. (addendum)

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(in the proof of Lemma 13, page 228 of PJM, page 11 of the author's copy)

It is noted by the author that with regard to the special case of an injection from the curve graph of the three-holed torus to the curve graph of six-holed sphere (or of the closed surface of genus two), the ϕ-image of the pants decomposition P does not seem adequately constrained - even using small intersection. There is a straightforward alternative:

"…The remaining cases, namely from the curve complex of the three-holed torus to the curve complex of the six-holed sphere or of the closed surface of genus two, are covered as follows: For any pants decomposition P of Σ_1 comprising only non-separating curves, choose a pair of distinct outer curves δ_1 and δ_2 each of small intersection with a single but different curve from P. Let α be a non-separating curve disjoint from both δ_1 and δ_2 intersecting every curve in P. When Σ_2 is the six-holed sphere, according to Lemma 8 each curve in P can only go to an outer curve. By Lemma 6, small intersection is preserved under ϕ. In particular, $\phi(\alpha)$ is a curve intersecting all three curves from $\phi(P)$. In order to do so, $\phi(P)$ must intersect at least one of $\phi(\delta_1)$ and $\phi(\delta_2)$, and this is a contradiction. This simultaneously deals with Σ_2 the closed surface of genus two. $\diamondsuit$"