

Correction : Ward identity for diffeomorphism.

I said in the class that Ward id for a diffeomorphism $f: \Sigma \rightarrow \Sigma$ is

$$\langle f^* \mathcal{O} \rangle_{f^*g} = \langle \mathcal{O} \rangle_g \quad \text{--- (W)}$$

But it is WRONG! The correct one is

$$\langle \mathcal{O} \rangle_{f^*g} = \langle f^* \mathcal{O} \rangle_g \quad \text{--- (R)}$$

Proof ~~Ward~~ The diffeo-invariance of theory means

$$D_{f^*g} f^* X e^{-S_E(f^*g, f^*X)} = D_g X e^{-S_E(g, X)}.$$

From this it follows

$$\int D_{f^*g} f^* X e^{-S_E(f^*g, f^*X)} \mathcal{O}(f^*X) = \int D_g X e^{-S_E(g, X)} \mathcal{O}(f^*X)$$

$$\parallel \text{put } f^*X = X'$$

$$\int D_{f^*g} X' e^{-S_E(f^*g, X')} \mathcal{O}(X')$$

$$\therefore \langle \mathcal{O} \rangle_{f^*g} = \langle f^* \mathcal{O} \rangle_g.$$

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Check of Consistency

First, despite the notation, $f^* \circ$ has the following property under composition of diffeos:

$$(f_1 f_2)^* \circ = f_1^* (f_2^* \circ)$$

proof $((f_1 f_2)^* \circ)(x) = \circ((f_1 f_2)^* x) = \circ(f_2^* (f_1^* x))$
 $= (f_2^* \circ)(f_1^* x) = (f_1^* (f_2^* \circ))(x) \quad //$

[* Perhaps $f^* \circ$ is a wrong notation. We may simply write $f \circ$. (so $(f \circ)(x) = \circ(f^* x)$).
But we stick to $f^* \circ$ since what it means is obvious]

$$\begin{aligned} \langle \circ \rangle_{(f_1 f_2)^* g} &= \langle (f_1 f_2)^* \circ \rangle_g = \langle f_1^* (f_2^* \circ) \rangle_g \\ &\parallel \qquad \qquad \qquad \parallel \\ \langle \circ \rangle_{f_2^* f_1^* g} &= \langle f_2^* \circ \rangle_{f_1^* g} = \langle f_1^* (f_2^* \circ) \rangle_g \end{aligned}$$

Consistent!

NB I first posted the wrong version (W) in the note for Nov. 22 class. Later, I revised — (R) is included.

Please be careful if you downloaded the first version!