

Homework 3 (MAT 1739, CFT II)

We have seen that the partition function of a GSO projected massless Majorana fermion system is

$$\begin{aligned}
 Z &= \frac{1}{2}Z_{\text{AP,AP}} + \frac{1}{2}Z_{\text{AP,P}} + \frac{1}{2}Z_{\text{P,AP}} \mp \frac{1}{2}Z_{\text{P,P}} \\
 &= \frac{1}{2} \left| q^{-\frac{1}{48}} \prod_{r>0} (1+q^r) \right|^2 + \frac{1}{2} \left| q^{-\frac{1}{48}} \prod_{r>0} (1-q^r) \right|^2 + \frac{1}{2} \cdot 2 \left| q^{\frac{1}{24}} \prod_{n>0} (1+q^n) \right|^2 \mp \frac{1}{2} \cdot 0 \\
 &= \left| \chi_0(q) \right|^2 + \left| \chi_{\frac{1}{2}}(q) \right|^2 + \left| \chi_{\frac{1}{16}}(q) \right|^2 \tag{1}
 \end{aligned}$$

where

$$\begin{aligned}
 \chi_0(q) &= \frac{1}{2}q^{-\frac{1}{48}} \prod_{r>0} (1+q^r) + \frac{1}{2}q^{-\frac{1}{48}} \prod_{r>0} (1-q^r) \\
 &= q^{-\frac{1}{48}} \left(1 + \sum_{r_1<r_2} q^{r_1+r_2} + \sum_{r_1<r_2<r_3<r_4} q^{r_1+r_2+r_3+r_4} + \dots \right), \\
 \chi_{\frac{1}{2}}(q) &= \frac{1}{2}q^{-\frac{1}{48}} \prod_{r>0} (1+q^r) - \frac{1}{2}q^{-\frac{1}{48}} \prod_{r>0} (1-q^r) \\
 &= q^{\frac{1}{2}-\frac{1}{48}} \left(\sum_r q^{r-\frac{1}{2}} + \sum_{r_1<r_2<r_3} q^{r_1+r_2+r_3-\frac{1}{2}} + \dots \right), \\
 \chi_{\frac{1}{16}}(q) &= q^{\frac{1}{24}} \prod_{n>0} (1+q^n) \\
 &= q^{\frac{1}{16}-\frac{1}{48}} \left(1 + \sum_n q^n + \sum_{n_1<n_2} q^{n_1+n_2} + \dots \right),
 \end{aligned}$$

Expand the three big parentheses up to order q^7 .