Final Homework (MAT 1711, CFT I)

Deadline: December 20.
Submit to: Helen Iyer at MP1109.

Solve the problems listed below, as many as you wish. Or, find some problem by yourself and solve it.

1. Partition function of the sigma model on a circle

   (a) Show that the torus partition function of the sigma model with $S^1_{2\pi R}$ target (obtained in the class of October 23) is modular invariant.

   (b) Compute the same partition function using path integral. You can use the zeta function regularization, if that is convenient.

2. Correlation functions in some free CFTs

   (a) Compute the correlation function $\langle :\mathrm{e}^{\sqrt{2}i\widehat{X}(1)}::\mathrm{e}^{-\sqrt{2}i\widehat{X}(2)}::\mathrm{e}^{\sqrt{2}iX(3)}::\mathrm{e}^{-\sqrt{2}iX(4)}:\rangle$ in the sigma model on the circle with $R=\sqrt{2}$ using boson fermion correspondence. ($\widehat{X}$ is the T-dual coordinate.)

   (b) Discuss the property of that correlator as the points 1,2,3,4 circle around each other. Interpret the result.

3. OPE in some free CFTs

   (a) Expand the operator product $T_{zz}(z)T_{ww}(w)$ in the massless Dirac fermion system.

   (b) Do the same for the general first order system introduced at the end of the class of November 22. What is the central charge? (You can use $c^{z\cdots}(z)b_{w\cdots}(w)\sim\frac{1}{z-w}$.) Is the system with $\lambda=1$ or $\lambda=2$ a unitary CFT?

   (c) Consider the operator $:\mathrm{e}^{ikX}:$ in the free massless scalar theory. Is it a primary operator? What is its conformal weight?

   (d) Consider the operator $:\overline{\psi}_-\psi_-:$ in the Dirac fermion system. Is it a primary operator? What is its conformal weight?

4. L_n's and $\widetilde{L}_n$'s in the sigma model on the circle $S^1_{2\pi R}$.

   (a) Find the expression of L_n's and $\widetilde{L}_n$'s (as operators acting on states) in terms of α_n's, $\widetilde{\alpha}_n$'s, $\alpha_0:=\frac{1}{\sqrt{2}}(p-w)$ and $\widetilde{\alpha}_0:=\frac{1}{\sqrt{2}}(p+w)$.

   (b) Consider an operator $\mathcal{O}_{l,m}$ corresponding to the state $|l,m\rangle$ of momentum $\frac{l}{R}$ winding number m, introduced in the October 18 class. Is it a primary operator? What is the dimension and the spin of that operator?