Right vs left actions

Suppose a Lie group G acts on a manifold M, and it is a right action: $x(gh) = (xg)h$ for $x \in M$, $g, h \in G$.

Variation of a function f on M by $X \in \mathfrak{g} = \text{Lie}(G)$ is a function $\delta_X f$ on M defined by

$$(\delta_X f)(x) = \frac{d}{dt} f(xe^{tX}) \Big|_{t=0}.$$

Show that, for $X \& Y \in \mathfrak{g}$,

$$\delta_X \delta_Y f - \delta_Y \delta_X f = \delta_{[X,Y]} f$$

Suppose it is a left action instead, $(gh)x = g(hx)$.

The variation is $(\delta_X f)(x) = \frac{d}{dt} f(e^{tX} x) \Big|_{t=0}$.

Show that, for $X \& Y \in \mathfrak{g}$

$$\delta_X \delta_Y f - \delta_Y \delta_X f = -\delta_{[X,Y]} f$$