

渦状腕の線形摂動理論：
渦状腕分裂によるクランプ形成

Beyond Toomre's Q

MNRAS submitted

arXiv: 1706.01895 (←昨日！！)

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Spiral or Clumpy?

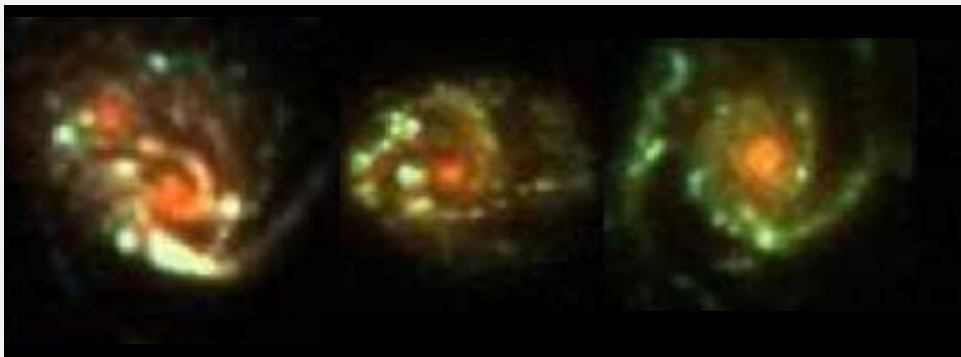
at low- z



- Spiral galaxies
 - Low redshifts
 - Gas-rich ($f_{\text{gas}} \sim 10\%$)

← Toomre instability

at high- z



- Clumpy galaxies
 - **Giant clumps**
 - High redshifts (mainly)
 - Gas-rich ($f_{\text{gas}} \sim 30\%$)

← Toomre instability

with HST Guo et al. (2014)

really...?

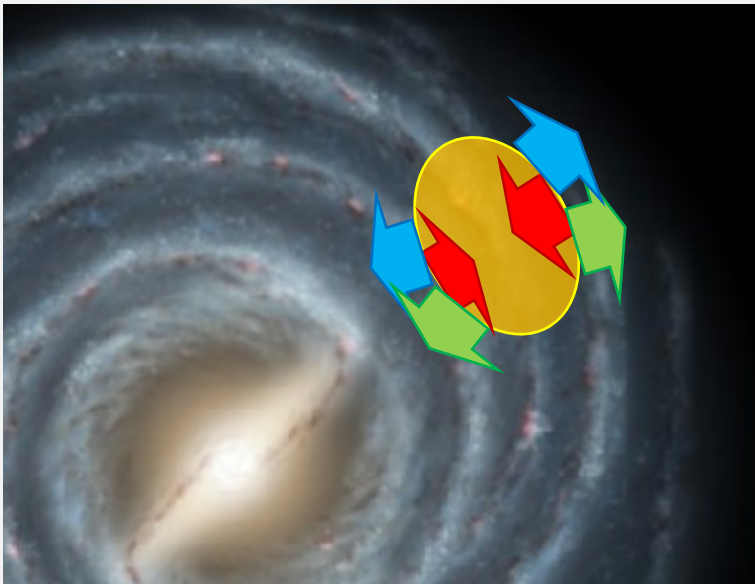
What was Toomre instability???

- From **local** and **linear** perturbation theory for **axisymmetric** perturbations in **smooth discs**,

Velocity dispersion
or sound speed
(pressure)

Epicyclic frequency
(Coriolis force)

The stability condition: $Q \equiv \frac{\sigma K}{\pi G \Sigma} > 1$

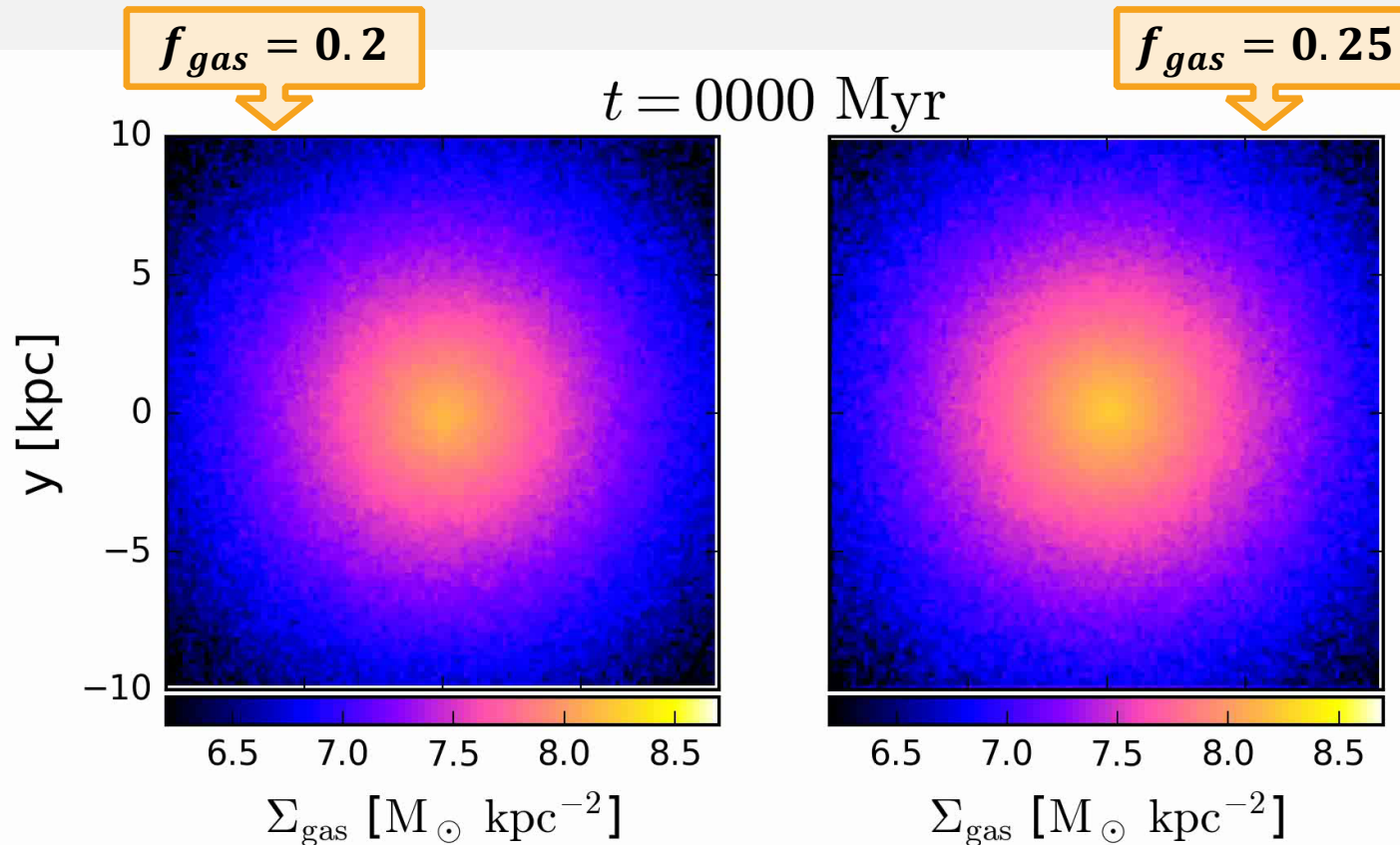


Surface density
(self-gravity)

If $Q < 1$, the local region is gravitationally unstable, going to collapse.

Spiral or Clumpy?

- Isolated disc galaxy simulations
 - Gas + stellar discs
 - Isothermal gas (no star formation, no feedback)
 - Moving-mesh code: Arepo



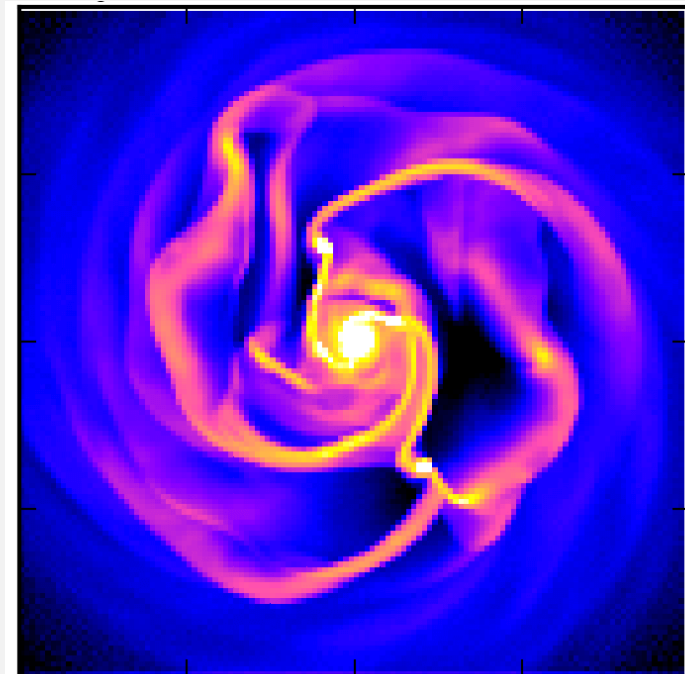
Spiral-arm fragmentation as a clump formation mechanism

- Spiral arms can fragment into clumps,
 - if a gas fraction is high and/or a disc is kinematically cold.
- Spiral-arm fragmentation is not Toomre instability!
- Spiral-arm fragmentation could be a possible mechanism of giant clump formation.

Beyond Toomre's Q

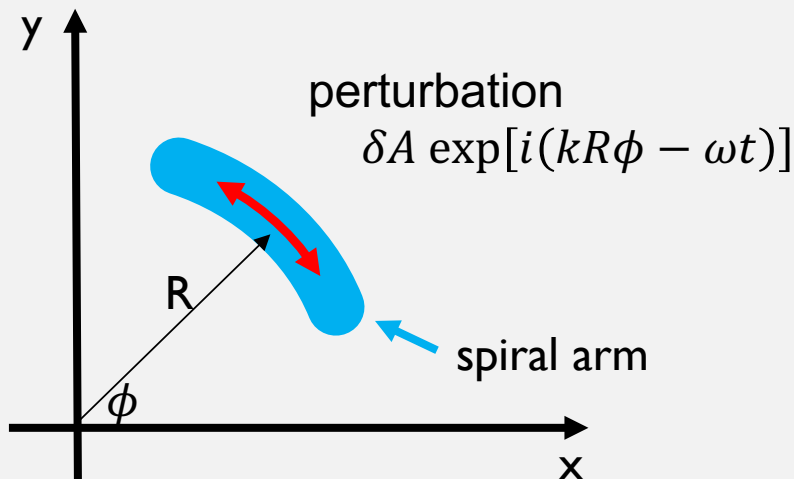
- The aims of this study:
 - How spiral-arm fragmentation occurs?
 - Derive **an instability parameter and its criterion**
 - Discuss **if the fragmentation can form high-z clumps**

Let's go to linear perturbation theory for a spiral arm!!
(線形擾動理論)



Set-up for the linear perturbation theory

- Now considering...
 - Gravitational instability for **azimuthal** perturbations on an **axisymmetric** spiral (ring).



Assuming:

- The spiral has **a rigid rotation** since self-gravitating.

$$\Omega = -B$$

- Replace surface density Σ with line-mass $\Upsilon = 1.4W\Sigma$ (**Gaussian**).

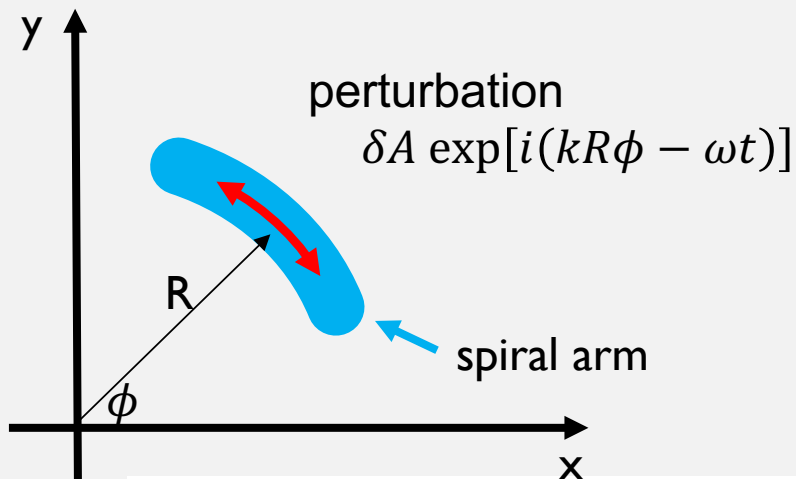
continuity:
$$\frac{\partial}{\partial t} \delta \Sigma + \frac{1}{R} \frac{\partial}{\partial R} (R \Sigma_0 \delta v_R) + \Omega \frac{\partial}{\partial \phi} \delta \Sigma + \frac{\Sigma_0}{R} \frac{\partial}{\partial \phi} \delta v_\phi = 0,$$

R-momentum:
$$\frac{\partial}{\partial t} \delta v_R + v_R \frac{\partial}{\partial R} \delta v_R + \Omega \frac{\partial}{\partial \phi} \delta v_R - 2\Omega \delta v_\phi = -\frac{\partial}{\partial R} \left(c_s^2 \frac{\delta \Sigma}{\Sigma_0} + \delta \Phi \right),$$

ϕ -momentum:
$$\frac{\partial}{\partial t} \delta v_\phi + v_R \frac{\partial}{\partial R} \delta v_\phi + \Omega \frac{\partial}{\partial \phi} \delta v_\phi - 2B \delta v_R = -\frac{1}{R} \frac{\partial}{\partial \phi} \left(c_s^2 \frac{\delta \Sigma}{\Sigma_0} + \delta \Phi \right).$$

Set-up for the linear perturbation theory

- Now considering...
 - Gravitational instability for **azimuthal** perturbations on an **axisymmetric** spiral (ring).



Assuming:

- The spiral has **a rigid rotation** since self-gravitating.

$$\Omega = -B$$

- Replace surface density Σ with line-mass $\Upsilon = 1.4W\Sigma$ (**Gaussian**).

continuity: $\omega\delta\Upsilon = k\Upsilon\delta v_\phi,$

R-momentum: $-i\omega\delta v_R = 2\Omega\delta v_\phi,$

φ-momentum: $-i\omega\delta v_\phi = -2\Omega\delta v_R - ik\frac{c_s^2}{\Upsilon}\delta\Upsilon - ik\delta\Phi.$

A dispersion relation for a single-component model

- One can obtain the dispersion relation for the perturbations,

$$\omega^2 = \left(c_s^2 + \frac{\Upsilon}{\delta\Upsilon} \delta\Phi \right) k^2 + 4\Omega^2.$$

- The Poisson equation for the perturbations is

$$\delta\Phi = \int_{-W}^W -G\delta\Upsilon K_0(|kx|)/W dx$$

$$= -\pi G\delta\Upsilon \left[\underbrace{K_0(kW)L_{-1}(kW) + K_1(kW)L_0(kW)}_{f(kW)} \right]$$

K : Bessel function

L : Struve function

$f(kW)$

W : half width of arm

A dispersion relation for a single-component model

- One can obtain the dispersion relation for the perturbations,

$$\omega^2 = \left(c_s^2 - \pi G f(kW) \Upsilon \right) k^2 + 4\Omega^2.$$

(cf. Takahashi, Tsukamoto & Inutsuka 2016)

- This can be transformed as

$$\frac{c_s^2 k^2 + 4\Omega^2 - \omega^2}{\pi G f(kW) \Upsilon k^2} = 1.$$

- When $\omega^2 < 0$, the spiral is unstable.
- Hence, the new instability parameter and its criterion can be defined as

$$S \equiv \frac{c_s^2 k^2 + 4\Omega^2}{\pi G f(kW) \Upsilon k^2} < 1.$$

A dispersion relation for a two-component model

- A galaxy usually has gas and stars. The dispersion relations of gas and stars are,

$$\text{gas:} \quad \omega^2 = \left(c_s^2 + \frac{\Upsilon_g}{\delta\Upsilon_g} \delta\Phi \right) k^2 + 4\Omega^2, \quad \delta\Upsilon_g = k^2 \frac{\Upsilon_g}{\omega^2 - 4\Omega^2 - c_s^2 k^2} \delta\Phi,$$

$$\text{stars:} \quad \omega^2 = \left(\sigma_\phi^2 + \frac{\Upsilon_s}{\delta\Upsilon_s} \delta\Phi \right) k^2 + 4\Omega^2, \quad \delta\Upsilon_s = k^2 \frac{\Upsilon_s}{\omega^2 - 4\Omega^2 - \sigma_\phi^2 k^2} \delta\Phi,$$

- Because gas and stars interact only through gravity, they are connected in the Poisson eq.,

$$\delta\Phi = -\pi G [\delta\Upsilon_g f(kW_g) + \delta\Upsilon_s f(kW_s)]$$

- Then, one can obtain the two-component dispersion relation,

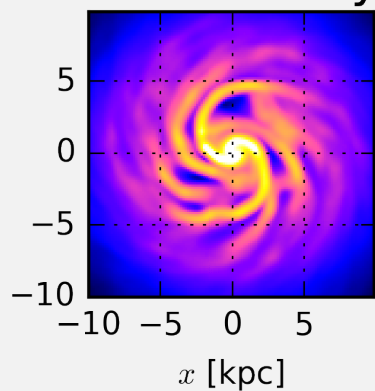
$$\left[\frac{\pi G k^2 \Upsilon_g f(kW_g)}{c_s^2 k^2 + 4\Omega^2 - \omega^2} + \frac{\pi G k^2 \Upsilon_s f(kW_s)}{\sigma_\phi^2 k^2 + 4\Omega^2 - \omega^2} \right] = 1,$$

- Finally, I obtain the new instability condition for 2-comp. models,

$$S \equiv \frac{1}{\pi G k^2} \left[\frac{\Upsilon_g f(kW_g)}{c_s^2 k^2 + 4\Omega_g^2} + \frac{\Upsilon_s f(kW_s)}{\sigma_\phi^2 k^2 + 4\Omega_s^2} \right]^{-1} < 1.$$

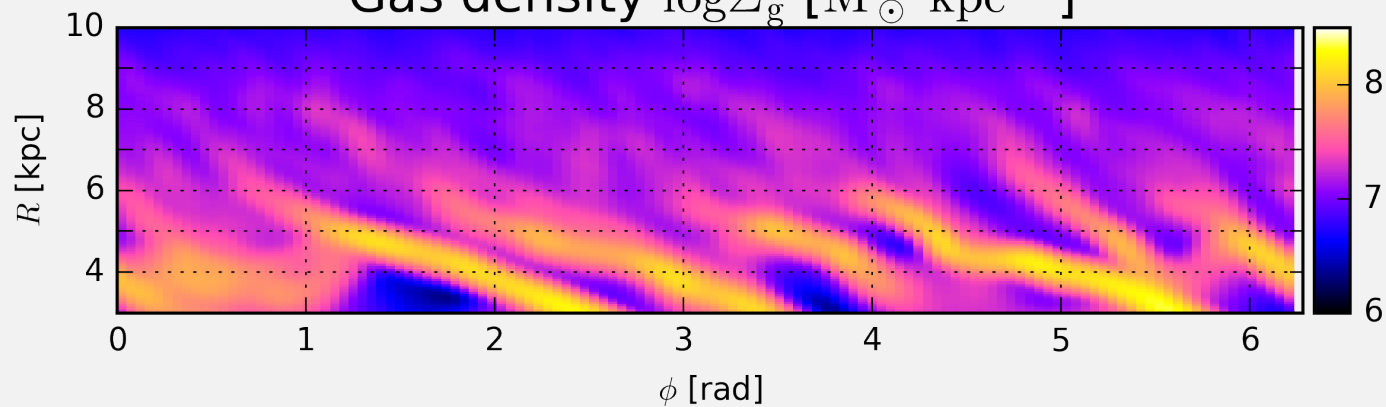
Demonstration

Gas density

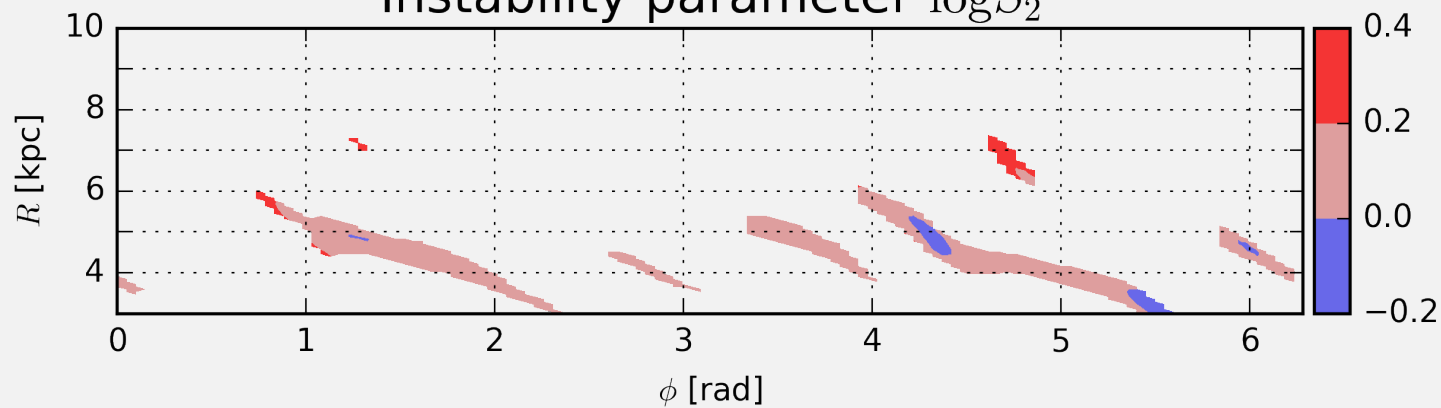


$t=150$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

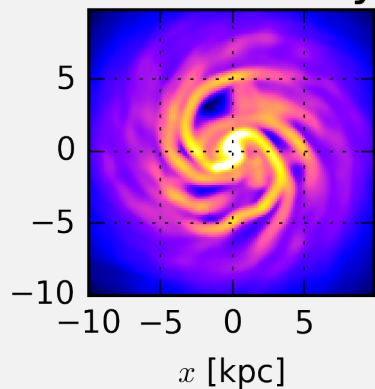


Instability parameter $\log S_2$



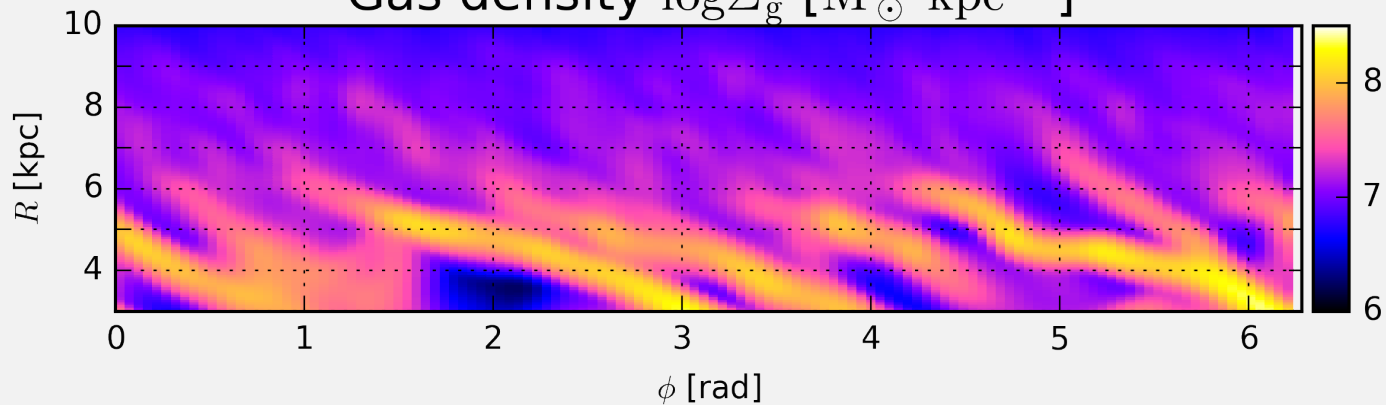
Demonstration

Gas density

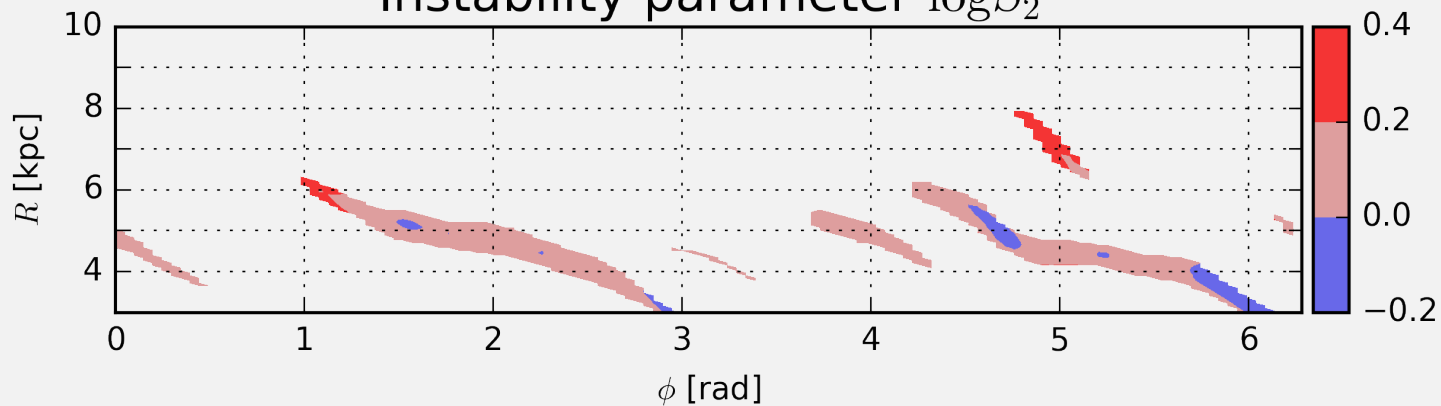


$t=160$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

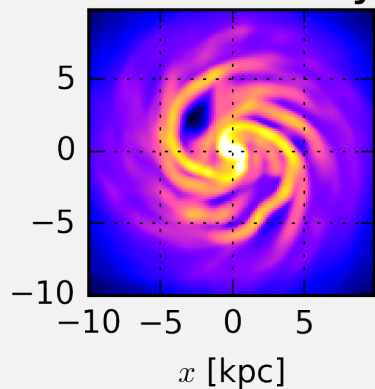


Instability parameter $\log S_2$



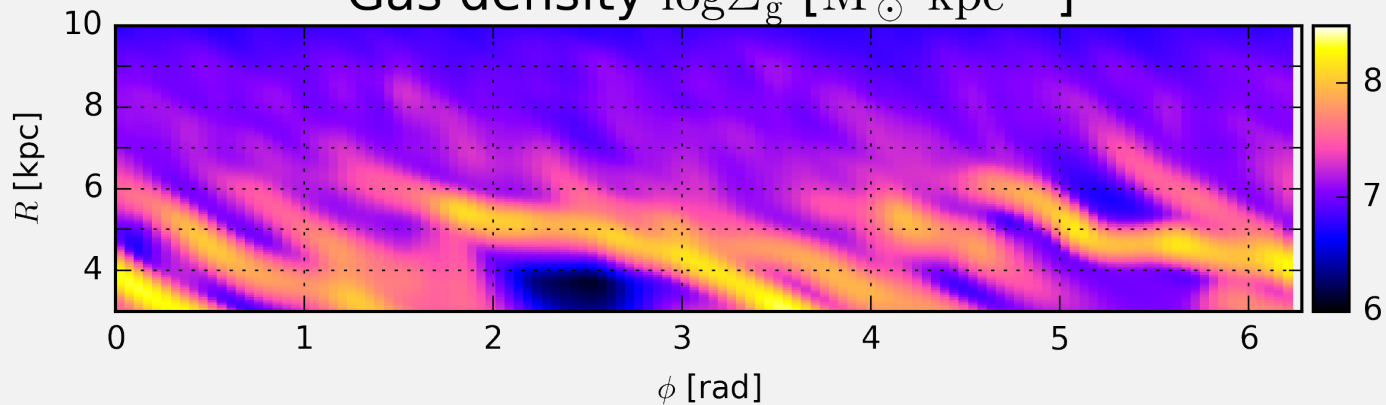
Demonstration

Gas density

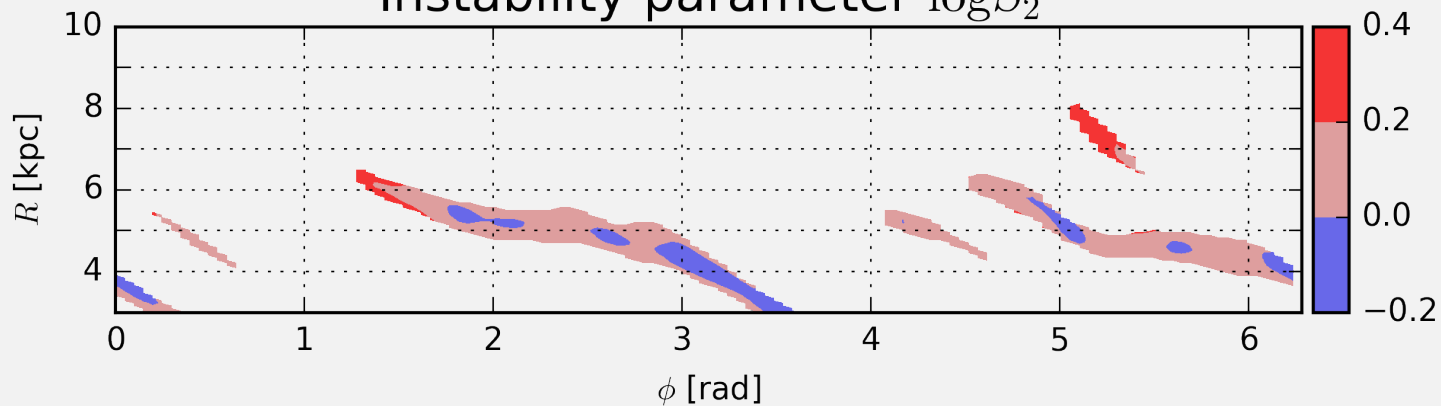


$t=170$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

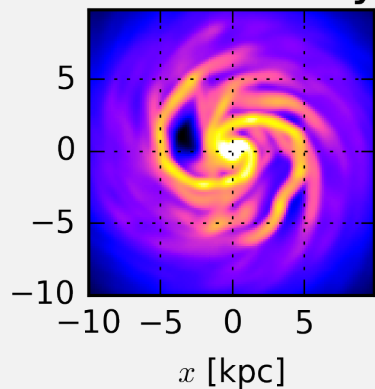


Instability parameter $\log S_2$



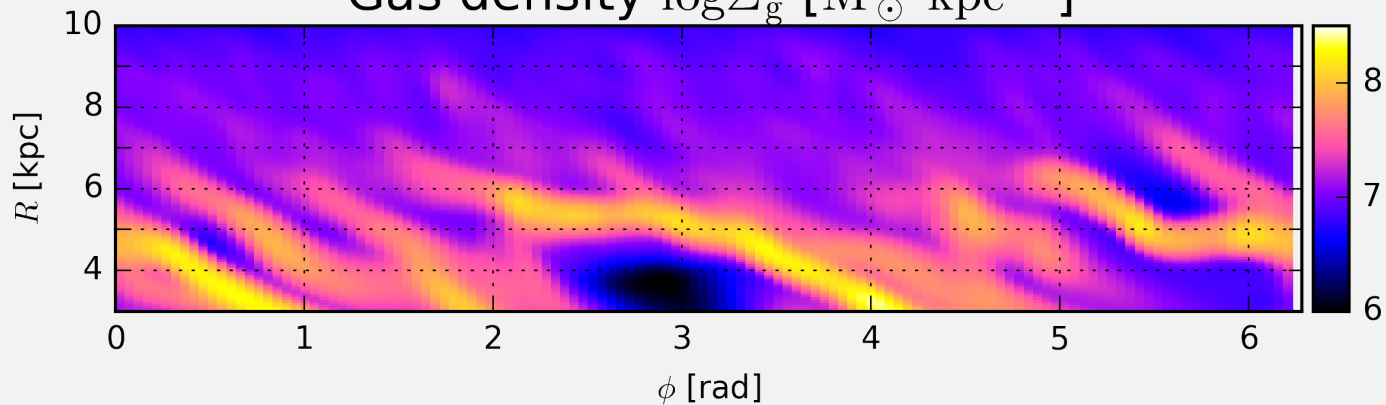
Demonstration

Gas density

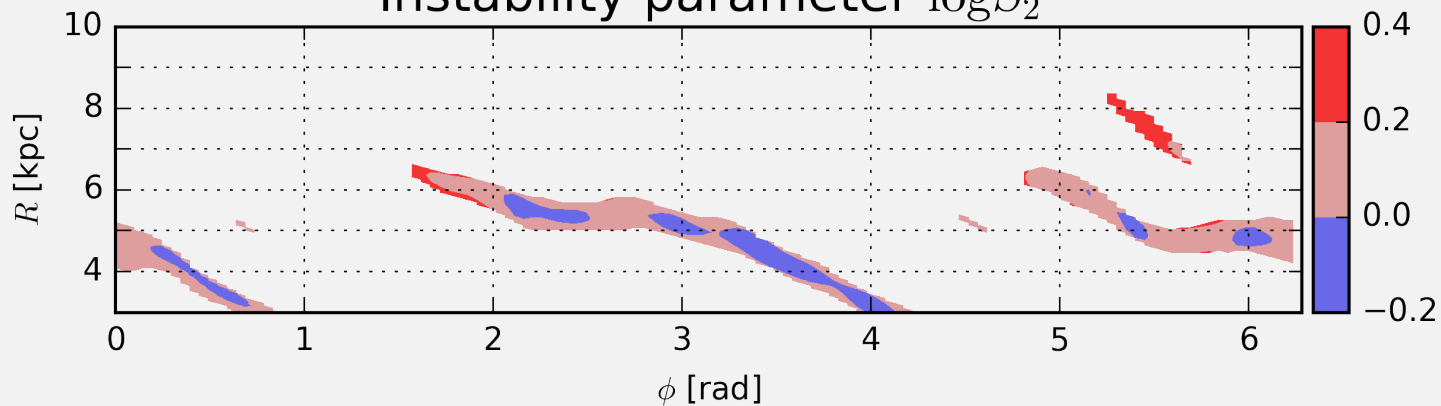


$t=180$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

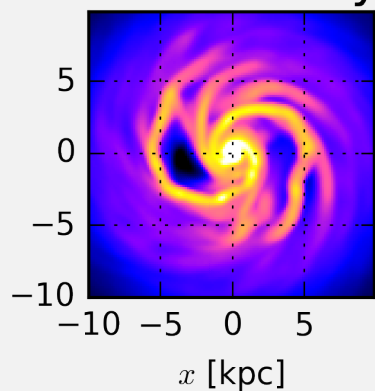


Instability parameter $\log S_2$



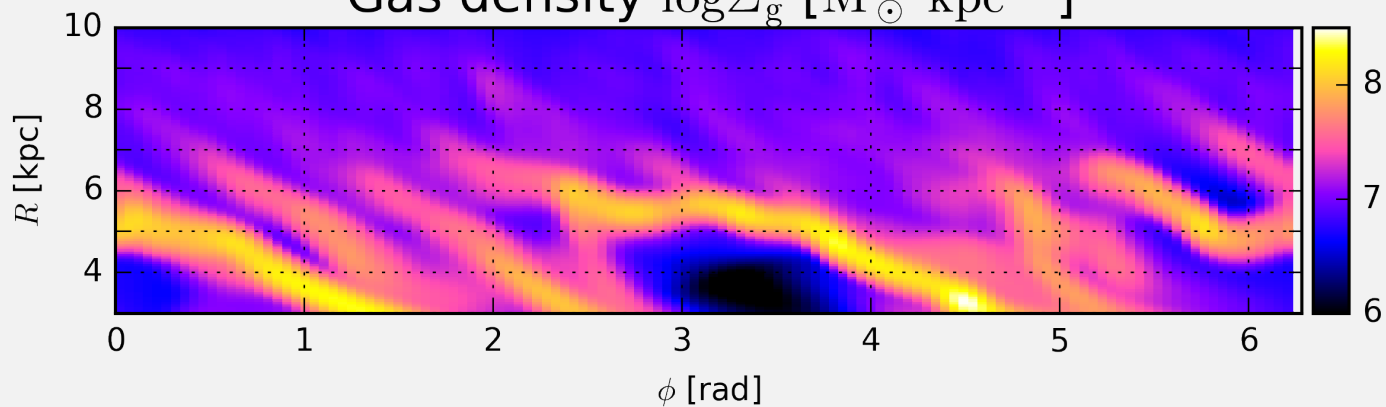
Demonstration

Gas density

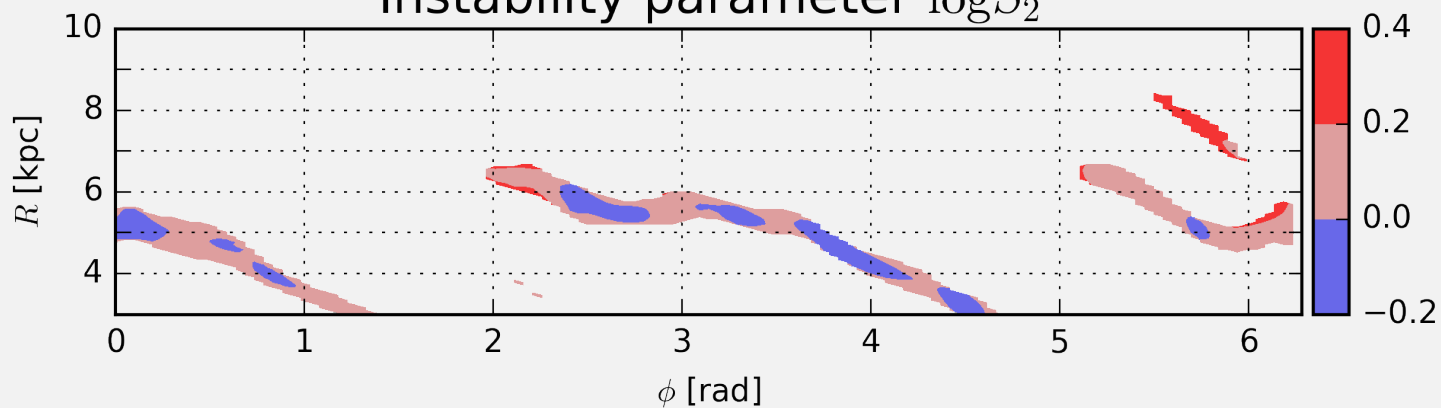


$t = 190$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

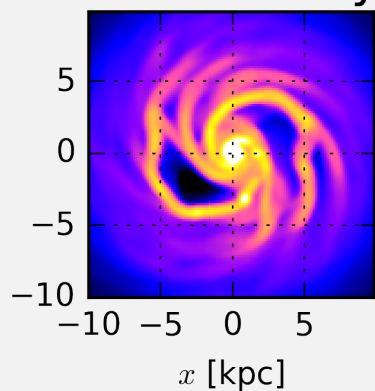


Instability parameter $\log S_2$

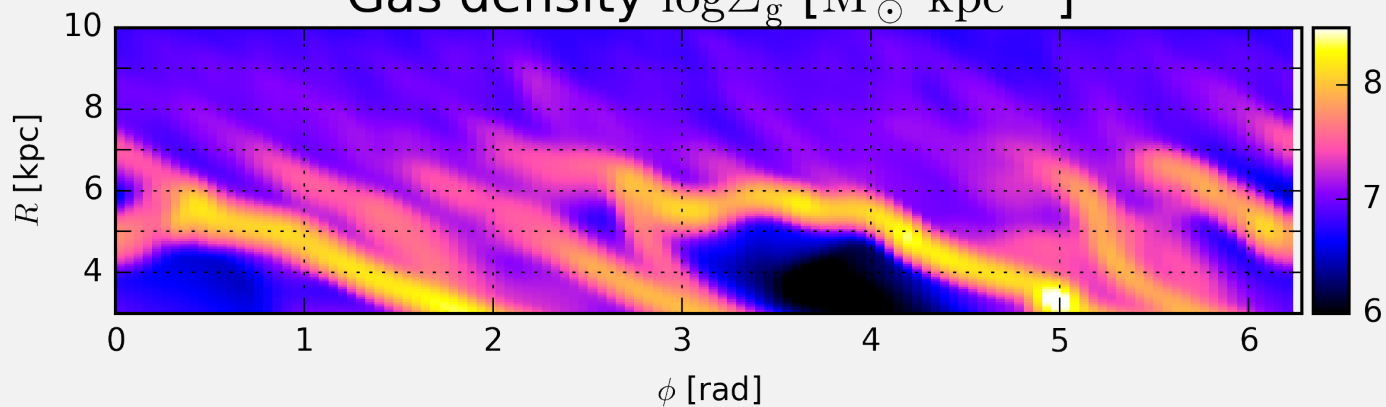


Demonstration

Gas density

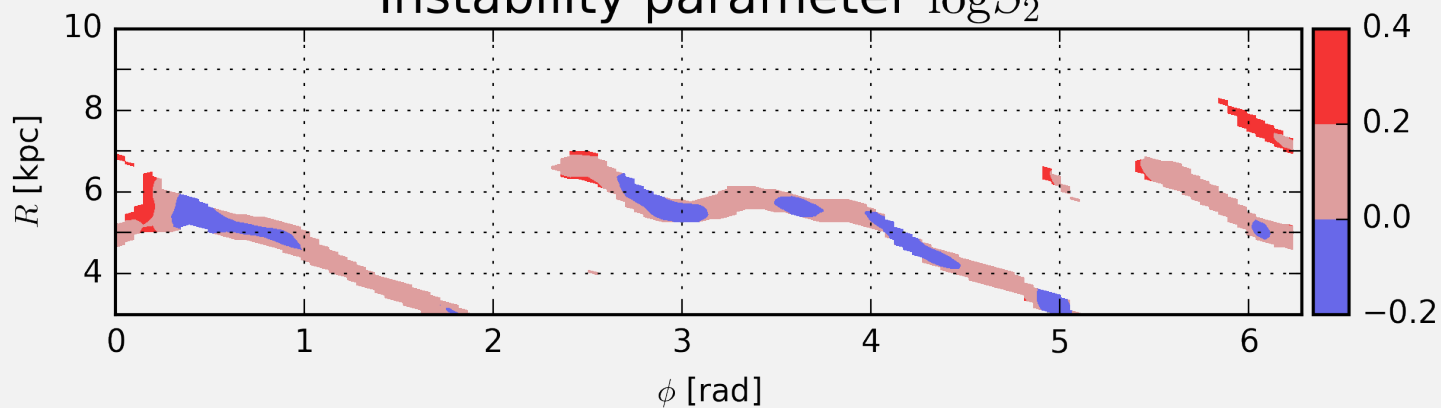


Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]



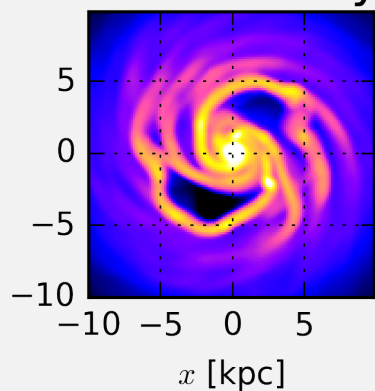
$t=200$ Myr

Instability parameter $\log S_2$

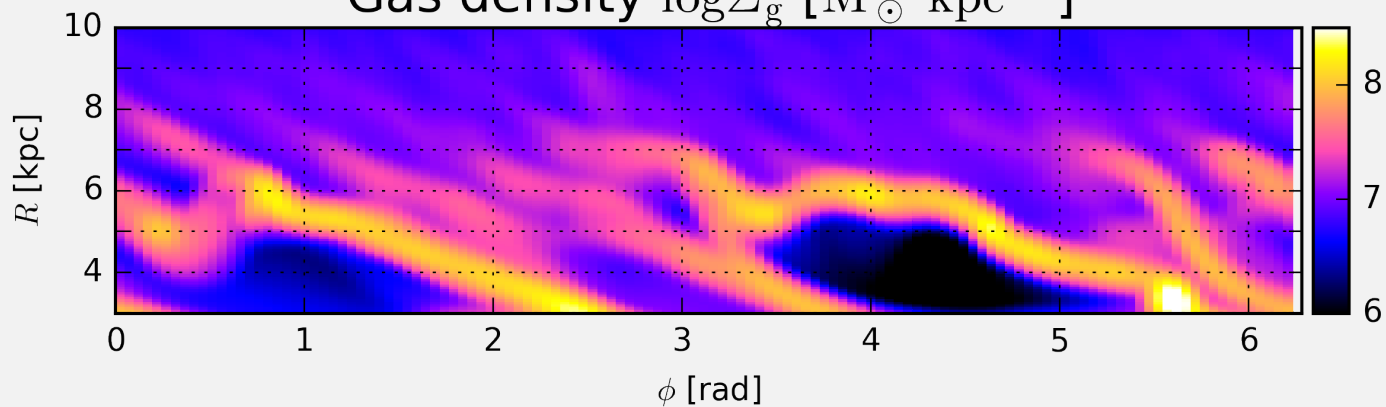


Demonstration

Gas density

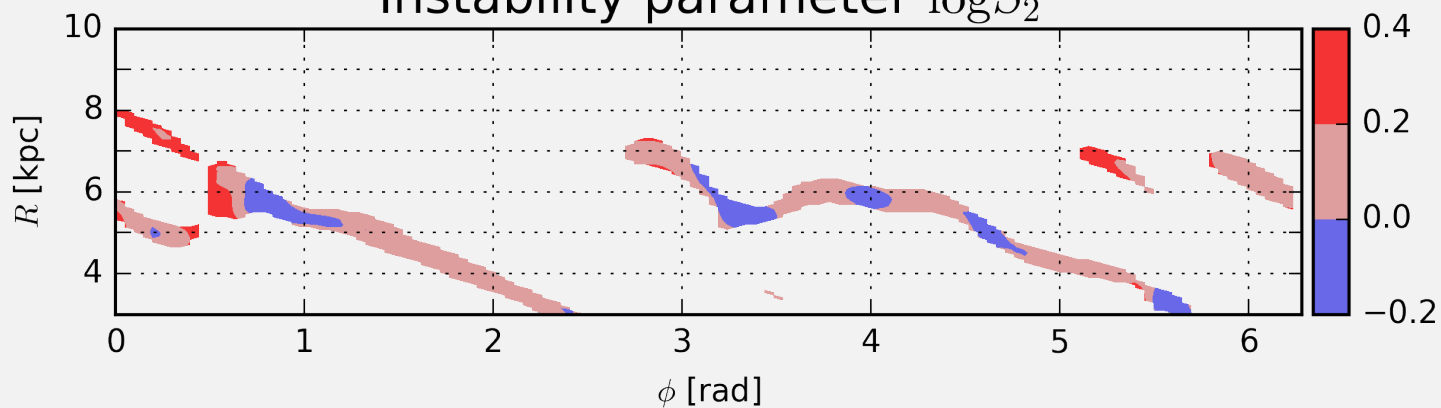


Gas density $\log \Sigma_g \text{ [M}_\odot \text{ kpc}^{-2}]$



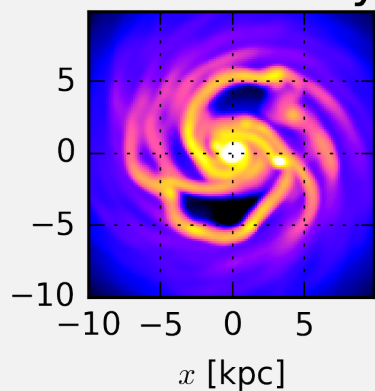
$t=210 \text{ Myr}$

Instability parameter $\log S_2$



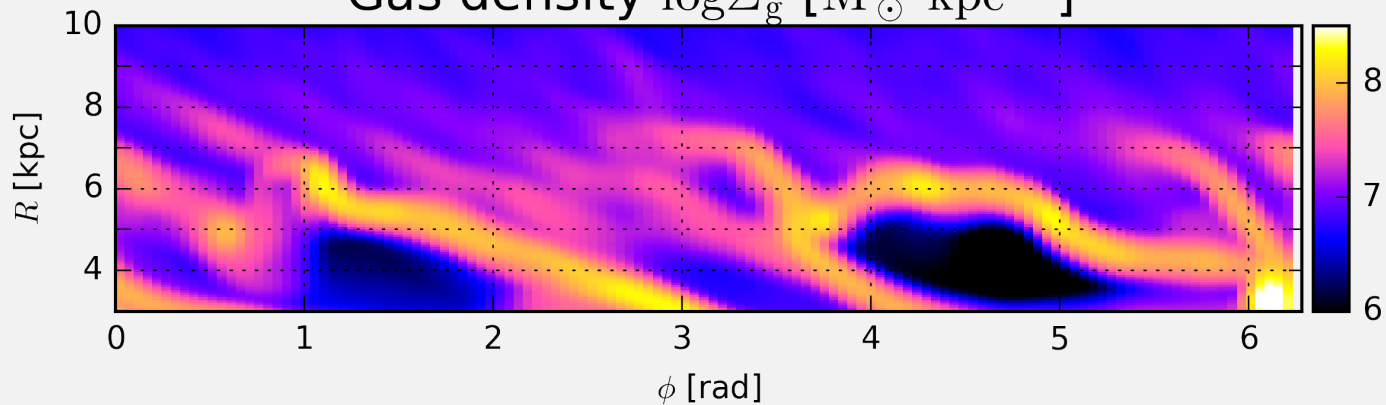
Demonstration

Gas density

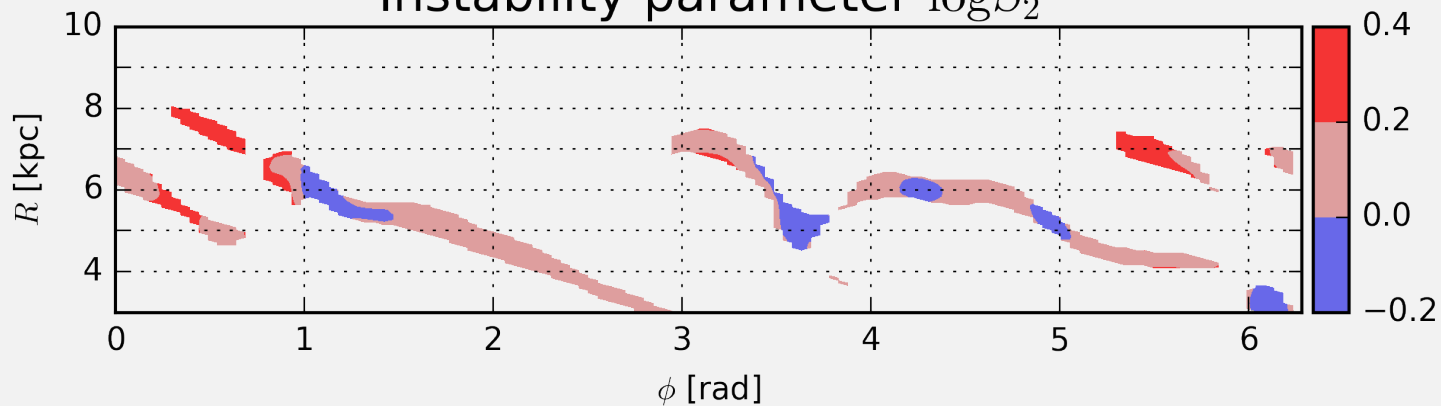


$t=220$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

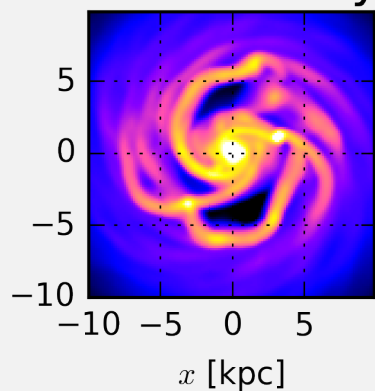


Instability parameter $\log S_2$



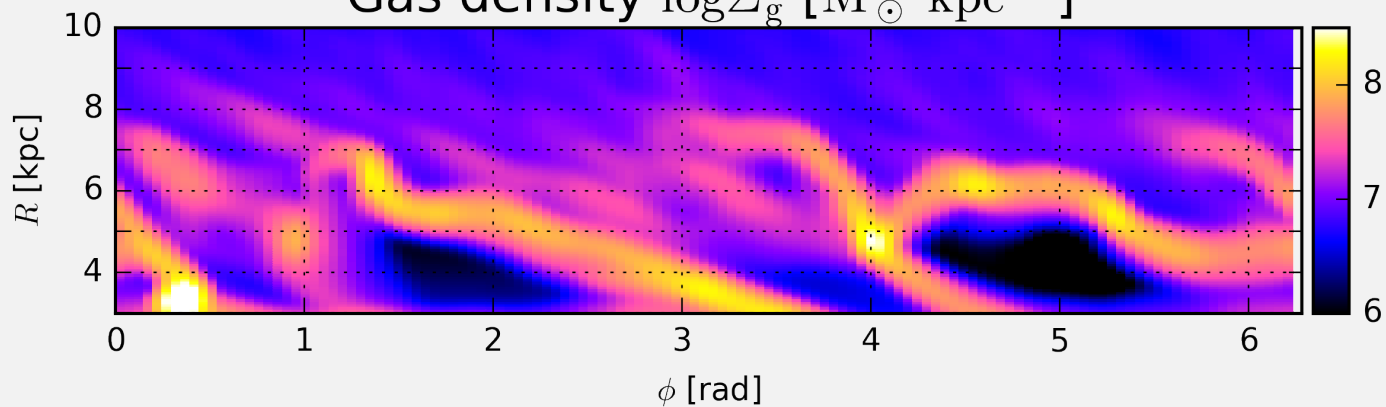
Demonstration

Gas density

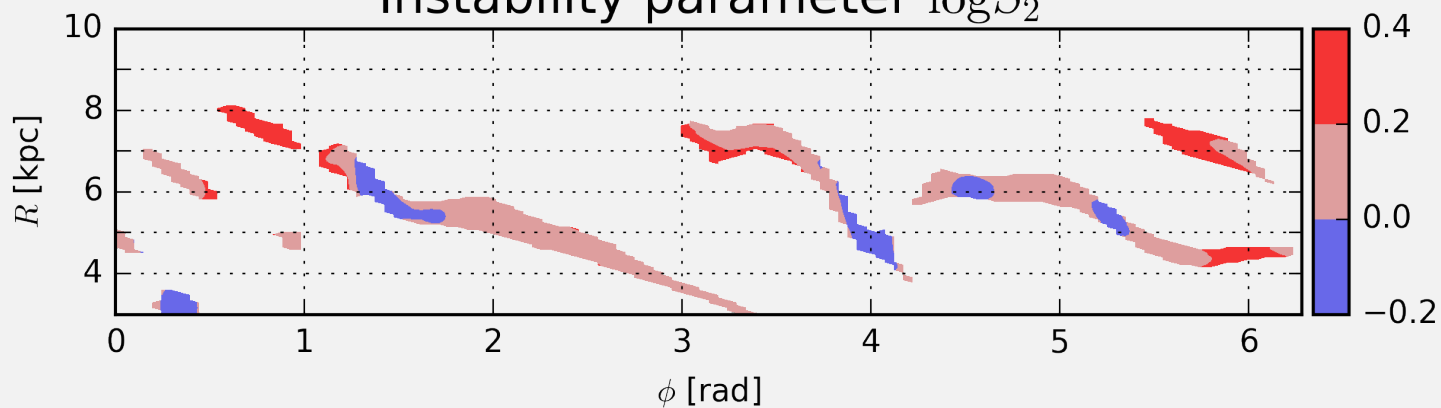


$t=230$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

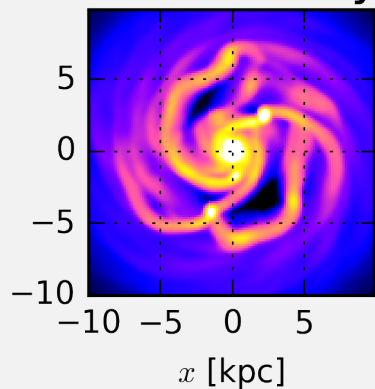


Instability parameter $\log S_2$



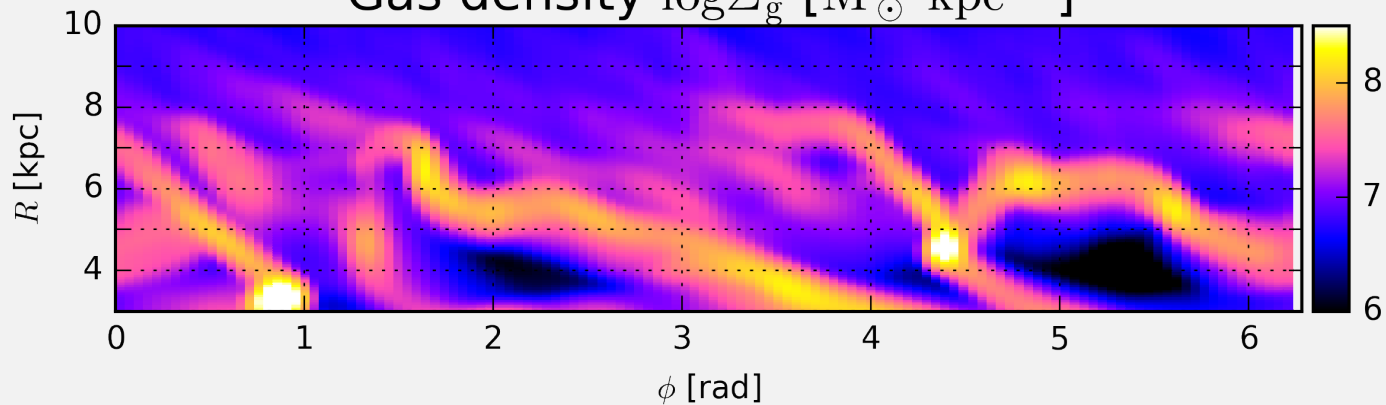
Demonstration

Gas density

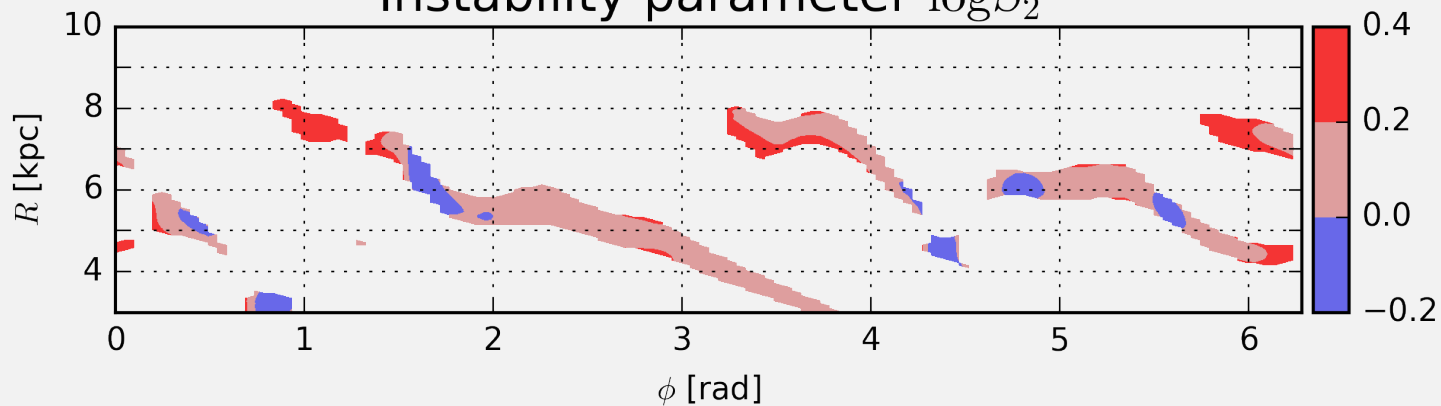


$t=240$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

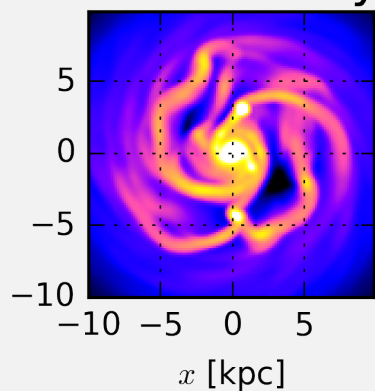


Instability parameter $\log S_2$



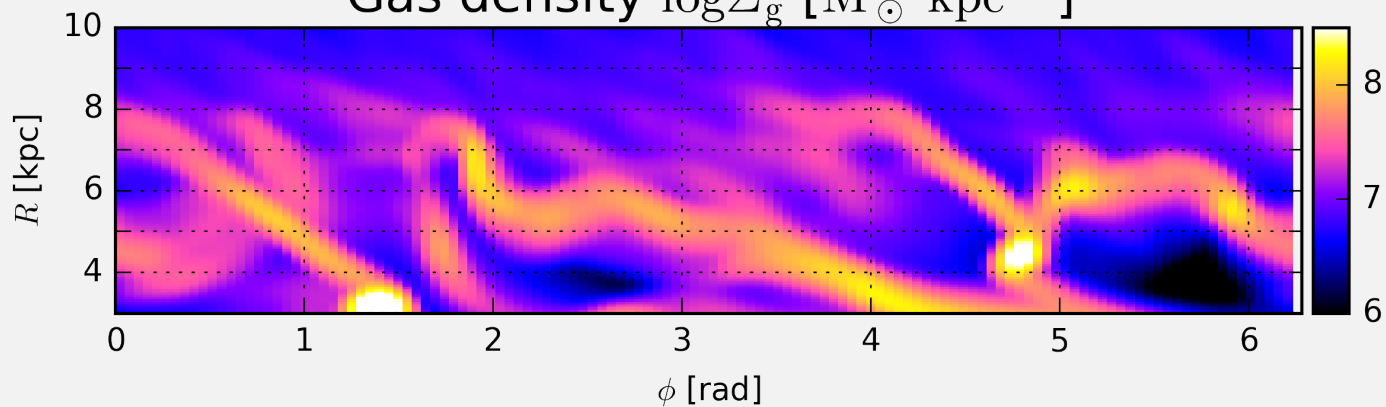
Demonstration

Gas density

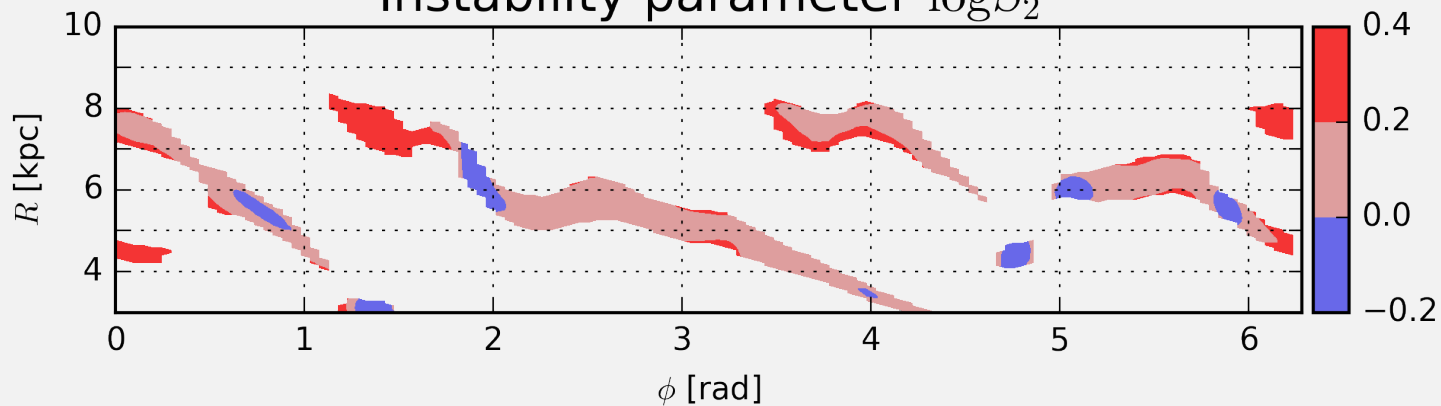


$t=250$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

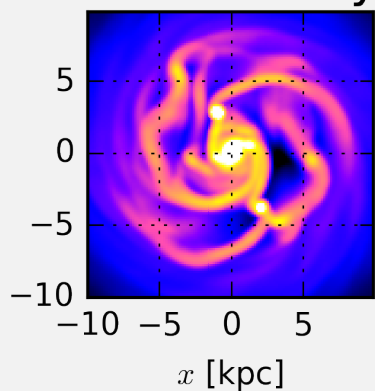


Instability parameter $\log S_2$



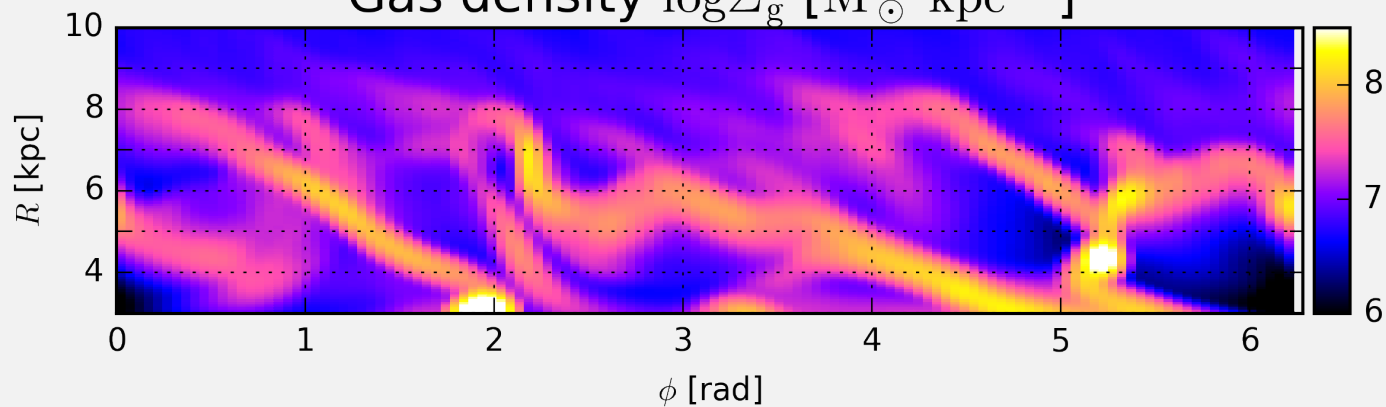
Demonstration

Gas density

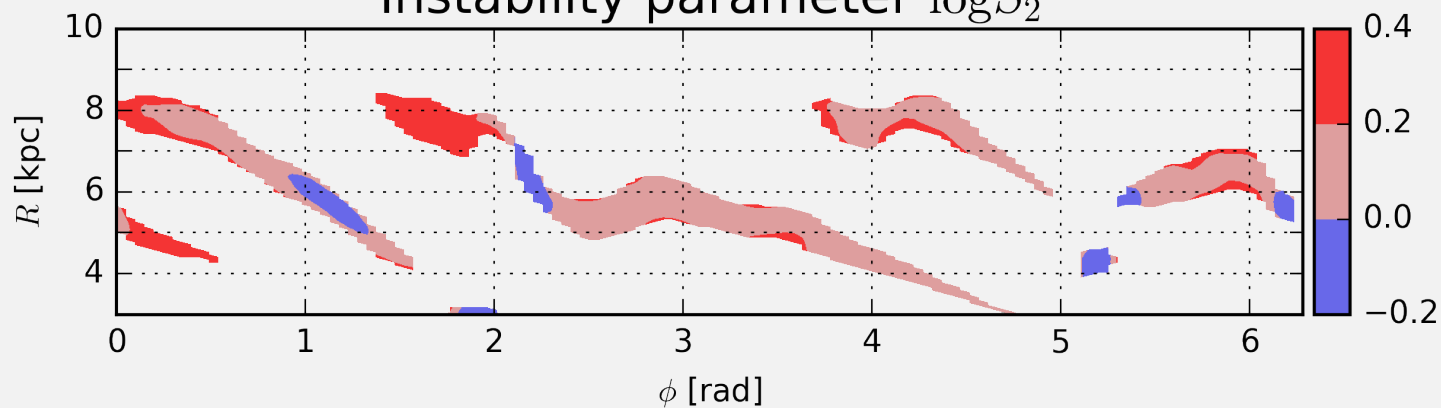


$t=260$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

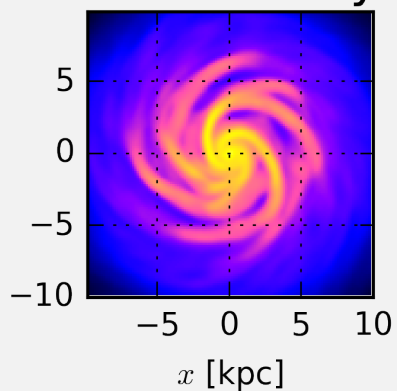


Instability parameter $\log S_2$



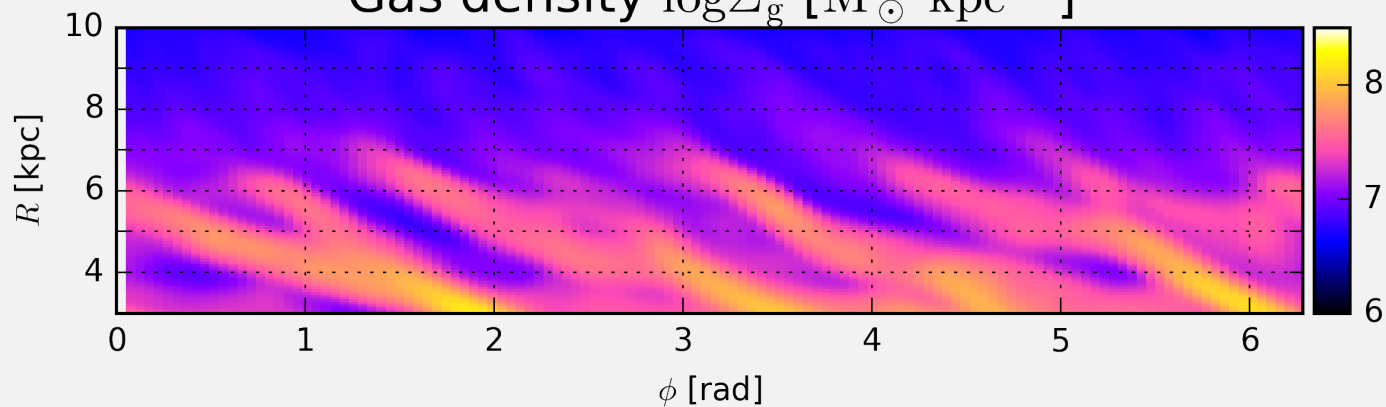
Demonstration

Gas density

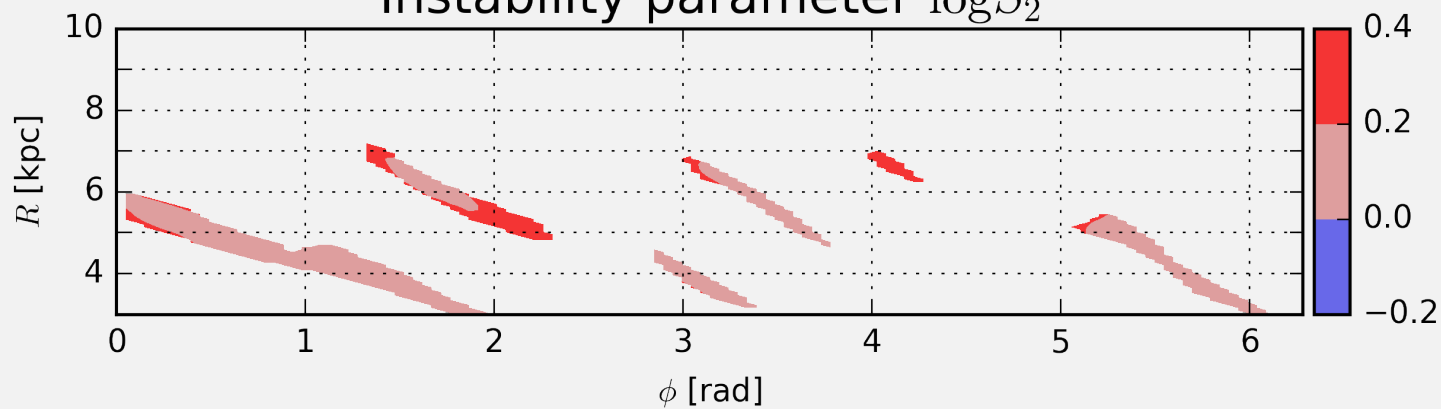


$t=200$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

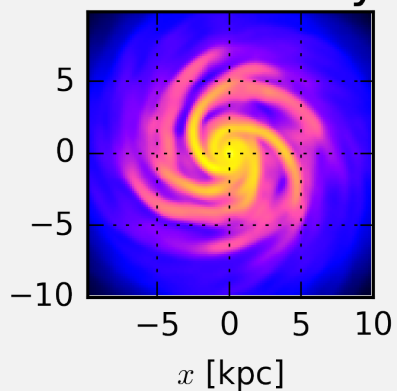


Instability parameter $\log S_2$



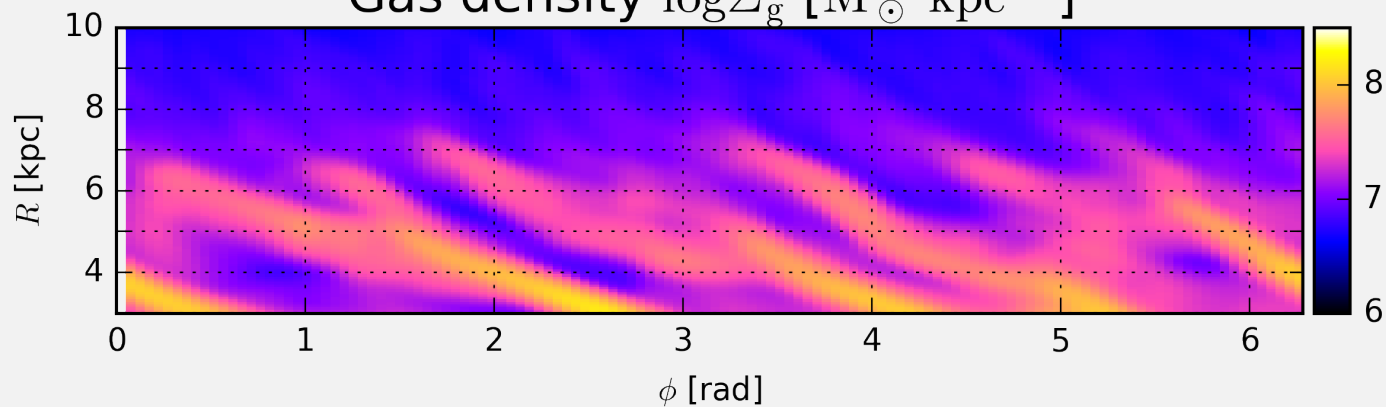
Demonstration

Gas density

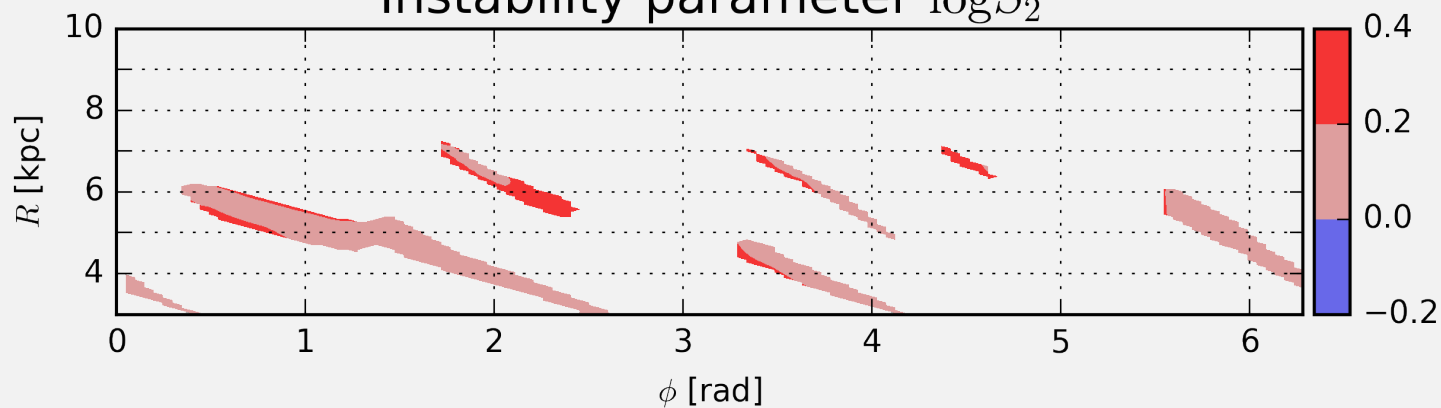


$t=210$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

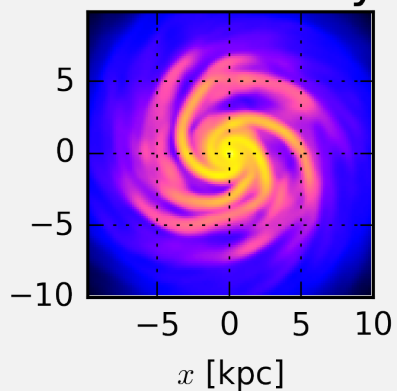


Instability parameter $\log S_2$



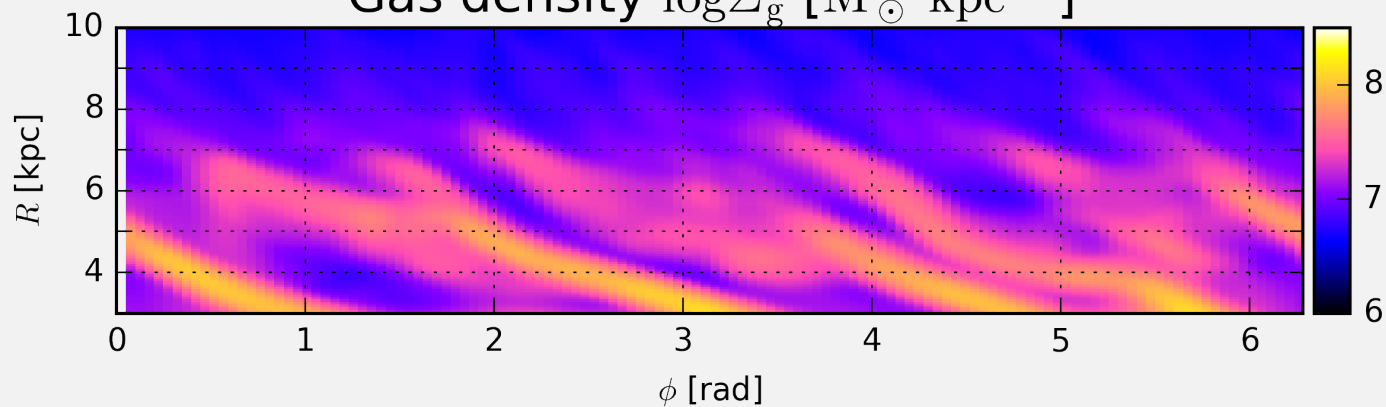
Demonstration

Gas density

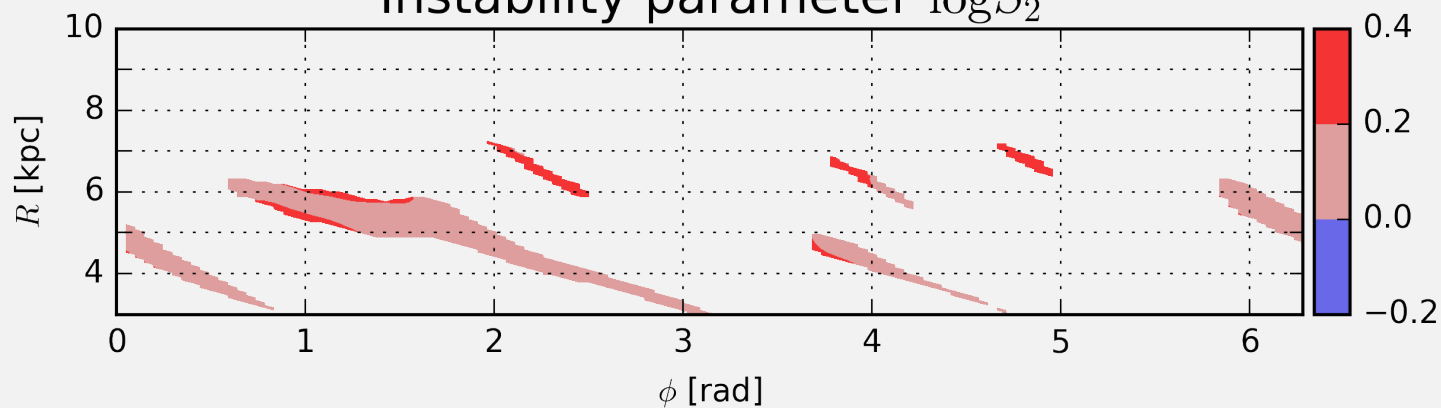


$t=220$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

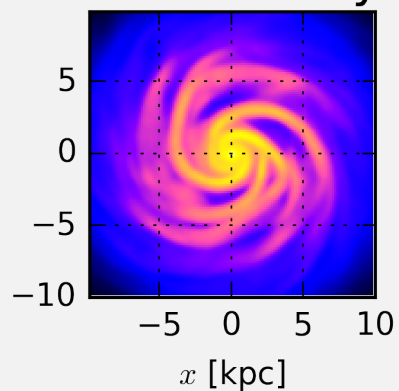


Instability parameter $\log S_2$



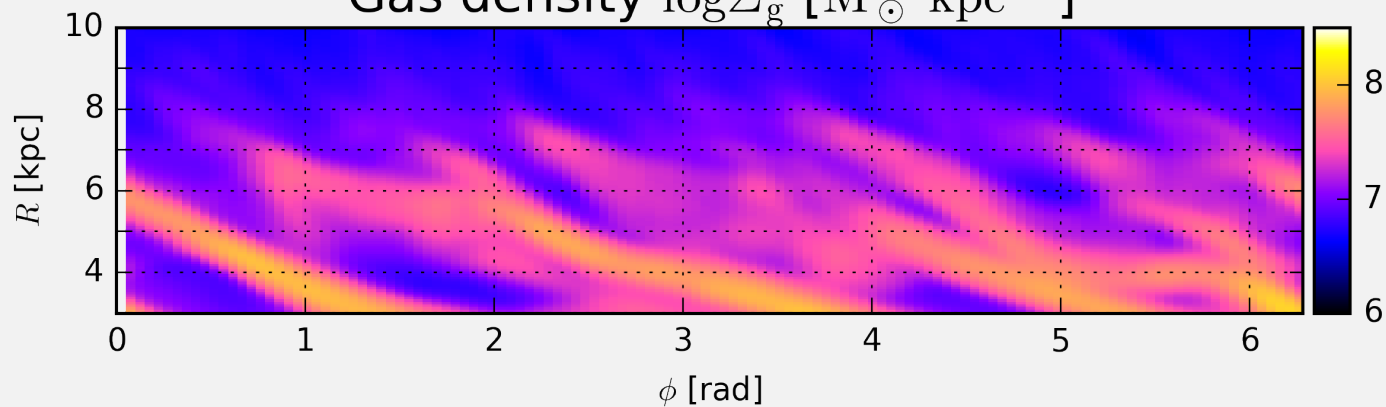
Demonstration

Gas density

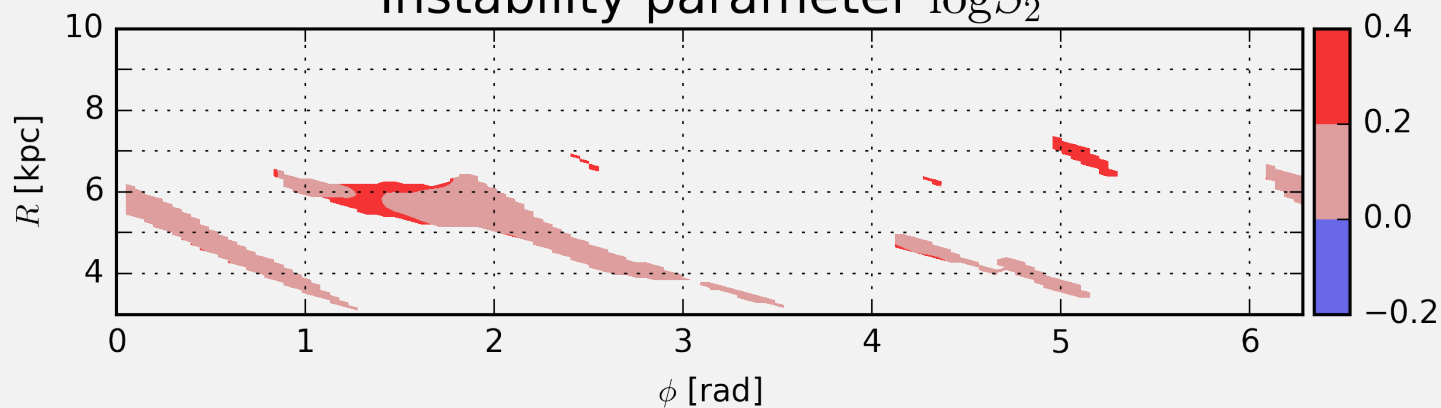


$t=230$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

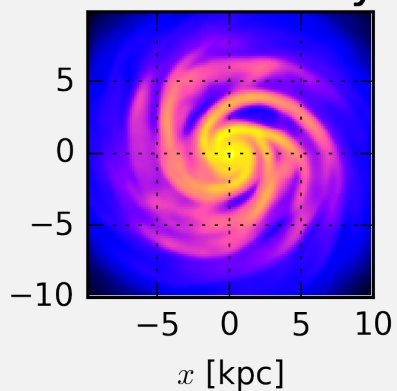


Instability parameter $\log S_2$



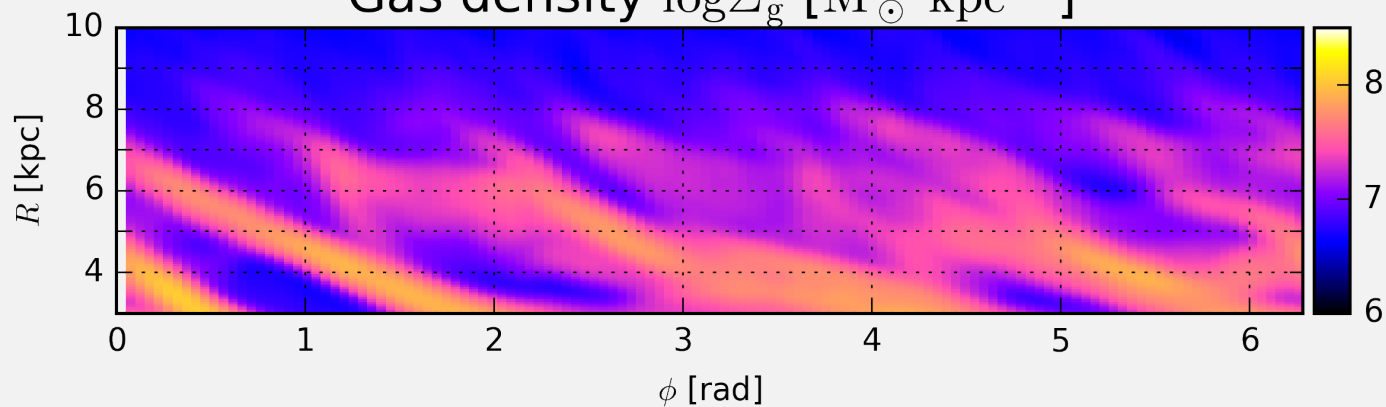
Demonstration

Gas density

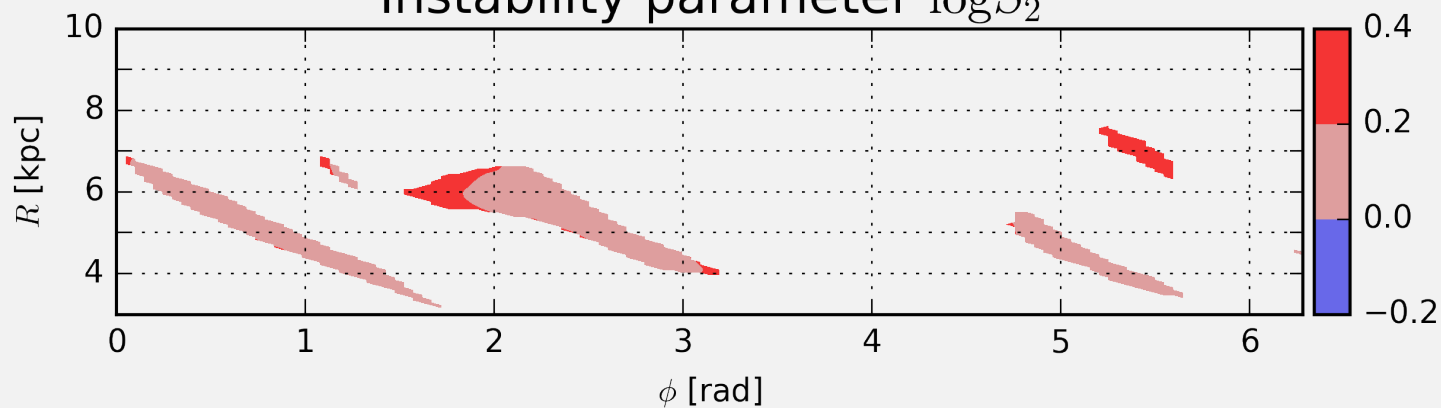


$t=240$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

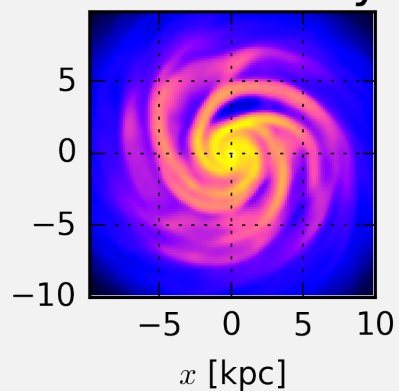


Instability parameter $\log S_2$



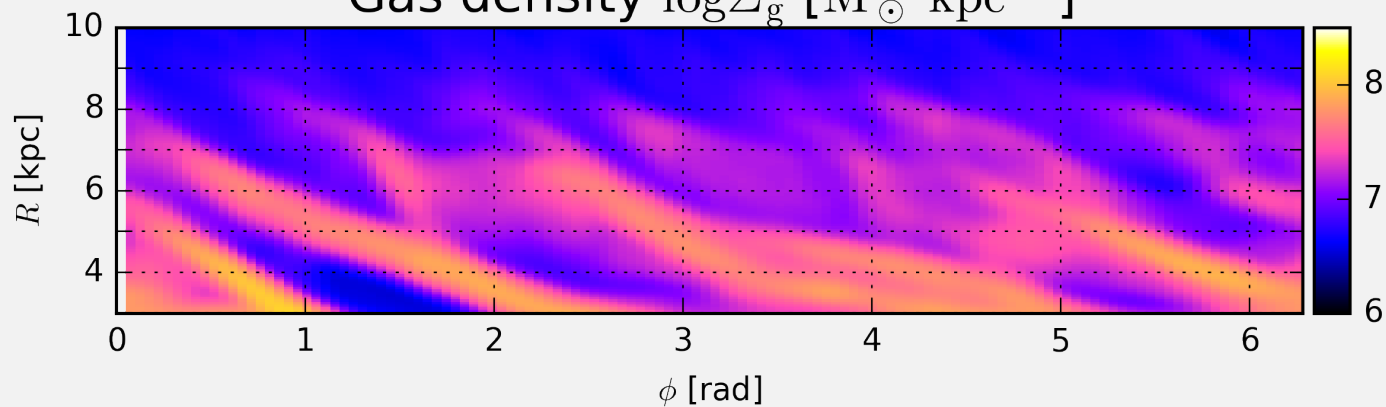
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Gas density

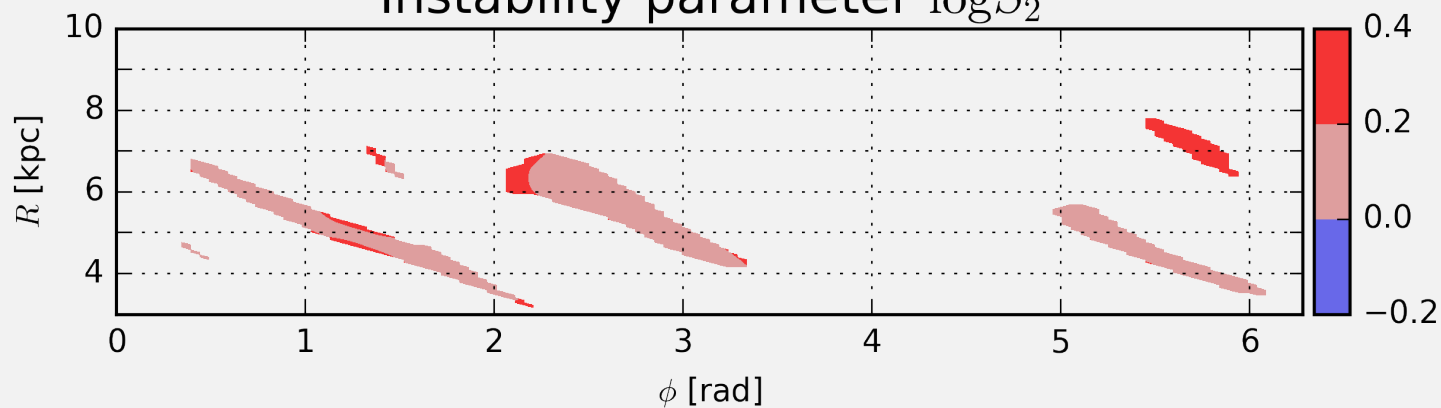


$t=250$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

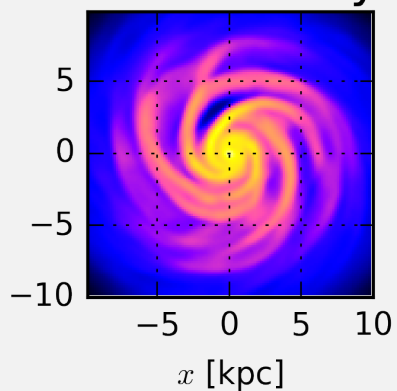


Instability parameter $\log S_2$



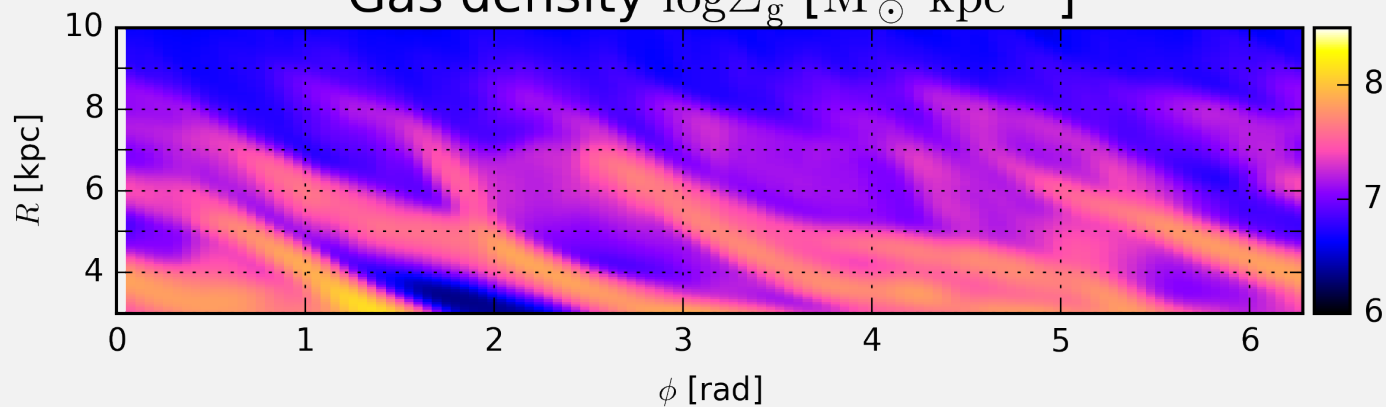
Demonstration

Gas density

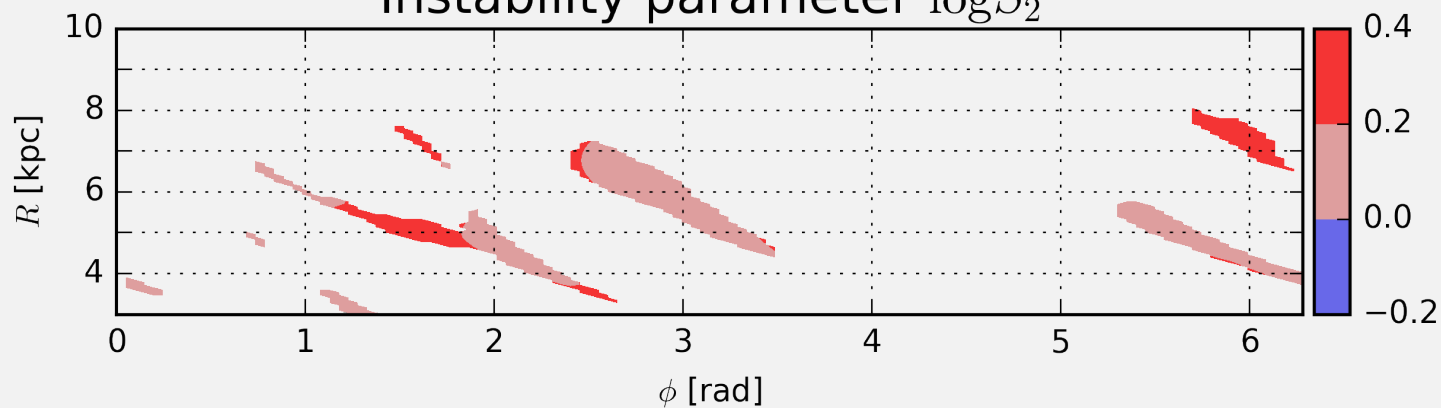


$t=260$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

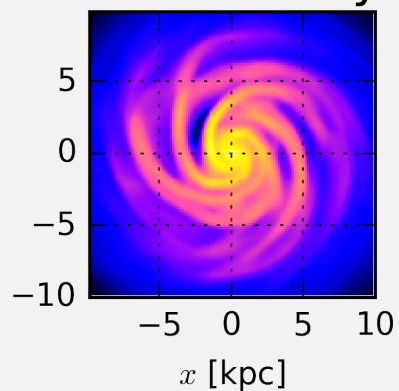


Instability parameter $\log S_2$



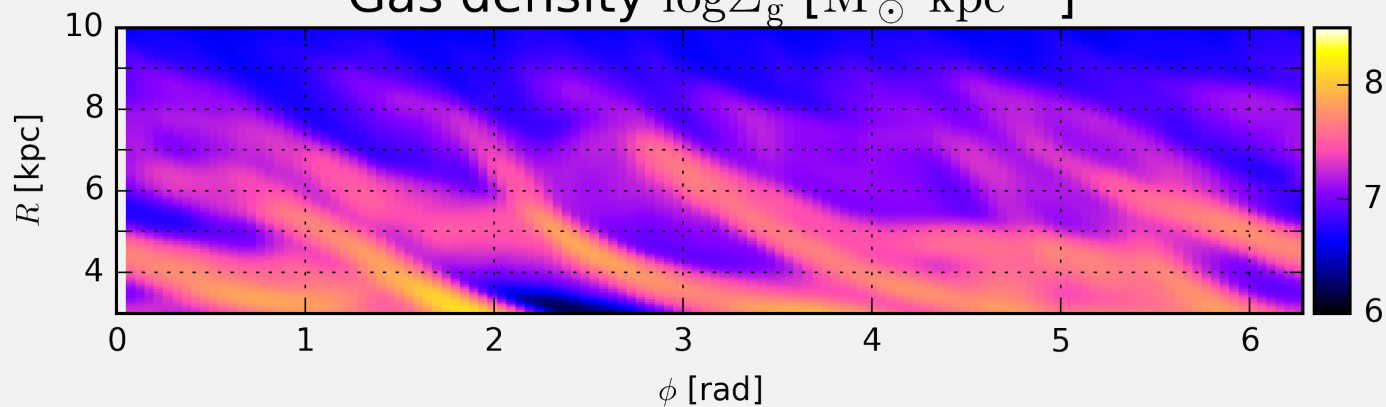
Demonstration

Gas density

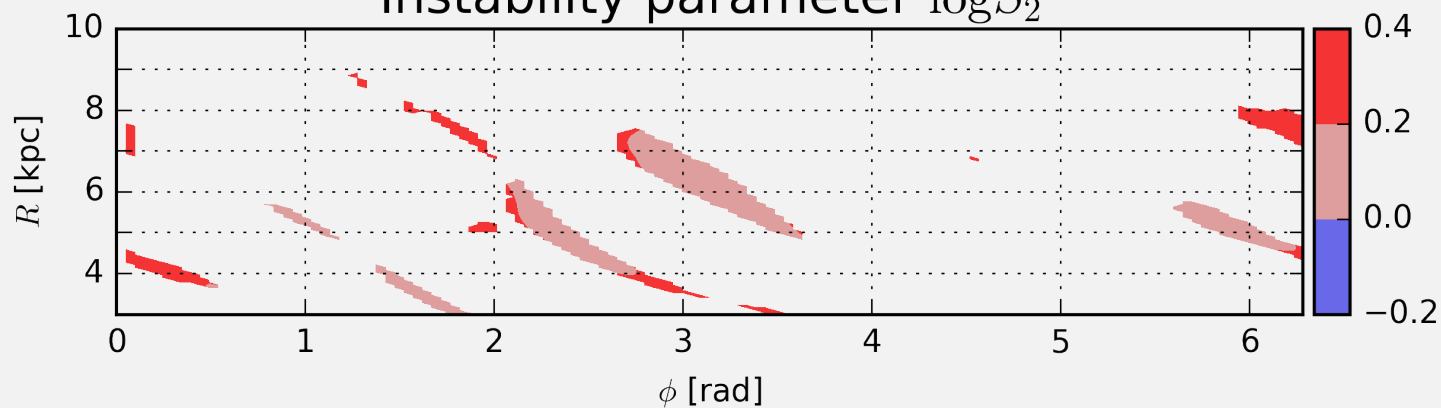


$t=270$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

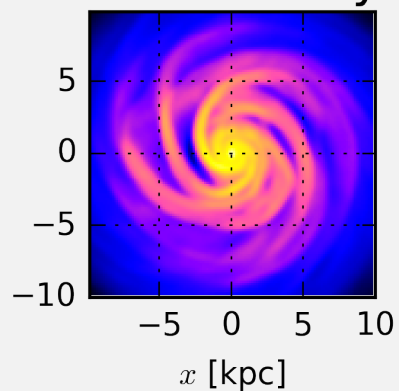


Instability parameter $\log S_2$



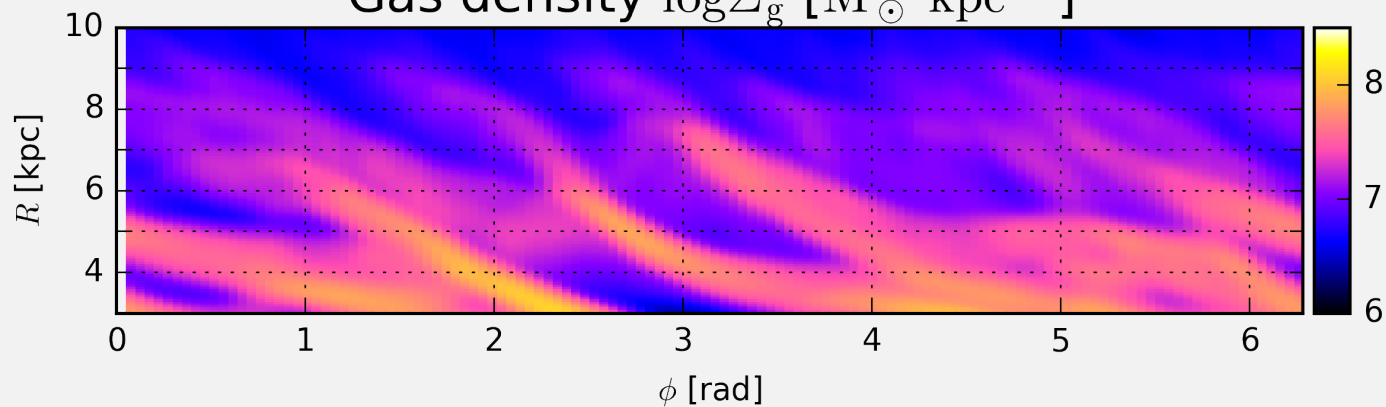
Demonstration

Gas density

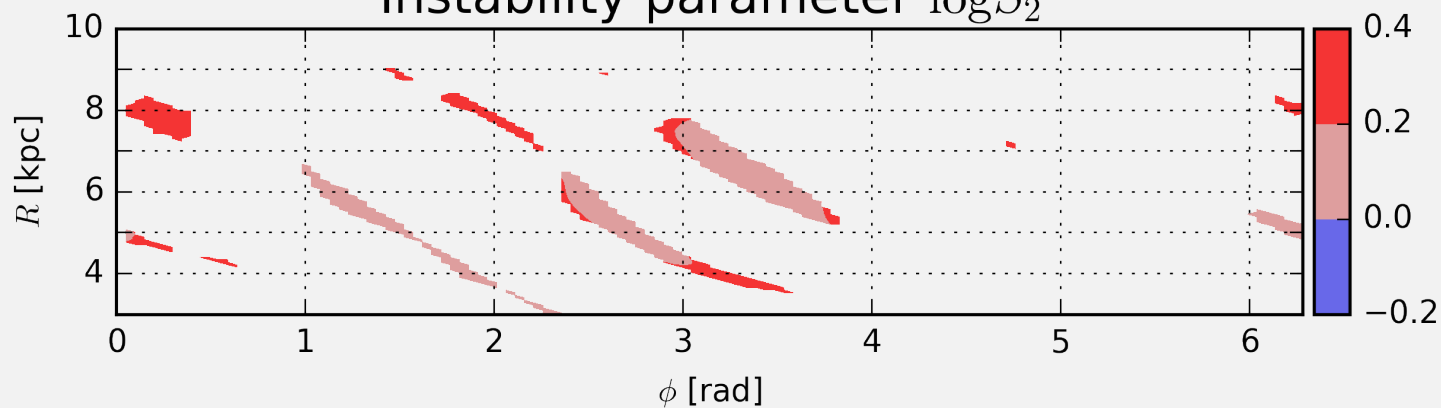


$t=280$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

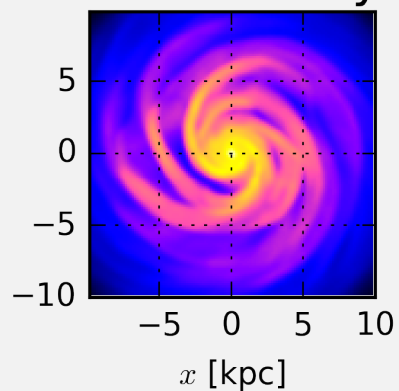


Instability parameter $\log S_2$



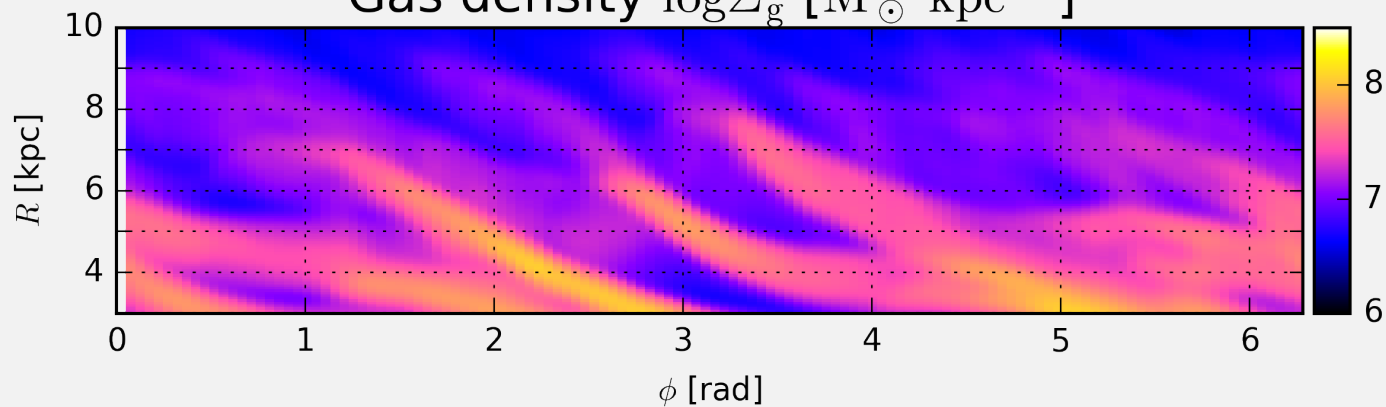
Demonstration

Gas density

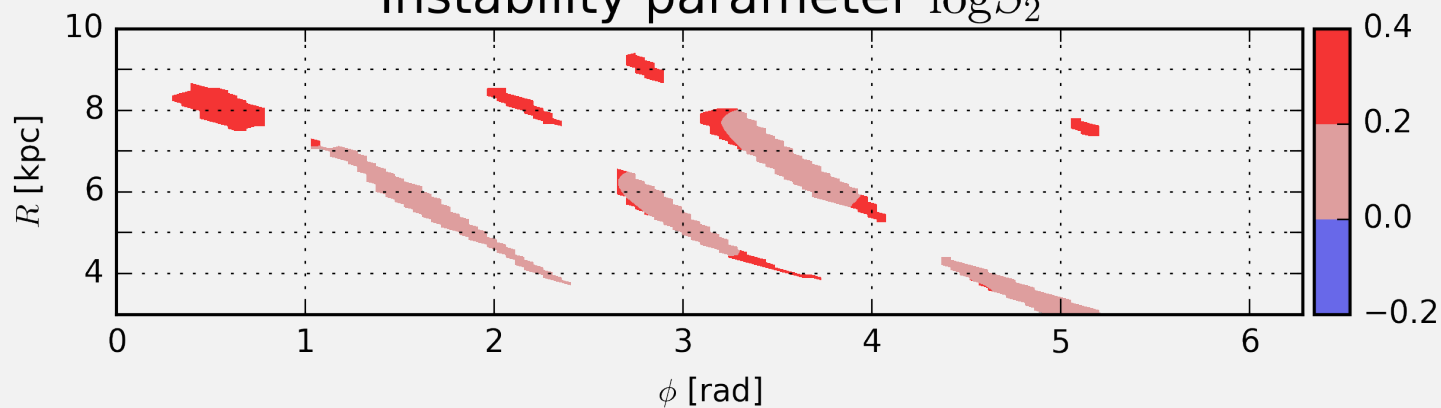


$t=290$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

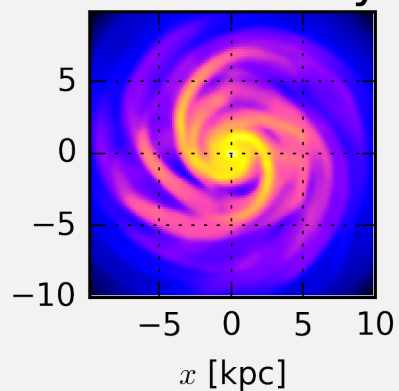


Instability parameter $\log S_2$



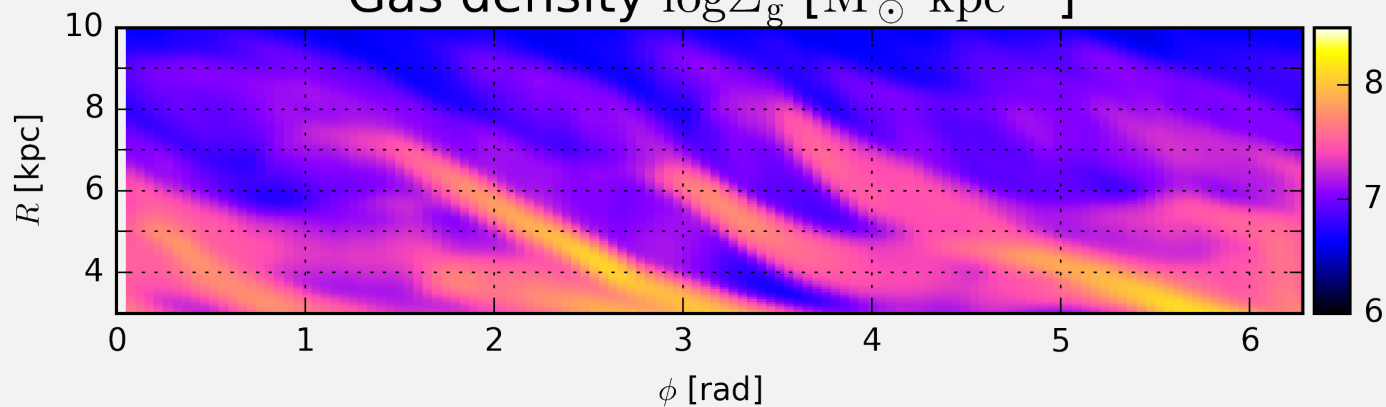
Demonstration

Gas density

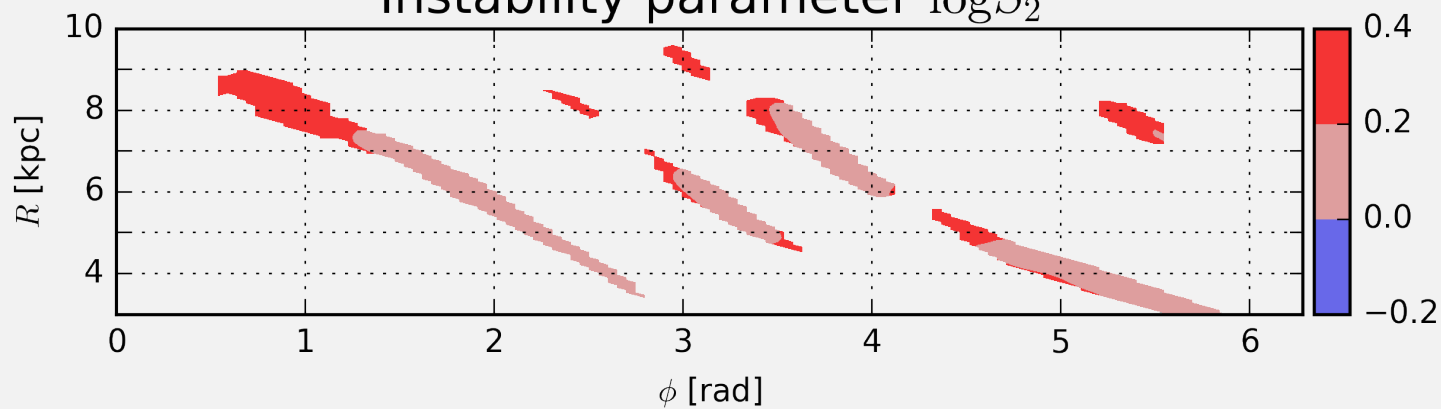


$t=300$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

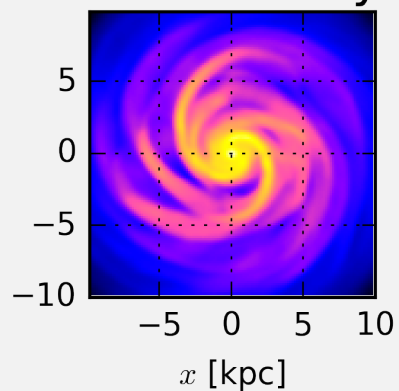


Instability parameter $\log S_2$



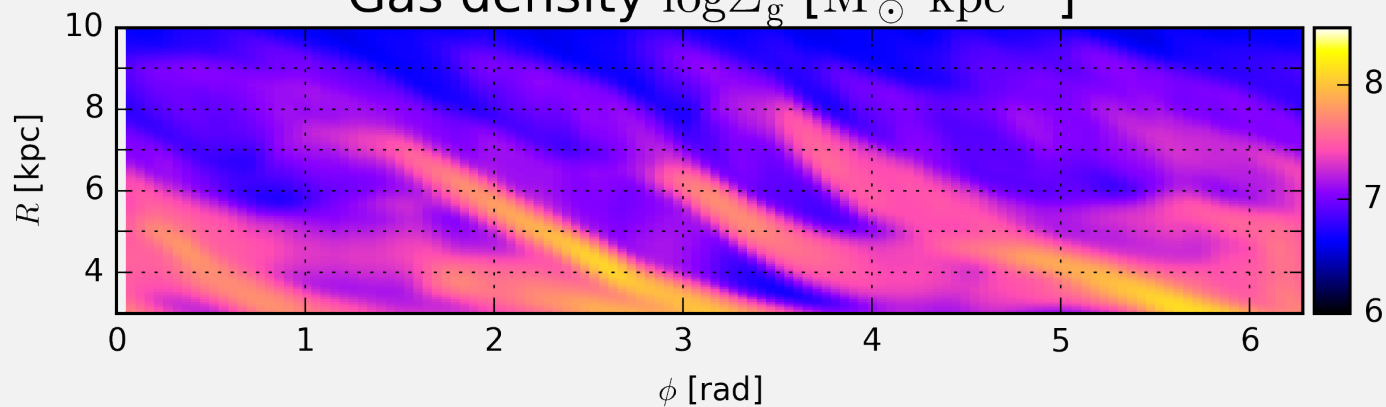
Demonstration

Gas density

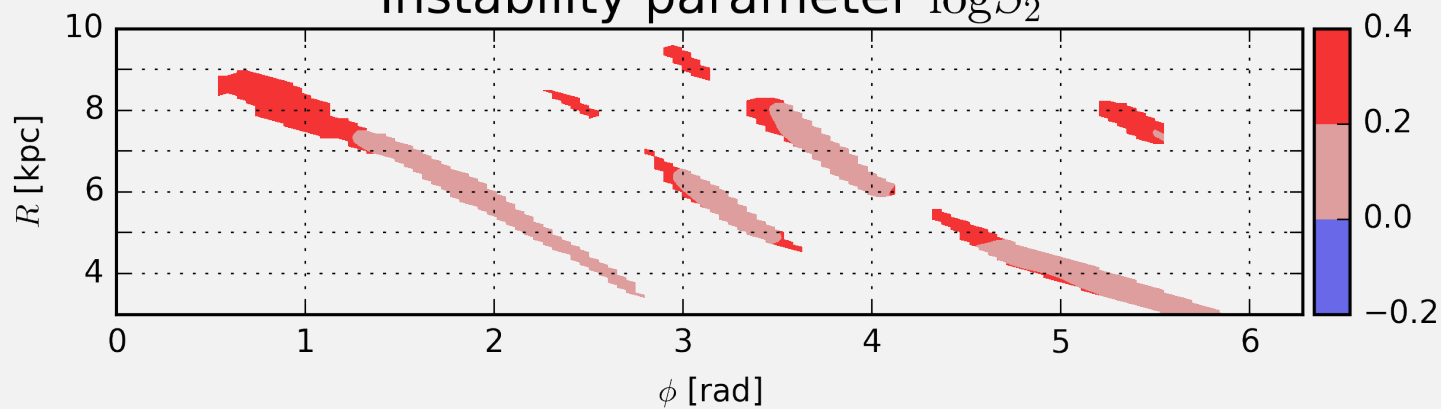


$t=300$ Myr

Gas density $\log \Sigma_g$ [$M_\odot \text{ kpc}^{-2}$]

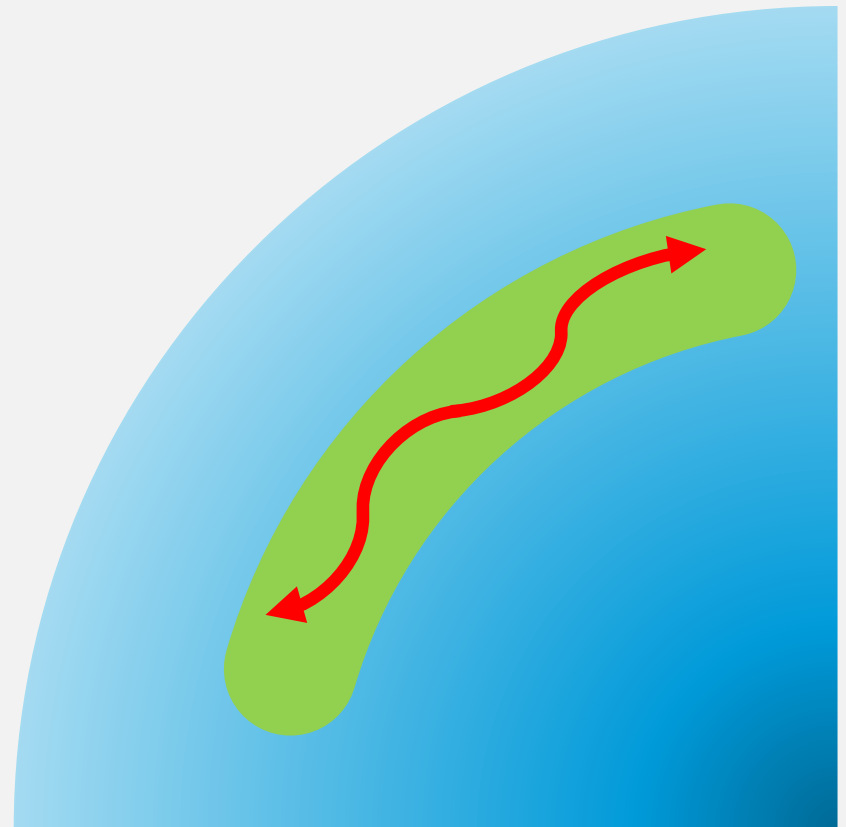


Instability parameter $\log S_2$



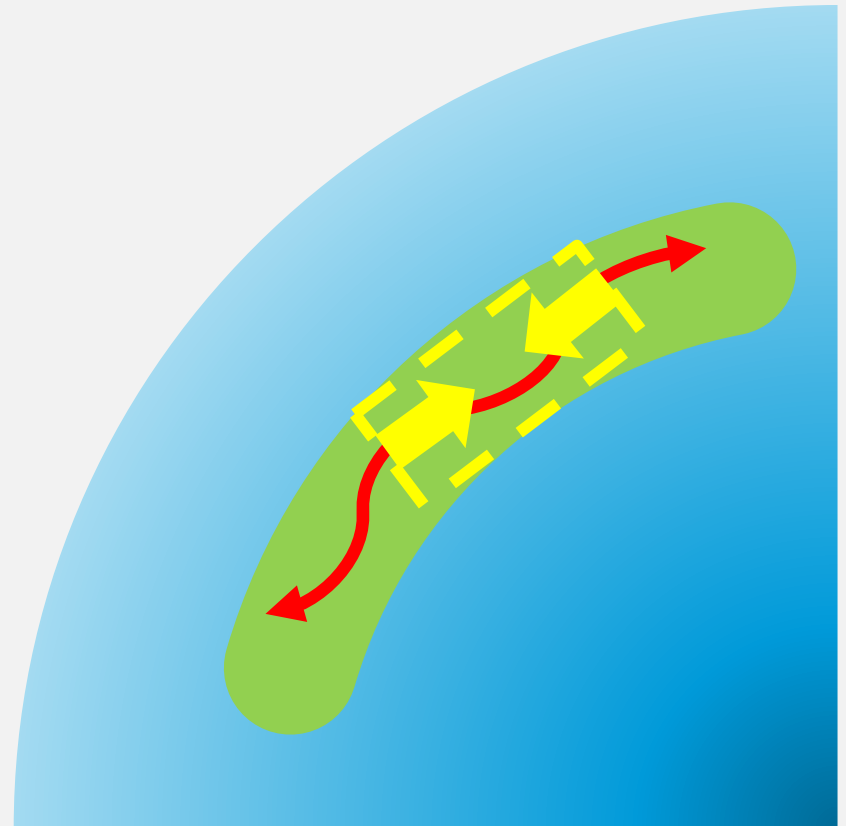
Clump mass estimation

- An unstable perturbation is expected to collapse along the arm (1D collapse)



Clump mass estimation

- An unstable perturbation is expected to collapse along the arm (1D collapse)
- $M_{cl} \sim \Sigma W \lambda$



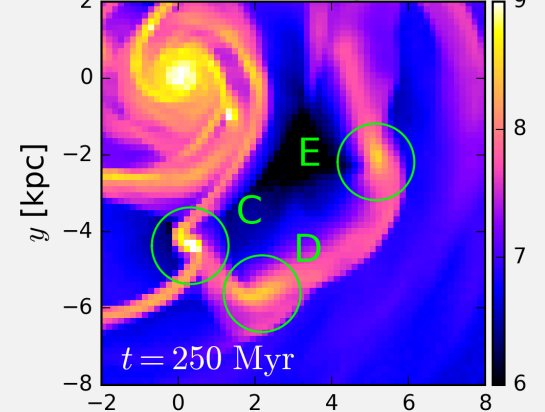
Clump-mass prediction

- An unstable perturbation is expected to collapse along the arm (1D collapse)

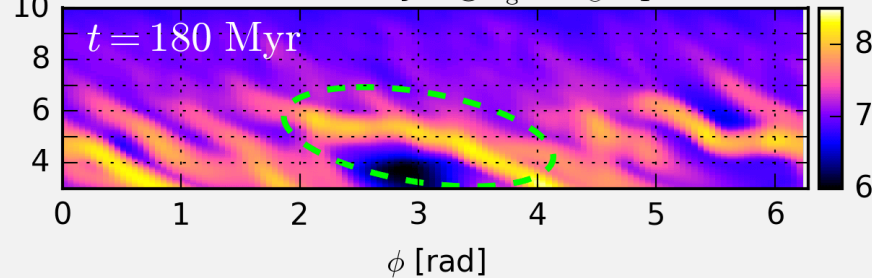
- $M_{cl} \sim \Sigma W \lambda$

- $M_{cl} \sim 10^8 - 10^{8.5} M_{\odot}$ (predicted)

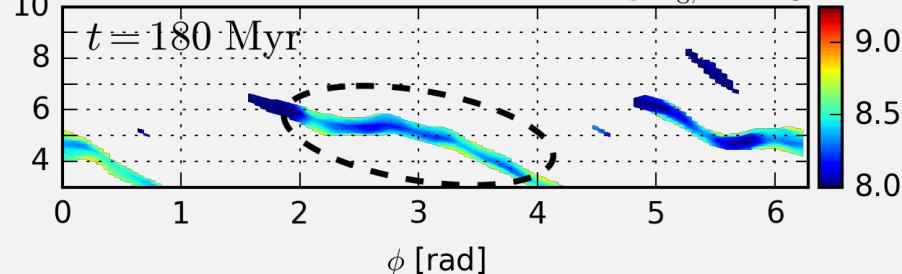
(a) Gas density $\log \Sigma_g [M_{\odot} \text{ kpc}^{-2}]$



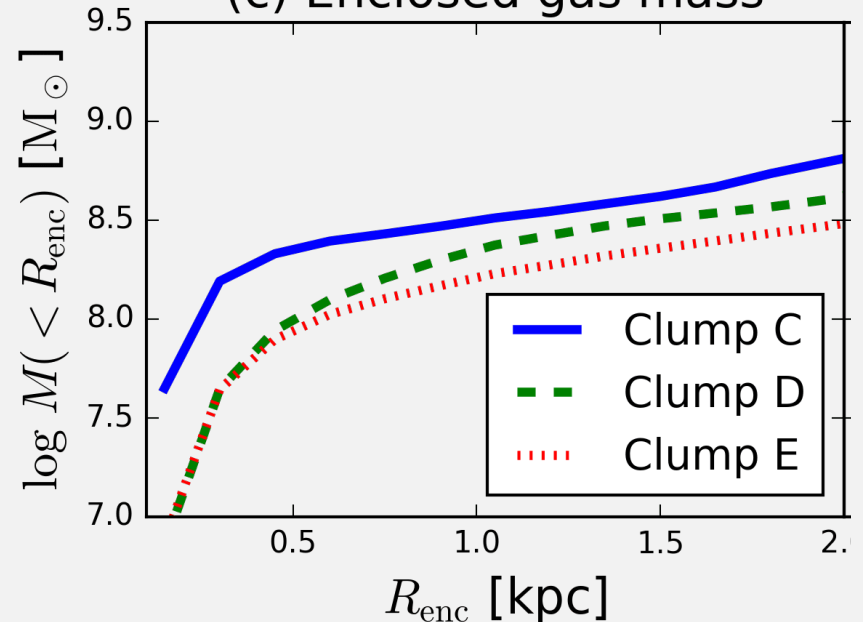
(a) Gas density $\log \Sigma_g [M_{\odot} \text{ kpc}^{-2}]$



(b) Predicted gas clump mass $\log M_{g,cl} [M_{\odot}]$



(c) Enclosed gas mass



Limitations of our models

- Our simulations are ideal
 - Isolated models
 - Isothermal gas
 - No star formation or stellar feedback
 - Very thin discs
- Our analytic model is also ideal
 - Razor-thin spiral arms
 - Tight-winding approximation
 - Equilibrium for spiral arms
 - Gaussian density distribution assumed
- We also find a few non-linear fragmentation
 - Clump formation can occur even if the instability condition is not satisfied.
 - But, non-linear fragmentation is quite rare in our simulations.

Violent Disc Instability (VDI)

(based on Toomre instability; Dekel et al. 2009)

V.S.

Spiral-Arm Instability (SAI)

VDI vs SAI

- Violent Disc Instability

Disc formation



Toomre instability
 $Q < 1$

Giant clump formation

- Spiral-Arm Instability

Disc formation



Toomre instability and/or
swing amplification

Spiral arm formation

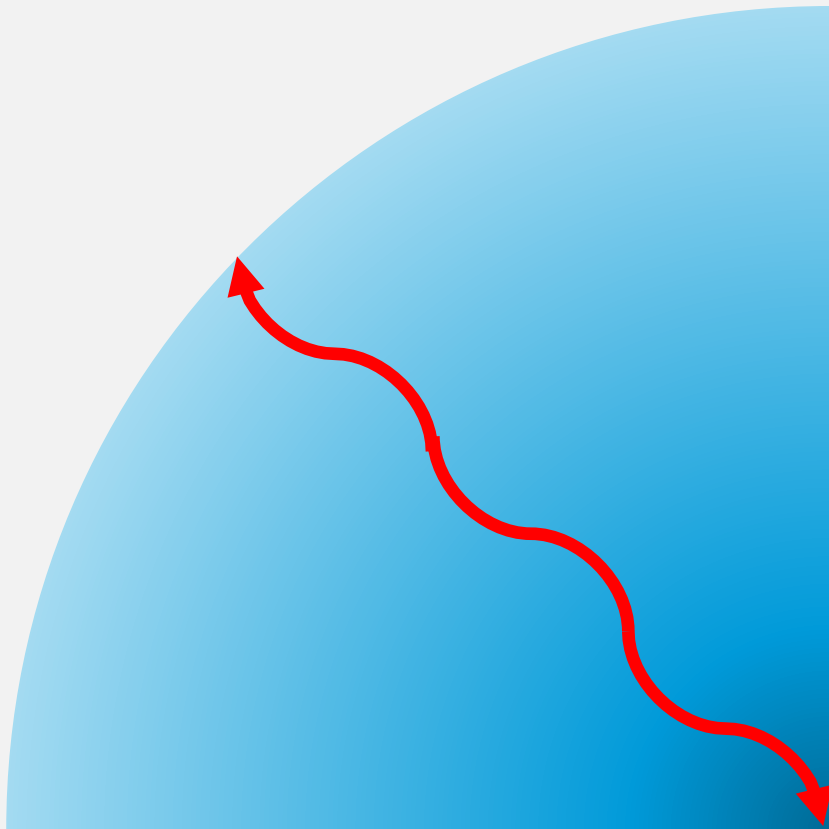


Spiral-Arm Instability
 $S < 1$

Giant clump formation

VDI vs SAI

- Violent Disc Instability



- Spiral-Arm Instability

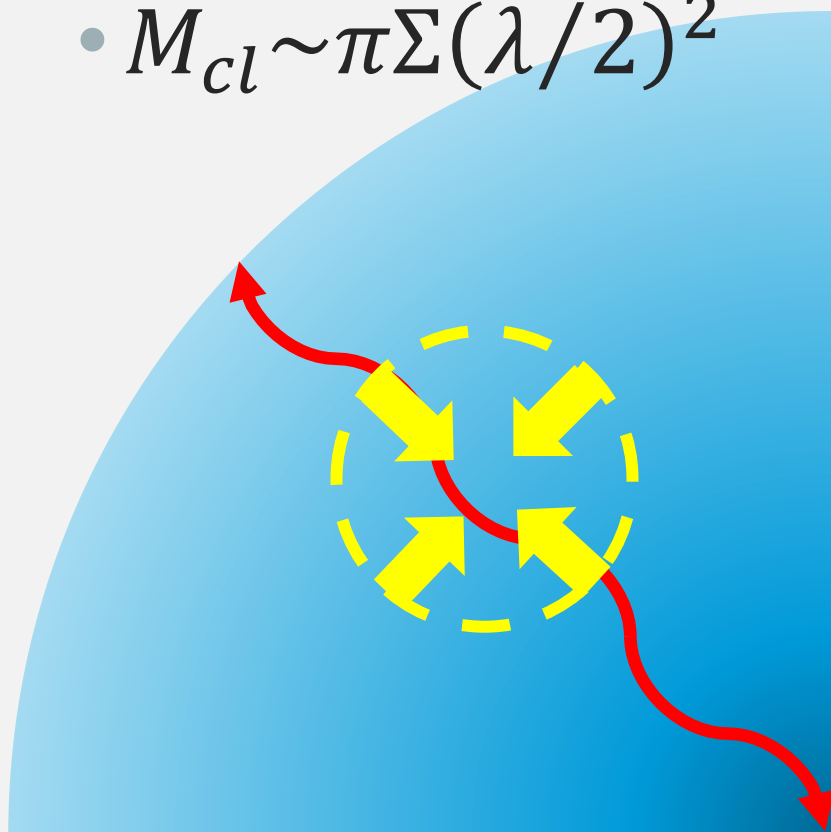


VDI vs SAI

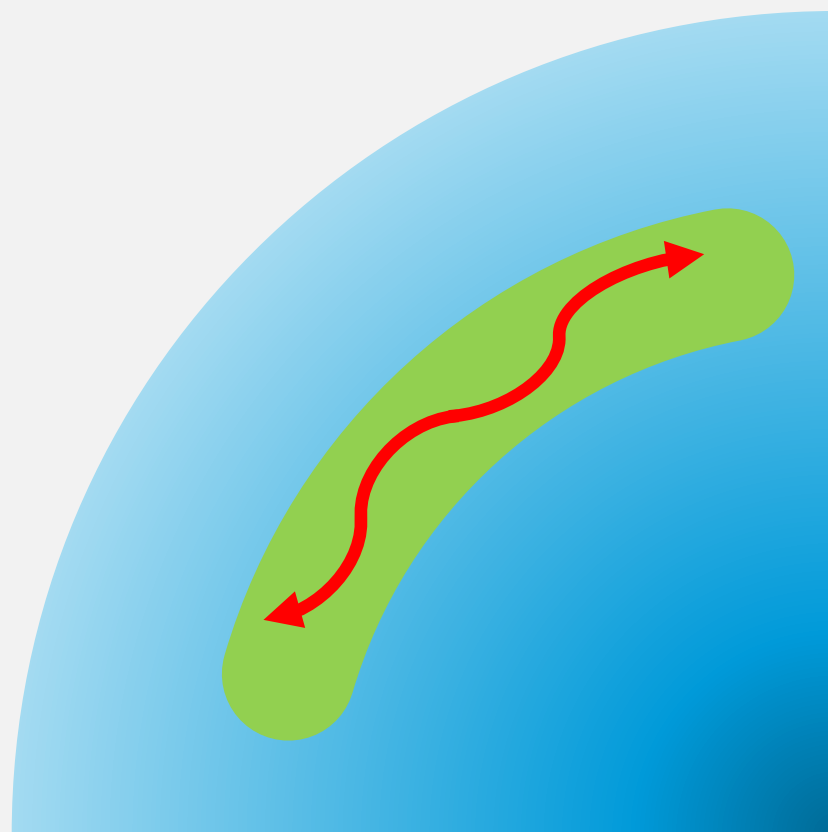
- Violent Disc Instability

- **2D collapse**

- $M_{cl} \sim \pi \Sigma (\lambda/2)^2$



- Spiral-Arm Instability

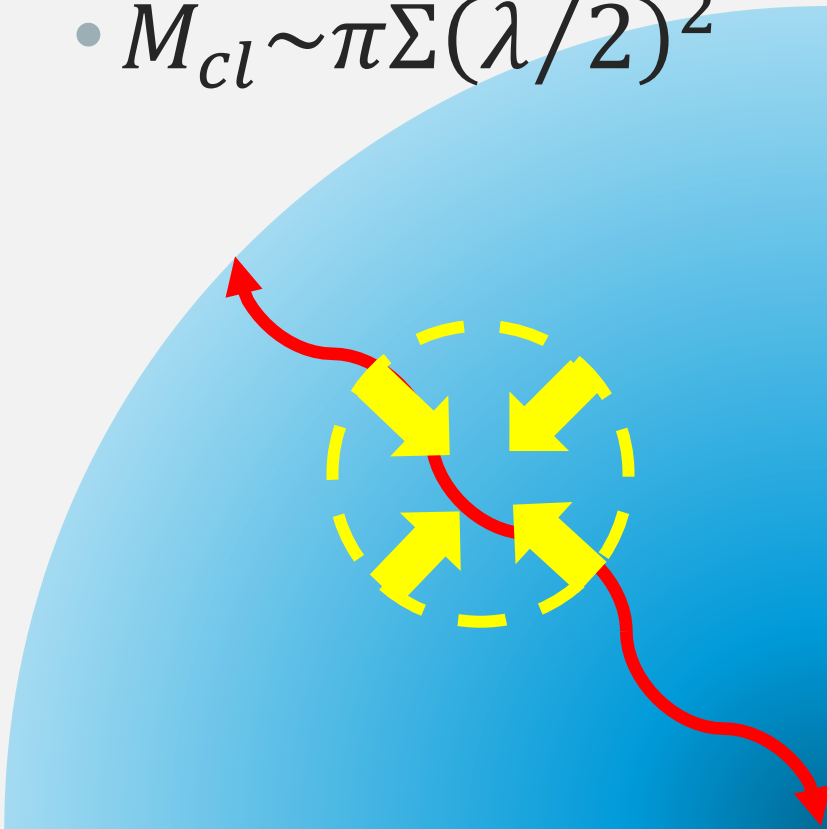


VDI vs SAI

- Violent Disc Instability

- **2D collapse**

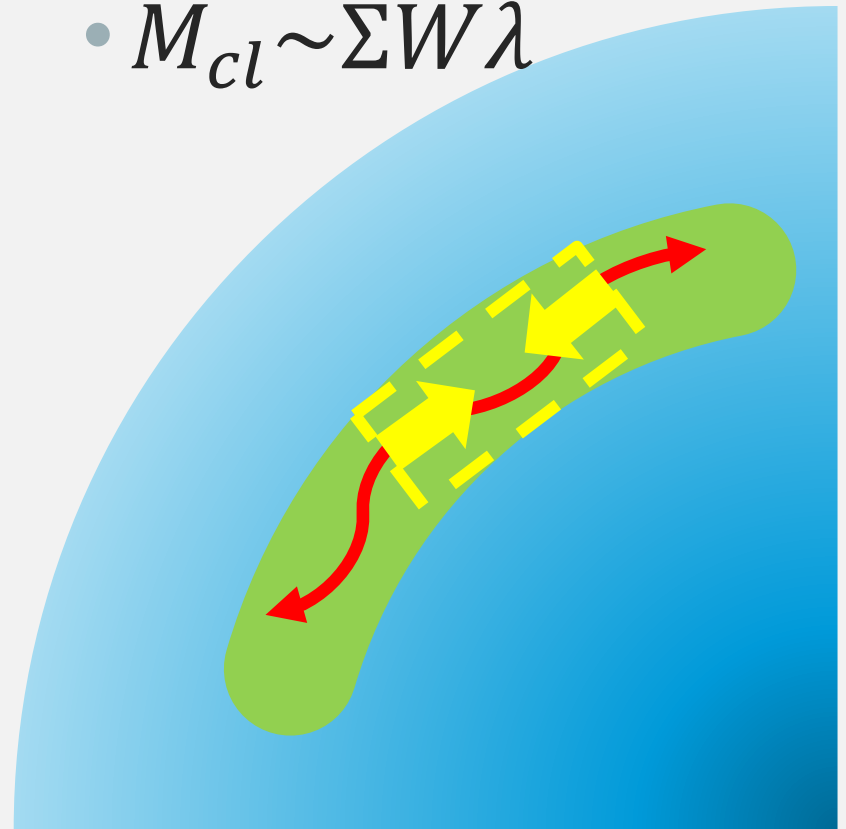
- $M_{cl} \sim \pi \Sigma (\lambda/2)^2$



- Spiral-Arm Instability

- **1D collapse**

- $M_{cl} \sim \Sigma W \lambda$



Scaling relations of high-z clumps

- From our analysis, we can obtain scaling relations of properties of giant clumps.

the most unstable wavelength: $\lambda_{\text{MU}} = 2\pi \left(\frac{\pi \alpha G F_0 A \Sigma W^{1-\alpha}}{8\Omega^2} \right)^{\frac{1}{2-\alpha}}.$

expected clump mass: $M_{\text{cl}} \sim \lambda_{\text{MU}} \frac{\Upsilon}{f_g} = \lambda_{\text{MU}} A \frac{\Sigma}{f_g} W.$

expected velocity dispersion within a clump: $\sigma_{\text{cl}}^2 \simeq \frac{1}{3} \frac{GM_{\text{cl}}}{R_{\text{cl}}} = \frac{2}{3} G \frac{\Sigma}{\varepsilon f_g} W A \simeq \frac{2}{3\pi} \eta \beta A (\varepsilon f_g)^{-1} \frac{W}{R_d} V^2.$

Spiral-arm instability

expected scaling relation:

$$R_{\text{cl}} \propto \left(\frac{\sigma_{\text{cl}}}{V} R_d \right)^{1.3}$$

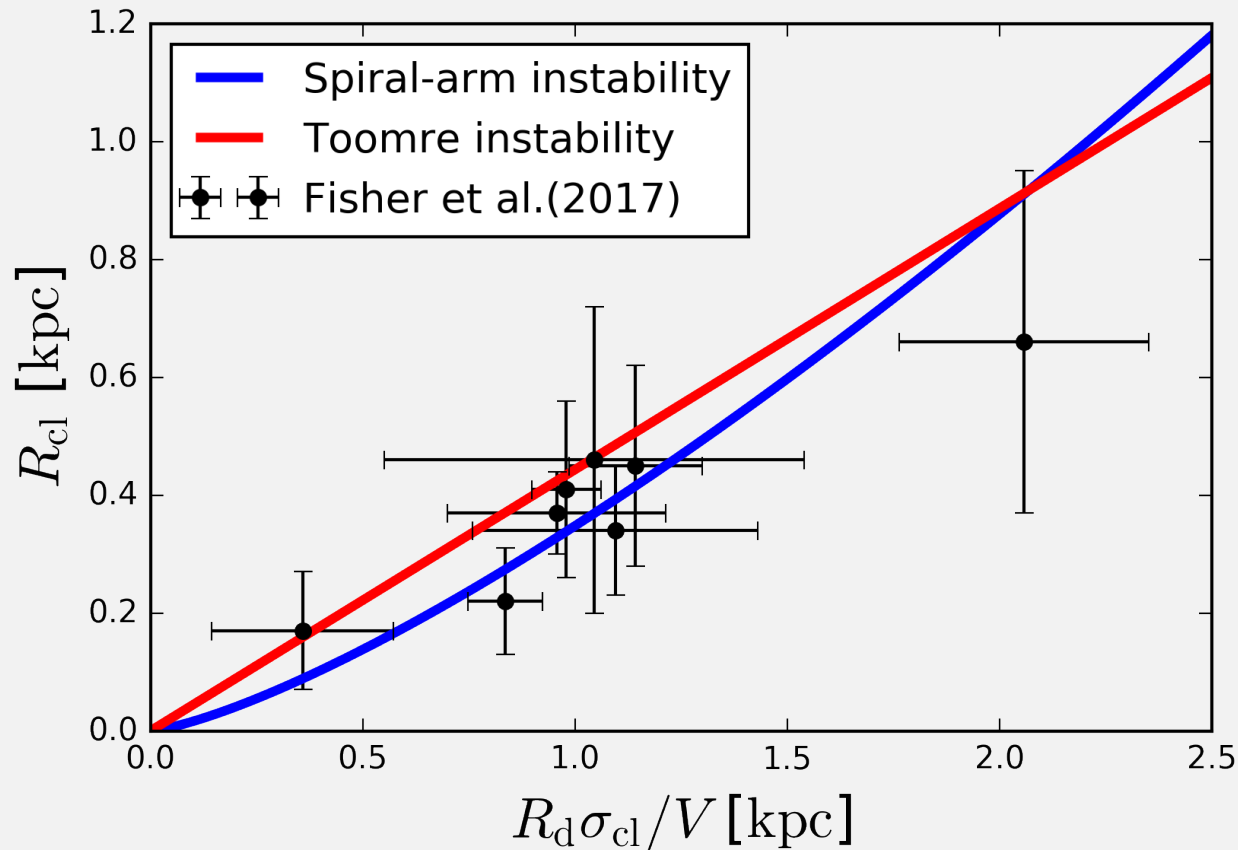
Toomre instability

expected scaling relation:

$$R_{\text{cl}} \propto \frac{\sigma_{\text{cl}}}{V} R_d$$

R_{cl} : clump radius, σ_{cl} : vel. disp. with in clump, R_d : disc radius, V : disc rot. vel.

Scaling relations of high-z clumps



- Both models of the spiral-arm and Toomre instability can explain the observations.
 - Neither is rejected by the observations.
- Our spiral-arm instability theory can be a possible mechanism of giant clump formation in high-z galaxies.

Scaling relations of high-z clumps

- From our analysis, we can obtain scaling relations of properties of giant clumps.

$$\frac{M_{\text{cl}}}{M_{\text{d,g+s}}} \simeq 2 \left[\frac{1}{8} \alpha F_0 (A\beta)^{3-\alpha} \eta \left(\frac{W}{R_{\text{d}}} \right)^{3-2\alpha} \right]^{\frac{1}{2-\alpha}}$$

Spiral-arm instability
expected scaling relation:

$$\frac{M_{\text{g,cl}}}{M_{\text{g,d}}} \propto f_{\text{g}}^{0.7} R_{\text{d}}^{-1.3},$$

$$\frac{M_{\text{cl}}}{M_{\text{d,g+s}}} \simeq \pi^2 a^{-4} \eta^2.$$

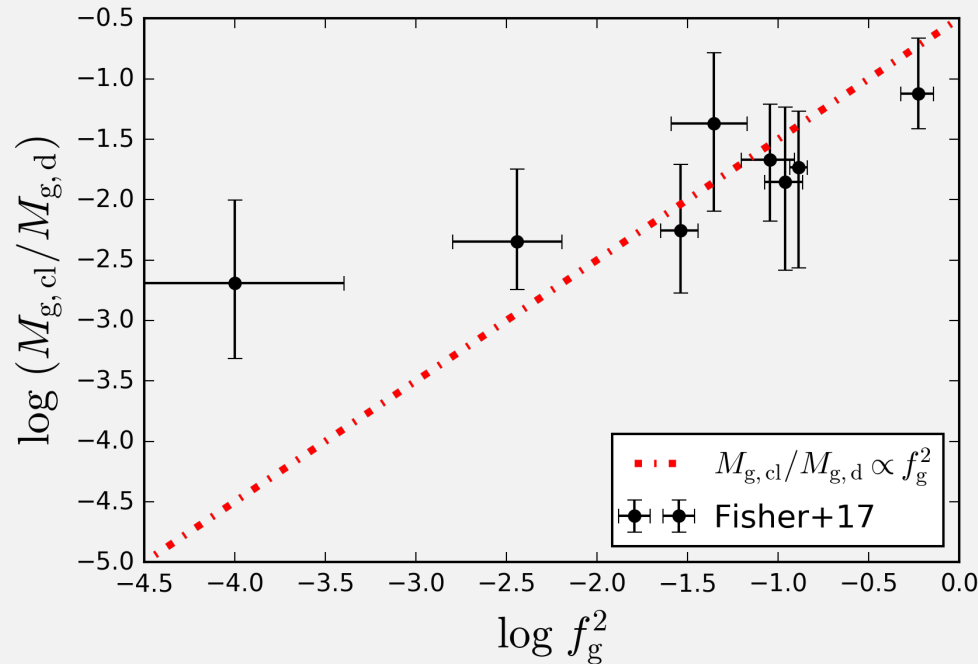
Toomre instability
expected scaling relation:

$$\frac{M_{\text{g,cl}}}{M_{\text{g,d}}} \propto f_{\text{g}}^2,$$

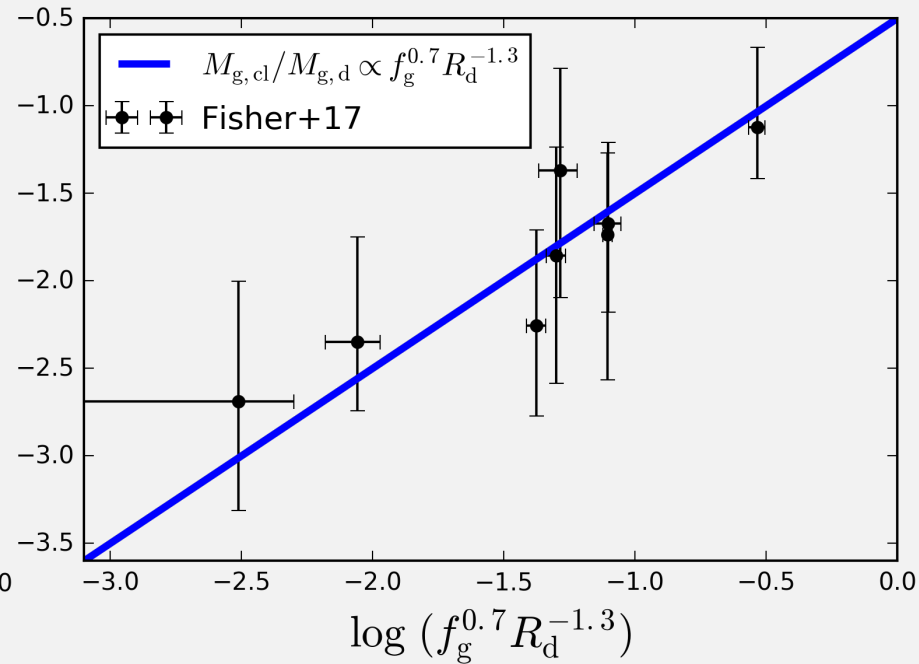
R_{cl} : clump radius, σ_{cl} : vel. disp. with in clump, R_{d} : disc radius, V : disc rot. vel.

Scaling relations of high- z clumps

- Violent Disc Instability



- Spiral-Arm Instability



- The observations may prefer our SAI model.
 - But, the sample size is too small.
 - If the most gas-poor one is excluded, the fittings become comparable in both models.

Summary

- We analytically derived an instability parameter and its criterion for spiral-arm fragmentation.

$$S \equiv \frac{1}{\pi G k^2} \left[\frac{\Upsilon_g f(kW_g)}{c_s^2 k^2 + 4\Omega_g^2} + \frac{\Upsilon_s f(kW_s)}{\sigma_\phi^2 k^2 + 4\Omega_s^2} \right]^{-1} < 1.$$

- Our novel instability parameter can characterize remarkably well fragmentation of spiral arms and clump formation following.
- Neither model of our spiral-arm nor Toomre instability is inconsistent with current observations.
 - **The spiral-arm instability could be a possible mechanism of giant clump formation in high-redshift galaxies.**

- **It is also interesting to adopt our instability analysis to local spiral galaxies.**