Higher-order moments

Masahiro Takada (IPMU)

INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

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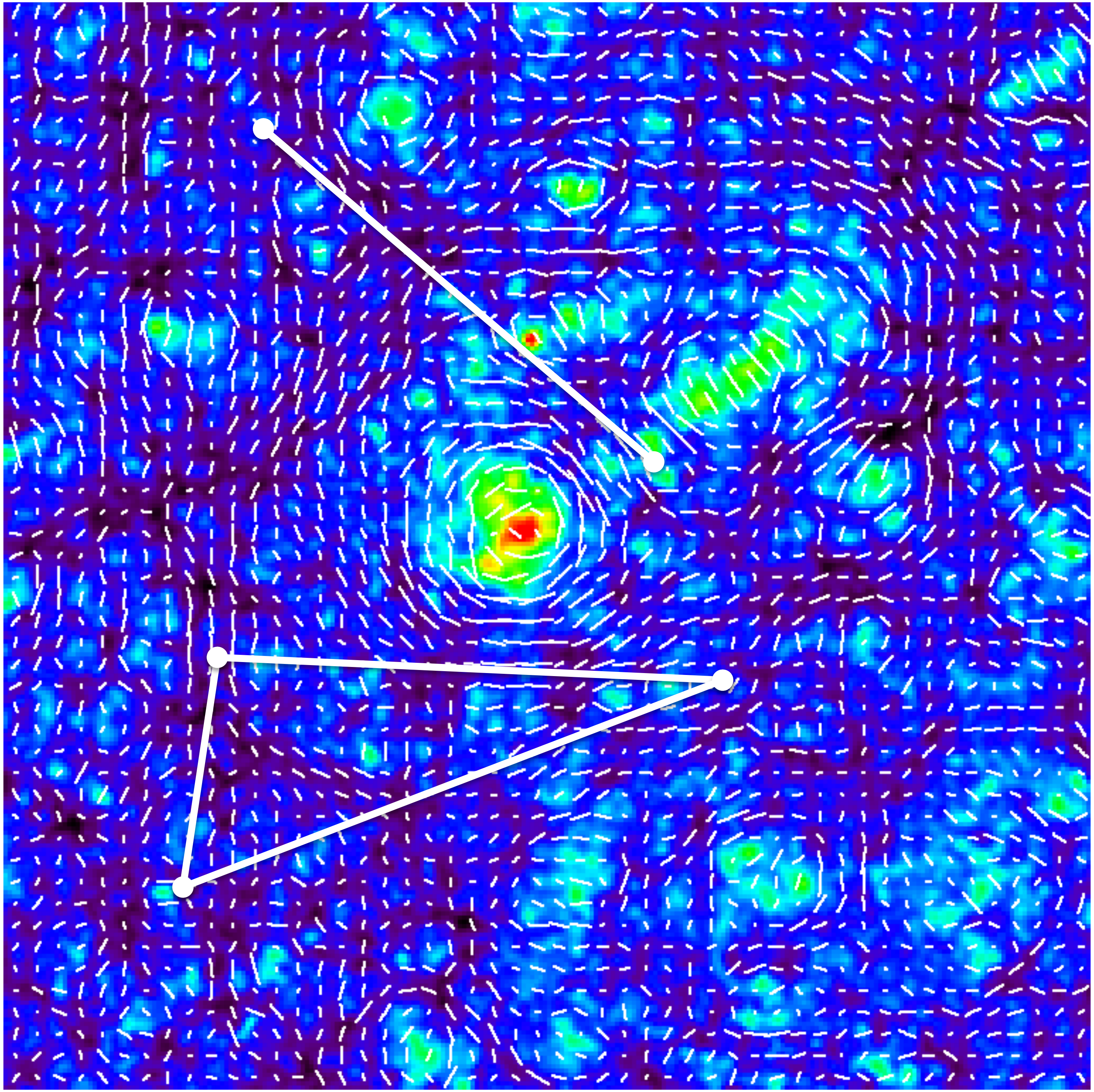
What are higher-order moments?

$$\langle \gamma\gamma\gamma\gamma \rangle$$

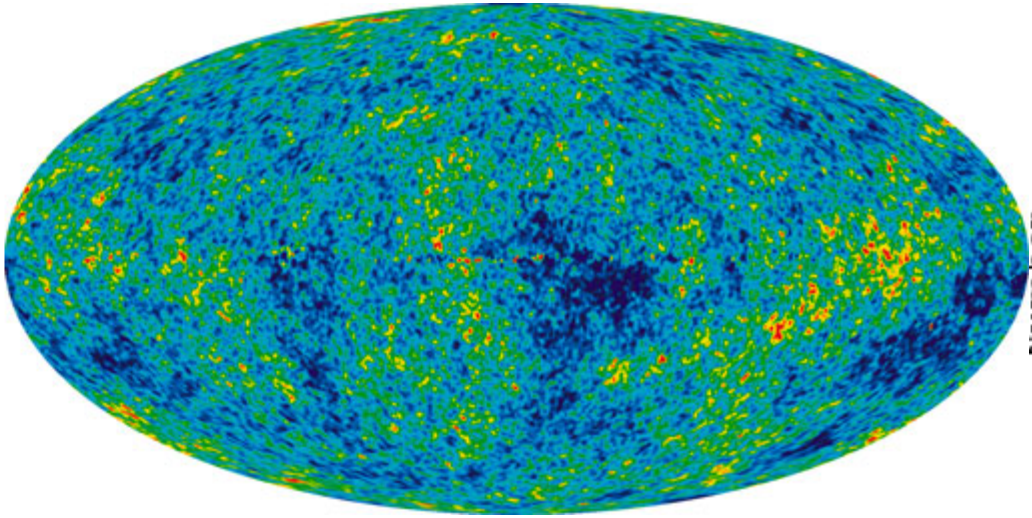
$$\langle \gamma\gamma\gamma \rangle$$

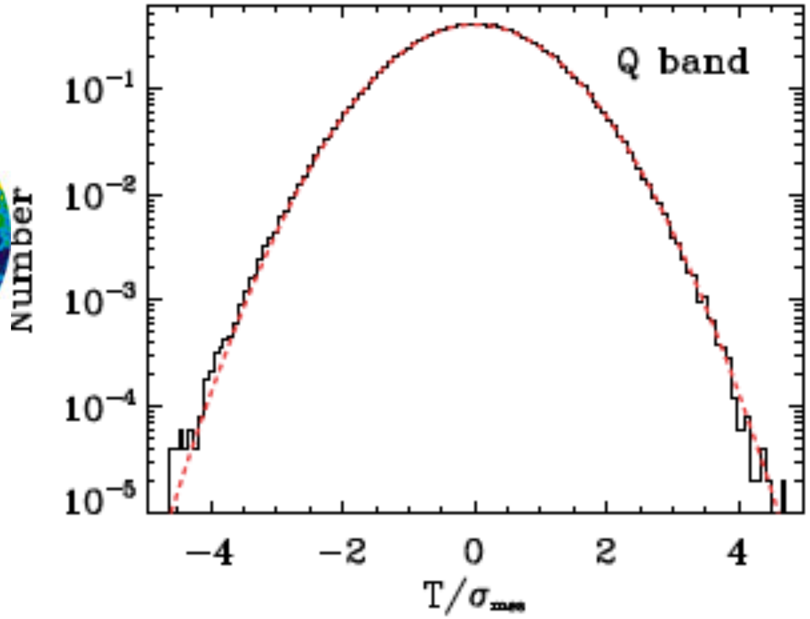
$$\langle \gamma\gamma h \rangle \qquad \cdots\cdots$$

- Anything beyond 2pt statistics $\langle \gamma\gamma \rangle$ or $\langle \gamma h \rangle$ …
- Note: the peak statistics may carry complementary information to the 2pt statistics, but I don't cover here



Why?

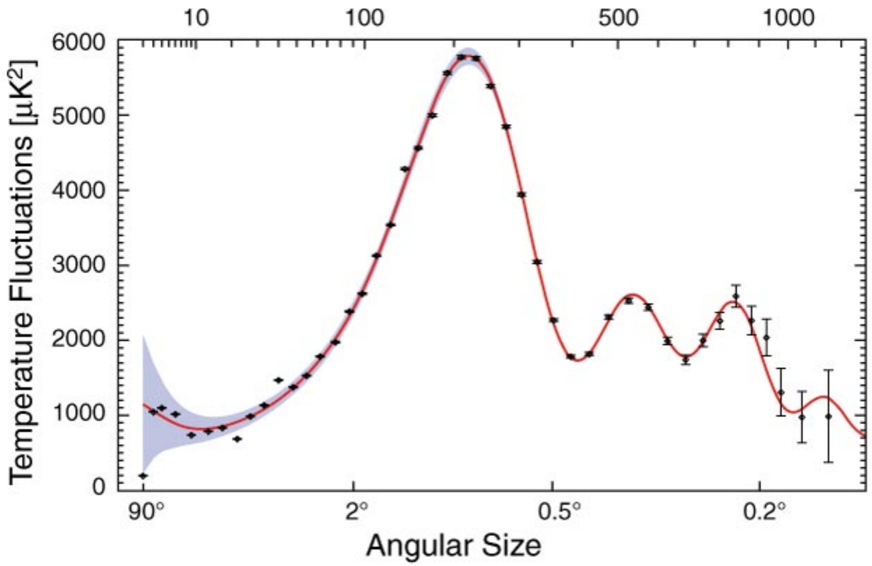




- We know the seed fluctuations are Gaussian
- The power spectrum carries the full information contained in a Gaussian field

$$\langle xxx \rangle = 0$$

$$\langle xxxx \rangle \rightarrow \langle xx \rangle \langle xx \rangle$$



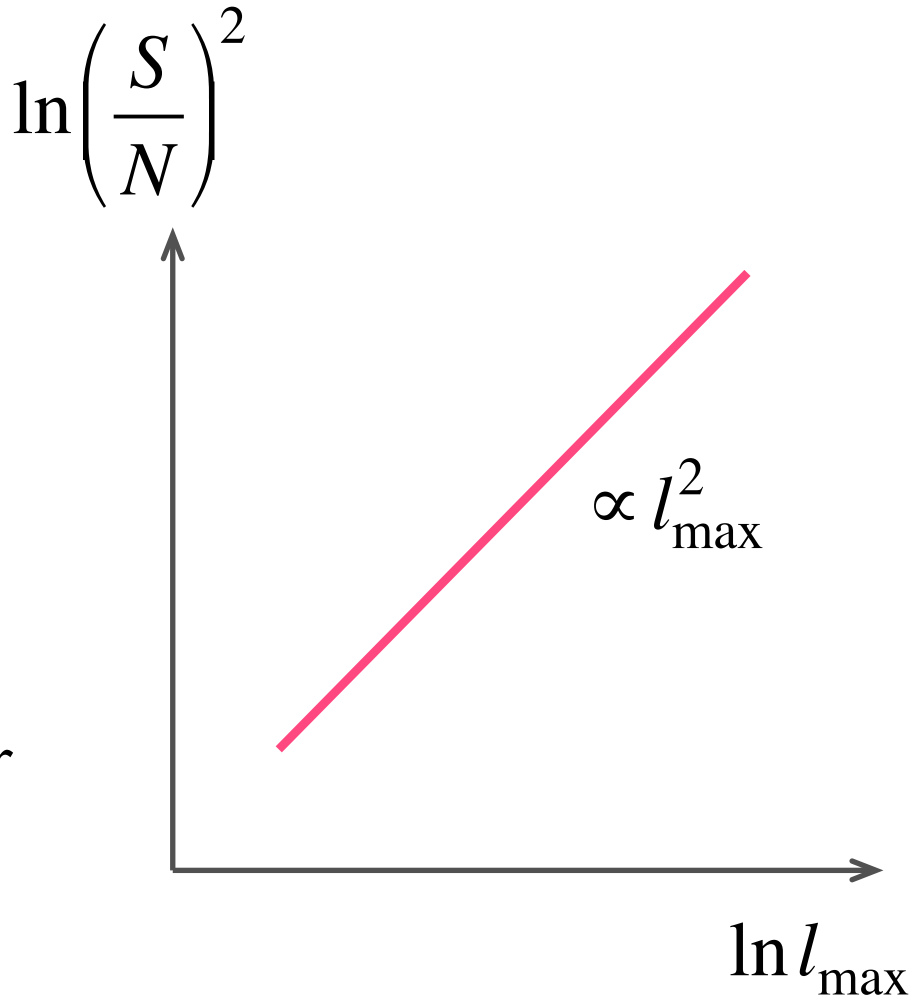
Information content

- The error of band power measurement for a Gaussian field (shot noise ignored)

$$\frac{\sigma^2(C_l)}{C_l^2} = \frac{2}{N_l} \propto \frac{1}{l \Delta l f_{\text{sky}}}$$

- The information content of power spectrum = the sum of the number of Fourier modes

$$\left(\frac{S}{N}\right)^2 \equiv \sum_{l=l_{\min}}^{l_{\max}} \left(\frac{C_l}{\sigma(C_l)}\right)^2 \propto l_{\max}^2 f_{\text{sky}}$$

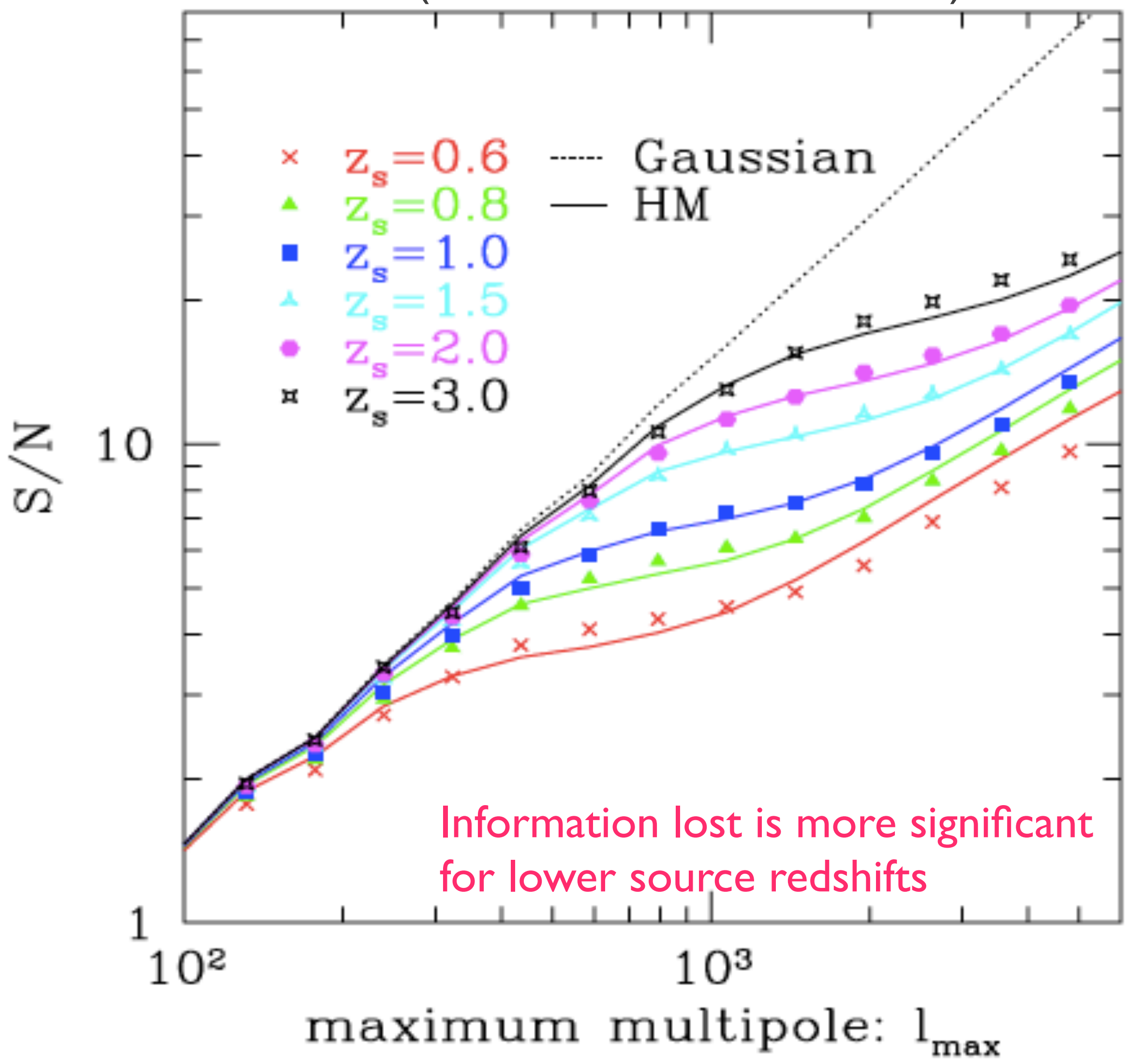


WMAP: lmax~1000
Planck: lmax~3000 (the improvement equivalent to 9 different universes)

Weak lensing case

- The information content of WL power spectrum is (significantly) smaller than the Gaussian expectation (also see Lee & Pen 08; MT & Jain 09; Yu+09)
- The power spectrum is not enough in WL case
- Where is the information contained in the initial field gone? The initial information is lost?

Sato+09 (used 1000 realizations)



Information lost is more significant for lower source redshifts

Cosmological complementarity

- Let's consider convergence field

$$\kappa(\boldsymbol{\theta}) = \int d\chi W(\chi)\delta(\boldsymbol{\theta})$$

$$\text{where } \; W(\chi) \propto \Omega_{m0}$$

- Consider the 3-point function of convergence, which is the lowest-order function to extract the non-Gaussian signal
  - The lensing 3pt function arises from the 3pt function of density fluctuation field

$$\langle \kappa\kappa\kappa \rangle = \int d\chi\, d\chi'\, d\chi'' W(\chi) W(\chi') W(\chi'') \langle \delta\delta\delta \rangle$$

- The perturbation theory for nonlinear structure formation (e.g., Bernardeau et al. 02 Phys. Rep.) predicts

$$\delta = \delta^{(1)} + \underbrace{\delta^{(2)}}_{\left(\delta^{(1)}\right)^2} + \underbrace{\delta^{(3)}}_{\left(\delta^{(1)}\right)^3} + \ldots.$$

- The 3pt function (bispectrum) of density fluctuation field is found to be

$$\left\langle \delta\delta\delta \right\rangle = \left\langle \left(\delta^{(1)} + \delta^{(2)} + \delta^{(3)} + ...\right)\left(\delta^{(1)} + \delta^{(2)} + \delta^{(3)} + ...\right)\left(\delta^{(1)} + \delta^{(2)} + \delta^{(3)} + ...\right) \right\rangle$$
$$= \left\langle \delta^{(1)}\delta^{(1)}\delta^{(1)} \right\rangle + \left\langle \delta^{(1)}\delta^{(1)}\delta^{(2)} + ... \right\rangle + ...$$
$$= \left\langle \delta^{(1)}\delta^{(1)}\delta^{(2)} + ... \right\rangle + ...$$

non-vanishing lowest-order term

$$\left\langle \tilde{\delta}(\boldsymbol{k}_1)\tilde{\delta}(\boldsymbol{k}_2)\tilde{\delta}(\boldsymbol{k}_3) \right\rangle = (2\pi)^3 B_\delta(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\delta_D^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)$$

$$B_\delta(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = F(\boldsymbol{k}_1,\boldsymbol{k}_2)P(k_1)P(k_2) + (\mathrm{cyc.})$$

$$\text{where} \quad F(\boldsymbol{k}_1,\boldsymbol{k}_2) \equiv \frac{10}{7} + \left(\frac{1}{k_1^2} + \frac{1}{k_2^2}\right)\left(\boldsymbol{k}_1 \cdot \boldsymbol{k}_2\right) + \frac{4}{7}\frac{\left(\boldsymbol{k}_1 \cdot \boldsymbol{k}_2\right)^2}{k_1^2 k_2^2}$$

- Hence, using the Limber's approximation, the lensing 3pt function can be given in terms of the mass bispectrum as

$$B_\kappa(\boldsymbol{l}_1,\boldsymbol{l}_2,\boldsymbol{l}_3) = \int d\chi W^3(\chi)\chi^{-4} B_\delta(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\Big|_{k_i = l_i/\chi}$$

$$\text{where} \quad \left\langle \tilde{\kappa}(\boldsymbol{l}_1)\tilde{\kappa}(\boldsymbol{l}_1)\tilde{\kappa}(\boldsymbol{l}_1) \right\rangle \equiv (2\pi)^2 B_\kappa(\boldsymbol{l}_1,\boldsymbol{l}_2,\boldsymbol{l}_3)\delta_D^2(\boldsymbol{l}_1+\boldsymbol{l}_3+\boldsymbol{l}_3)$$

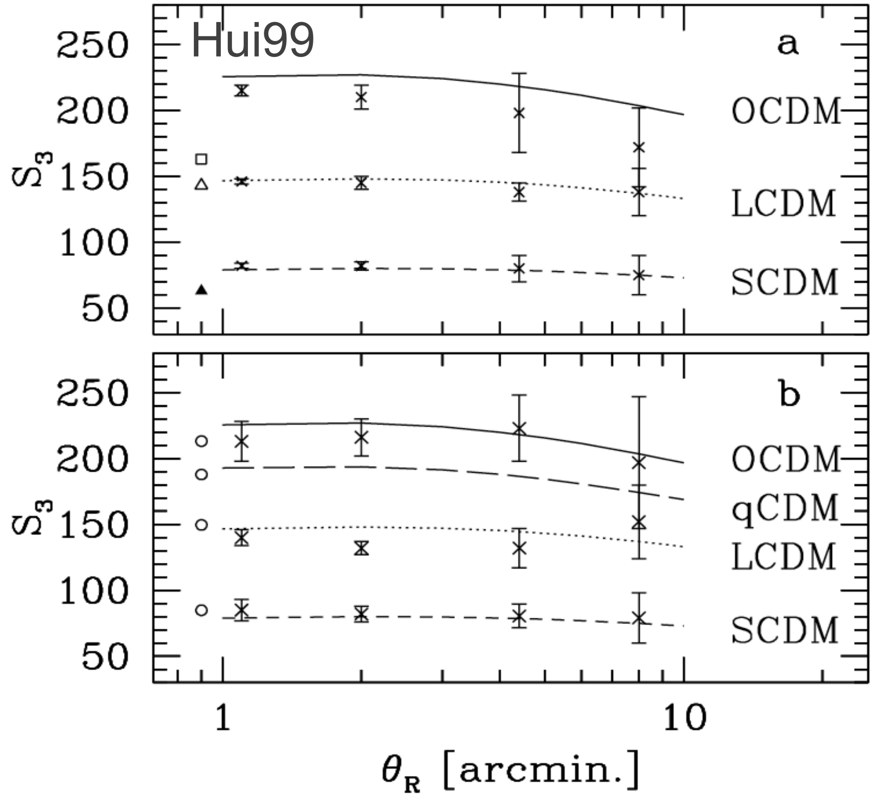
- The lensing power spectrum (2pt) and bispectrum (3pt) depend on the lensing efficiency function and the mass power spectrum amplitude

$$P_\kappa(l) = \int d\chi W^2(\chi)\chi^{-2} P_\delta(k = l/\chi) \propto W^2(\chi) P_\delta \propto \Omega_{m0}^2 \sigma_8^2$$

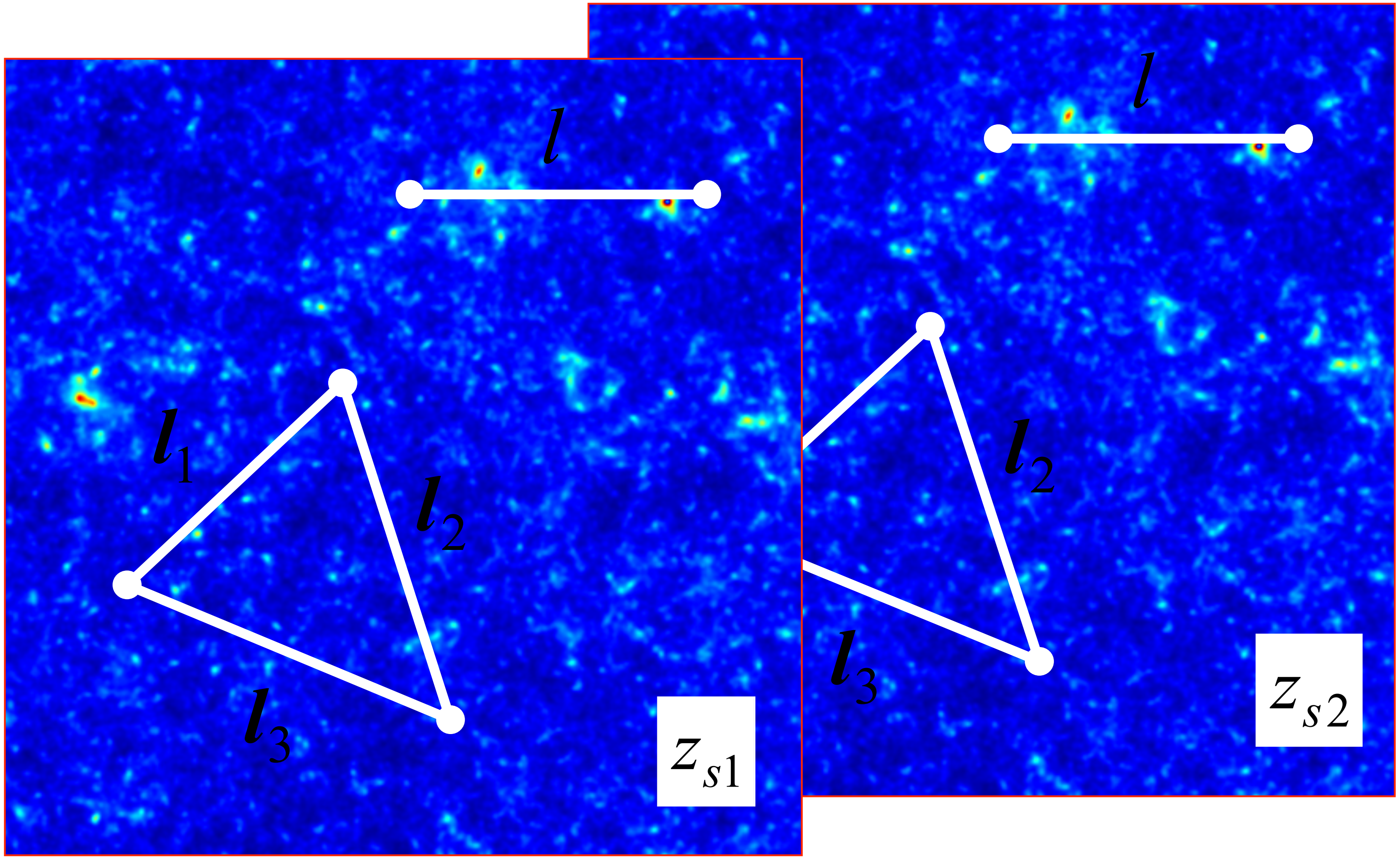
$$B_\kappa(\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{l}_3) = \int d\chi W^3(\chi)\chi^{-4} B_\delta(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\Big|_{k_i = l_i/\chi} \propto W^3 P_\delta^2 \propto \Omega_{m0}^3 \sigma_8^4$$

- Hence combining the 2pt and 3pt lensing correlations can break parameter degeneracies (Hui 99; MT & Jain 04): e.g.,

$$\frac{B_\kappa}{P_\kappa^2} \propto \Omega_{m0}^{-1}$$



WL 2pt+3pt tomography



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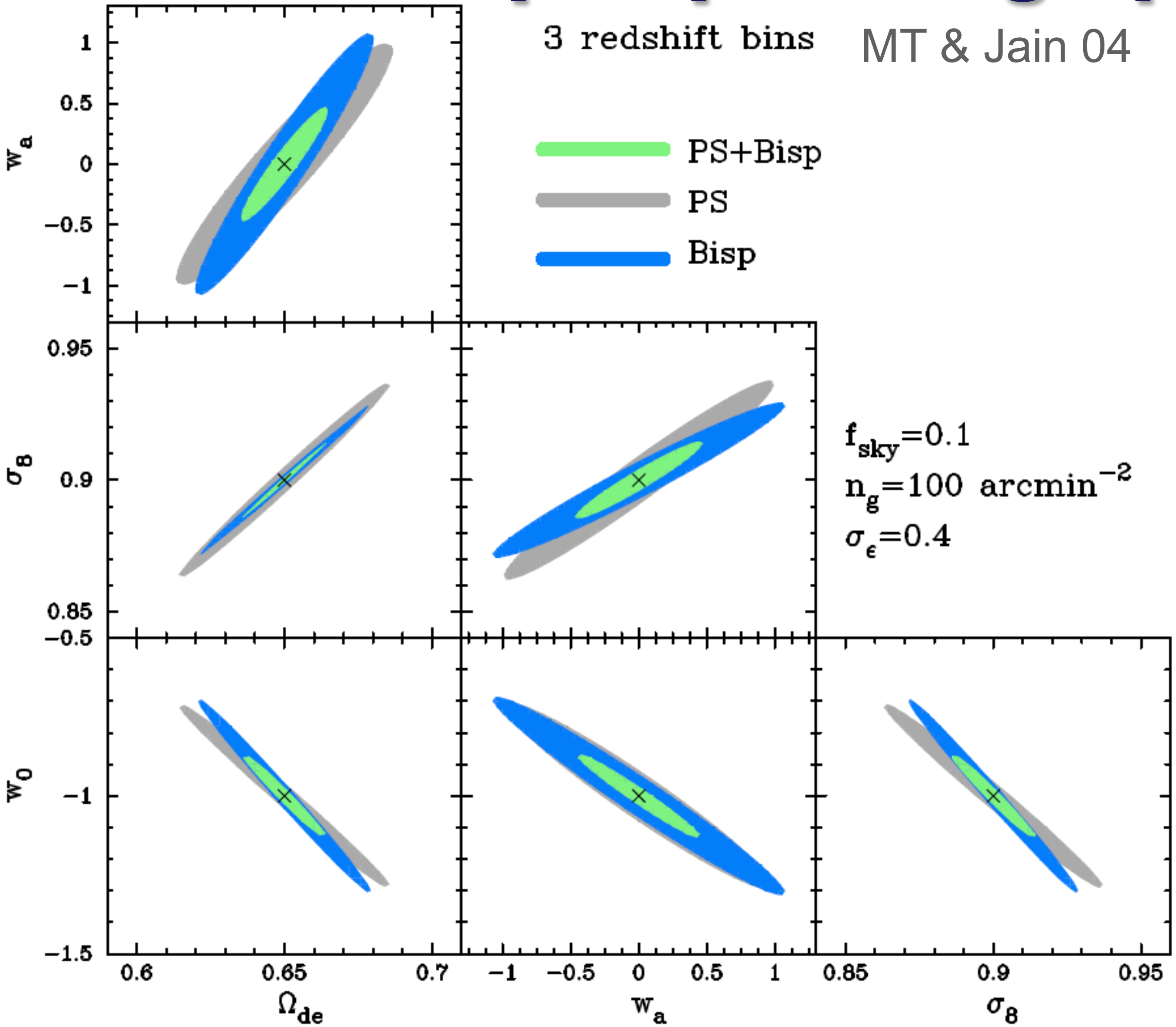


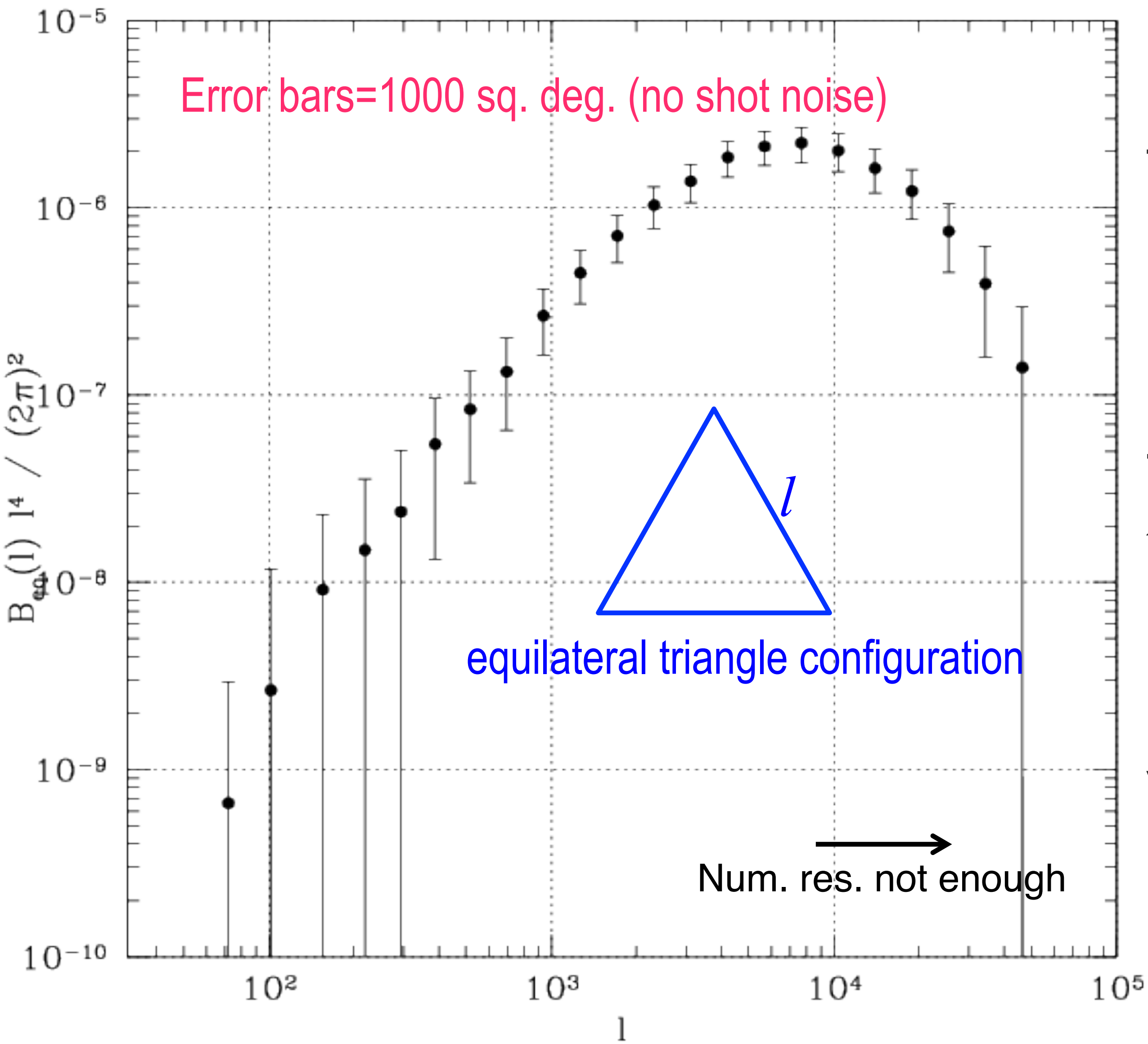
MT & Jain 04

A factor 2-3 improvement (by recovering the initial information content via 3pt function)

This forecast includes all the triangle configurations for a given lmax=3000

Non-Gaussian errors need to be included

Prospect for WL 3pt measurement



Estimate the lensing bispectrum as a function of triangle configurations, using 1000 ray-tracing simulations

The left panel shows the example for equilateral triangle configuration

What is the S/N?

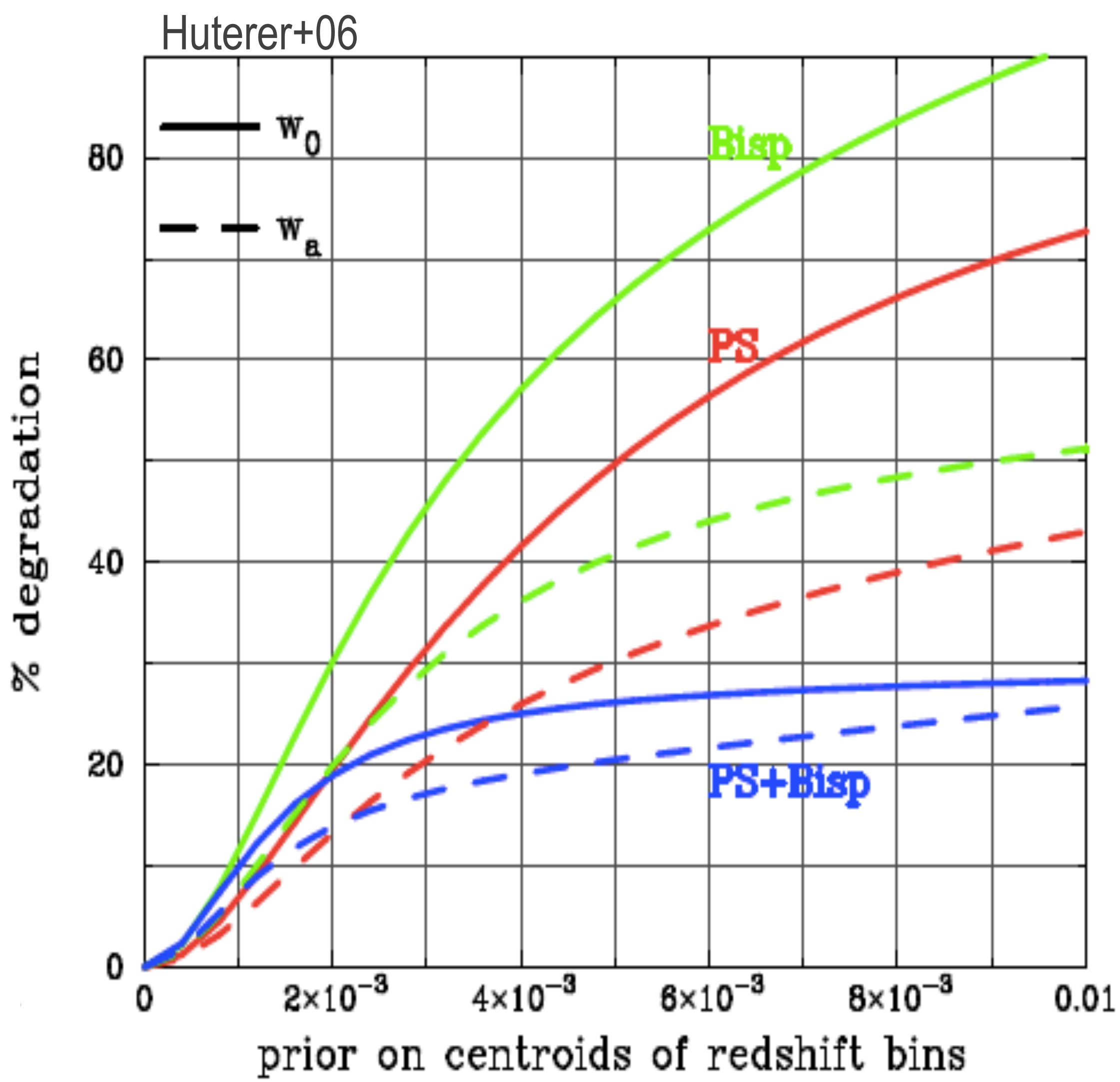
Kayo & MT in prep.

Self-calibration of systematic errors

$$P_\kappa \propto W^2(\chi) P_\delta$$

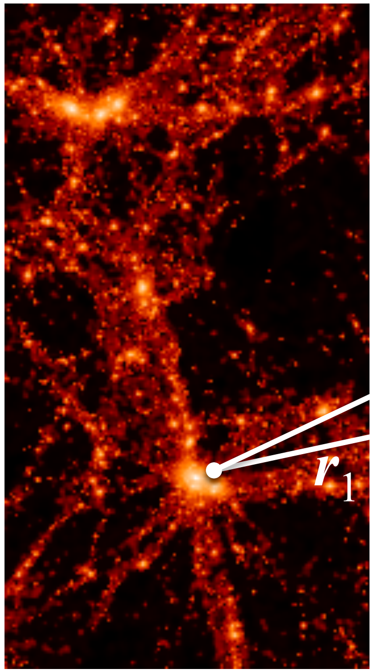
$$B_\kappa \propto W^3(\chi) P_\delta^2$$

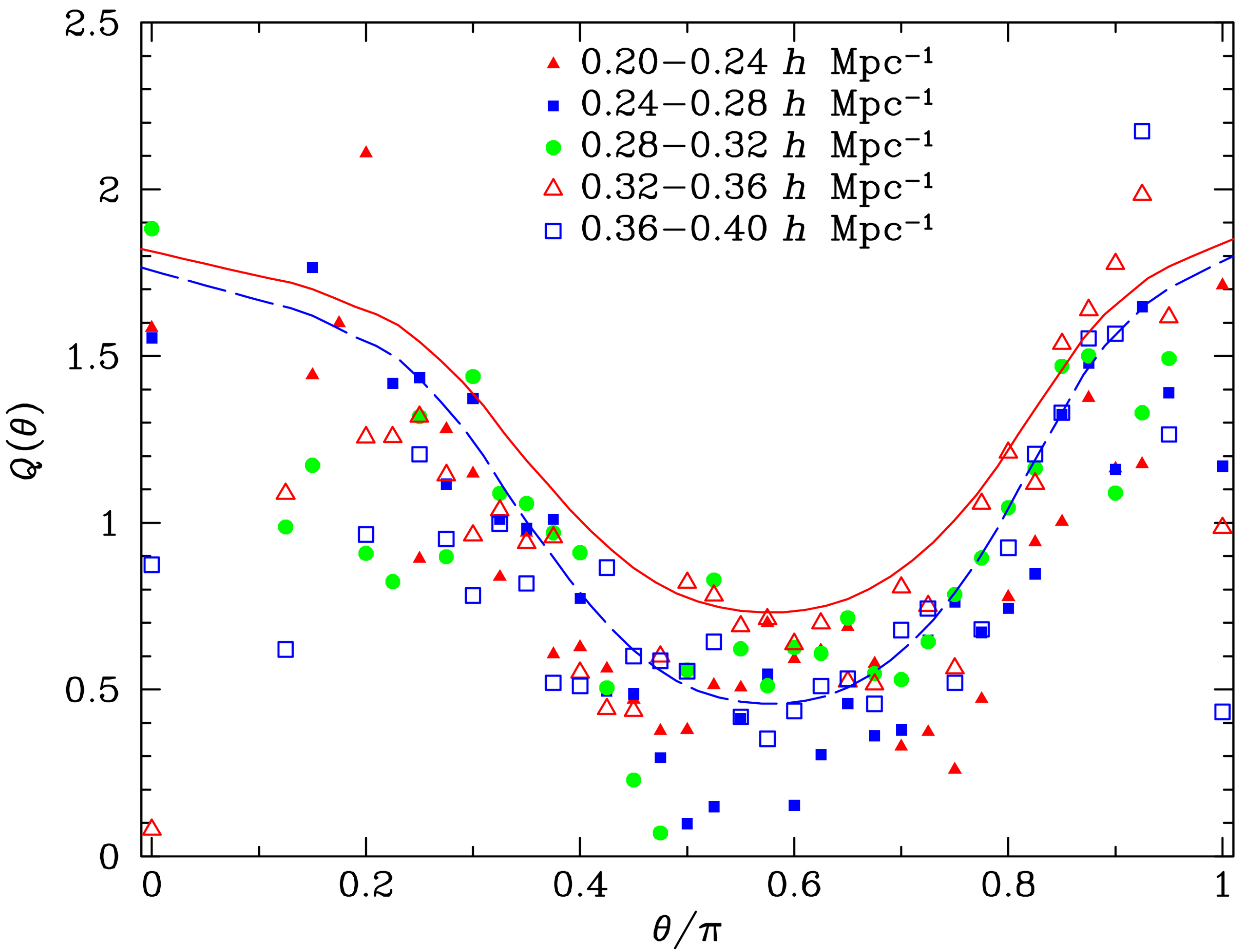
- The lensing PS and Bisp *depend* on cosmological parameters as well as on the WL systematic errors, if exist, in a different way
- The theory tells the cosmological dependences
- Hence combining the 2pt and 3pt allows for a self-calibration of the systematic errors from the same data set



Another usefulness of 3pt

- The bispec
- Can explor
- One charac appearanc
- Also for pri





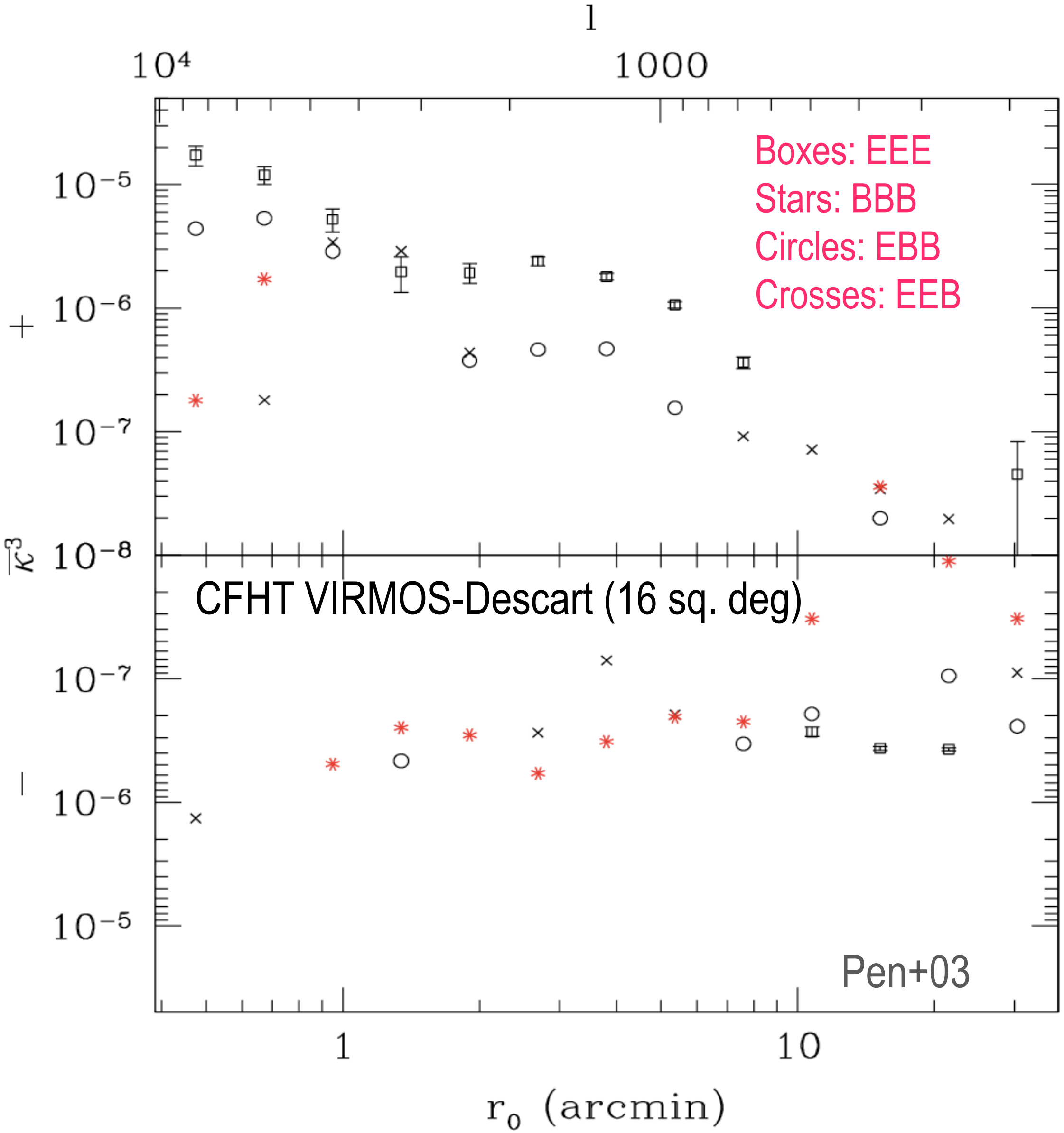
Feldman et al. PRL 01

Status of the measurement

- Pen et al. (03) reported the first detection of the WL skewness from CFHT data with 16 sq. deg.
- The skewness contains the collapsed information of the bispectrum

$$\text{Skewness} \equiv \left\langle \left(\kappa \otimes F(r_0)\right)^3 \right\rangle$$

- The next search is from CFHT 170 sq. deg. data?

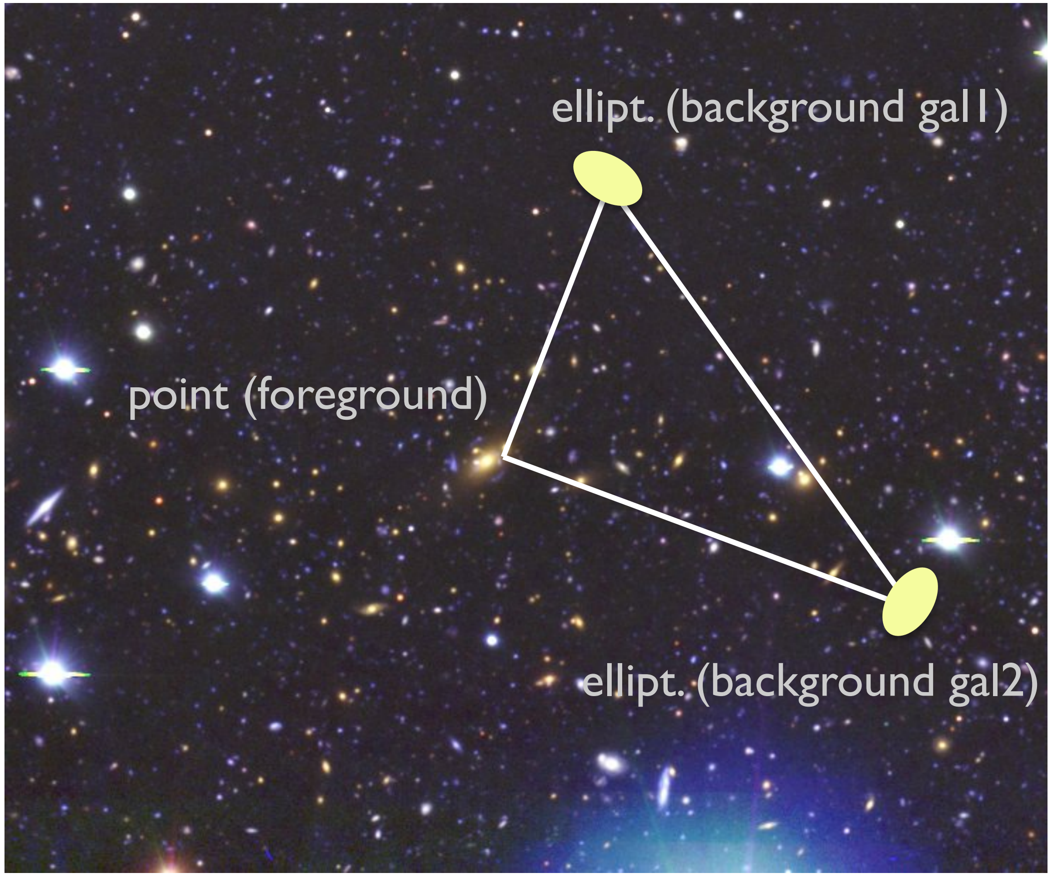


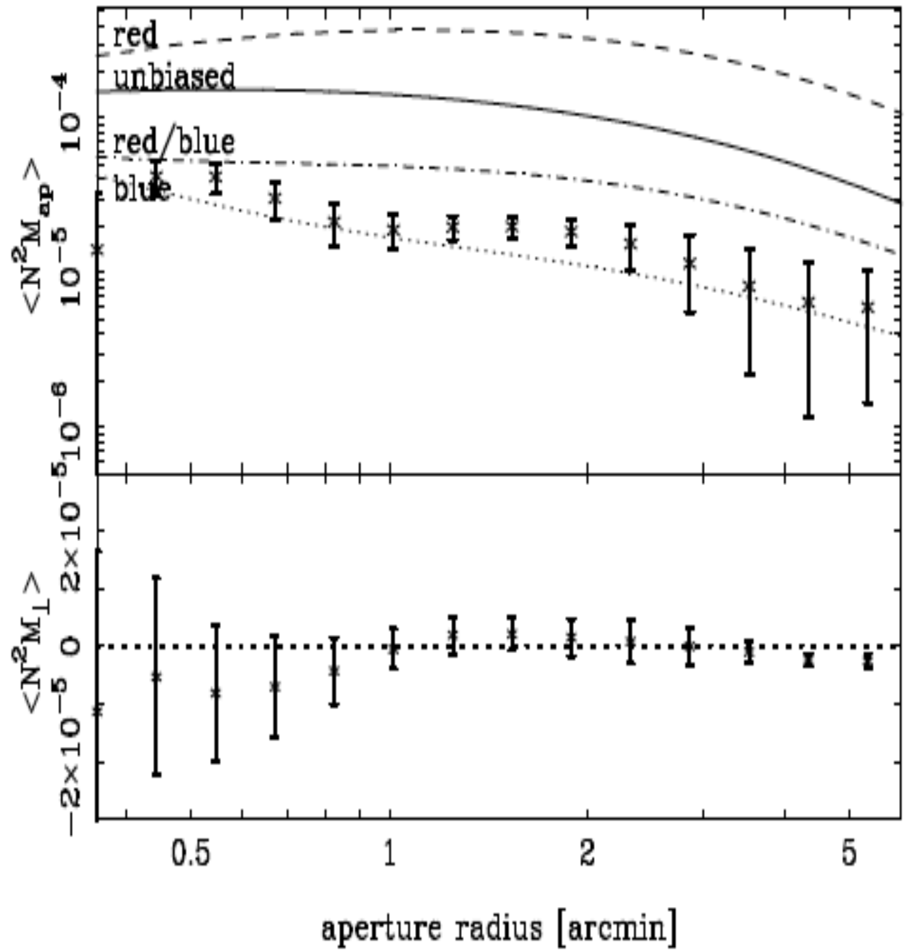
Issues need to be studied (possible subjects for students)

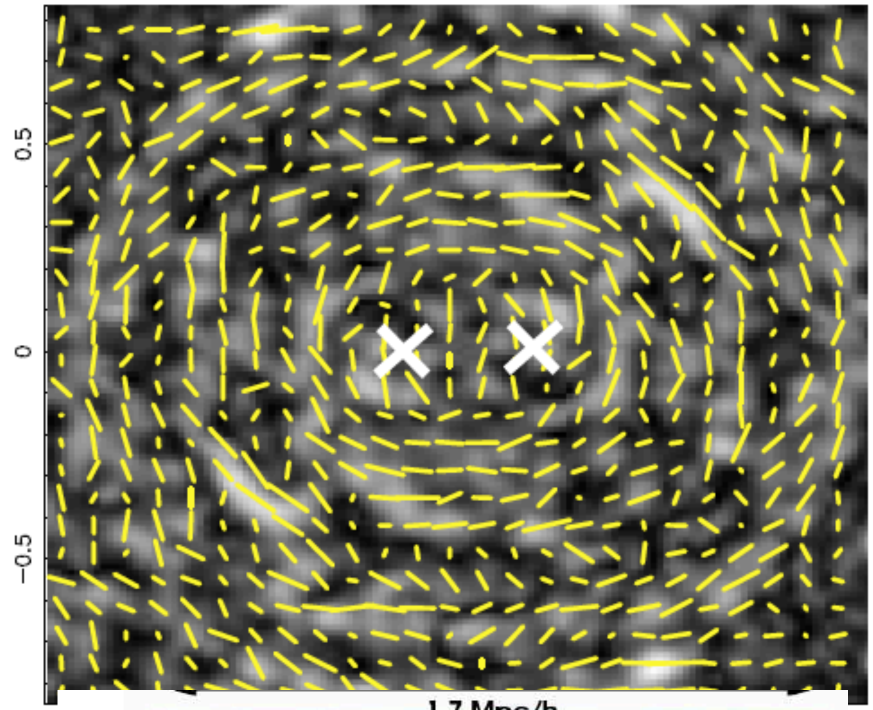
- The E/B mode decomposition of lensing 3pt function
  – Doable by applying the CMB-used methods to lensing fields (e.g., see Hikage+11 for the E/B-mode decomposition of power spectrum)
- An optimal, reasonably fast, measurement method of measuring the 3pt functions of all the configurations
  – Even for the SDSS, the galaxy bispectrum hasn't been fully explored to improve the cosmological constraints (BAO and cosmo paras)
- Develop an accurate model of the theoretical predictions
  – Need to use a sufficient number of simulation realizations
- The error covariances
  – The bispectrum covariance arises from up to the 6-pt correlation functions
- What about the 4pt function or the higher?

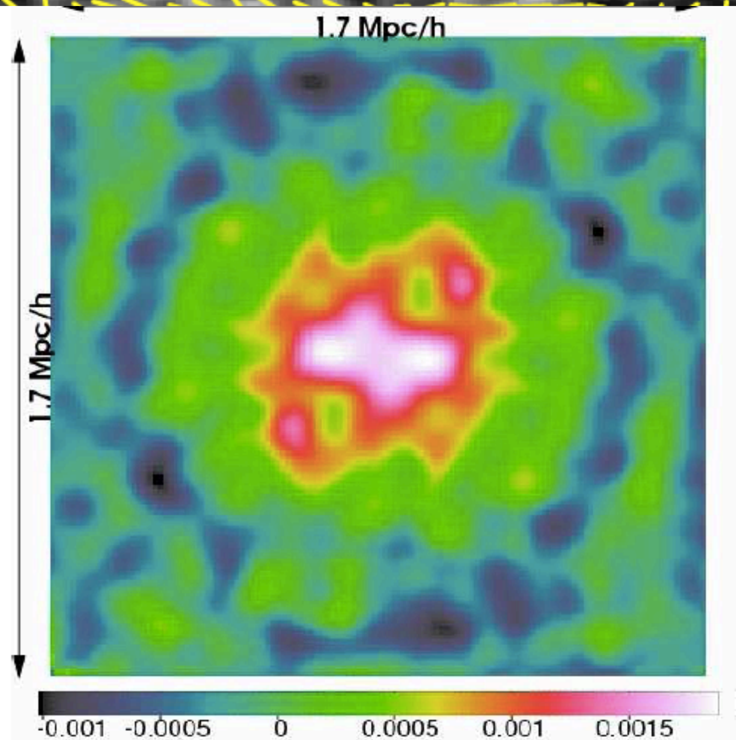
These are not the full story...

- Other various n-point functions are available from the same data; halo-shear-shear, halo-halo-shear, halo-halo-halo, …









- Simon et al (08) used the CFHT data of 34 sq. deg., and then reported a detection of the galaxy-galaxy-shear correlations

Hope

2pt functions

$$\langle \gamma\gamma \rangle \propto W_{\mathrm{gl}}^2 P_\delta(k)$$

$$\langle \gamma h \rangle \propto b W_{\mathrm{gl}} P_\delta(k)$$

$$\langle hh \rangle \propto b^2 P_\delta(k)$$

3pt functions

$$\langle \gamma\gamma\gamma \rangle \propto W_{\mathrm{gl}}^3 P_\delta^2(k)$$

$$\langle h\gamma\gamma \rangle \propto b W_{\mathrm{gl}}^2 P_\delta^2(k)$$

$$\langle hh\gamma \rangle \propto b^2 W_{\mathrm{gl}} P_\delta^2(k)$$

$$\langle hhh \rangle \propto b^3 P_\delta^2(k)$$

- The different correlation functions depend on halo bias, matter power spectrum and lensing efficiency in different ways
- The comprehensive 2pt analysis of WL (Bernstein 09)
- A comprehensive joint analysis of 2pt and 3pt functions for a survey (not yet done)
- This combination is more powerful? If so, how much improvement?

Summary

- For any large-scale structure probe, the power spectrum is not sufficient to extract the full information
- Definitely should explore the higher-order moments for coming/future surveys that are very expensive surveys
- Combining the higher-order functions with the 2pt information can be useful for improving the constraining power or calibrating the systematic errors
- Open up a new window to study the primordial non-Gaussianity
- The full use of the higher-order moments haven't yet been fully explored; projects for you!