WL simulations

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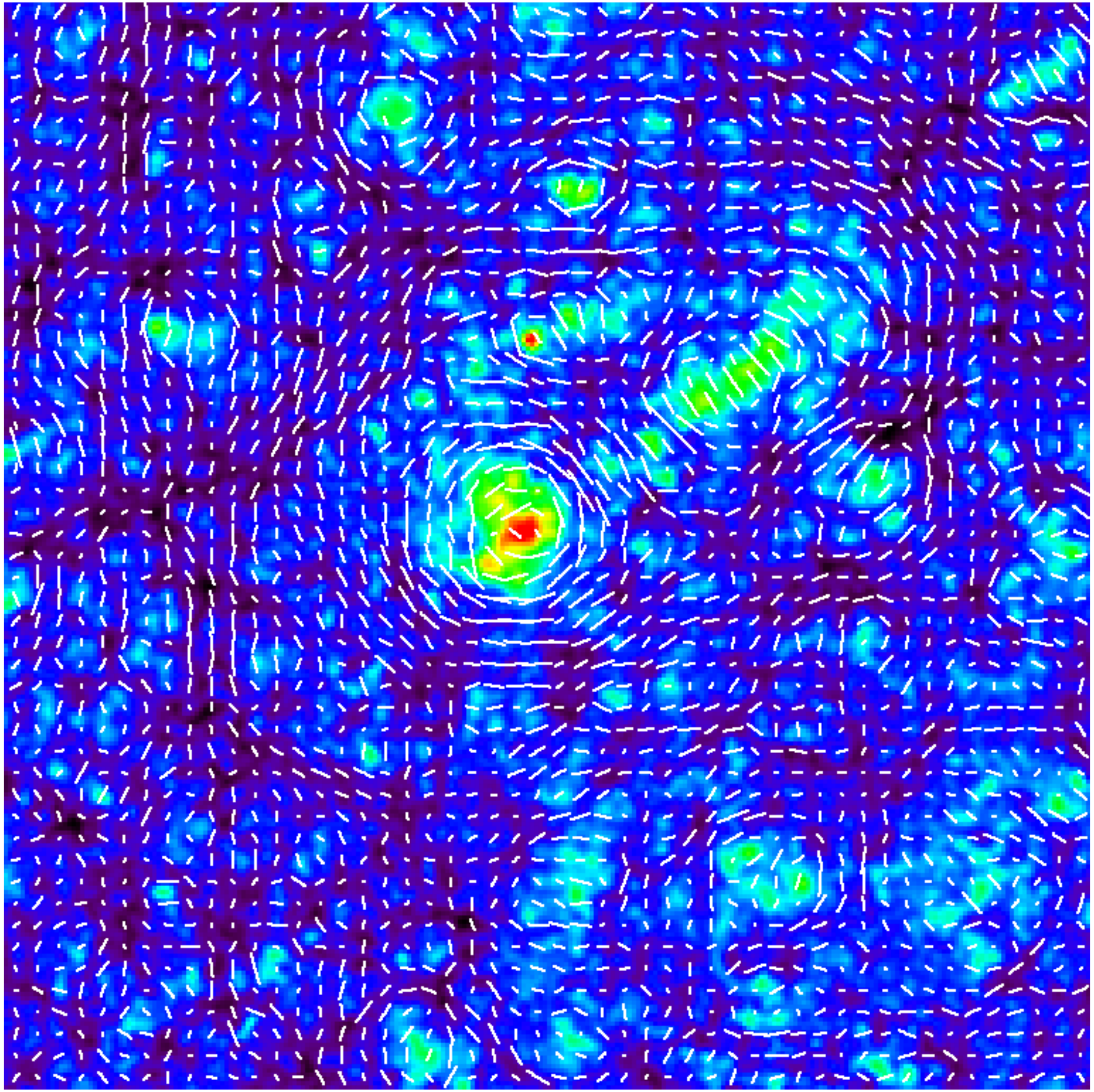


Courtesy of T. Hamana (NAOJ)

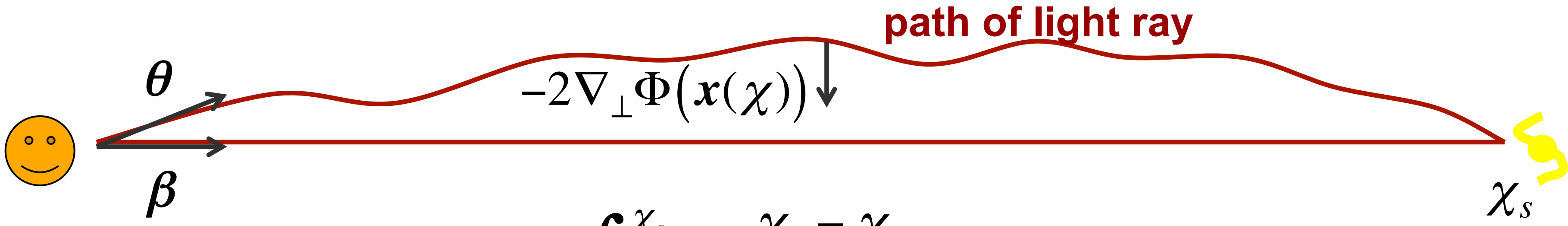
Why ray-tracing simulations?

ray-tracing simulation = simulating deflection of light ray, emitted from a distant galaxy, via simulated large-scale structures

- WL observables affected by nonlinear structures that are very difficult to analytically model
- Useful for constructing mock/simulation catalogs (e.g., for testing methods/pipeline or studying the effect of survey geometry)
- Enable to study various cross-correlations of WL observables with other observables; e.g., cluster-shear correlation, and combined method of WL and galaxy clustering
- Enable to study error (co)variances of WL observables or between different observables
- To study the effects of survey geometry and masks (E/B mode decomposition; Hikage et al. 11)

Lens equation

See Alan's lectures



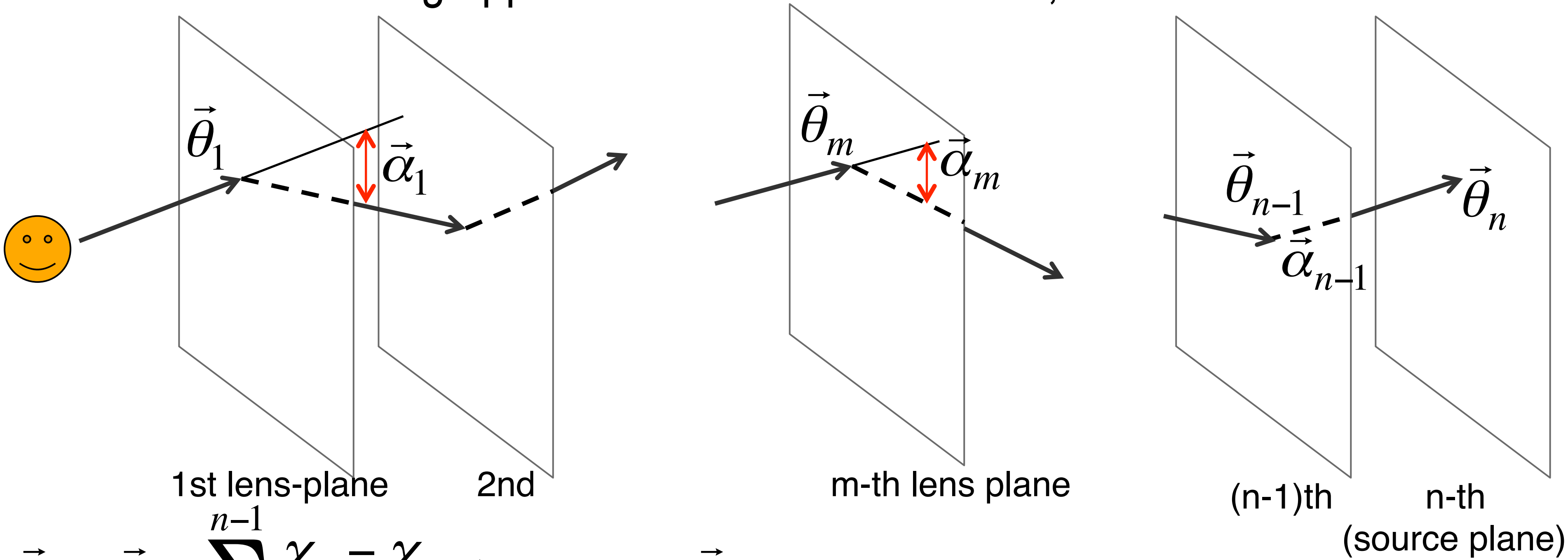
$$\boldsymbol{\beta} = \boldsymbol{\theta} - 2\int_0^{\chi_s} d\chi \frac{\chi_s - \chi}{\chi_s} \nabla_\perp \Phi(\boldsymbol{x}(\chi))$$

$$\nabla^2\Phi(\boldsymbol{x}) = 4\pi G\bar{\rho}_m a^2 \delta(\boldsymbol{x})$$

- The integration is along the *perturbed* light ray path
- The light deflection at each redshift is given by $-2\nabla_\perp\Phi(\boldsymbol{x}(\chi))$, the component perpendicular to the line-of-sight direction

Multi-lens plane algorithm

discrete lensing approximation: Jain et al. 2000; Hamana & Mellier 00

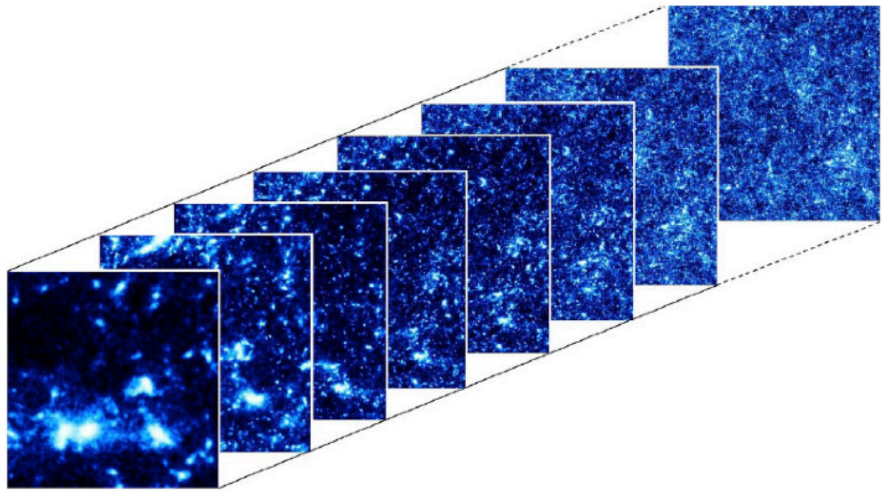


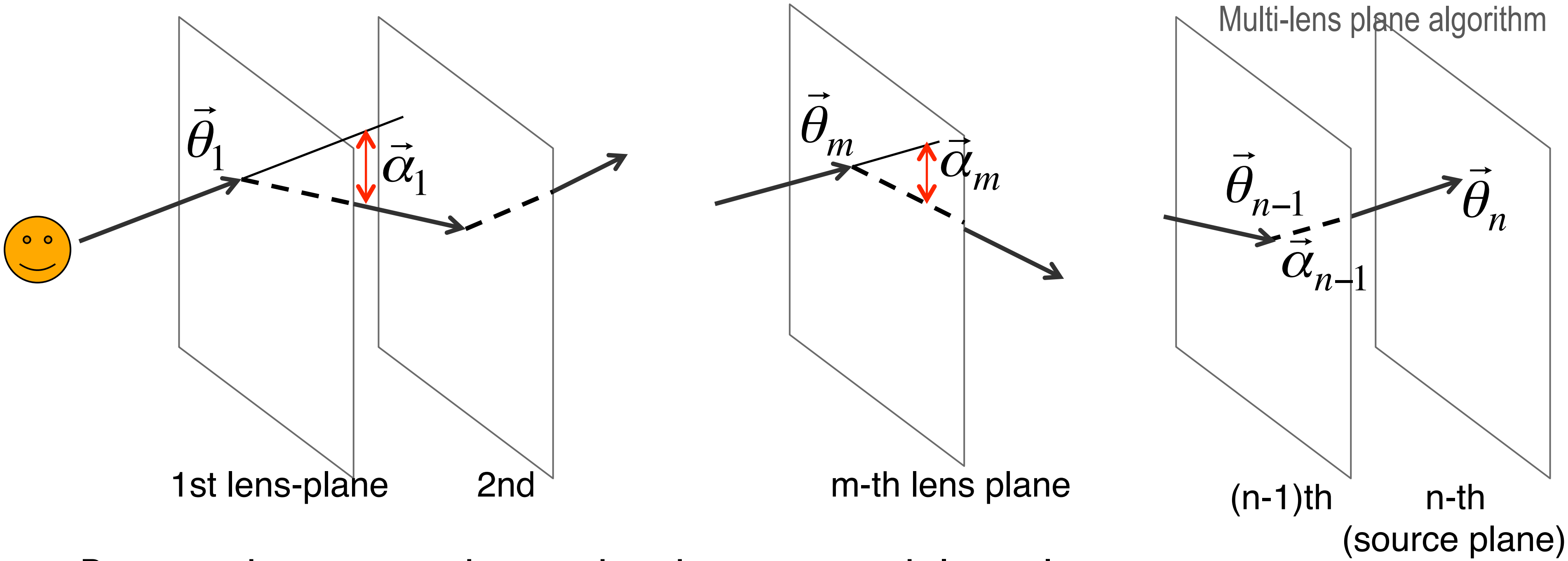
$$\vec{\theta}_n = \vec{\theta}_1 - \sum_{m=1}^{n-1} \frac{\chi_n - \chi_m}{\chi_m} \vec{\alpha}_m(\chi_m, \chi_m \vec{\theta}_m)$$

where $\vec{\alpha}_m = \nabla \psi_{(m)}$

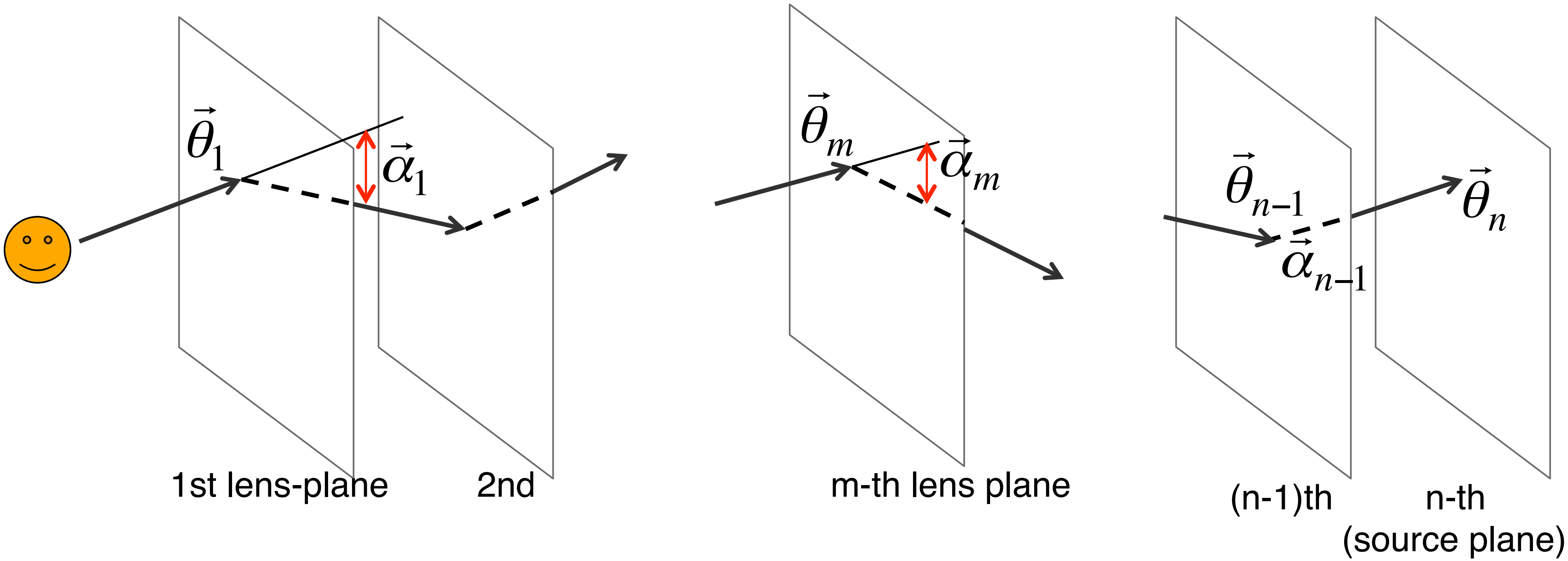
$$\nabla^2 \psi_{(m)}(\vec{\theta}) \equiv 3H_0^2 \Omega_{m0} \Sigma_{(m)}(\vec{\theta})$$

$$\Sigma_{(m)}(\vec{\theta}) \equiv \int_{\chi_m - \Delta\chi/2}^{\chi_m + \Delta\chi/2} d\chi \delta$$





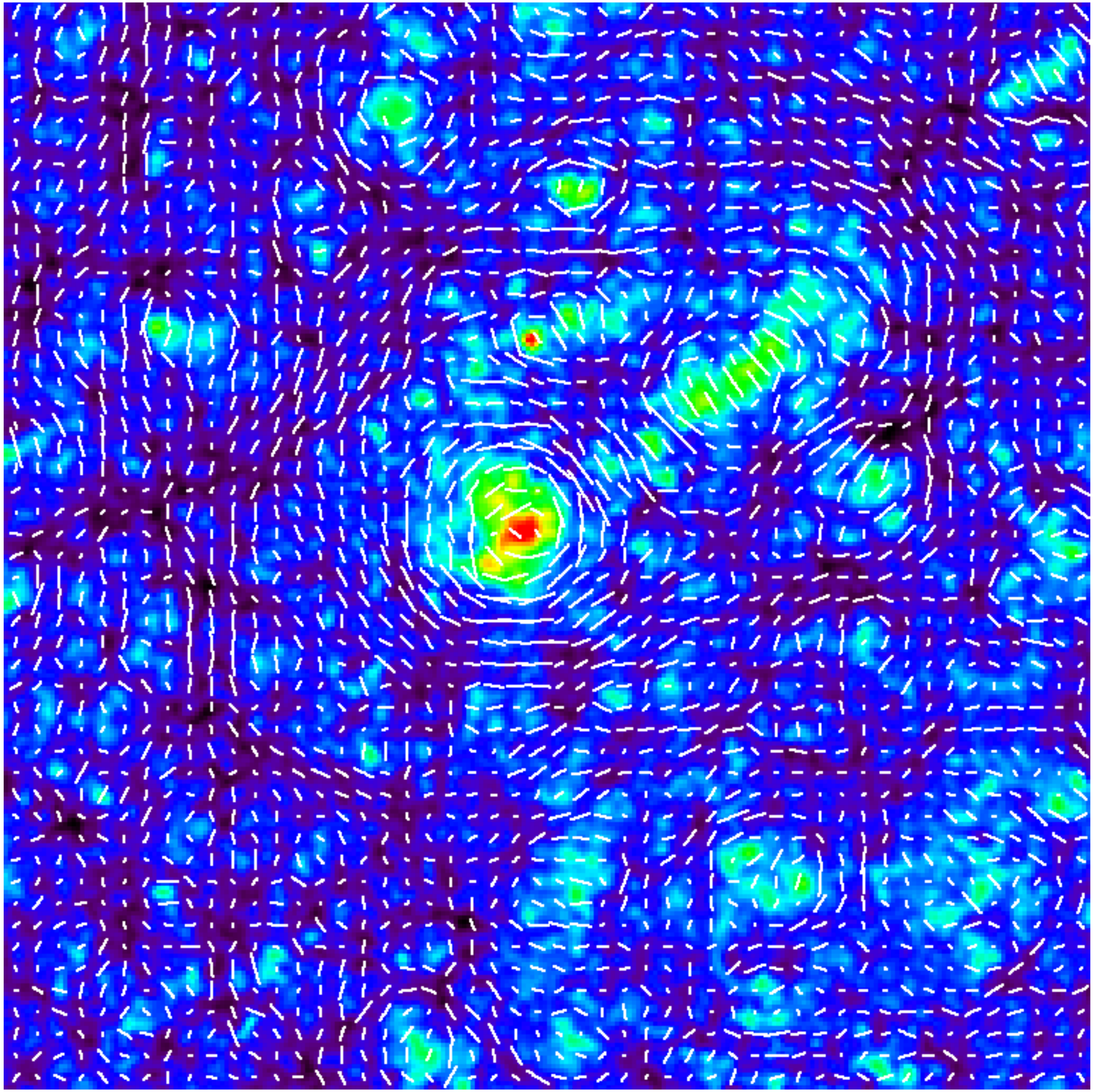
- Prepare the projected mass distribution in each lens plane
  - Mass distribution taken from a snapshot of N-body simulation at the lens redshift
  - Separation of lens planes > the correlation length of mass; usually ~50-100Mpc sep.
  - (Ideally) use different realizations of N-body simulations at different lens planes for a given cosmology; if not available, a randomization of N-body realization used (shift, rotation, projection along random direction)
- Calculate the 2D lens potential at each lens plane
- Trace the propagation of light from an observer to the source plane
- The light rays at source plane form distorted grids; use interpolation to get the regular-grid lens mapping
- Repeat these to make a lot of realizations



- Lensing distortion matrix can be obtained similarly

$$\boldsymbol{A} \equiv \frac{\partial \vec{\theta}_n}{\partial \vec{\theta}_1} = \boldsymbol{I} - \sum_{m=1}^{n-1} \frac{\chi_n - \chi_m}{\chi_m} \underline{\frac{\partial \vec{\alpha}_m}{\partial \vec{\theta}_m} \frac{\partial \vec{\theta}_m}{\partial \vec{\theta}_1}}$$

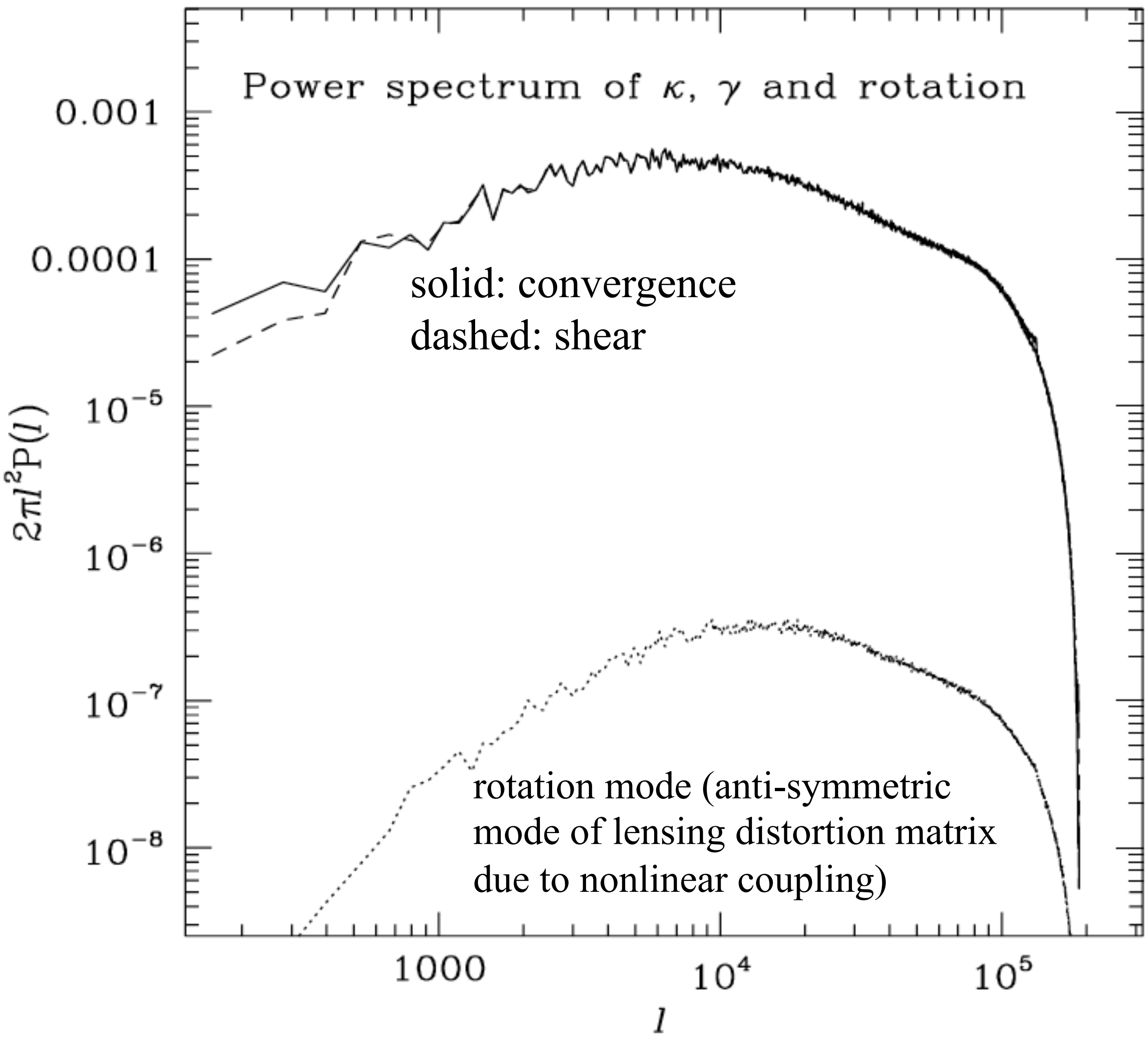
calculate for each lens plane

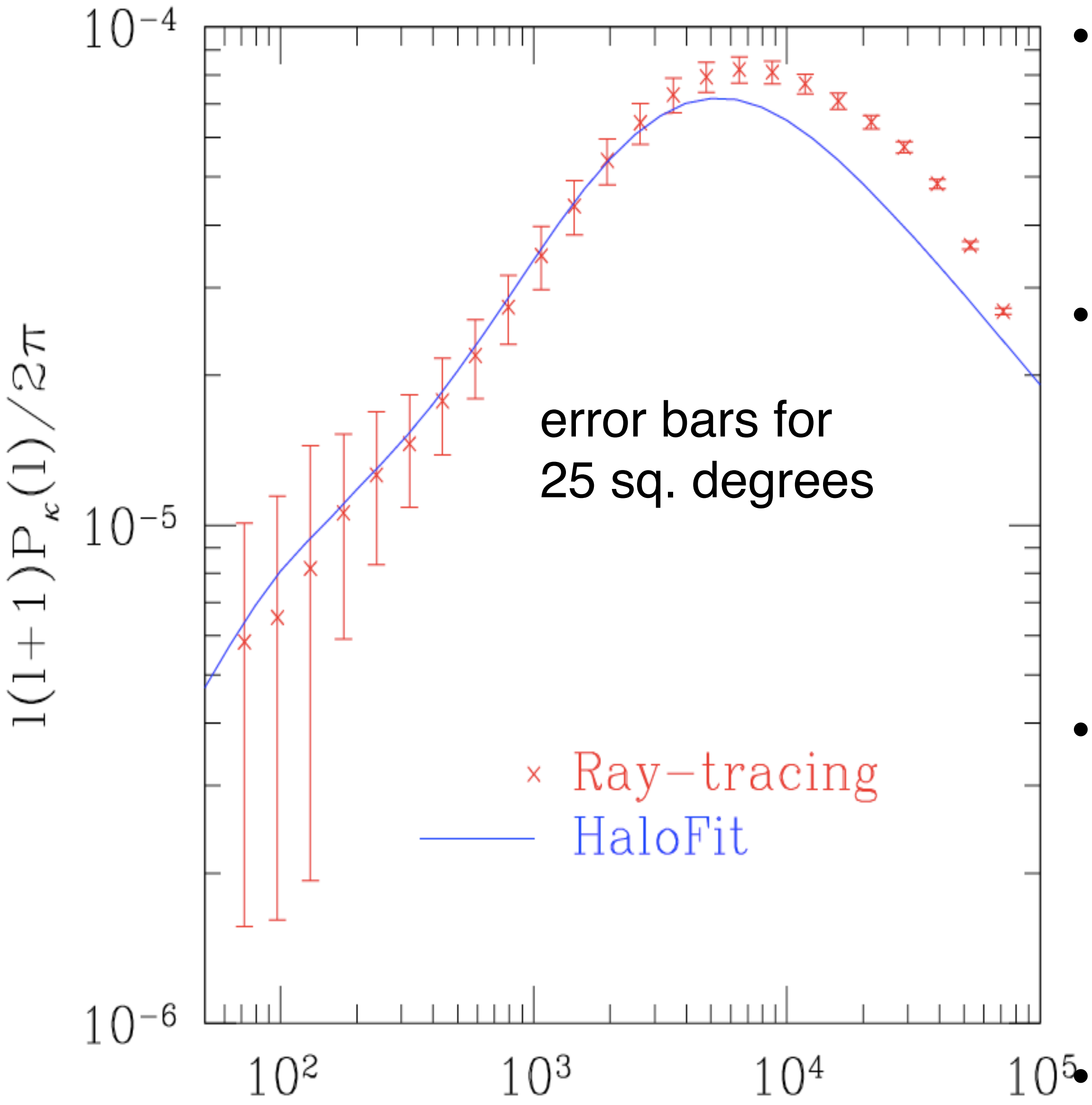


Courtesy of T. Hamana (NAOJ)

Lens power spectra

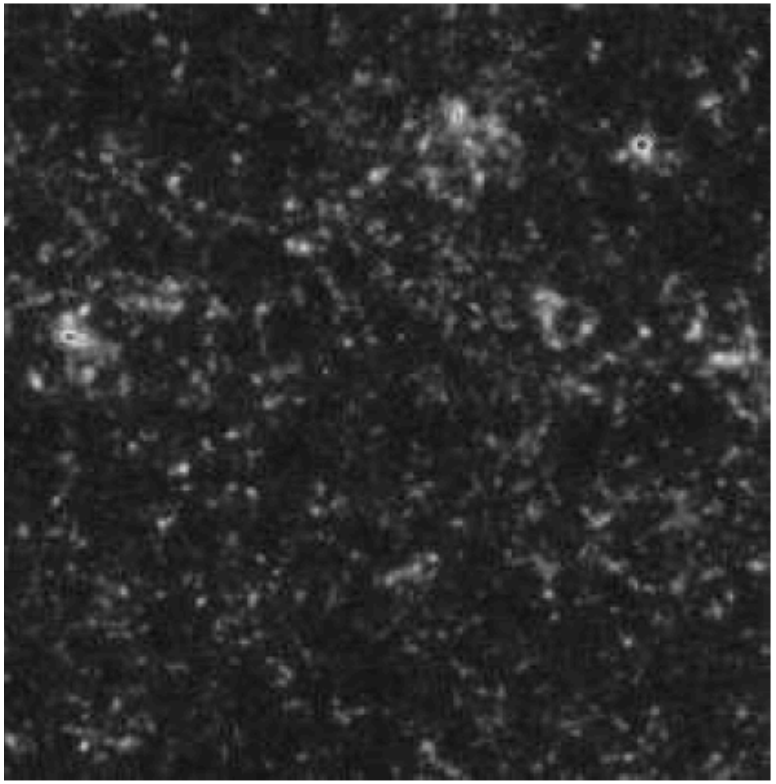
Jain et al 00



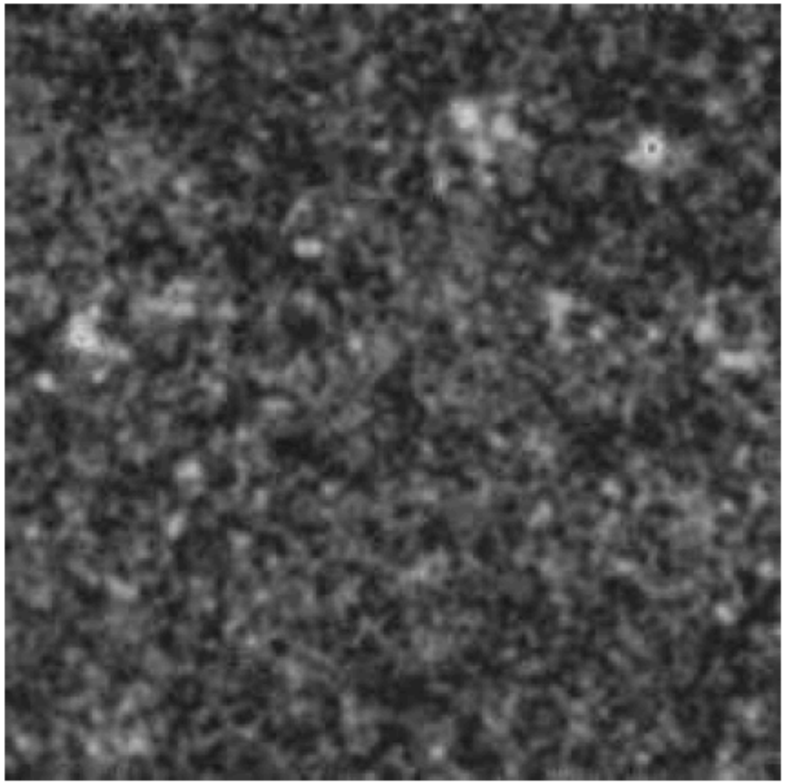


Sato et al 09

- With the advent of numerical resources, now easy to implement ray tracing simulations
- Sato et al. (09) generated 1000 realizations of ray-tracing simulations for ΛCDM model, using 1000 N-body simulation realizations
- The simulation power spectra show sizable difference from the analytical fitting formula (e.g., Halofit)
- The PS error covariance (see later)



w/o noise



wish shape noise

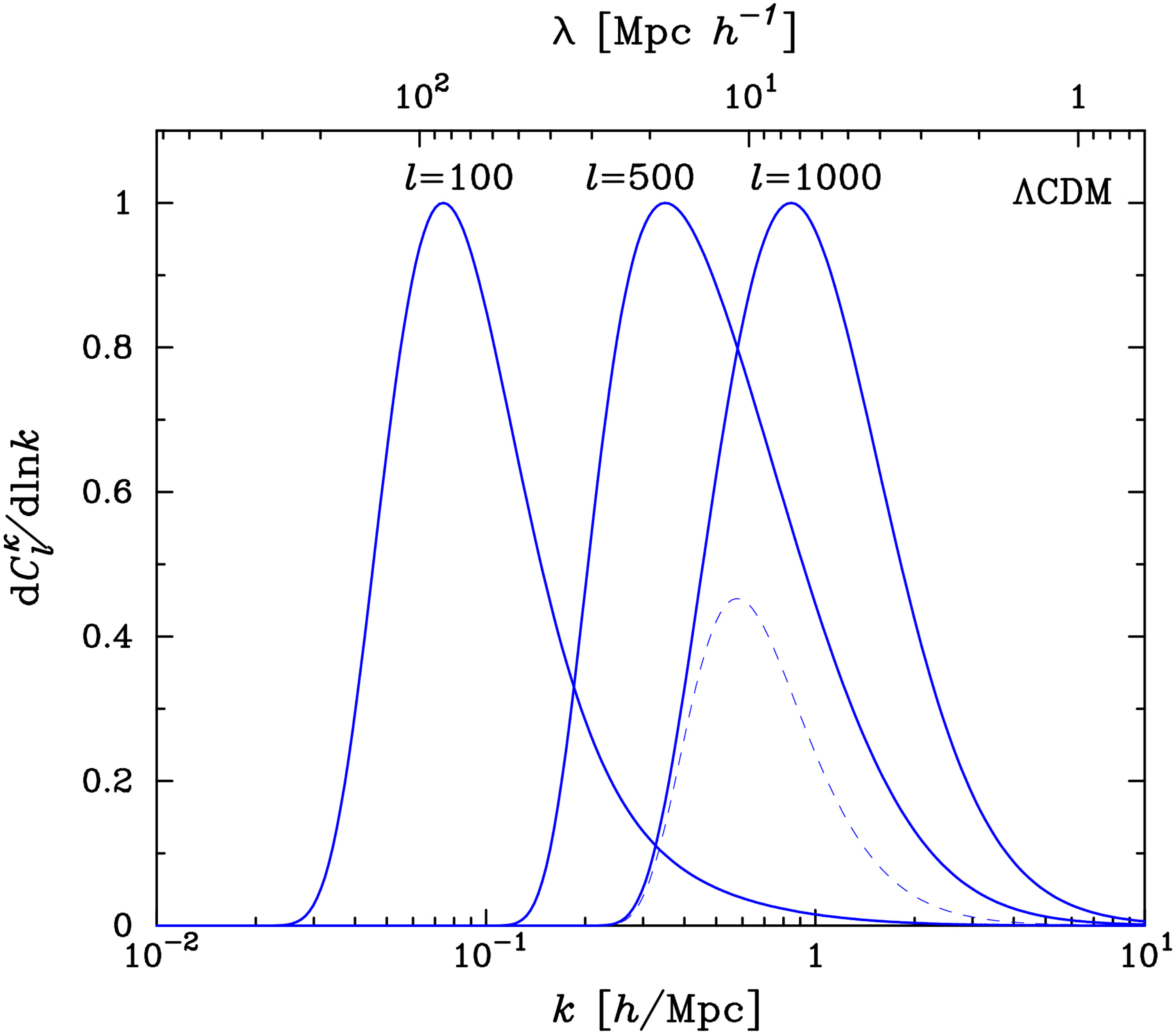
Can generate the lensing fields with shape noise contamination assuming the Gaussian noise with the power spectrum

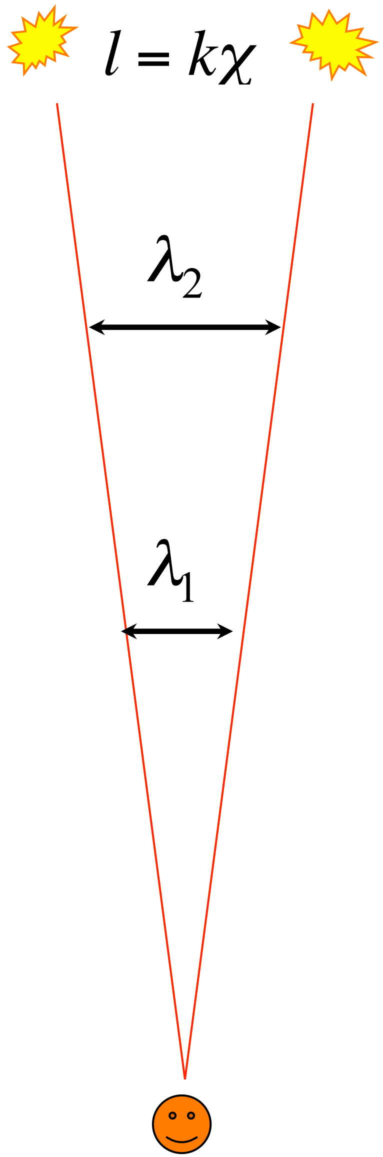
$$P_n = \frac{\sigma_e^2}{\bar{n}_g} \longrightarrow \kappa_n(\theta_{\mathrm{pix},i})$$

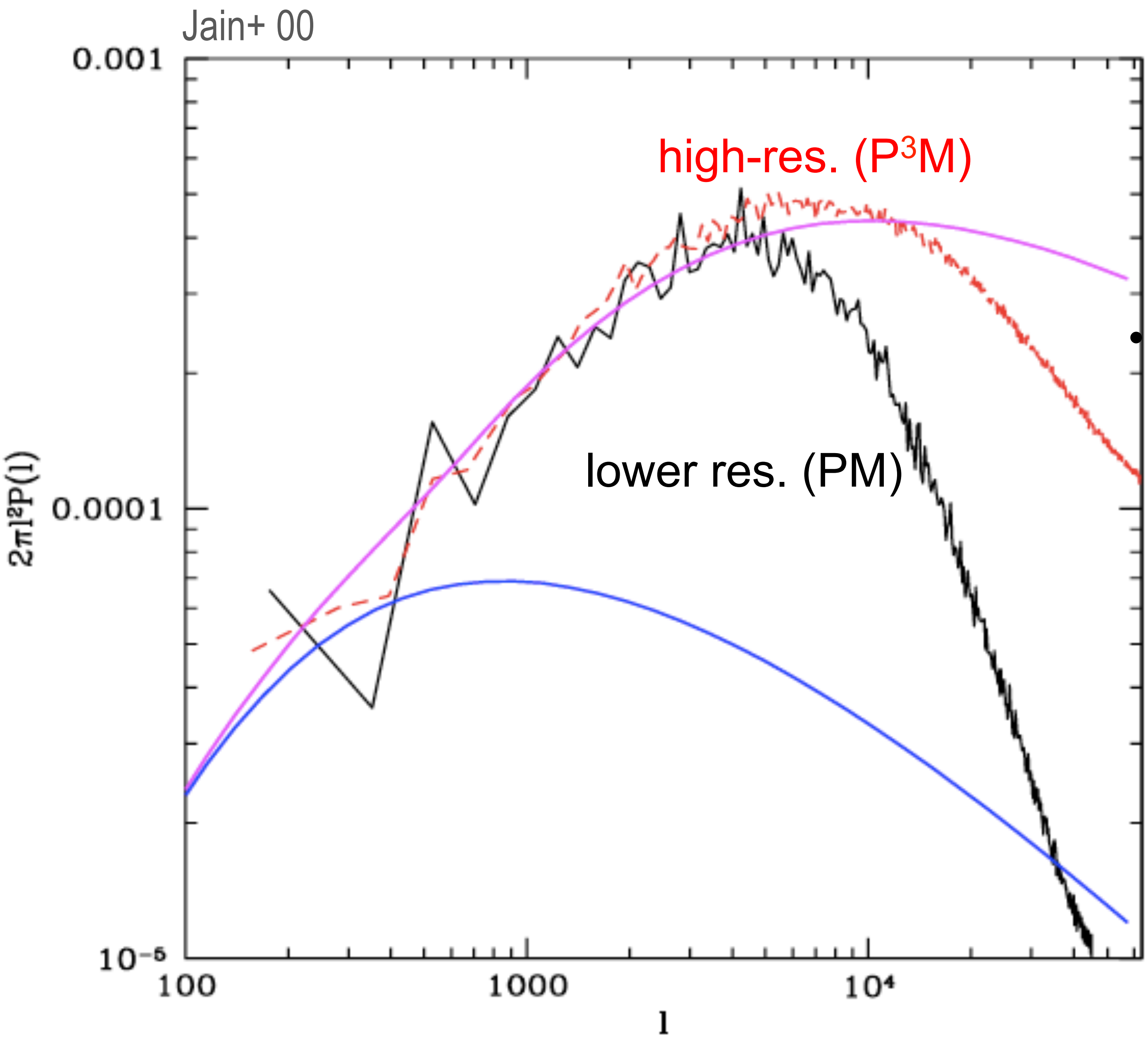
Numerical issues

- N-body simulations need to be sufficiently accurate
  - Box size, initial redshifts, resolution, …
  - The number of simulation outputs
  - Baryonic effects eventually need to be included
  - Effect of neutrinos
- Discrete lens planes
  - To build the lensing potential at each redshift, one usually uses the fast Fourier transform (FFT) algorithm; the effect of grid sizes in the FFT algorithm
  - Minimize the effect of discrete lens planes
- Other issues
  - 3D lensing vs. lensing: $\nabla_{\perp}\Phi\left(\boldsymbol{x}(\chi)\right)$
  - The effect of sky curvature (flat sky vs. full-sky calculations)

Lensing projection (2D⇔3D)

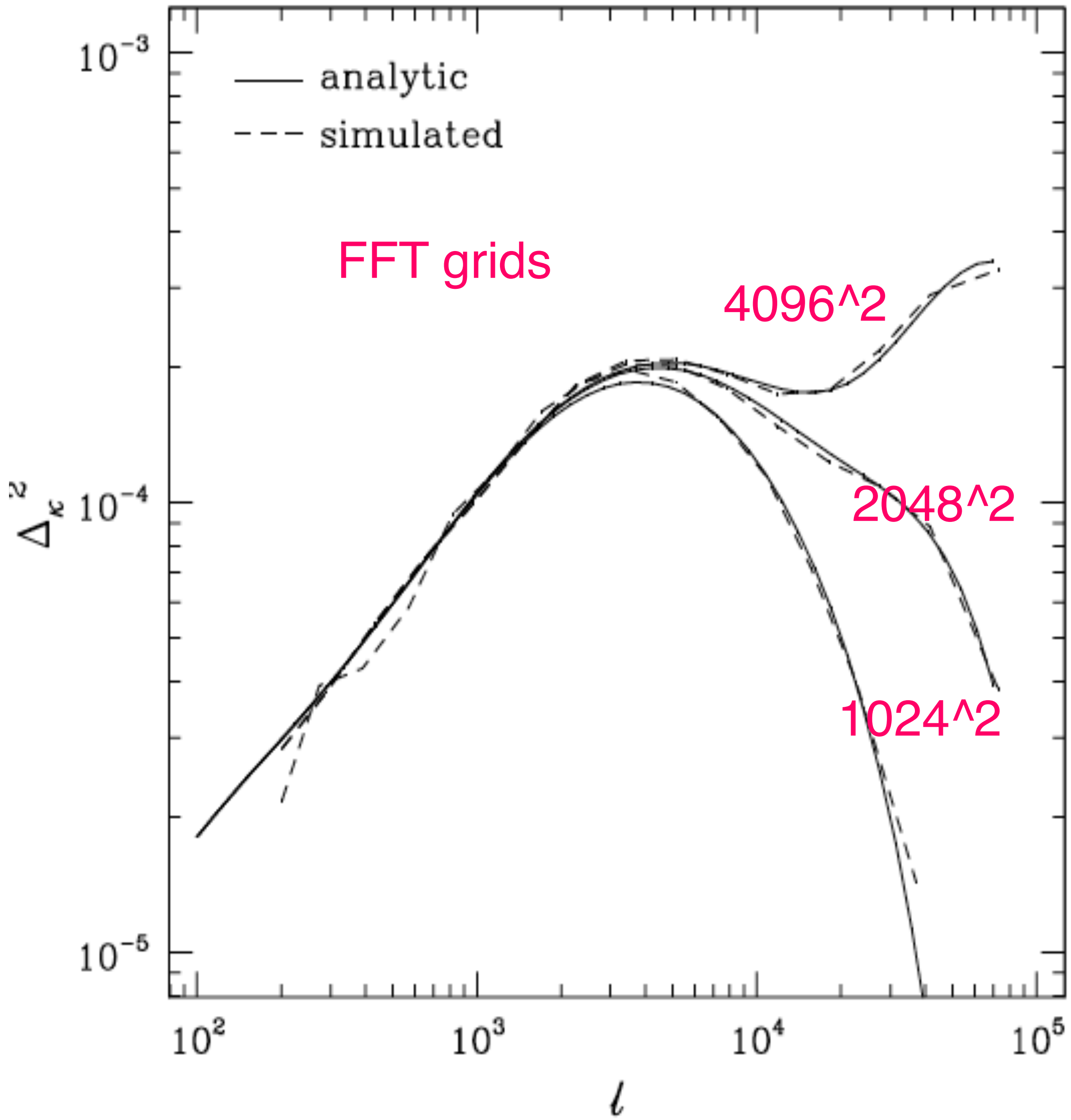






- Need to be careful the resolution of N-body simulation to have a sufficient accuracy of lensing PS on relevant angular scales

Vale & White 03



3D mass power spectrum

$$\tilde{P}_m(k) = \left(P_m(k) e^{-\sigma_n^2 k^2} + \frac{1}{\bar{n}_g} \right) e^{-\sigma_g^2 k^2}$$

$\bar{n}_g$: mean particle num. density

σ_n : chara. scale of N-body res.

σ_n : chara. scale of FFT grids

projection

2D lensing spectrum

C_l^κ

Again need proper grid size to have the sufficient accuracy on relevant angular scales

Born approximation

- The lensing PS involves the Born approximation (Gary's lecture)

$$C_l^{\kappa} = \int_0^{\chi_s} d\chi W_{GL}^2(\chi) \frac{l^4}{\chi^6} P_{\Phi}\left(k = \frac{l}{\chi}; z\right)$$

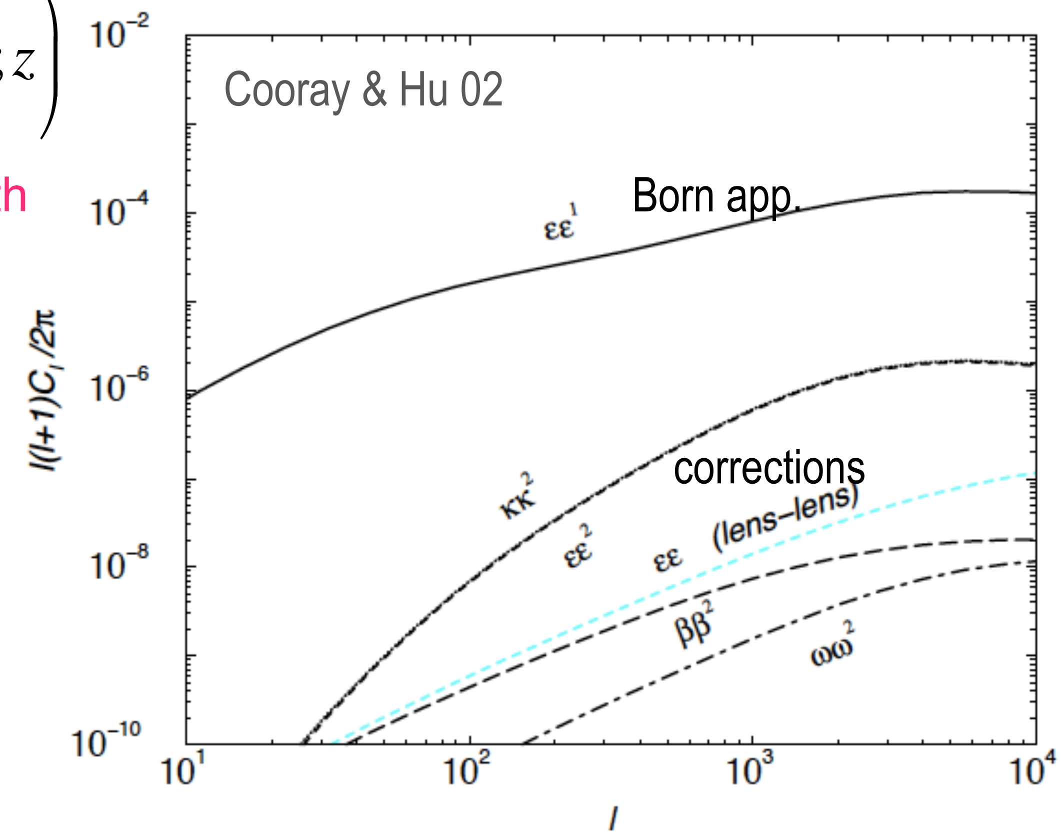
along the unperturbed path

- The higher-order and lens-lens coupling contributions

$$\Phi(\boldsymbol{\theta} + \boldsymbol{\alpha}) \approx \Phi(\boldsymbol{\theta}) + \alpha_i \Phi_{,i} + (1/2)\alpha_i \alpha_i \Phi_{,ij} + \ldots$$

$$\longrightarrow \delta C_l^{\kappa} = O(\Phi^4) + \ldots$$

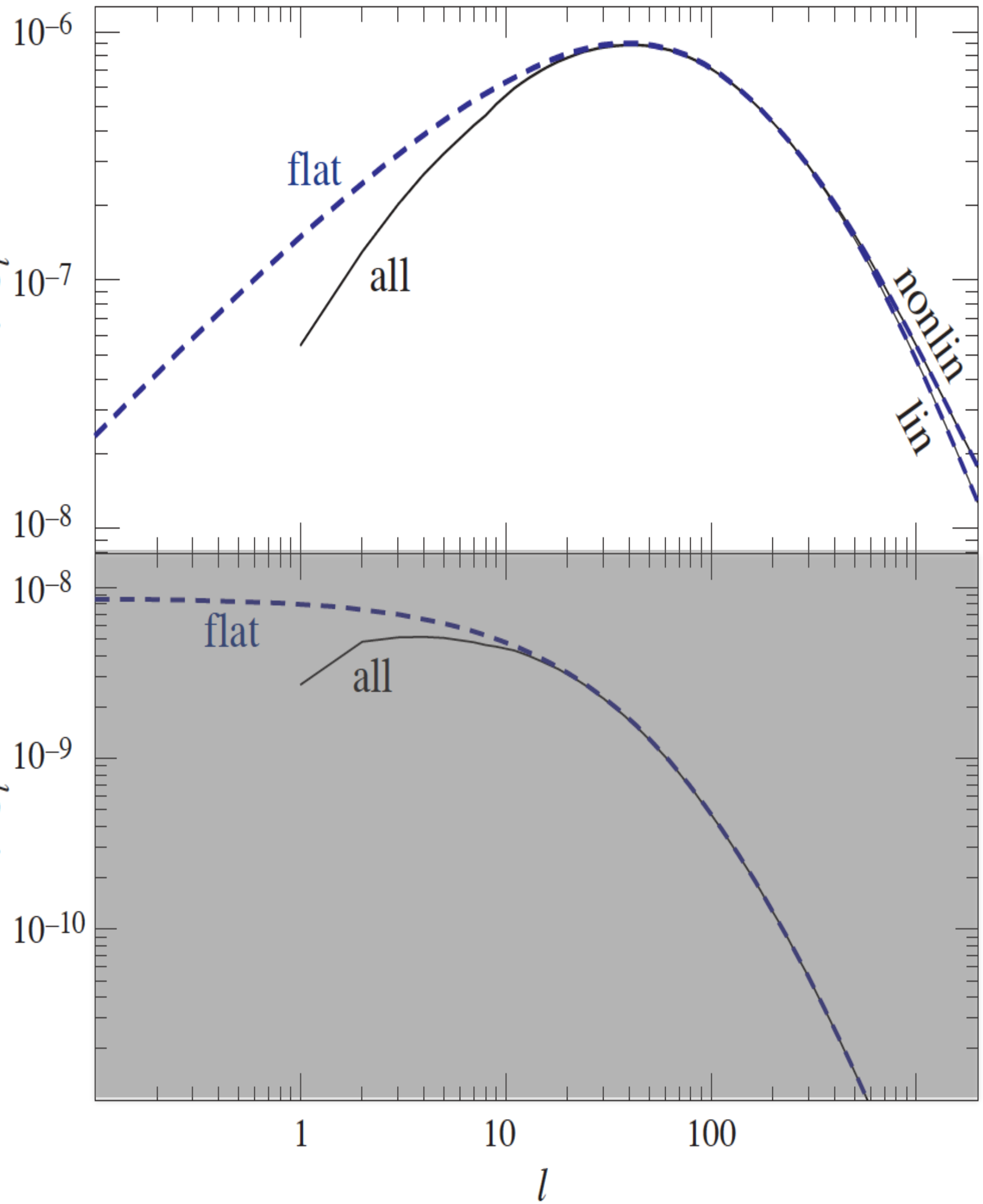
  – The correction is smaller by two orders of magnitudes



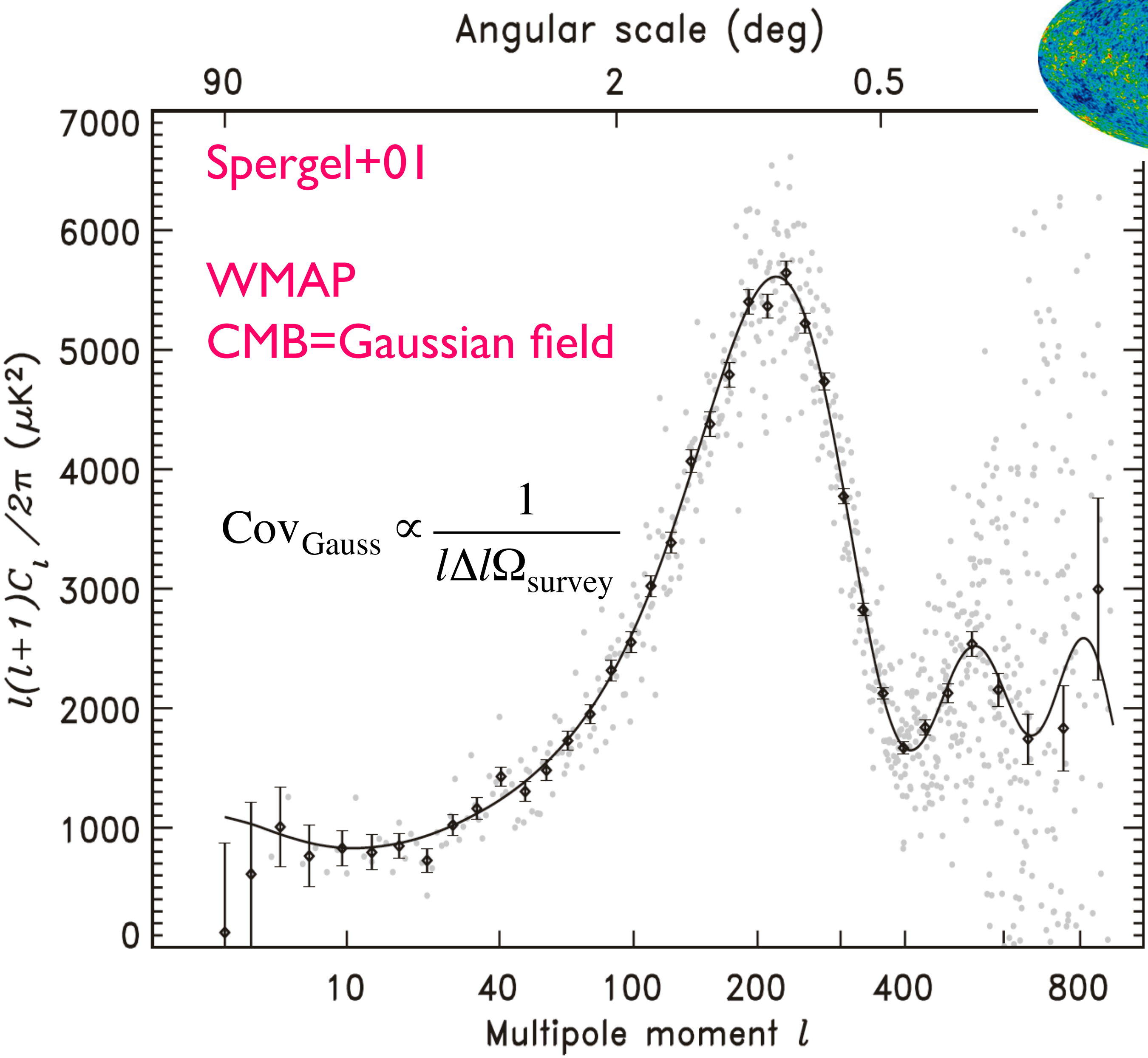
Full vs. flat-sky

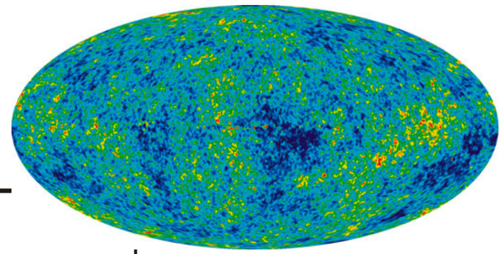
- Straightforward to extend the WL power spectrum formula to a formula including the full-sky effect (beyond the Limber's approximation)
- The full-sky effect becomes important at angular scales greater than a degree scale (ell<100 or angle>a degree scale)

Hu 00

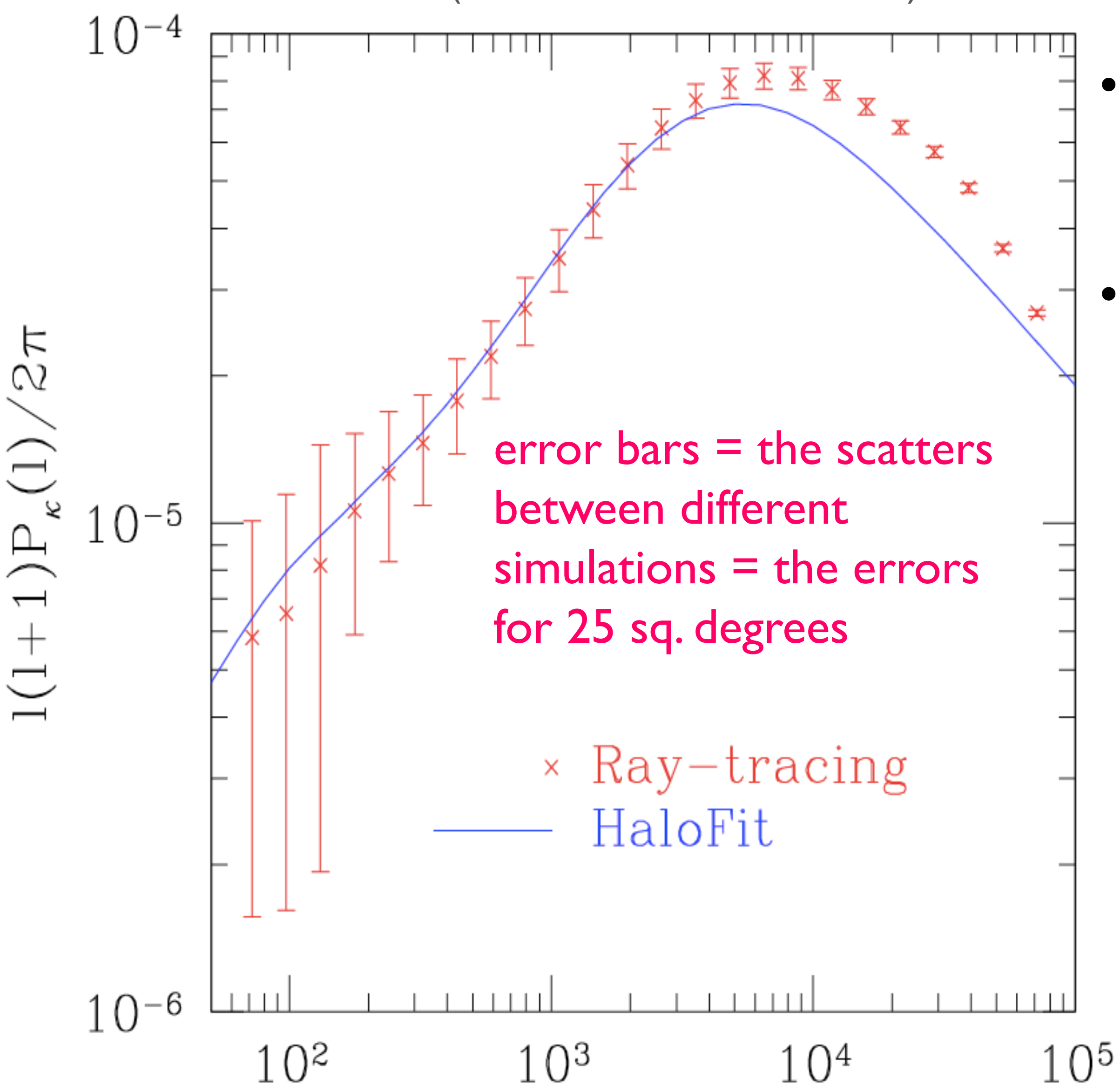


Recap: CMB covariance





Sato+ 09 (used 1000 simulations)



- Simulations can also be used to estimate the PS covariance
- WL power spectrum measurements are affected also by non-Gaussian errors

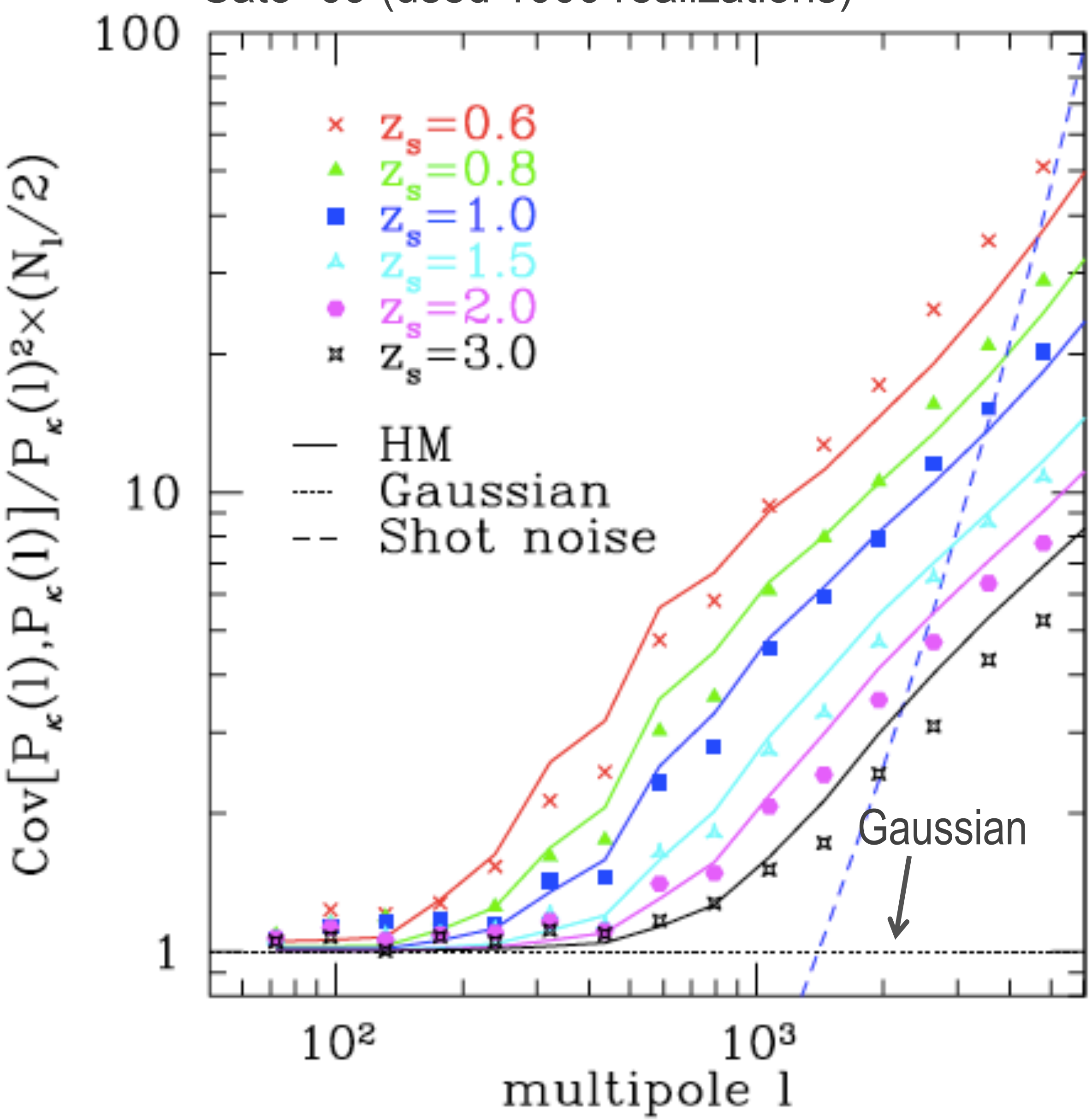
$$\mathrm{Cov}_{\mathrm{Gauss}} \propto \frac{1}{l\Delta l \Omega_{\mathrm{survey}}}$$

$$\mathrm{Cov}_{\mathrm{Non\text{-}Gauss}} \propto \frac{1}{\Omega_{\mathrm{survey}}}$$

Non-Gaussian errors are significant ...

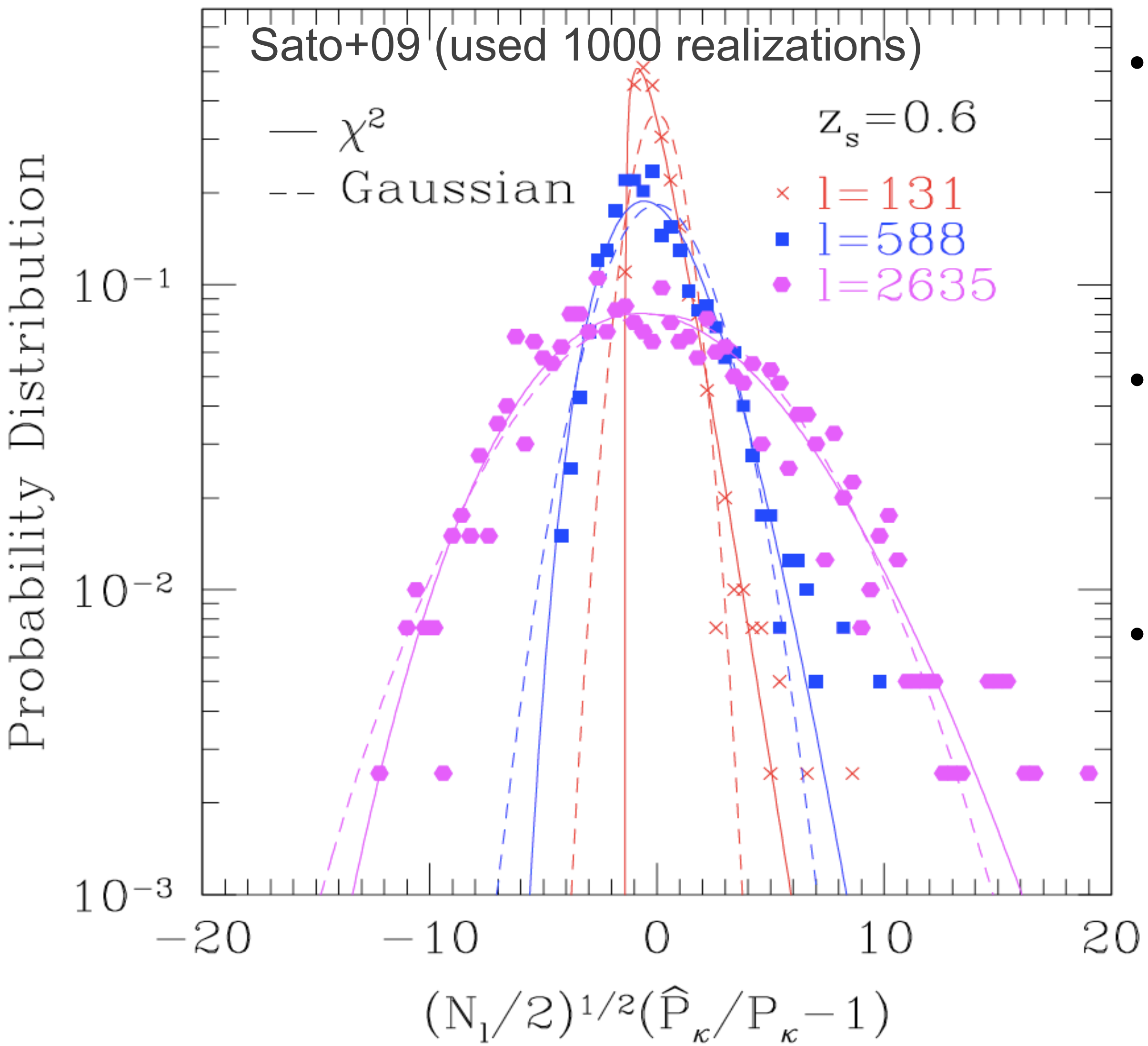
- Nonlinear large-scale structures induce significant non-Gaussian errors in the power spectrum covariance
- The non-Gaussian errors arise mainly from the halo sampling variance (not from the trispectrum)
- This effects needs to be properly taken into account for future surveys

Sato+09 (used 1000 realizations)



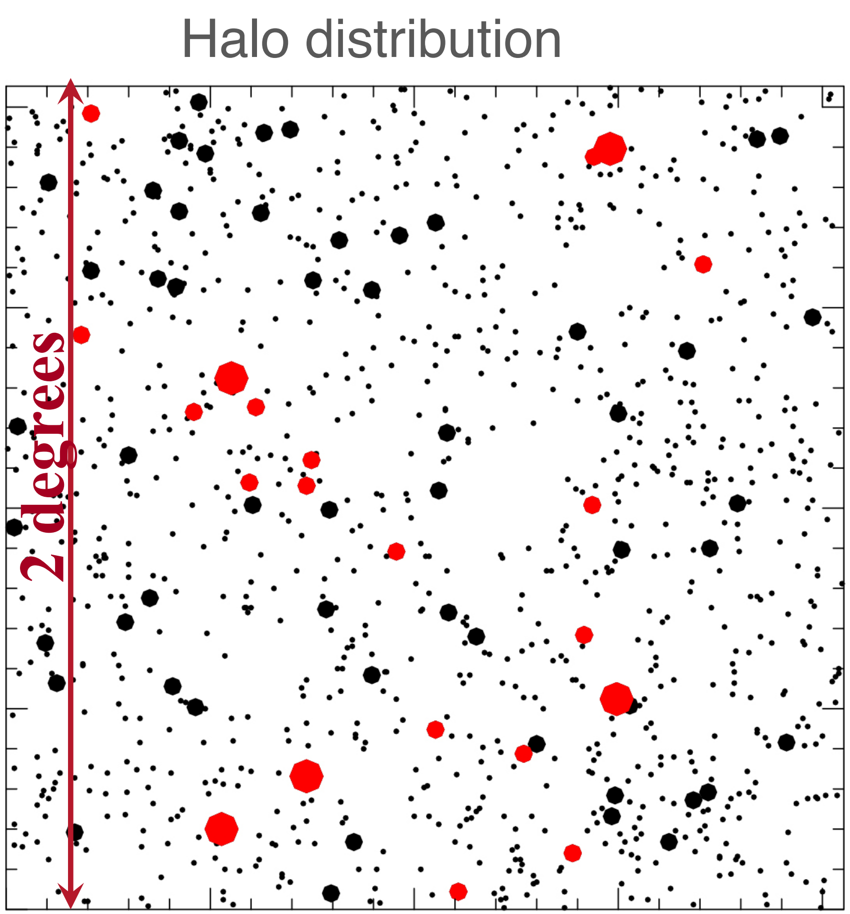
PdF of PS band powers

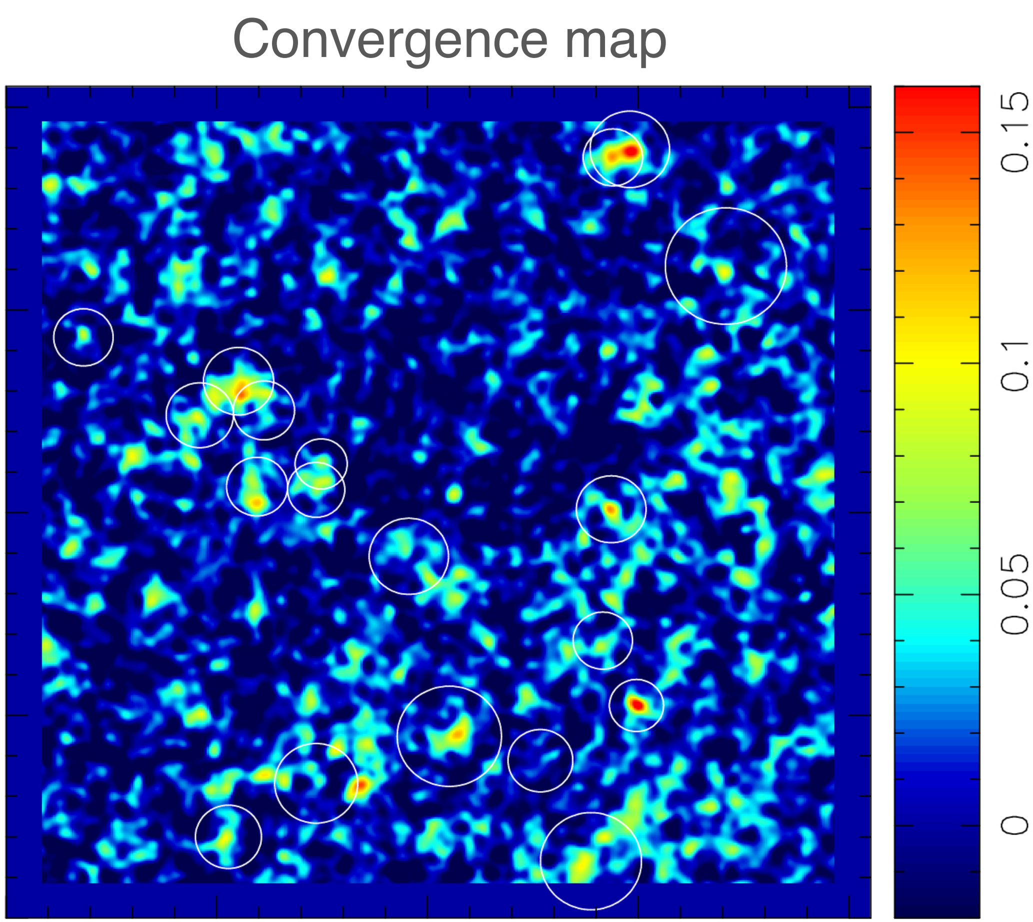
Parameter estimation: $\chi^2(\boldsymbol{p}) = \left(C_l^{\rm obs} - C_l^{\rm theory}(\boldsymbol{p})\right)\boldsymbol{C}_{ll'}^{-1}\left(C_{l'}^{\rm obs} - C_{l'}^{\rm theory}(\boldsymbol{p})\right)$



- Exactly speaking, need to know the PDF of power spectrum powers at each multipole bin, in order not to have any parameter bias
- Sato et al (09) used the 1000 realizations to study the PDF, and then found the skewness is small
- However, note that the width of the PDF around ell's (l~1000) is significantly widened by non-Gaussian errors

Halo-lensing connection





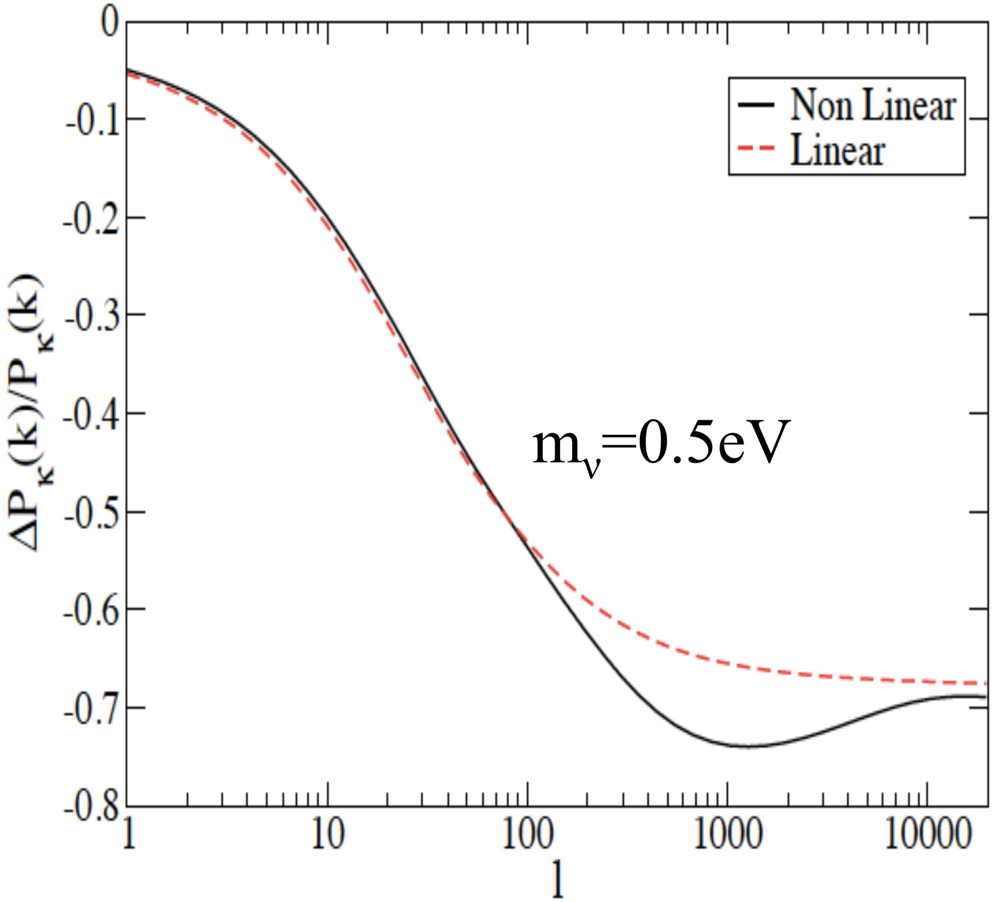
Haman+00

- Peak statistics; halo-shear correlation, …
- Mock catalogs of the observables; galaxies, SZ, X-ray, …

Things need to be done (possible subjects for students!)

- Baryonic effect
  - Should be there: baryon cooling, AGN, SN feedbacks
  - Just hard to accurately model
- Neutrino effect
  - Neutrinos are not massless; the experiments show m_nu>0.05 or 0.1eV
  - The massive neutrinos lead to suppression in the clustering of CDM
  - Again hard to simulate the nonlinear clustering (the initial attempts have just started; e.g., Viel+10)

Ichiki+09



Summary

- Simulations are just NEEDED for high-precision cosmology
  - Nonlinear clustering; analytical modeling impossible (in my opinion)
  - Study the impact of various systematics (non-Gaussian errors, intrinsic alignments, E/B-mode decomposition, lens-lens coupling, …)
  - Enable to estimate the PdFs of data vector and the error covariance matrix
  - Mock catalogs of survey you want to work on
- The current status is …
  - The required accuracy of N-body simulations is already met if we focus on the lensing fields at ell<a few 1000
  - Numerical issues needed to be carefully understood, but are straightforward to take into account
- Still need to more carefully study …
  - Baryonic effects (cooling, star formation, AGN, SN feedback…)
  - Effects of massive neutrinos