3d Seiberg Dualities with Tensor Matter

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based on work with J. Park and H. Kim

arXiv: 1110.2547

Outline

- Seiberg and Kutasov-Schwimmer-Seiberg dualities in 4d
- 3d analogs of Seiberg duality (unitary gauge group)
- 3d analogs of KSS duality (unitary gauge group)
- Nonperturbative truncation of the chiral ring
- Other gauge groups and matter representations

Seiberg duality in 4d, I

Seiberg duality (1994) is a conjectured IR duality between certain pairs of $\mathcal{N}=1$ $d=4$ gauge theories.

Electric theory: $SU(N_c)$ gauge theory with N_f fundamental flavors $(Q^a, \tilde{Q}_{\tilde{a}})$, $a, \tilde{a} = 1, \ldots, N_f$,

Magnetic theory: $SU(N_f - N_c)$ gauge theory with N_f fundamental flavors $q_a, \tilde{q}^{\tilde{a}}$, a singlet "meson" superfield $M^a_{\tilde{a}}$, and a superpotential

$$W = \tilde{q}^{\tilde{a}} q_a M^a_{\tilde{a}}.$$

Checks: symmetries, 't Hooft anomaly matching, chiral ring matching, consistent behavior under defomations and along the moduli space.
Generalizes to $SO(N_c)$ and $USp(2N_c)$ theories with fundamental matter.

Seiberg duality in 4d, II

Chiral ring matching:

$$\tilde{Q}Q \mapsto M,$$
$$Q^{N_c} \mapsto q^{N_f - N_c},$$
$$\tilde{Q}^{N_c} \mapsto \tilde{q}^{N_f - N_c}.$$

NB. The electric and magnetic theories flow to a nontrivial IR fixed point for $3N_c/2 < N_f < 3N_c$. If N_f is in the range $N_c \leq N_f \leq 3N_c/2$, the magnetic theory is IR free. If $N_f < N_c$, the magnetic theory does not make sense, while the electric theory does not have a stable vacuum.

KSS duality in 4d, I

Electric theory: $SU(N_c)$ gauge theory with N_f fundamental flavors $(Q^a, \tilde{Q}_{\tilde{a}})$, $a, \tilde{a} = 1, \ldots, N_f$, an adjoint X and a superpotential

$$W = \mathrm{Tr}\, X^{n+1}.$$

Magnetic theory: $SU(nN_f - N_c)$ gauge theory with N_f fundamental flavors $q_a, \tilde{q}^{\tilde{a}}$, an adjoint Y, n singlet "meson" superfields M_j, $j = 1, \ldots, n$,and a superpotential

$$W = \mathrm{Tr}\, Y^{n+1} + \sum_{j=1}^{n} \tilde{q} Y^{n-j} q \, M_j.$$

KSS duality in 4d, II

Chiral ring matching:

$$\tilde{Q}X^{j-1}Q \mapsto M_j, \quad \mathrm{Tr}\, X^{j-1} \mapsto \mathrm{Tr}\, Y^{j-1}, \quad j = 1, \ldots, n.$$

Baryon-like chiral operators also match.

Note the chiral ring relation $X^n = 0$ following from $\partial W/\partial X = 0$.

- KSS duality holds if the perturbation $W = \mathrm{Tr} X^{n+1}$ is relevant. The condition for this is $R_X < 2/(n+1)$, where R_X can be computed by a-maximization in the $W = 0$ theory (Intriligator, Wecht). This gives upper bound on N_f.
- There are additional relations in the electric (resp. magnetic) chiral ring coming from the characteristic equation for X (resp. Y). These relations do not match. KSS proposed that the magnetic characteristic equation appears in the electric theory as a nonperturbative effect, and vice versa.

KSS duality in 4d, III

KSS duality generalizes to other classical groups and matter representations (Intriligator, Leigh, Strassler, 1995):

- $SO(N_c)$ theory with N_f fundamentals, an adjoint X and a superpotential $W = \mathrm{Tr}\, X^{2n+2}$.
- $USp(2N_c)$ theory with $2N_f$ fundamentals, an adjoint X and a superpotential $W = \mathrm{Tr}\, X^{2n+2}$.
- $SO(N_c)$ theory with N_f fundamentals, a symmetric traceless X and a superpotential $W = \mathrm{Tr}\, X^{n+1}$.
- $USp(2N_c)$ theory with $2N_f$ fundamentals, an antisymmetric traceless X and a superpotential $W = \mathrm{Tr}\, (XJ)^{n+1}$.
- $SU(N_c)$ theory with N_f fundamentals, a symmetric or antisymmetric tensor flavor $X^{ij}, \tilde{X}_{ij}$, and a superpotential $W = \mathrm{Tr}(X\tilde{X})^{n+1}$.

KSS duality in 4d, IV

Further remarks:

- Some of these dualities can be “derived” using suitable brane constructions of $\mathcal{N} = 1$ gauge theories and brane moves which amount to continuation past infinite coupling. Not clear why and when this is allowed.
- No dual description is known for the $SU(N_c)$ theory with N_f fundamentals, an adjoint X, and $W = 0$. Naive attempt to take the limit $n \to \infty$ of the KSS duality fails because for fixed N_f, N_c and large enough n the operator $\mathrm{Tr}\, X^{n+1}$ becomes irrelevant, and the KSS duality does not apply.

Aharony dualities in 3d, I

Seiberg dualities have 3d counterparts with $\mathcal{N}=2$ $d=3$ SUSY (Aharony, 1997, Aharony-Shamir, 2011).

- $U(N_c)$ theory with N_f fundamental flavors is dual to $U(N_f-N_c)$ theory with N_f fundamental flavors, N_f^2 singlets $M^a_{\tilde{a}}$, two extra singlets v_+, v_-, and a superpotential

$$W=\tilde{q}qM+V_+v_-+V_-v_+,$$

where $V_\pm$ are monopole operators with magnetic charge $(\pm 1,0,\ldots,0)$.

There are no gauge-invariant baryon operators, mesons match as before. Monopole operators $U_\pm$ in the electric theory map to singlet fields $v_\pm$ in the magnetic theory.

Aharony dualities in 3d, II

There are similar dualities for gauge groups $O(N_c)$ and $USp(2N_c)$.

- $O(N_c) \mapsto O(N_f - N_c + 2)$
- $USp(2N_c) \mapsto USp(2N_f - 2N_c - 2)$.

There are no baryons to match, while basic monopole operators map to singlets. The chiral ring seems to be generated by mesons and basic monopole operators (Bashkirov, 2011).

3d dualities with Chern-Simons terms, I

In 3d one can turn on Chern-Simons couplings. There are 3d Seiberg-like dualities for $\mathcal{N} = 2$ $d = 3$ theories with Chern-Simons terms (Giveon, Kutasov, 2008, A.K. 2011).

Recipe: take Aharony duality, on the electric side turn on CS coupling k, on the magnetic side turn on CS coupling $-k$, replace $N_f \mapsto N + f + |k|$ and drop singlets coupled to monopole operators.

Thus $U(N_c)_k$ theory with N_f fundamental flavors is dual to $U(N_f + |k| - N_c)_{-k}$ theory with N_f flavors, a singlet meson M and $W = \tilde{q}qM$.

For $|k| \geq N_c$ it makes sense to set $N_f = 0$, and we get:

$U(N_c)_k$ $\mathcal{N} = 2$ Chern-Simons theory is dual to $U(|k| - N_c)_{-k}$ $\mathcal{N} = 2$ Chern-Simons theory.

3d dualities with Chern-Simons terms, II

Further remarks:

- $\mathcal{N} = 2\ U(N_c)_k$ CS theory is equivalent to bosonic $U(N_c)$ CS theory at CS level $k_b = k - N_c \cdot \mathrm{sign} k$ (for $|k| \geq N_c$). Thus for $N_f = 0$ Seiberg-like duality with CS terms is level-rank duality of bosonic $U(N)_k$ CS theories .
- If $N_f + |k| - N_c < 0$, the electric theory appears to break SUSY spontaneously.
- Similar remarks apply to gauge groups $O(N_c)$ and $USp(2N_c)$.

Testing 3d dualities

- Chiral ring matching (but matching monopole operators is not straightforward, see Bashkirov 2011).
- Parity anomaly matching
- S^3 partition function
- Superconformal index (twisted partition function on $S^2 \times S^1$).

The lasts two tests are very powerful (one compares functions of $r+1$ variables, where r is the rank of the global symmetry group). Sometimes one can prove equality of S^3 partition functions analytically (A.K., Willett, Yaakov, 2010, Willett, Yaakov, 2011).

3d analogs of KSS duality, I

KSS duality also appears to have 3d analogs (Niarchos, 2008, A.K., Kim and Park, 2011). There are such analogs both with and without Chern-Simons couplings. I will discuss the case $k \neq 0$.

Electric theory: $U(N_c)_k$ theory with N_f fundamental flavors, an adjoint X, and $W = \mathrm{Tr}\, X^{n+1}$.

Magnetic theory: $U(n(N_f + |k|) - N_c)_{-k}$ theory with N_f fundamental flavors, an adjoint Y and

$$W = \mathrm{Tr}\, Y^{n+1} + \sum_{j=1}^{n} \tilde{q} Y^{n-j} q\, M_j .$$

3d analogs of KSS duality, II

This duality is supposed to hold for N_f and k such that $\mathrm{Tr}\, X^{n+1}$ is a relevant perturbation. The condition for this is $R_X < 2/(n+1)$.

R_X as a function of N_f, N_c, k can be obtained in principle using F-maximization (Jafferis, 2010). That is, one computes $|Z(S^3)|$ in the $W = 0$ theory as a function of R_X and R_Q and minimizes it with respect to these parameters.

This is easy to do numerically only for $N_f = 0$, when one does not have R_Q at all.

3d analogs of KSS duality, III

We tested the 3d KSS duality by computing the S^3 partition function and superconformal index of dual theories.

For example, consider the "self-dual" case: electric $U(2)_1$ theory with $N_f = 1$, and $W = \mathrm{Tr}\, X^3$. For both electric and magnetic theory we got (here $r = R_Q$):

$r - 1/3$	-0.3	-0.2	-0.1	0
$\log\lvert Z\rvert$	-0.423782	-1.66927	-1.94454	-1.91804

$r - 1/3$	0.1	0.2	0.3
$\log\lvert Z\rvert$	-1.73155	-1.45191	-1.12236

Index $I(x) = \mathrm{Tr}\,(-1)^F x^{E+j_3} \exp(-\beta(E - R - j_3))$:

$$I(x) = 1+x^{2/3}+x^{4/3}+x^{2r}(1+2x^{2/3}+x^{4/3})+x^{-2r}(-x^{8/3}-x^{10/3})+x^{4r}+\ldots$$

3d analogs of KSS duality, IV

- Setting $N_f = 0$, we already get an interesting statement: $U(N_c)_k$ theory with an adjoint and $W = \mathrm{Tr}\, X^{n+1}$ is dual to $U(n|k| - N_c)_{-k}$ theory with an adjoint and $W = \mathrm{Tr}\, Y^{n+1}$.
- If in addition $N_c > n|k|$, the magnetic rank is negative. We interpret this as a sign that the electric theory breaks SUSY.
- If $N_c = n|k|$, the magnetic theory is trivial, so the electric theory must flow to a trivial fixed point.

How is all this compatible with the chiral rings of electric and magnetic theories? They seem to be spanned by $\mathrm{Tr}\, X^j$ and $\mathrm{Tr}\, Y^j$, but the range over which j runs is not the same.

This is again the issue of nonperturbative relations in the chiral ring.

Chiral ring truncation

Some insight is gained by considering the superconformal index.

Take $U(2)_1$ with an adjoint X and $W = \mathrm{Tr}\, X^3$. The magnetic gauge group is $U(0)$, so the theory must be trivial.

Classically, the electric chiral ring has a single generator $v = \mathrm{Tr}\, X$ and a single relation $v^3 = 0$. The magnetic chiral ring is trivial.

The indices agree ($I = 1$), because the contribution of v ($\sim x^{2/3}$) is canceled by a monopole operator with magnetic charge $(1, -1)$.

Thus at strong coupling $\mathrm{Tr}\, X$ is either not closed or becomes exact, thanks to the presence of a monopole operator.

Duality without the superpotential, I

For $k = 1$ and $N_f = 0$ we can do better: we can find the dual description of the theory with $W = 0$, and explain the chiral ring truncation in terms of this dual description.

Consider $U(N_c)_1$ theory with an adjoint X, but now with $W = 0$.

Chiral ring is freely generated by $u_i = \mathrm{Tr} X^i$, $i = 1, \ldots, N_c$. The VEVs of u_i parameterize the moduli space.

Along the moduli space the gauge group is Higgsed to $(U(1)_1)^{N_c}$, which is a trivial TQFT.

Conjecture: this theory is dual to a free theory of N_c chiral superfields $u_1, \ldots, u_{N_c}$.

Duality without the superpotential, II

We tested this by computing the S^3 partition function (as a function of $r = R_X$) and the index, and found perfect agreement.

For example, for $N_c = 3$ we got for both theories:

r	0.2	0.3	0.4	0.5	0.6
$\log \vert Z \vert$	-0.613634	-0.635679	-0.318126	0.000000	-0.163086

The partition function is hard to compute for higher N_c, the index is easier.

Relation with Jafferis-Yin duality

For $N_c = 2$ this duality is almost the same as the "Duality Appetizer" (Jafferis, Yin, 2010). They proposed that $SU(2)_1$ theory with an adjoint and $W = 0$ is dual to a single free chiral superfield u_2.

The S^3 partition functions differ by a factor $1/\sqrt{2}$. They proposed that this factor can be attributed to a decoupled $U(1)_2$ CS theory on the “magnetic" side. This agrees with the low-energy behavior along the moduli space (Higgsing $SU(2)_1$ by an adjoint VEV gives $U(1)_2$ CS theory).

Alternatively, we can move this factor to the electric side and interpret it as coming from $U(1)_{1/2}$ CS theory. This makes sense because $U(1)_{1/2} \times SU(2)_1 = U(2)_1$.

For higher N_c we get a generalization of the "Duality Appetizer".

Chiral ring truncation revisited, I

Now we can see why for $W = \mathrm{Tr}\, X^{n+1}$ additional relations in the chiral ring appear on the quantum level (at least for $N_f = 0$ and $k = 1$).

Consider again $U(2)_1$ with an adjoint X and $W = 0$. The chiral ring is freely generated by $v = \mathrm{Tr}\, X$ and $u = \mathrm{Tr}\, X^2$.

Example 1.
$W = \mathrm{Tr}\, X^4$. The magnetic gauge group is $U(1)_{-1}$, the magnetic chiral ring is generated by $v = Y$ with a relation $Y^3 = 0$.

In the electric theory the classical chiral ring has two generators $v = \mathrm{Tr}\, X$ and $u = \mathrm{Tr}\, X^2$ and relations $u^2 = uv^2, v^3 = 3uv$.

Now note that $\mathrm{Tr}\, X^4 = u^2/2 + uv^2 - v^4/2$, so if we treat u and v as free, we get an F-term relation $u = -v^2$. Then classical relations become $v^4 = 0$ and $v^3 = 0$.

Chiral ring truncation revisited, II

Example 2.
$W = \mathrm{Tr}\, X^3$. The magnetic theory is trivial. In the electric theory the classical chiral ring has a single generator v with a relation $v^3 = 0$ (we have $u = 0$ thanks to the superpotential).

Now note that $\mathrm{Tr}\, X^3 = 3uv/2 - v^3/2$, so if u, v are regarded as free, we get F-term relations $u = v^2, v = 0$. So the quantum chiral ring is trivial, as predicted by duality.

Generalizations I

The 3d KSS duality with Chern-Simons terms has analogs for other gauge groups and matter representations.

- $O(N_c)_k$ theory with N_f fundamentals, an adjoint X and a superpotential $W = \mathrm{Tr}\, X^{2n+2}$.
- $USp(2N_c)_k$ theory with $2N_f$ fundamentals, an adjoint X and a superpotential $W = \mathrm{Tr}\, X^{2n+2}$.
- $O(N_c)_k$ theory with N_f fundamentals, a symmetric traceless X and a superpotential $W = \mathrm{Tr}\, X^{n+1}$.
- $USp(2N_c)_k$ theory with $2N_f$ fundamentals, an antisymmetric traceless X and a superpotential $W = \mathrm{Tr}\,(XJ)^{n+1}$.
- $U(N_c)_k$ theory with N_f fundamentals, a symmetric or antisymmetric tensor flavor $X^{ij}, \tilde{X}_{ij}$, and a superpotential $W = \mathrm{Tr}(X\tilde{X})^{n+1}$.

Generalizations II

All these dualities are very similar to 4d dualities found by Intriligator, Leigh and Strassler. We tested them by computing $Z(S^3)$ and the index for many dual pairs.

Is there is a dual description of the electric theories with the superpotential turned off, similar to the generalized Jafferis-Yin duality for $U(N_c)_1$ theories with an adjoint?

We found two more classes of theories for which this works.

- $SO(2N_c+2)_1$ theory with an adjoint and $W=0$ is dual to the theory of N_c+1 free fields $\sigma_{2j}, j=1,\ldots,N_c$ and p (dual to $\mathrm{Tr}X^{2j}$ and $\mathrm{Pf}\,X$)
- $USp(2N_c)_2$ theory with an antisymmetric tensor X and $W=0$ is dual to the theory of N_c free fields $\sigma_j, j=1,\ldots,N_c$ (dual to $\mathrm{Tr}\,(XJ)^j$).

Generalizations III

These dualities are plausible because along the moduli space both theories reduce to trivial TQFTs $(U(1)_1)^{N_c+1}$ or $(SU(2)_2)^{N_c}$ plus moduli.

We tested these dualities for several low values of N_c by computing $Z(S^3)$ and the index.

For other gauge groups and matter representations this does not work. For example, $USp(2N_c)_k$ theory with an adjoint is not dual to a free theory, for any k, because the TQFT along the moduli space $(U(1)_{2k})^{N_c}$ is not trivial for all $k \in \mathbb{Z}$.