Supercurrents

Nathan Seiberg

IAS

2011

Zohar Komargodski and NS arXiv:0904.1159, arXiv:1002.2228
Tom Banks and NS arXiv:1011.5120
Thomas T. Dumitrescu and NS arXiv:1106.0031

Summary

- The supersymmetry algebra can have brane charges.
- Depending on the brane charges, there are different supercurrent multiplets.
- The nature of the multiplet is determined in the UV but is valid also in the IR. This leads to exact results about the renormalization group flow.
- Different supermultiplets are associated with different off-shell supergravities. (They might be equivalent on shell.)
- Understanding the multiplets leads to constraints on supergravity and string constructions.

The $4d$ $\mathcal{N}=1$ SUSY Algebra (imprecise)

[Ferrara, Porrati; Gorsky, Shifman]:

$$\{\overline{Q}_{\dot\alpha}, Q_\alpha\} = 2\sigma^\mu_{\alpha\dot\alpha}\left(P_\mu + Z_\mu\right) \ ,$$
$$\{Q_\alpha, Q_\beta\} = \sigma^{\mu\nu}_{\alpha\beta} Z_{\mu\nu} \ .$$

- Z_μ is a string charge.
- $Z_{[\mu\nu]}$ is a complex domain wall charge.
- They are infinite – proportional to the volume.
- They are not central.
- They control the tension of BPS branes.
- Algebraically P_μ and Z_μ seem identical. But they are distinct...

The $4d$ $\mathcal{N} = 1$ SUSY Current Algebra

$$\{\overline{Q}_{\dot{\alpha}}, S_{\alpha\mu}\} = 2\sigma^{\nu}_{\alpha\dot{\alpha}} (T_{\nu\mu} + C_{\nu\mu}) + \cdots \ ,$$
$$\{Q_{\beta}, S_{\alpha\mu}\} = \sigma^{\nu\rho}_{\alpha\beta} C_{\nu\rho\mu} \ .$$

- $T_{\mu\nu}$ and $S_{\alpha\mu}$ are the energy momentum tensor and the supersymmetry current.
- $C_{[\mu\nu]}$, $C_{[\mu\nu\rho]}$ are conserved currents associated with strings and domain-walls. Their corresponding charges are Z_{μ}, $Z_{[\mu\nu]}$ above.
- Note that $T_{\{\mu\nu\}}$ and $C_{[\mu\nu]}$ are distinct.

Properties of the Supercurrent Multiplet

- $T_{\mu\nu}$ is conserved and symmetric. It is subject to improvement (actually more general)

$$T_{\mu\nu} \to T_{\mu\nu} + \left(\partial_\mu \partial_\nu - \eta_{\mu\nu}\partial^2\right) u \ .$$

- $S_{\alpha\mu}$ is conserved. It is subject to improvement

$$S_{\alpha\mu} \to S_{\alpha\mu} + \left(\sigma_{\mu\nu}\right)_\alpha{}^\beta \partial^\nu \eta_\beta \ .$$

- We impose that $T_{\mu\nu}$ is the highest spin operator in the multiplet.
- We consider only well-defined (gauge invariant) local operators.

The S-Multiplet

The most general supercurrent satisfying our requirements is the S-multiplet $\mathcal{S}_{\alpha\dot{\alpha}}$ (real)

$$\overline{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = \chi_\alpha + \mathcal{Y}_\alpha \ ,$$
$$\overline{D}_{\dot{\alpha}}\chi_\alpha = 0 \ , \qquad D^\alpha\chi_\alpha = \overline{D}_{\dot{\alpha}}\overline{\chi}^{\dot{\alpha}} \ ,$$
$$D_\alpha\mathcal{Y}_\alpha + D_\beta\mathcal{Y}_\alpha = 0 \ , \qquad \overline{D}^2\mathcal{Y}_\alpha = 0 \ .$$

Equivalently,

$$\overline{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = \overline{D}^2 D_\alpha V + D_\alpha X,$$
$$\overline{D}_{\dot{\alpha}}X = 0 \ , \qquad V = V^\dagger$$

but V and X do not have to be well defined.

Components the S-Multiplet

$$\overline{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = \chi_\alpha + \mathcal{Y}_\alpha$$

$$\mathcal{S}_\mu = j_\mu + \theta^\alpha S_{\alpha\mu} + \overline{\theta}_{\dot{\alpha}}\overline{S}^{\dot{\alpha}}_\mu + (\theta\sigma^\nu\overline{\theta})T_{\mu\nu} + \ldots$$

It includes 16+16 operators:

- Energy momentum tensor $T_{\mu\nu}$ (10 - 4 = 6)
- String current $\epsilon_{\mu\nu\rho\sigma}\mathcal{F}^{\rho\sigma}$ in $\chi_\alpha = \overline{D}^2 D_\alpha V$ (3)
- A complex domain wall current $\epsilon_{\mu\nu\rho\sigma}\partial^\sigma x$ in $\mathcal{Y}_\alpha = D_\alpha X$ (2)
- A non-conserved R-current j_μ (4)
- A real scalar (1)
- 16 fermionic operators

Improvements of the S-Multiplet

The S-multiplet is not unique. The defining equation

$$\overline{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = \chi_\alpha + \mathcal{Y}_\alpha$$

is invariant under the transformation

$$\begin{aligned}
&\mathcal{S}_{\alpha\dot{\alpha}} \to \mathcal{S}_{\alpha\dot{\alpha}} + [D_\alpha, \overline{D}_{\dot{\alpha}}]U \ , \\
&\chi_\alpha \to \chi_\alpha + \frac{3}{2}\overline{D}^2 D_\alpha U \ , \\
&\mathcal{Y}_\alpha \to \mathcal{Y}_\alpha + \frac{1}{2}D_\alpha \overline{D}^2 U \ ,
\end{aligned}$$

with real U (well-defined up to an additive constant).

This changes $S_{\alpha\mu}$, $T_{\mu\nu}$, the string and domain wall currents by improvement terms.

Special Cases

$$\overline{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = \chi_\alpha + \mathcal{Y}_\alpha$$

In special cases the S-multiplet is decomposable:

Special Cases

$$\overline{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = \chi_\alpha + \mathcal{Y}_\alpha$$

In special cases the S-multiplet is decomposable:

- If $\chi_\alpha = \frac{3}{2}\overline{D}^2 D_\alpha U$ with a well defined U, we can set it to zero. This happens when there is no string charge (the string current can be improved to zero). Then, the S-multiplet is decomposed to a real (vector superfield) U and the Ferrara-Zumino multiplet (below).

Special Cases

$$\overline{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = \chi_\alpha + \mathcal{Y}_\alpha$$

In special cases the S-multiplet is decomposable:

- If $\chi_\alpha = \frac{3}{2}\overline{D}^2 D_\alpha U$ with a well defined U, we can set it to zero. This happens when there is no string charge (the string current can be improved to zero). Then, the S-multiplet is decomposed to a real (vector superfield) U and the Ferrara-Zumino multiplet (below).
- If $\mathcal{Y}_\alpha = D_\alpha X = \frac{1}{2}D_\alpha\overline{D}^2 U$ with a well defined U, we can set it to zero. This happens when there is no domain wall charge. Here the S-multiplet is decomposed to a real (vector superfield) U and the R-multiplet (below).

Special Cases

$$\overline{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = \chi_\alpha + \mathcal{Y}_\alpha$$

In special cases the S-multiplet is decomposable:

- If $\chi_\alpha = \frac{3}{2}\overline{D}^2 D_\alpha U$ with a well defined U, we can set it to zero. This happens when there is no string charge (the string current can be improved to zero). Then, the S-multiplet is decomposed to a real (vector superfield) U and the Ferrara-Zumino multiplet (below).
- If $\mathcal{Y}_\alpha = D_\alpha X = \frac{1}{2}D_\alpha\overline{D}^2 U$ with a well defined U, we can set it to zero. This happens when there is no domain wall charge. Here the S-multiplet is decomposed to a real (vector superfield) U and the R-multiplet (below).
- If both are true with the same U, we can set $\chi_\alpha = \mathcal{Y}_\alpha = 0$. This happens when the theory is superconformal.

The Ferrara-Zumino (FZ) Multiplet

When $\chi_\alpha = \frac{3}{2}\overline{D}^2 D_\alpha U$ we find the most familiar supercurrent – the FZ-multiplet

$$\overline{D}^{\dot\alpha} \mathcal{J}_{\alpha\dot\alpha} = D_\alpha X \ ,$$
$$\overline{D}_{\dot\alpha} X = 0 \ .$$

- It contains 12+12 independent real operators: j_μ (4), $T_{\mu\nu}$ (6), x (2) and $S_{\alpha\mu}$ (12).
- It exists only if there are no string currents – it does not exist if there are FI-terms ζ or if the Kähler form is not exact. Nontrivial

$$C_{\mu\nu} \sim \zeta \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$
$$C_{\mu\nu} \sim i\epsilon_{\mu\nu\rho\sigma} K_{i\bar{i}} \partial^\rho \phi^i \partial^\sigma \overline{\phi}^{\bar{i}}$$

are obstructions to its existence.

The R-Multiplet

When $\mathcal{Y}_\alpha = D_\alpha X = \frac{1}{2} D_\alpha \overline{D}^2 U$ we find the R-multiplet

$$\overline{D}^{\dot\alpha} \mathcal{R}_{\alpha\dot\alpha} = \chi_\alpha \ ,$$
$$\overline{D}_{\dot\alpha} \chi_\alpha = 0 \ , \qquad D^\alpha \chi_\alpha = \overline{D}_{\dot\alpha} \overline{\chi}^{\dot\alpha} \ .$$

- This multiplet includes 12+12 operators. Among them is a string current. But there is no domain wall current.
- $j_\mu = \mathcal{R}_\mu \big|$ is a conserved R-current – the theory has a $U(1)_R$ symmetry.
- This multiplet exists even when the Kähler form is not exact or the theory has FI-terms.
- $S_{\alpha\mu}$, $T_{\mu\nu}$ differ from those in the FZ-multiplet by improvement terms.

Example: Wess-Zumino Models

Every theory has an S-multiplet.
Example: a WZ theory

$$\mathcal{S}_{\alpha\dot{\alpha}} = 2K_{i\bar{j}} D_\alpha \Phi^i \overline{D}_{\dot{\alpha}} \overline{\Phi}^{\bar{j}} \ ,$$
$$\chi_\alpha = \overline{D}^2 D_\alpha K \ ,$$
$$X = 4W \ .$$

- All the operators are globally well defined.
- Since X has to be well defined up to adding a constant, we can allow multi-valued W.

Example: Wess-Zumino Models

- If the Kähler form is exact, there are no strings and the S-multiplet can be improved to the FZ-multiplet

$$\mathcal{J}_{\alpha\dot{\alpha}} = 2K_{i\bar{j}} D_\alpha \Phi^i \overline{D}_{\dot{\alpha}} \overline{\Phi}^{\bar{j}} \ - \frac{2}{3}[D_\alpha, \overline{D}_{\dot{\alpha}}]K \ ,$$

$$X = 4W - \frac{1}{3}\overline{D}^2 K \ .$$

Example: Wess-Zumino Models

- If the Kähler form is exact, there are no strings and the S-multiplet can be improved to the FZ-multiplet

$$\mathcal{J}_{\alpha\dot{\alpha}} = 2K_{i\bar{j}} D_\alpha \Phi^i \overline{D}_{\dot{\alpha}} \overline{\Phi}^{\bar{j}} \; - \frac{2}{3}[D_\alpha, \overline{D}_{\dot{\alpha}}]K \; ,$$

$$X = 4W - \frac{1}{3}\overline{D}^2 K \; .$$

- If the theory has an R-symmetry, the S-multiplet can be improved to the R-multiplet (even when the Kähler form is not exact)

$$\mathcal{R}_{\alpha\dot{\alpha}} = 2K_{i\bar{j}} D_\alpha \Phi^i \overline{D}_{\dot{\alpha}} \overline{\Phi}^{\bar{j}} - [D_\alpha, \overline{D}_{\dot{\alpha}}] \sum_i R_i \Phi^i \partial_i K \; ,$$

$$\chi_\alpha = \overline{D}^2 D_\alpha \left(K - \frac{3}{2} \sum_i R_i \Phi^i \partial_i K \right) \; ,$$

Constraints on RG-Flow

- Consider a SUSY field theory, which has an FZ-multiplet in the UV (e.g. a gauge theory without FI-terms). Hence, the FZ-multiplet exists at every energy scale. This constrains the low-energy theory:
  - No string charge in the SUSY algebra
  - No FI-terms, even for emergent gauge fields (previous arguments by [Shifman, Vainshtein; Dine; Weinberg]).
  - The Kähler form of the target space is exact (previous argument by [Witten]).

Constraints on RG-Flow

- Consider a SUSY field theory, which has an FZ-multiplet in the UV (e.g. a gauge theory without FI-terms). Hence, the FZ-multiplet exists at every energy scale. This constrains the low-energy theory:
  - No string charge in the SUSY algebra
  - No FI-terms, even for emergent gauge fields (previous arguments by [Shifman, Vainshtein; Dine; Weinberg]).
  - The Kähler form of the target space is exact (previous argument by [Witten]).
- Consider a SUSY field theory with a $U(1)_R$ symmetry. It has an R-multiplet at every energy scale. Hence, there are no domain wall charges in the supersymmetry algebra and in particular, no BPS domain walls.

The S-Multiplet in $3d$

- The S-multiplet for $\mathcal{N}=2$ in $3d$ is given by

$$\begin{aligned} &\overline{D}^{\beta}\mathcal{S}_{\alpha\beta} = \chi_\alpha + \mathcal{Y}_\alpha \ , \\ &\overline{D}_\alpha \chi_\beta = \frac{1}{2} C \varepsilon_{\alpha\beta} \ , \qquad D^\alpha \chi_\alpha = -\overline{D}^{\alpha} \overline{\chi}_\alpha \ , \\ &D_\alpha \mathcal{Y}_\beta + D_\beta \mathcal{Y}_\alpha = 0 \ , \qquad \overline{D}^{\alpha}\mathcal{Y}_\alpha = -C \ , \end{aligned}$$

  where C is a complex constant.

- It leads to a new term in the SUSY current algebra:

$$\{Q_\alpha, S_{\beta\mu}\} = \frac{1}{4}\overline{C}\gamma_{\mu\alpha\beta} + \cdots \ .$$

  We interpret it as a space-filling brane current (not affected by improvements). This is consistent with dimensional reduction from $4d$.

Application: Partial SUSY-Breaking

- If $C \neq 0$, the vacuum preserves at most two of the four supercharges. SUSY can be partially broken from $\mathcal{N} = 2$ to $\mathcal{N} = 1$.
- It happens because of a deformation of the current algebra [Hughes, Polchinski].
- This is fundamentally different from spontaneous breaking, where the current algebra is not modified.
- The nature of the multiplet and the value of C are determined in the UV. Hence, if $C = 0$ in the UV (e.g. in conventional SUSY gauge theories), there cannot be partial SUSY breaking.

Partial SUSY-Breaking

Other places with the same phenomenon:

- In the $2d\ \mathcal{N} = (0, 2)$ $\mathbf{CP}^1$ model instantons generate nonzero C (earlier work by [Witten; Tan, Yagi]).
- $\mathcal{N} = (2, 2)$ in $2d$
  - Three different space-filling brane currents can break $(2, 2) \to (1, 1)$, $(2, 0)$, or $(0, 2)$.
  - Simple models realize these possibilities [Hughes, Polchinski; Losev, Shifman].
- $\mathcal{N} = 2 \to \mathcal{N} = 1$ breaking in $4d$ [Hughes, Liu, Polchinski; Antoniadis, Partouche, Taylor; Ferrara, Girardello, Porrati].

Constraints on Linearized SUGRA

Linearized SUGRA is obtained by adding to the flat space Lagrangian

$$\mathcal{L}_{flat\ space} + \int d^4\theta H^\mu \mathcal{S}_\mu + \mathcal{O}(H^2)$$

where $\mathcal{S}_\mu$ is the supercurrent and H^μ is a superfield containing the deformation of the metric.

Constraints on Linearized SUGRA

Linearized SUGRA is obtained by adding to the flat space Lagrangian

$$\mathcal{L}_{flat\ space} + \int d^4\theta H^\mu \mathcal{S}_\mu + \mathcal{O}(H^2)$$

where $\mathcal{S}_\mu$ is the supercurrent and H^μ is a superfield containing the deformation of the metric.

Standard SUGRA ("old-minimal SUGRA") uses the FZ-multiplet

$$\begin{aligned}\mathcal{L} =& \mathcal{L}_{\text{flat space}} + h^{\mu\nu} T_{\mu\nu} + \psi^{\mu\alpha} S_{\mu\alpha} + \overline{\psi}^{\mu\dot{\alpha}} \overline{S}_{\mu\dot{\alpha}} \\ &+ b^\mu j_\mu + M x + \overline{M}\overline{x} + \ldots\end{aligned}$$

It exists only when the FZ-multiplet exists; i.e. when the Kähler form is exact [Witten and Bagger] and when there is no FI-term.

Constraints on Linearized SUGRA

If the theory has a global $U(1)_R$ symmetry we can use "new-minimal SUGRA", which is based on the R-multiplet.

- On shell it is equivalent to the "old-minimal" formalism.
- Even though the $U(1)_R$ symmetry of the matter theory is gauged, the resulting theory has a global $U(1)_R$ symmetry.

Constraints on Linearized SUGRA

If the theory has a global $U(1)_R$ symmetry we can use "new-minimal SUGRA", which is based on the R-multiplet.

- On shell it is equivalent to the "old-minimal" formalism.
- Even though the $U(1)_R$ symmetry of the matter theory is gauged, the resulting theory has a global $U(1)_R$ symmetry.
- We conclude that theories without an FZ-multiplet can be coupled to SUGRA only if they have a global $U(1)_R$ symmetry. This is consistent with earlier work of [Freedman; Barbieri, Ferrara, Nanopoulos, Stelle; Kallosh, Kofman, Linde, Van Proeyen].

Constraints on Linearized SUGRA

If the theory has a global $U(1)_R$ symmetry we can use "new-minimal SUGRA", which is based on the R-multiplet.

- On shell it is equivalent to the "old-minimal" formalism.
- Even though the $U(1)_R$ symmetry of the matter theory is gauged, the resulting theory has a global $U(1)_R$ symmetry.
- We conclude that theories without an FZ-multiplet can be coupled to SUGRA only if they have a global $U(1)_R$ symmetry. This is consistent with earlier work of [Freedman; Barbieri, Ferrara, Nanopoulos, Stelle; Kallosh, Kofman, Linde, Van Proeyen].
- Excluding gravity theories with global symmetries, such models are not acceptable.

This constrains many supergravity and string constructions.

Constraints on Linearized SUGRA

A rigid theory without an FZ-multiplet and without a $U(1)_R$ symmetry can be coupled to linearized SUGRA using the S-multiplet.

Constraints on Linearized SUGRA

A rigid theory without an FZ-multiplet and without a $U(1)_R$ symmetry can be coupled to linearized SUGRA using the S-multiplet.

- The resulting SUGRA (16/16 SUGRA) has more degrees of freedom.
- One way to think about it is to add to the matter system another propagating chiral superfield such that it has an FZ-multiplet and then use the standard formalism [Siegel].
- This is familiar from heterotic compactifications, where the additional propagating degrees of freedom are the dilaton, the dilatino and the two-form B.

Conclusions

- The supersymmetry current and the energy-momentum tensor are embedded in a supermultiplet.
- The S-multiplet is the most general supercurrent multiplet.
  - It has 16+16 components.
  - It always exists.
  - In special situations it is decomposable – can be improved to a smaller multiplet.
- The most common supercurrent is the FZ-multiplet.
  - It has 12+12 components.
  - It exists when there are no string charges in the SUSY algebra.
  - This happens when the Kähler form is exact and there are no FI-terms.

Conclusions

- If the theory has a $U(1)_R$ symmetry it has an R-multiplet
  - It has 12+12 components.
  - It does not admit domain wall charges in the algebra.

Conclusions

- If the theory has a $U(1)_R$ symmetry it has an R-multiplet
  - It has 12+12 components.
  - It does not admit domain wall charges in the algebra.
- This discussion constrains the dynamics:
  - If the UV theory has an FZ-multiplet, the low-energy theory has an exact Kähler form and no FI-terms – it does not have string charges in the SUSY algebra.
  - If the theory has a $U(1)_R$ symmetry, it has an R-multiplet and then it does not have charged domain walls.
  - Space-filling brane currents give rise to partial SUSY breaking (fundamentally different from spontaneous breaking).
  - If the corresponding current is not present in the UV, SUSY cannot be partially broken.

Conclusions

- This also constrains linearized supergravity
  - Only theories with an FZ-multiplet can be coupled to "old-minimal supergravity."
  - Theories with nontrivial Kähler form or FI-terms can be coupled to "new-minimal supergravity", but then they must have a continuous global $U(1)_R$ symmetry.
  - More general theories can be coupled through their S-multiplet. But the resulting theory has more degrees of freedom.
  - Constraints on SUGRA/string constructions.
  - There conclusions are limited to linearized supergravity. Additional consistent possibilities exist in intrinsic gravitational theories with Planck size coupling constants [Witten and Bagger; NS].