Superstring Perturbation Theory Revisited

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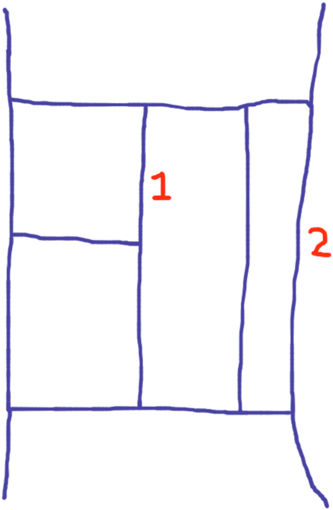
The role of modular invariance in string perturbation theory was discovered initially by J. Shapiro about forty years ago, after C. Lovelace had shown the special role of 26 dimensions. Although it took time for this to be fully appreciated, modular invariance eliminates the ultraviolet region from string and superstring perturbation theory, and consequently there is no issue of ultraviolet divergences. I will have nothing new to say about this today.

However, the literature from the 1980's has left some small unclarity about the infrared behavior of superstring perturbation theory, and this is what I want to revisit. First of all, the general statement one wants to establish is simply that the infrared behavior of superstring perturbation theory is the same as that of a field theory with the same massless particles and low energy interactions. There is some slight unclarity about aspects of this and that is what I want to reconsider. It is just a question of some details since at least 98% of the work was done 25 years ago. (Before I get too far, I should say that I wish this conference had been a little later, to give me more time, but anyway there won't really be time for some of the issues I am still working on.)

I want to give a couple of examples of what I mean in saying that the infrared behavior of string theory is the same as that of a corresponding field theory. Let us consider a Feynman diagram. A very simple question of infrared behavior is to consider what happens when a single propagator goes on shell. First I'll consider a propagator whose "cutting" does not separate a diagram in two.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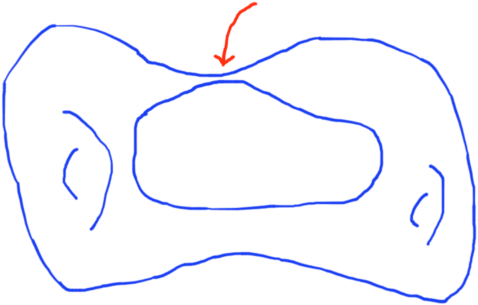
Let us assume our particles are massless so the propagator is $1/k^2$. In D noncompact dimensions, the infrared behavior when the momentum in a single generic propagator goes to zero is

$$\int \mathrm{d}^D k \, \frac{1}{k^2}$$

and this converges if $D > 2$. (For an exceptional internal line, such as the one labeled 2 in the diagram, the infrared behavior when a single momentum goes to zero is worse, because this forces other propagators to go on shell. In the case shown in the sketch, the condition to avoid a divergence is actually $D > 4$.)

All this has a close analog in string theory. First of all, a nonseparating line that goes to zero momentum is analogous to a nonseparating degeneration of a Riemann surface.



A degeneration of a Riemann surface – separating or not – can be described by an equation

$$xy = \varepsilon,$$

where x is a local parameter on one side, y is one on the other, and ε measure the narrowness of the neck – or, by a conformal transformation, the length of the tube separating the two sides. The contribution of a massless particle running through the neck is

$$\int \frac{|\mathrm{d}^2\varepsilon|}{|\varepsilon|^2 \log|\varepsilon|^{D/2}},$$

and this converges near $\varepsilon = 0$ if $D > 2$.

Not only is the answer the same, but we can match the two computations. The string theory answer came from

$$\int d^D k \int |d^2\varepsilon| \varepsilon^{L_0 - 1} \overline{\varepsilon}^{\overline{L}_0 - 1} = \int d^D k \int |d^2\varepsilon| |\varepsilon\overline{\varepsilon}|^{k^2/2-1}$$

where I use $L_0 = \overline{L}_0 = k^2/2$. Performing the integral over k gives the factor of $1/\log|\varepsilon|^{D/2}$. Instead of doing the integral, let us introduce the analog of the Schwinger parameter by $\varepsilon = \exp(-(t + is))$ where s is an angle and t plays the same role as the Schwinger parameter of field theory. The integral over s just gives a factor of 2π, giving

$$2\pi \int d^D k \int^{\infty} \mathrm{d}t \exp(-tk^2).$$

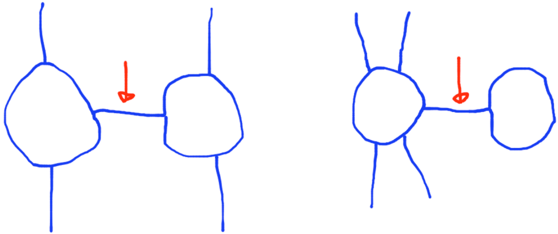
(Note that I indicated the upper limit of the t integral but not the lower limit, which is affected by modular invariance.)

This agrees perfectly with field theory even before doing the k or t integral, bearing in mind that the Schwinger representation of the Feynman propagator is

$$\frac{1}{k^2} = \int_0^\infty \mathrm{d}t \, \exp(-tk^2).$$

Just as in field theory, we could also consider a situation in which one momentum going to zero puts other lines on-shell. This gives an infrared divergence if $D \leq 4$, whether in field theory or string theory.

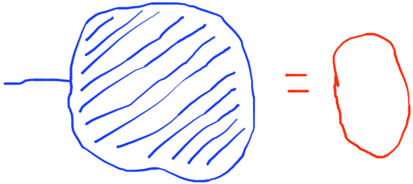
There are many other questions that match simply between string theory and field theory, for example "cutting" a diagram to probe unitarity. For something where the match is less straightforward, let us consider a separating line. Here are two cases in field theory.



The difference is that in the second case the external lines are all on one side.

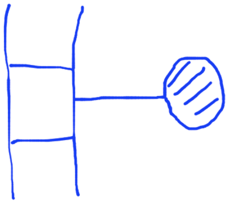
We don't integrate over the momentum that passes through the separating line; it is determined by momentum conservation. On the left, this momentum is generically nonzero so for typical external momenta, we don't sit on the $1/k^2$ singularity; when we vary the external momenta, the $1/k^2$ gives a pole in the S-matrix (at least in this approximation). This is physically sensible and we do not try to get rid of it. On the right, it is different. The momentum passing through the indicated line is 0 and hence we will get $1/0$ unless the matrix element on the right vanishes.

So a field theory with a massless scalar has a sensible perturbation expansion only if the "tadpole" or one point function of the scalar vanishes:



We have to impose this condition for all massless scalars. However, it is non-trivial only for the ones that are invariant under all (local or global) symmetries.

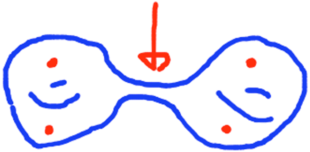
If the tadpoles do vanish, we just throw away all the corresponding diagrams

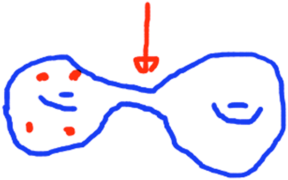


and evaluate the S-matrix by summing the others.

All this is relevant to perturbative string theory, since whenever we do have a perturbative string theory, there is always at least one massless neutral scalar field that might have a tadpole, namely the dilaton. So perturbative string theory will only make sense if the dilaton tadpole vanishes (along with other tadpoles, if there are more massless scalars). In either field theory or string theory, the usual way to show vanishing of the tadpole of a massless scalar (neutral under all symmetries) is to use supersymmetry. Indeed, without supersymmetry, it is unnatural to have a massless neutral scalar.

Just as in field theory, we can distinguish different kinds of diagrams with separating degenerations:





As one should anticipate from what I have said, it is the one on the right that causes trouble. There are two reasons that this problem is harder to deal with than in field theory:

1) Technically, it is harder to understand spacetime supersymmetry in string theory than in field theory, and to use it to show that the integrated massless tadpoles vanish.

2) In field theory, the tadpoles are the contributions of certain diagrams and if they vanish, one just throws those diagrams away. String theory is more subtle because it is more unified; the tadpole is part of a diagram that also has nonzero contributions. Vanishing tadpoles makes the diagrams of string perturbation theory infrared convergent but only conditionally so and so there is still some work to do to define them properly. (This is a point where I believe I've improved what was said in the 80's, but I won't explain it today.)

Since we can only hope for the tadpoles to vanish in the supersymmetric case, we have to do supersymmetric string theory. This means that our Riemann surfaces are really super Riemann surfaces. A super Riemann surface is a rather subtle sort of thing and it takes practice to get any intuition about them, so there is a limit to how much I can explain today.

A super Riemann surface (with $N = 1$ SUSY) is a supermanifold of dimension $(1|1)$, but it has much more structure than that. There are far too many $(1|1)$ supermanifolds. One way to describe what is a super Riemann surface is that it is a $(1|1)$ supermanifold Σ endowed with an odd (or fermionic) line sub-bundle $L \subset T\Sigma$ such that if s is an odd section of L, then $s^2 = \{s, s\}/2$ generates the quotient $T\Sigma/L$. If so it is always possible to pick local coordinates z, θ such that L has a section

$$s = \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial z}.$$

The superconformal generators (i.e., the vector fields that preserve L) are then locally of the form $f(z)s$ and

$$(f(z)s)^2 = g \frac{\partial}{\partial z} + \frac{g'}{2} \theta \frac{\partial}{\partial \theta}, \quad g = f^2,$$

which are familiar formulas.

A Neveu-Schwarz vertex operator is a field $\Phi(z, \theta)$ that is inserted at a point $z, \theta \in \Sigma$. But a Ramond vertex operator lives at a singularity in the superconformal structure. Here we still endow Σ with a subbundle $L \subset T\Sigma$, but now a generating section s of L has the property that s^2 vanishes on a divisor in Σ; the divisor is where the Ramond vertex operators are inserted. The local structure is

$$s = \frac{\partial}{\partial \theta} + z\theta \frac{\partial}{\partial z}$$

so $s^2 = z\partial/\partial z$ and vanishes on the divisor $z = 0$.

Friedan, Martinec, and Shenker in 1985 explained what kind of vertex operators are inserted at such superconformal singularities – they are often called spin fields – and how to compute their operator product expansions. In particular, the operators that generate spacetime supersymmetry are of this kind, so their work made it possible to see spacetime supersymmetry in a covariant way in superstring theory. As regards practical calculations, their work also made it possible to compute arbitrary tree amplitudes with bosons and fermions in a covariant way, and many loop amplitudes of low order. Moreover, in the intense period of effort in the 1980's – some of the prime contributors are here today – the main ingredients of a systematic, all-orders algorithm were assembled. My reconsideration of the problem has aimed at simplifying and extending the understanding of a few details.

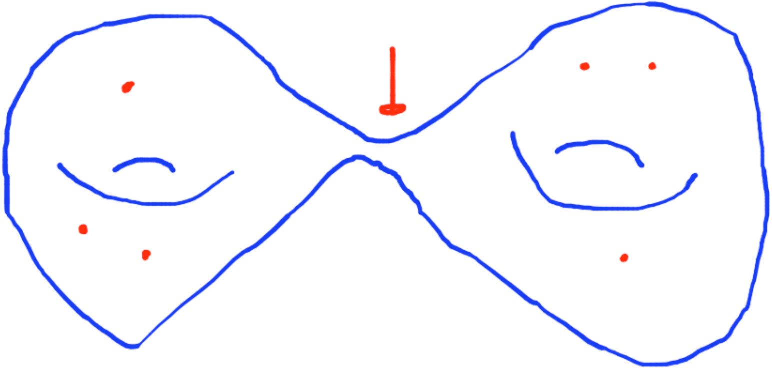
It turns out that this problem requires greater sophistication in understanding supermanifolds and how to integrate over them than is needed in any other problem that I know of in supersymmetry and supergravity. That is probably the main reason for any unclarity that surrounds it.

Some low order cases are deceptively simple and really don't give a good idea of a general algorithm for superstring perturbation theory. For example, in genus $g = 1$, the dilaton tadpole vanishes in $\mathbb{R}^{10}$ by summing over spin structures, but the fact that this makes sense depends upon the fact that in $g = 1$ (with no punctures) there are no fermionic moduli. As soon as there are odd moduli, there is no meaningful notion of two super Riemann surfaces being the same but with different spin structures. In particular, in genus $g > 1$, there is no meaningful operation of summing over spin structures without integrating over supermoduli. In genus $g = 2$, D'Hoker and Phong found an effective and very beautiful way to integrate over fermionic moduli first (after which the sum over spin structures makes sense and could be used to show the vanishing of the dilaton tadpole) and then integrate over bosonic moduli. This calculation is currently the gold standard, but it is more or less clear that for generic g their procedure has no analog and the only natural operation is the combined integral over all bosonic and fermionic moduli.

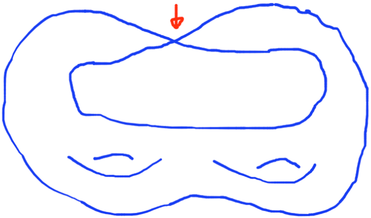
Instead of talking more about what doesn't work in general, let us discuss what does work. First of all, there is a natural measure on supermoduli space, which I will call $\widetilde{\mathcal{M}}_{g,n}$. This was constructed in the 1980's via conformal field theory (in varied approaches by Moore, Nelson, and Polchinski; E. & H. Verlinde; and D'Hoker and Phong) by adapting the analogous formulas for the bosonic string. Also, though less well known, there is for the important case of strings in $\mathbb{R}^{10}$ a slightly abstract but very elegant – and completely rigorous mathematically – construction of the measure by Rosly, Schwarz and Voronov (1985) via algebraic geometry.

Another key point is that integration of a bounded function on a compact supermanifold is a well-defined operation just as on an ordinary manifold. For today I will assume that you all know or will believe that.

Supermoduli space is not compact – or if we take its Deligne-Mumford compactification, then the function we want to integrate has singularities – precisely because of the infrared effects that we have been talking about.



Although supermoduli space is very subtle, if one asks precisely the questions whose answers one needs, those particular questions tend to have simple answers. For instance, although a sum over spin structures (independent of the integration over supermoduli) does not make sense in general, a very small piece of it makes sense when a node develops



and this leads to the GSO projection on the physical states that propagate through the node.

For another example, the description of the moduli space near a node is just as simple as for a bosonic Riemann surface. In the bosonic case, the gluing of a surface with local parameter x to one with local parameter y is by

$$xy = \varepsilon'.$$

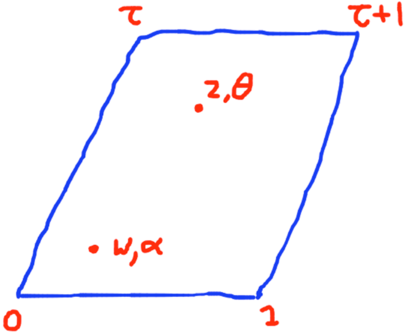
For the super case, the gluing of local parameters x, θ to y, ψ is by an almost equally simple formula

$$xy = \varepsilon^2, \quad y\theta = \varepsilon\psi, \; x\psi = \varepsilon\theta.$$

Importantly, the gluing depends in both cases on only one bosonic parameter ε or ε'. In the super case, there are no odd moduli for the gluing. The locus $\varepsilon = 0$ in $\widetilde{\mathcal{M}}_{g,n}$ is a product of spaces of the same type $\widetilde{\mathcal{M}}_{g_1,n_1+1} \times \widetilde{\mathcal{M}}_{g_2,n_2+1}$ with $g_1 + g_2 = g$, $n_1 + n_2 = n$.

In order to integrate over supermoduli space – say for the heterotic string – one has to find the gluing parameters ε, ε' for right-movers and left-movers near each component at infinity, and then set $\varepsilon^2 = \overline{\varepsilon'}$ before integrating. (For Type II the analog is $\varepsilon = \overline{\varepsilon}'$.) With this recipe – once one knows that there are no tachyons and that the massless tadpoles vanish – integration on supermoduli space is just as well-defined as it would be on a compact supermanifold. Instead of more fully explaining in the abstract what the recipe means and why it is needed and works, I will explain it in a concrete example (studied by Dine, Ichinose and Seiberg (1987), Atick, Dixon, and Sen (1987), and Green and Seiberg (1987) following work on the low energy effective field theory by Dine, Seiberg and EW). The example will also show some of the subtleties of supermanifolds and integration on supermoduli space more effectively than I could do in an abstract explanation.

The example involves a genus 1 two-point function of Neveu-Schwarz vertex operators, with an even spin structure. The genus 1 surface Σ has a bosonic modulus τ but this plays no important role and we will just view it as a fixed complex number. (As mentioned before, the genus 1 surface with even spin structure has no fermionic moduli until we introduce vertex operators.)



The two NS vertex operators are inserted at points that we will call (w, α) and (z, θ). Using the translation symmetry of Σ, we can set $w = 0$, so the moduli space $\widetilde{\mathcal{M}}$ has dimension $1|2$ with parameters z, α, θ (as explained more precisely presently). Infinity in $\widetilde{\mathcal{M}}$ is the region near $z = w$, which means $z = 0$, since we have set $w = 0$.

Now let us describe $\widetilde{\mathcal{M}}$ as a complex supermanifold. It is a natural supergeneralization of an ordinary torus, parametrized by z, α, θ subject to the equivalences

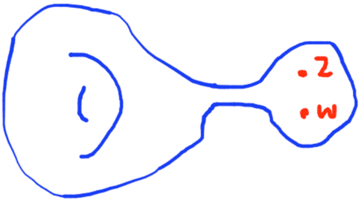
$$z \to z + 1,\ \alpha \to \alpha,\ \theta \to -\theta$$

and

$$z \to z + \tau,\ \alpha \to \alpha,\ \theta \to \theta.$$

The minus sign for θ in the first line is important and puts us in an even spin structure.

This is $\widetilde{\mathcal{M}}$, but we need to find the good parameter for gluing. What gluing means here is explained in this picture:



The natural gluing parameter is not $\varepsilon^2 = z$ but $\varepsilon^2 = z^*$ with

$$z^* = z + \alpha\theta.$$

Since we are doing a heterotic string calculation, if we call the supersymmetric degrees of freedom right-moving, then the left-moving degrees of freedom live on an ordinary Riemann surface Σ' (which roughly speaking is the complex conjugate of Σ – soon you will see why this statement is rough). We take Σ' to be the torus with parameter $\tau' = \overline{\tau}$. For the two-point function, we take one of the two points to be at the origin and the other at z', where the equivalences for z' are just

$$z' \cong z' + 1 \cong z' + \overline{\tau}.$$

This defines the left-moving moduli space $\mathcal{M}'$. The gluing parameter for the left-movers is just

$$\varepsilon' = z'.$$

Now we need to decide what we want to integrate. For the right-moving degrees of freedom, we take the integration measure to be just $\mathrm{d}z\,\mathrm{d}\alpha\,\mathrm{d}\theta$. This coincides with $\mathrm{d}z^*\,\mathrm{d}\alpha\,\mathrm{d}\theta$, so it is globally well-defined except for one problem: it is odd under $z \to z+1$, $\alpha \to \alpha$, $\theta \to -\theta$. We will compensate for this by taking the left-moving measure to be similarly odd under $z' \to z'+1$, which with a suitable choice of integration cycle will make the combined measure well-defined. So the left-moving measure can't just be $\mathrm{d}z'$; it must be $\mathrm{d}z'\,P(z')$ where the function $P(z')$ is odd under $z' \to z'+1$. Moreover $P(z')$ can only have a pole at infinity, that is at $z'=0$. We pick the most obvious choice:

$$P(z') = \sum_{m,n\in\mathbb{Z}} \frac{(-1)^m}{z'-m-n\overline{\tau}}.$$

So the integral we are going to do is

$$\int_\Gamma \mathrm{d}z\,\mathrm{d}\alpha\,\mathrm{d}\theta\,\mathrm{d}z'\;P(z')$$

where Γ is a suitable integration cycle in the product $\widetilde{\mathcal{M}} \times \mathcal{M}'$. What will Γ be? The Riemann surfaces that the left- and right-movers live on are complex conjugates of each other *modulo nilpotents*. Given only this knowledge, the most general possibility for Γ would be

$$z + \alpha\theta h(z, \overline{z}) = \overline{z}',$$

for some function $h(z, \overline{z})$. To be consistent with the shifts $z \to z + 1$, $z \to z + \tau$ (where $\alpha\theta$ is odd under the first shift) we can, for example, set $h = 0$, but we can't set $h = 1$.

However, there is one more constraint, which has to do with what happens at infinity, that is at $z = z' = 0$. The gluing parameters are $\varepsilon^2 = z^*$, $\varepsilon' = z'$. So the relation $\varepsilon^2 = \overline{\varepsilon}'$ that we are supposed to impose at infinity tells us that near $z = z' = 0$, we must have $z^* = \overline{z}'$ or $z + \alpha\theta = \overline{z}'$. This means $h(z = 0) = 1$.

So in short, the integration cycle Γ is defined by

$$z + \alpha\theta h(z, \overline{z}) = \overline{z}'$$

where $h(0,0) = 1$ and otherwise h is only constrained to be odd under $z \to z + 1$ and even under $z \to z + \tau$. With these restrictions, the integral we are trying is independent of the choice of h, because we are integrating a closed form $\mathrm{d}z\,\mathrm{d}\alpha\,\mathrm{d}\theta\,\mathrm{d}z' P(z')$, so the integral is invariant under infinitesimal "wiggling" of the integration cycle. (More exactly, the form is closed except at $z' = 0$ where the pole in $P(z')$ comes into play; but the condition on h doesn't allow "wiggling" at $z' = 0$. It is because the form is singular at infinity, i.e. at $z' = 0$, that we need a condition on how the integration cycle behaves at infinity.)

Now to evaluate the integral, we are going to express everything in terms of z', α, θ, using the relation $z + \alpha\theta h(z, \bar{z}) = \bar{z}'$ to solve for z:

$$z = \bar{z}' - \alpha\theta h(\bar{z}', z')$$

So

$$\mathrm{d}z = \cdots - \alpha\theta \frac{\partial h}{\partial \bar{z}'} \mathrm{d}\bar{z}'$$

where I dropped terms that won't contribute to the integral. So our integral becomes

$$\int \mathrm{d}\bar{z}' \, \mathrm{d}z' \, \mathrm{d}\alpha \, \mathrm{d}\theta \left(-\alpha\theta \frac{\partial h}{\partial \bar{z}'} \right) P(z').$$

Now we integrate over the odd variables, using $\int \mathrm{d}\alpha \, \mathrm{d}\theta \, \alpha\theta = 1$, and we integrate by parts with respect to $\bar{z}'$, using the fact that $\partial_{\bar{z}'} P(z')$ is a delta function supported at $z' = 0$, coming from the pole in $P(z')$. So the integral is just $2\pi i$.

Hopefully it is now clearer why I said that superstring perturbation theory requires a little more sophistication with supermanifolds and superintegration than we usually need in studying supersymmetry and supergravity. Note, for example, that although there was a well-defined answer, there was no sensible answer to the question of where in moduli space the answer came from, since this depended on the unnatural choice of h. (After integration by parts, the answer seems to come from a delta function at $z' = 0$, but I believe that this is special to this low genus example, analogous to the proof that the genus 1 dilaton tadpole vanishes by summing over spin structures.)

The next topic that I want to discuss is integration by parts. We need this to prove the decoupling of pure gauge degrees of freedom and also to prove spacetime supersymmetry and vanishing of tadpoles. This is actually one place where what was done in the 1980's can be improved. Traditonally, arguments involving integration by parts have been made by first integrating over odd moduli and then using the bosonic version of Stokes's theorem to integrate by parts on a purely bosonic manifold. However, this introduces many technicalities and complications. There is a perfectly good super-analog of Stokes's theorem and it is best to use this.

There isn't time to systematically explain the super-analog of Stokes's theorem, so I will just rely on the fact that you probably all know the basic idea of fermionic integration by parts, which is that for an odd variable α and any function $f(\alpha)$, one has

$$\int \mathrm{d}\alpha \frac{\mathrm{d}}{\mathrm{d}\alpha} f = 0.$$

Indeed the Berezin integral

$$\int \mathrm{d}\alpha \cdot 1 = 0, \int \mathrm{d}\alpha \cdot \alpha = 1$$

is defined to make this true.

It takes some explanation to describe what the symbols mean, but the form of the super-Stokes's theorem (attributed to Bernstein and Leites, 1979) is what one would guess:

$$\int_X \mathrm{d}\Lambda = \int_{\partial X} \Lambda.$$

Here $\mathrm{d}\Lambda$ is the analog of a "volume form" and Λ is the analog of a "form of codimension 1."

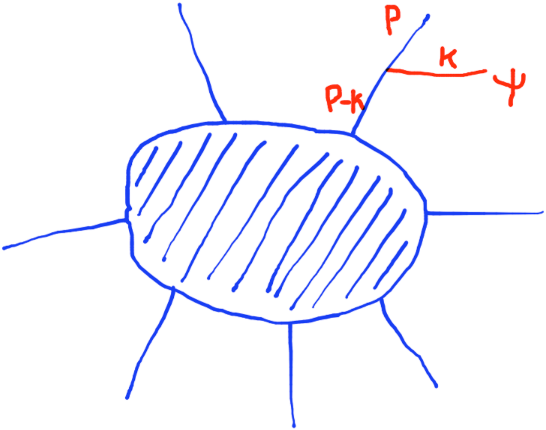
Now a scattering amplitude $\langle V_1 V_2 \ldots V_n \rangle$ is associated with a "volume form" Υ that must be integrated over, roughly speaking, supermoduli space. Actually, as we've seen, it must be integrated over a certain cycle Γ in a product $\widetilde{\mathcal{M}} \times \mathcal{M}'$. Just as for the bosonic string, if, say, $V_1 = \{Q, W\}$ for some W, then the volume form Υ is $\mathrm{d}\Lambda$ for some Λ. Then in checking decoupling of $\{Q, W\}$, we get

$$\langle \{Q, W\} V_2 \ldots V_n \rangle = \int_\Gamma \Upsilon = \int_{\partial\Gamma} \Lambda.$$

If Λ has a good behavior on $\partial\Gamma$, then the right hand side vanishes and so therefore does the left hand side. For vanishing of $\int_{\partial\Gamma} \Lambda$, one needs to know (i) vanishing of tadpoles, otherwise none of the integrals converge and (ii) a certain condition about mass renormalization that I have been suppressing though we will incorporate it shortly. (This condition has a field theory analog: the condition on which modes are supposed to decouple can depend on the particle masses so it can be affected by mass renormalization.)

This argument is much simpler than any argument using the bosonic version of Stokes's theorem. It has an important corrolary. If one knows that the massless tadpoles vanish, then spacetime supersymmetry is a special case of the decoupling of pure gauge modes.

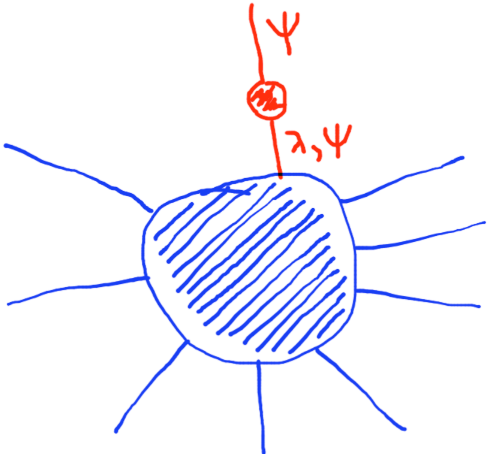
We consider a scattering amplitude involving a soft gravitino. We take its wavefunction to be $\Psi_{I\alpha} = \exp(ik \cdot x)\eta_{I\alpha}$ where I is a vector index and α is a spinor index. A matrix element for emission of a soft gravitino has singular terms where the gravitino is attached to an external leg:



I've drawn this as a field theory picture, but I hope you all understand that there is an analogous string theory picture. The line that emits the gravitino is just slightly off shell, with momentum $P-k$. If $P^2=M^2$ and $k^2=0$, then $(P-k)^2=M^2-2P\cdot k$, so the propagator of this line is $1/((P-k)^2-M^2)=-1/2P\cdot k$ (or something similar if the line represents a particle with spin). This is singular at $k\to 0$. The amplitude also comes with a numerator which is a matrix element of the supercurrent S, via which the gravitino couples, between the two states $\langle (P-k)'|S|P\rangle$ (the prime in $\langle (P-k)'|$ is meant to remind us that S has acted on the particle spin). In all, this soft emission amplitude is essentially $\langle (P-k)'|S|P\rangle/(-2P\cdot k)$ times an amplitude with the external gravitino and particle $|P\rangle$ replaced by an external state $|(P-k)'\rangle$.

Now if we set the gravitino polarization vector-spinor $\eta_{I\alpha}$ to be $k_I \zeta_\alpha$ (for some spinor ζ_α), then the whole amplitude must vanish. This is a special case of the decoupling of states $\{Q, W\}$ for any W. It is hard to evaluate this condition exactly, but its leading behavior as $k \to 0$ can be evaluated, and is the sum of terms of the form $\langle (P-k)'|k \cdot S|P\rangle/(-2P \cdot k)$ times an amplitude with one of the external particles $|P\rangle$ replaced by $|(P-k)'\rangle$. The sum of all these terms must vanish and this is the Ward identity of spacetime supersymmetry.

This type of argument may be familiar from field theory. It works the same way in string theory, except that we have to know that the massless tadpoles vanish (or none of the amplitudes are defined). However, in either field theory or string theory, I have left something out so far. Potentially, the supersymmetric Ward identity can contain another term if the coupling of a soft gravitino has a singular contribution like this:



This happens if loops generate a term in the effective action that is of the form $\overline{\Psi}_I \Gamma^I \lambda$, with some previously massless fermion λ, or a term $\overline{\Psi}_I \Gamma^{IJ} \Psi_J$. In the first case, supersymmetry is spontaneously broken, with λ as a Goldstone fermion; in the second case, we land in AdS space with unbroken supersymmetry. (The only example of the first type known in the literature is the one that motivated the genus 1 integral that we studied. No example of the second type seems to be known in the literature in any dimension, though it seems to me that it might be possible for this to happen in three spacetime dimensions.)

In many classes of string vacua, it is straightforward to prove that $\overline{\Psi}_I \Gamma^I \lambda$ and $\overline{\Psi}_I \Gamma^{IJ} \Psi_J$ terms are not generated by loops. For example in all of the ten-dimensional superstring theories except Type IIA, this follows from considerations of spacetime chirality which make it impossible to write the interactions in question. For Type IIA, the result follows if one also uses the fact that perturbation theory has $(-1)^{F_L}$ as a symmetry. (This excludes the Romans mass term.)

So all we need in order to land in a happy place is an extension of this type of reasoning to show that the massless tadpoles vanish. Though this is expected to follow from spacetime supersymmetry, I believe that the type of argument I have given is not quite powerful enough to prove it.

Given the experience from the old literature (see for example Martinec (1986), Atick, Moore and Sen (1988)), one expects that what one should do is to make a similar argument but with k set to 0 at the beginning. We used the fact that the vertex operator $V_{\Psi,k}$ for a gravitino of polarization $\eta_{I\alpha} = k_I\zeta_\alpha$ is $\{Q, W_k\}$ for some W_k. If we set $k = 0$, then $V_{\Psi,k} = 0$ and the relation becomes $0 = \{Q, S\}$ where S, which is the limit of W_k for $k = 0$, is the fundamental spin field. S has ghost number 1 (while a vertex operator for particle emission such as V has ghost number 2) so by analogy with more simple cases, the condition $\{Q, S\} = 0$ should mean that S generates a symmetry in spacetime.

For practice, let us look at a correlation function $\langle SV_1 \ldots V_n \rangle$. This can't be integrated over the usual integration cycle Γ, since the ghost number is too small by 1. But it can be integrated over the codimension 1 cycle $\partial\Gamma$. Schematically, we have

$$0 = \int_\Gamma \langle \{Q, S\} V_1 \ldots V_n \rangle = \int_{\partial\Gamma} \langle SV_1 \ldots V_n \rangle.$$

This vanishing relation can be written as a sum of contributions from the many components of $\partial\Gamma$. Many of them don't contribute because the momentum flowing through the node is off-shell.

The following contributions do have on-shell momentum flowing through the node and definitely can contribute:

If these are the only nonzero boundary contributions, then again we get the supersymmetric Ward identity, much as before.

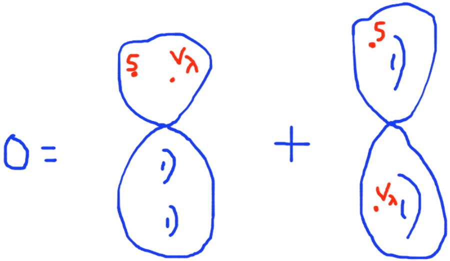
The other contributions that might appear (because they involve on-shell momentum flowing through the node) correspond to supersymmetry breaking (or a cosmological constant) or a massless tadpole. We'll draw them in a moment, in a slightly simpler situation.

To finally address the question of whether there is a massless tadpole, let us replace the product $V_1 \cdots V_n$ with a single vertex operator V_λ of a massless fermion that is a superpartner of a scalar ϕ whose tadpole we want to understand. The relation

$$0 = \int_{\partial\Gamma} \langle S V_\lambda \rangle$$

is now simple because $\partial\Gamma$ has only two types of components.

The relation is explicitly then



The first term is the dilaton tadpole, and the second – but this is what I do not fully understand – should be nonzero precisely when supersymmetry is spontaneously broken (or a cosmological constant is being generated).

When one can show that the gravitino cannot gain a mass in perturbation theory – for instance in $\mathbb{R}^{10}$ – this relation should (when combined with what was discovered in the 80's and a few details that we haven't had time for today) – remove the very slight unclarity that has surrounded superstring perturbation theory.