Non-Abelian Statistics
versus
The Witten Anomaly

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based on:
Interactions between hep-th and cond-mat have been very fruitful: SSB, higgs mechanism, topological solitons ...
More recently: hopes for many practical uses for string theory.
e.g. controllable examples of non-Fermi liquid fixed points
(possible states of fermions at finite density other than Landau’s nearly free
effective field theory).

QFT question for today: Is it possible to realize deconfined particles in 3+1 dimensions which exhibit non-abelian statistics?
There’s a recent set of ideas, inspired by work in cond-mat,
suggesting a route to doing this seemingly-impossible thing.
Its failure mode is interesting.
Particle statistics

In 3+1 dims particles are either bosons or fermions. Why: boring topology of configuration space:
\[ \pi_0 \text{ (paths)} = \pi_1(C_{n}^{3+1}) = S_n \]
\[ C_{n}^{d+1} \equiv \{ \text{config space of } n \text{ particles} \} \backslash \{ \text{close approaches} \} \]

In 2+1: \[ \pi_1(C_{n}^{2+1}) = B_n, \text{ braid group (infinite-dimensional)} \rightarrow \text{anyons.} \]
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Anyons

Abelian anyons: state of several anyons acquires a phase upon braiding.
Non-Abelian anyons: braiding acts by a unitary on degenerate statespace.

Abelian anyons exist and have been observed as quasiparticles in well-understood FQHE states.
∃ good evidence that non-Abelian anyons are also realized in FQHE states.

Non-Abelian anyons would make a great quantum computer

• Quantum state stored non-locally protected from decoherence to (local) environment.
• Do computations by adiabatically braiding anyons.

[Kitaev, Freedman]

[Hasan-Kane]
Majorana solitons

A framework for realizing a class of non-abelian anyons: Majorana zeromode localized on soliton

\[ \gamma_i = \gamma_i^\dagger \quad \{ \gamma_i, \gamma_j \} = 2\delta_{ij} \quad i, j = 1..n \]

Hilbert space of groundstates of \( n \) solitons represents this algebra.

\[ \Gamma_1 \equiv \gamma^1 + i\gamma^2, \ldots \quad \Gamma_1 |\downarrow\downarrow\rangle \equiv 0, \Gamma_1^\dagger |\downarrow\downarrow\rangle = |\uparrow\downarrow\rangle \ldots \]

\( n \) such ‘Ising anyons’ make a degenerate space of \( \dim \mathcal{H}_n \sim \sqrt{2^n} \).

Info about \( \mathcal{H}_n \) not localized on particles (despite realization in local QFT).

Realizations in 2 + 1d:

\( \nu = \frac{5}{2} \) QH states [Moore-Read, Nayak-Wilczek], \( p + ip \) superconductors [Ivanov, Read-Green],
surface states of TI [Fu-Kane], solvable toy models [Kitaev], many other proposals.
Majorana solitons, an example in 2+1 d

Fermionic quasiparticles in certain 2d superconductors:

\[ \chi \equiv \begin{pmatrix} c_\uparrow \\ c_\downarrow \\ c_\uparrow \\ c_\downarrow \end{pmatrix} \]

\[ \mathcal{L}_{\text{fermions}} = i\chi^T \left( \sigma^i \partial_i + \Phi \Gamma^+ + \overline{\Phi} \Gamma^- \right) \chi \]

Vortex: \( \Phi(r, \varphi) = e^{i\varphi} |\Phi(r)| \)

[Jackiw-Rossi, Ivanov, Read-Green] has a majorana zeromode.

Note: Ising anyons are a special case (not universal for quantum computation).

**Lesson:** All we need to do to realize non-Abelian (Ising) statistics is to find solitons with normalizable majorana zeromodes.
**Majorana hedgehogs**

Consider a 3+1d system with a global $SO(3)$ symmetry broken by an adjoint scalar vev

$$\langle \Phi^A \Phi^A \rangle = v^2 \quad A = 1, 2, 3.$$ 

Couple to a real 8-component spinor (two majorana doublets of $SU(2) \simeq SO(3)$):

$$H_{\text{fermions}} = i \chi^T \left( \gamma^i \partial_i + \lambda \Phi_A \Gamma^A \right) \chi$$

$\langle \Phi \rangle$ gaps fermions, $m_{\text{bulk}} \sim \lambda v$. 

Aside on motivation from topological insulators with superconductors attached: [Fu-Kane 08, Teo-Kane 09, Wilczek, unpublished]
**Majorana hedgehogs**

Consider a 3+1d system with a *global* $SO(3)$ symmetry broken by an adjoint scalar vev

$$\langle \Phi^A \Phi^A \rangle = \nu^2 \quad A = 1, 2, 3.$$ 

Couple to a real 8-component spinor

(two majorana doublets of $SU(2) \simeq SO(3)$):

$$H_{\text{fermions}} = i \chi^T \left( \gamma^i \partial_i + \lambda \Phi_A \Gamma^A \right) \chi$$

$$\langle \Phi \rangle \text{ gaps fermions, } m_{\text{bulk}} \sim \lambda \nu.$$ 

Hedgehog: $\Phi^A = \hat{r}^A \phi(r) \quad \phi(r) \xrightarrow{r \to \infty} \nu, \quad \phi(r) \xrightarrow{r \to 0} 0$ has a majorana zeromode.
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$\Phi^1 + i \Phi^2 = \text{supercond. order parameter (zero at vortex)}$

$\Phi^3 = \text{Dirac mass (changes sign at bdy of TI)}$
Problems of majorana hedgehogs

The hedgehogs are not quite particles: spatial var. of $\Phi$ is extra data.

Minimal data for topology:
preimage under $\Phi$ of north pole and nearby point
\[\Phi\] ribbon between hedgehog pairs.

[Freedman et al, 1005.0583]

“projective ribbon statistics”
Problems of majorana hedgehogs

The hedgehogs are not quite particles: spatial var. of $\Phi$ is extra data.
Minimal data for topology:
preimage under $\Phi$ of north pole and nearby point
$\rightarrow$ ribbon between hedgehog pairs.

[Freedman et al, 1005.0583]

“projective ribbon statistics”

Observation: Variation of $\Phi$ costs energy.
Hedgehogs are not finite-energy excitations

$$E = H[\Phi_{\text{hedgehog}}] \sim \int_0^L d^3x \left( \vec{\nabla} \Phi^A \cdot \vec{\nabla} \Phi_A + \ldots \right) \sim \nu^2 L$$

(like global SO(2) vortex in 2+1 dims).

Configurations with zero total hedgehog number have finite energy
But: $V_{\text{eff}}(R) \sim \int_0^R r^2 dr \cdot \left( \frac{\phi}{r} \right)^2 \sim R\nu^2$.
linear confinement.

Not so good for adiabatic motion.
Deconfined majorana solitons in $3+1$ dims?

Two apparently-different routes to models with *deconfined* majorana particles:

- Gauge the SU(2) symmetry
- Disorder the $\langle \Phi \rangle$. (*Zero stiffness, no gradient energy.*)
Gauge the SU(2)

- \( SU(2) \langle \Phi \rangle \in \text{adj} \rightarrow U(1) \)
- Sol’n with \( \Phi^A = \hat{r}^A \phi(r) \rightarrow \) ‘t Hooft-Polyakov monopole:

\[
A^A_i = \epsilon_{ij} \hat{r}^j A(r), \quad A^A_0 = 0
\]

\[\phi(r) \xrightarrow{r \to \infty} v, \quad A(r) \xrightarrow{r \to \infty} \frac{1}{r} \quad \implies \quad D_i \Phi \xrightarrow{r \to \infty} 0.\]

- carries magnetic charge = hedgehog \# 
  \[
  \implies \text{magnetic coulomb force } F \sim \frac{q_m q_m'}{r^2} \text{ (falls off!)}
  \]
Gauge the SU(2)

- $\text{SU}(2) \langle \Phi \rangle \xrightarrow{\text{adj}} U(1)$
- Sol’n with $\Phi^A = \hat{r}^A \phi(r) \rightarrow 't \text{ Hooft-Polyakov monopole:}$

\[
A_i^A = \epsilon_{ij} \hat{r}^j A(r), \ A_0^A = 0
\]

- $\phi(r) r \rightarrow \infty \sim \nu$, $A(r) r \rightarrow \infty \sim \frac{1}{r}$ $\Rightarrow D_i \Phi r \rightarrow \infty 0$.

- carries magnetic charge = hedgehog #

$\Rightarrow$ magnetic coulomb force $F \sim \frac{q_m q'_m}{r^2}$ (falls off!)

- \[ \mathcal{L}_{\text{fermions}} = \chi^\dagger i \vec{\sigma}^\mu D_\mu \chi - \frac{1}{2} \lambda \chi^\dagger \vec{\tau} \cdot \vec{\Phi} \chi + h.c. \]

- $\chi_{\alpha a} \text{ Weyl} \in (1, 2, 2) \text{ of } \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(2)_{\text{gauge}}$

- $\chi^\dagger \equiv \chi^T i \sigma^2 i \tau^2 \in (1, \bar{2}, \bar{2})$

- Two independent mass scales:

$m_W = g \nu$, and the mass of the fermion $\lambda \nu$. 

$\left[ \text{Does not exist: Witten anomaly} \right]_{\text{Witten 1982}}$
Gauge the SU(2)

- \(\text{SU}(2) \langle \Phi \rangle \xrightarrow{\text{adj}} U(1)\)
- Sol’n with \(\Phi^A = \hat{r}^A \phi(r) \rightarrow \text{t Hooft-Polyakov monopole:}\)

\[
A^A_i = \epsilon_{ij} \hat{r}^j A(r), \quad A^A_0 = 0
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- \(\phi(r) \xrightarrow{r \to \infty} v, \quad A(r) \xrightarrow{r \to \infty} \frac{1}{r} \implies D_i \Phi \xrightarrow{r \to \infty} 0.\)

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- \(\mathcal{L}_{\text{fermions}} = \chi^\dagger i \bar{\sigma}^\mu D_\mu \chi - \frac{1}{2} \lambda \chi \ schizophrenic \cdot \Phi \chi + h.c.\)

\(\chi_{\alpha a} \text{ Weyl} \in (1, 2, 2) \text{ of SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(2)_{\text{gauge}}\)

\(\chi^v \equiv \chi^T i \sigma^2 i \tau^2 \in (1, \bar{2}, \bar{2})\)

- Two independent mass scales:
  \(m_W = gv, \) and the mass of the fermion \(\lambda v.\)

[• Does not exist: Witten anomaly [Witten 1982] ]
Majorana zeromode

Momentarily treat $A, \Phi$ as classical background fields: Dirac equation

$$0 = \delta \bar{\chi} S_{\text{fermion}} = -i \bar{\sigma}^\mu D_\mu \chi + \lambda^\dagger i \sigma^2 \Phi \cdot \tau i \tau^2 \chi^*.$$  

ansatz from [Jackiw-Rebbi, 1976], with reality conditions.

$$\chi_{\alpha a} = i \tau^2_{\alpha a} g(r) \ (\alpha: \text{spin index, } a: \text{SU}(2) \text{ doublet index}).$$

$$(\partial_i + 2 \hat{r}_i A)g + i \lambda \phi \hat{r}_i g^* = 0.$$  

rephasing $\chi \implies \lambda > 0$ WLOG

$$g(r) = ce^{-\pi i/4} e^{-\int^r (\lambda \phi - 2A)}$$

$c$ is a real constant.

phase of the normalizable solution determined by normalizability at $r \to \infty$. 
Witten anomaly

\[ \int [D\chi] e^{iS_{\text{fermions}}[\chi, A, \Phi]} \equiv e^{i\Gamma[A, \Phi]} \times \text{non-universal stuff} \]

Fermion determinant represents \( \pi_4(\text{SU}(2)) = \mathbb{Z}_2 \):

\[ (\star) \quad e^{i\Gamma[A^g, \Phi^g]} = (-1)^g e^{i\Gamma[A, \Phi]} . \]

But \( A, A^g \) are continuously connected:

\[ \implies \int [DAD\Phi] e^{i\Gamma[A, \Phi]} \times (\text{anything gauge invariant}) = 0 \]

**One argument for** \((\star)\): Embed the theory in an SU(3) gauge theory with a perturbative gauge anomaly


Calculate the variation of the fermion measure between 1 and \( g \) by integrating the SU(3) anomaly.

**Claim:** The addition of the adjoint scalar \( \Phi \) doesn't change this.
Witten anomaly with adjoint scalar

More explicitly: consider the (perturbatively anomalous) SU(3) gauge theory with

- an adjoint scalar $\tilde{\Phi}$,
- an SU(3) triplet of Weyl fermions $\tilde{\chi}$
- an SU(3) triplet of scalars $\Upsilon$,

with the coupling

$$L_{SU(3)} \supset \tilde{\chi}_a^T \sigma^2 \epsilon_{abc} \tilde{\Phi}_{cd} \tilde{\chi}_d,$$

$a = 1, 2, 3$ is a triplet index.

$\langle \Upsilon \rangle = \lambda$ breaks the SU(3) down to SU(2), is the Yukawa coupling.

The form of the perturbative SU(3) anomaly is unaffected by the addition of scalars.

$\Gamma[A, \Phi]$ is a smooth functional for invertible $\Phi$

(integrate out massive fermions)

Ineffable: naive $\Gamma_{WZW}[A, \Phi] = 0$ for SU(2).
Canceling the Witten anomaly

\[ S[\chi, \Phi, A] \rightarrow S[\chi, \Phi, A] + \Gamma[\Phi, A] \]

But: if \( \Phi = 0 \) anywhere, \( \Gamma \) is ill-defined. \((\text{e.g. core of monopole.})\)

Requires UV completion.

Important point: presence of fermion zms is a UV sensitive question.

\[ L_{2 \text{ fermions}} = \chi^l l^\dagger i\bar{\sigma}^\mu D_\mu \chi^l - \lambda^{lJ} \chi^l \chi^J \cdot \vec{\Phi} \chi^J - m^{lJ} \chi^l \chi^J + h.c. \]

\( \chi^l_{\alpha a} \) a pair of (left-handed) Weyl doublets of SU(2):

\( l = 1, 2 \) a flavor index, \( \alpha = 1, 2 \): spin, \( a = 1, 2 \) gauge. \( 2^3 \) complex fermions

Same spectrum as [Jackiw-Rebbi 76] but more general couplings.

Three mass scales:

the mass of the \( \mathcal{W} \)-bosons, \( m_\mathcal{W} = g_\mathcal{V} \),
and the masses of the two Weyl fermions \( \lambda_{1,2} \mathcal{V} \)

For \( \lambda_1 \mathcal{V} \ll m_\mathcal{W} \ll \lambda_2 \mathcal{V} \)

large window of energies with same bulk spectrum as above.
Relation to Jackiw-Rebbi model

$\lambda$ is symmetric, $\lambda^{IJ} = \lambda^{JI}$ by Fermi statistics.
By field redefinitions, can diagonalize $\lambda$ with real eigenvalues $\lambda_{1,2}$.
Phase of $m$ is physical.
$m = m^\dagger$, preserves a CP symmetry $\chi \mapsto i\sigma^2 i\tau^2 \chi^*$. 

For $m = 0$, JR found in this model a complex zeromode of the monopole.
Quantizing this mode makes the monopole into a pair of bosons of charge $\pm e/2$ (under the 'extra' U(1)).
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Jackiw-Rebbi case: $\lambda_1 = \lambda_2 = \lambda_0 \implies$ extra $U(1)$ symmetry:

$$\chi_1 \mapsto e^{i\theta} \chi_1, \quad \chi_2 \mapsto e^{-i\theta} \chi_2$$

(in basis where $\lambda = \begin{pmatrix} 0 & \lambda_0 \\ \lambda_0 & 0 \end{pmatrix}$)

$$\psi \equiv \begin{pmatrix} \chi_1 \\ \chi_2^* i\tau^2 i\sigma^2 \end{pmatrix}, \quad \lambda_0 \equiv \lambda_0^R + i\lambda_0^I$$

$$L_{2\text{fermions}} = \bar{\psi}i\slashed{D}\psi - \bar{\psi} \left( \lambda_0^R + i\lambda_0^I\gamma^5 \right) \vec{\tau} \cdot \vec{\Phi} \psi + m\bar{\psi}\psi.$$ 

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Fermion zeromodes in the two-doublet model

For $m_{\text{Dirac}} = 0$:
In the basis where $\lambda$ is diagonal with real evals $\lambda_{1,2}$, zeromode equations for $\chi_{1,2}$ decouple. Two real solutions, like JR:

$$\chi_l \alpha a (r) = i \tau^2_{\alpha a} g_l, \quad g_l = c_l e^{-\pi i/4} e^{- \int^r (\lambda_l \phi - 2A)}.$$

For $m_{\text{Dirac}} \neq 0$:
Ansatz which decomposes $\chi \in (2,2)$ into irreps of the unbroken $\text{SU}(2) \subset \text{SU}(2)_{\text{gauge}} \times \text{SU}(2)_{\text{spin}}$:

$$\chi_{a \alpha l} = i \tau^2_{a \alpha} g_l + i (\tau^2 \tau^i)_{a \alpha} g^i_l.$$

Guess: $\vec{g} = \hat{r} g_r (r)$. Dirac equation becomes:

$$0 = i \vec{\nabla} g - 2 i \hat{r} A g - \lambda^\dagger g^* \phi \hat{r} - m^\dagger \vec{g}^*$$

$$0 = i \vec{\nabla} \cdot \vec{g} + 2 i A \vec{g} \cdot \hat{r} + \lambda^\dagger \vec{g}^* \cdot \hat{r} \phi + m^\dagger g^*.$$
Conclusions about zeromodes

- First assume $m = m^\dagger$.

For $\sqrt{\lambda_1 \lambda_2} < m$, both modes are non-normalizable. (Else, both normalizable.)

Check: $\det M_{\text{bulk}} = (\lambda_1 \lambda_2 v^2 - m^2)^2 \rightarrow 0$ precisely at marginal normalizibility.

Sizes of zms can be varied independently by $\lambda_{1,2}$.

The zeromode wavefunctions involve products of exponentials of the form $e^{mr} e^{-\lambda vr}$, one might have thought (pantingly) that one zeromode would become non-normalizable, e.g. for $\lambda_1 v < m < \lambda_2 v$.

This hope is not realized.

Remnant: sometimes zm profile is ring-like:

- For $m \neq m^\dagger$ no zero-energy solutions.
No-go arguments

1. If we gauge away or disorder the Station Q ribbon, the configuration space has $\pi_1(C_n) = S_n$.
2. Rough sketch of argument for inevitability of Witten anomaly:

| Witten anomaly | chiral anomaly mod two | majorana number of monopole |

In a Witten-anomalous theory, $(-1)^F = e^{i\pi j^\text{axial}} = e^{i\pi \tau^3}$ is a gauge symmetry. [Goldstone, 83]

$\Rightarrow$ chiral anomaly mod two is a gauge anomaly.

(In a normal theory: $\psi_L \rightarrow \bar{\psi}_R$.
Here: $\psi_L \rightarrow \text{vacuum}$.)

$$\text{ind}_{\mathbb{R}} \mathcal{D}[\text{monopole}] \equiv \oint_{S^2_{\infty}} \vec{\nabla} \cdot \vec{j}^\text{axial}$$

[Callias 78]: $\text{ind}_{\mathbb{C}} \mathcal{D}$

[Santos-Nishida-Chamon-Mudry, 09]: real index for vortex in 2d
5d model

Some theories are only realizable as the boundary of a higher-dimensional model. [Nielsen-Ninomiya, Kaplan]

\textit{e.g.:} domain-wall fermions in lattice QCD,
single dirac cones on surface of a topological insulator

Consider SU(2) gauge theory in 4+1 dimensions with a Dirac fermion doublet and adjoint Higgs.
On a circle: fourth spatial dimension \( y \approx y + 2\pi R \).

Kink of \( M(r) \)
supports a 4d massless Weyl fermion.

Bad features: 5d; needs UV completion (lattice, strings); kinks can annihilate.
Mass scales: \( M_W, R^{-1} \), the Dirac mass \( m \),
the inverse thickness of the kink, extreme UV cutoff
At energies \( E \ll 1/R \), this model reduces to the two-doublet theory above.
Majorana monopole strings

\(q \in \pi_2(S^2)\) supports monopole strings. (3d particles when stretched along \(y\)). Intersections between monopole strings and domain walls of 5d mass \(\rightarrow\) localized Majorana zeromodes.

- the two Majorana modes need not pair up. For \(m \gg R^{-1}\), their wavefunction overlap is exponentially small.
- low-energy braiding always exchanges majoranas in pairs
- dyon rotor \(\rightarrow\) 1+1 XY model along string. decoherence to local basis or linear confinement from monopole string tension:
Disordering the SU(2)

Imagine a state with hedgehogs but zero stiffness (no LRO). Effective field theories for such states are usefully studied using “slave particle” techniques (successful in similar problem of spin liquid states). Result: emergent U(1) gauge theory, under which the defects are magnetically charged. Again requires UV completion. One way to do it: SU(2) gauge theory with a Weyl doublet.

Another attempt [Freedman, Hastings, Nayak, Qi, 1107.2731]: a lattice model (majorana fermions with hopping amplitudes determined by a quantum dimer model configuration) They argue for a majorana zeromode on the defect. But: gapless bulk fermions. Reduces to *two-doublet* model with \( m = 0, \lambda_1 \neq 0, \lambda_2 = 0 \)!
Other possible loopholes?

- What if we just gauge the $U(1) \subset SU(2)$? Still linear tension.
- What if the 5th dimension ends? For some boundary conditions: gapless 4d mode [Station Q].
- Lorentz-breaking fermion kinetic terms [Tong]? Still anomalous?

Conclusion: It would be nice to tighten the no-go statement (prove the Callias I R index theorem, understand the functional $\Gamma$) and it will be interesting to see what other physics has to come in to save the world from non-Abelian statistics in 3+1 dimensions.

Final positive comment: These issues are important for understanding possible generalizations of the notion of flux attachment from 2+1 to 3+1.
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The end

Thanks for listening.