Alga (coamoeba)

Alga of Laurant polynomial W(x,y) is given by the image of the zero locus of W(x,y) by the argument map

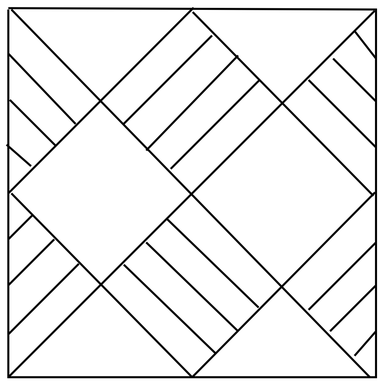
$$\begin{array}{ccc} (\mathbb{C}^\times)^2 & \to & T=(\mathbb{R}/\mathbb{Z})^2 \\ \cup & & \cup \\ (x,y) & \mapsto & \frac{1}{2\pi}(\arg(x),\arg(y)) \end{array}$$

cf. Amoeba (arg−>log| |)

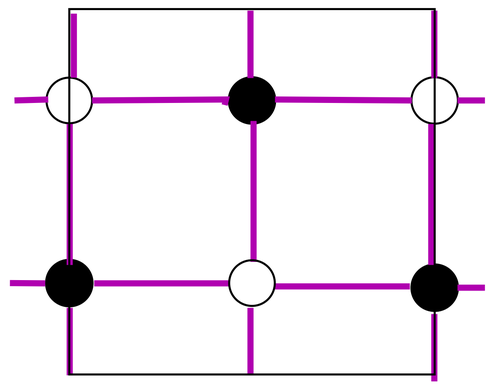
tropicalization of amoeba=(p,q) web

e.g $W = x - \frac{1}{x} + y + \frac{1}{y}$

Alga



Brane Tiling

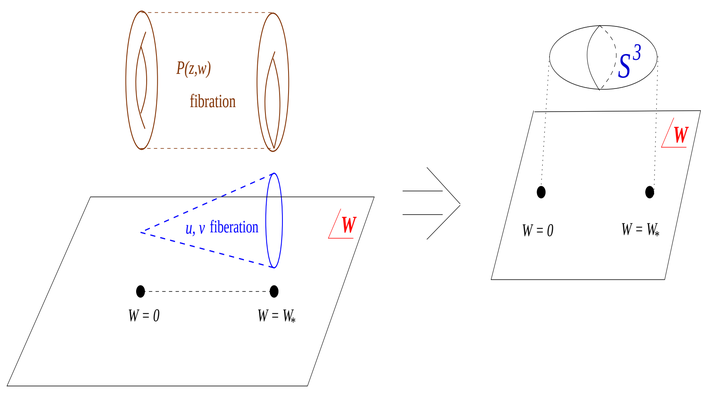


Brane tiling is essentially alga!!

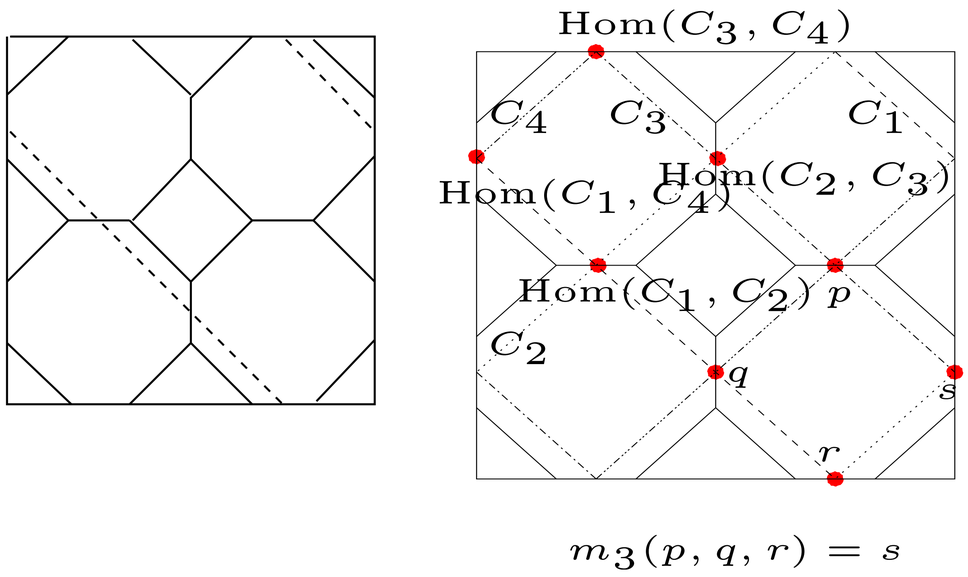
Disc: vertex
orientation: white/black

4. Application to Homological Mirror Symmetry

Take further T−dual : D6−brane (mirror of D3)
Double fibration over W−plane
W(x,y)=uv=W
W: Newton polynomial of toric diagram
D6−brane wraps vanishing cycles:



These vanish cycles can be read off in alga:



$m_3(p,q,r) = s$

Homological Mirror Symmetry: [Kontsevich]

$$\underbrace{D^b \operatorname{coh} Z}_{B-model} \cong \underbrace{D^b\, \mathfrak{Fuk}^{\to}\, W}_{A-model}$$

Z: toric Fano

W: superpotential (Newton Polynomial)

Proven for F0, toric del Pezzo etc.

LHS: calculated by Bondal's theorem

RHS: difficult to compute

but easily calculated using algae!!

Fukaya category

Object: vanishing cycles (D6−brane)

Morphism: Intersection point of vanishing cycles (open strings)

compositiion of morphism: (superpotential or disk amplitude in hol. CS)

Theorem [Ueda−Y]

Homological mirror symmetry is correct for abelian orbifolds of $\mathbb{P}^1 \times \mathbb{P}^1$

(It's almost trivial to include orbifold case in this proof)

5. Summary and Outlook

Summary

Brane tiling and alga: useful for understanding dual gauge theory
The correctness of the algorithm can be rigorously proven in some cases
Also useful for homological mirror symmetry

Future Works

Extension to other cases (e.g. toric del Pezzo, to appear)
Seiberg duality, "Phases" of N=1 theories? cascading?
Relation with derived category approach? Tachyon condensation?
Relation with topological strings, instanton counting, free fermion?
Beyond brane tiling : real shape of branes? beta function?
Application to phenomenology, e.g. dynamical SUSY breaking