Brane Tilings, Algae and Quiver Gauge Theories
- with Application to Homological Mirror Symmetry -

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By placing D3-brane as a probe at the tip of the Calabi-Yau cone, we have a 4-dimensional superconformal field theory on that D3-brane. Understanding which Calabi-Yau corresponds to which gauge theory is an important problem, especially in light of AdS/CFT correspondence. The case when Calabi-Yau is toric is analyzed most extensively, and it is long believed that the gauge theory in question is quiver gauge theory. However, converting fan to quiver and vice versa has been a difficult problem in practice.

Recently, Hanany and collaborators have come up with an ingenious method to do just that, using the notion of *brane tilings*. Their physical meaning is later clarified: by taking T-duality twice, probe D3-branes turn into D5-NS5 system. The projection of the shape of NS5-branes onto D5-branes gives "*alga*", and brane tilings are obtained from algae by certain reduction.

Despite these impressive developments, brane tiling algorithm has been checked only in several examples. We have thus in [2, 3] given a precise mathematical formulation and rigorous proof of Hanany-Vegh algorithm([1]; this gives quiver and superpotential from toric diagram) in the case where toric diagram is triangle or paralellogram. In the triangle case (that is, $\mathbb{C}^3/A$, A: abelian subgroup of $SL_3(\mathbb{C})$) it was moreover shown that the quiver we obtained by the algorithm is nothing but the McKay quiver of A.

We have also used the technology of brane tilings and algae to prove homological mirror symmetry. By taking further T-duality, we have D6-branes mirror to original D3-brane picture. These D6-brane wraps so-called vanishing cycles, and these cycles can be represented on alga. In this way, we have computed Fukaya category (category of A-branes), and proved homological mirror symmetry for abelian orbifolds of $\mathbb{P}^1 \times \mathbb{P}^1$ ([3]). This result is not only new in mathematics, but it also gives physically intuitive understanding of the proof.

These works are in collaboration with Kazushi Ueda (Osaka U.).

References

[1] A. Hanany and D. Vegh, "Quivers, tilings, branes and rhombi," arXiv:hep-th/0511063.

[2] K. Ueda and M. Yamazaki, "Brane tilings for parallelograms with application to homological mirror symmetry," arXiv:math.ag/0606548.

[3] K. Ueda and M. Yamazaki, "A Note on Brane Tilings and McKay Quivers," arXiv:math.ag/0605780.