

What is **Brane Tiling** ?

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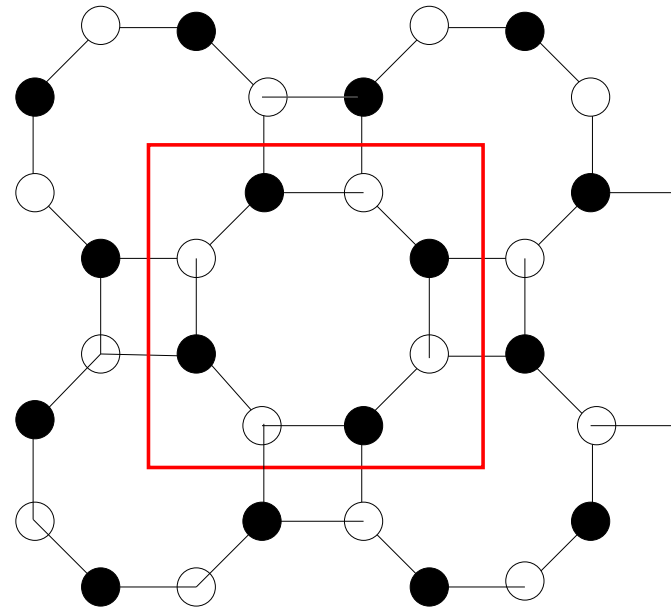
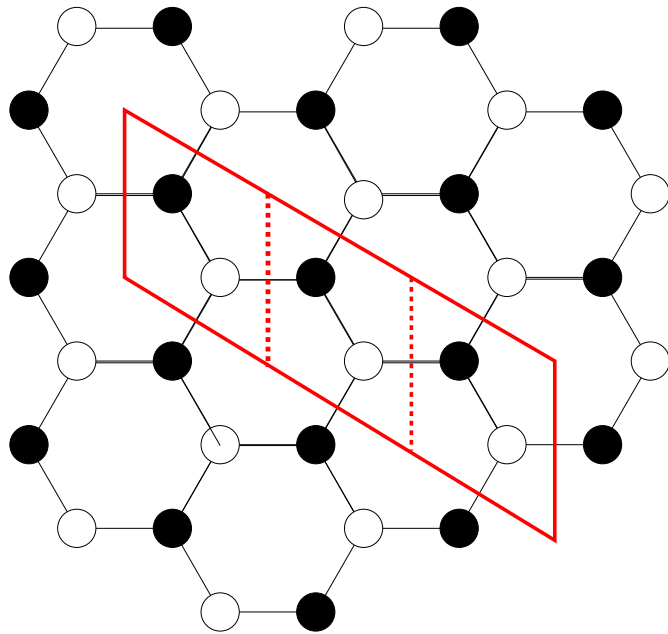
2007/03/06 @ Niseko

K. Ueda and M.Y., [math.AG/0605780](#), [0606548](#), [0703???](#)

Y. Imamura, H. Isono, K. Kimura, M.Y. [hep-th/0702049](#)

Brane Tilings as a Bipartite Graph

- Answer 1: "Brane Tiling" is a bipartite graph on torus (+ extra conditions):



Physical Meaning?

- Hanany and collaborators found that they are useful for studying certain kinds of gauge theories (4d $\mathcal{N} = 1$ superconformal quiver gauge theories).
- Checked in numerous examples, BUT we lack the **physical** explanation of why they work and what they really are.
- Recent advances in brane tilings have clarified the meaning of brane tilings, which we will now explain.

Plan of This Talk

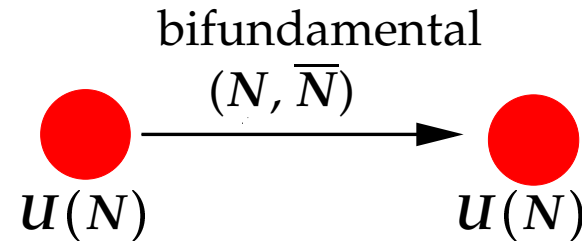
1. Introduction
2. **Physical Meaning of Brane Tiling**
3. Applications of Brane Tiling
4. Summary and Outlook

Quiver Gauge Theory

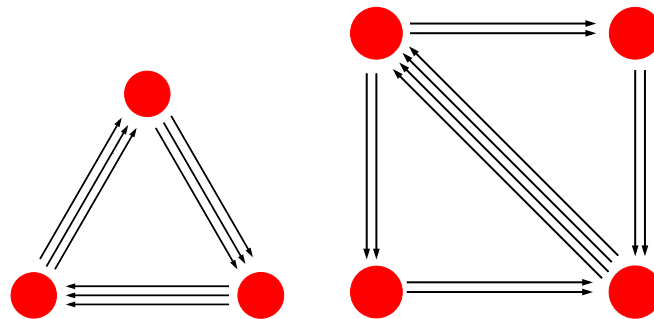
Quiver(**箭**): "portable case for holding arrows", an oriented graph

- vertex= gauge group

- oriented arrow= bifundamental



- each quiver specifies gauge theory



We are now going to consider 4d $\mathcal{N} = 1$ supersymmetric quiver gauge theories.

Quiver from String Theory?

Question

Can we realize quiver gauge theories in string theory?

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Answer

Yes! We use D5-branes and NS5-branes.

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cf. other settings for quiver gauge theory

- D3+Calabi-Yau

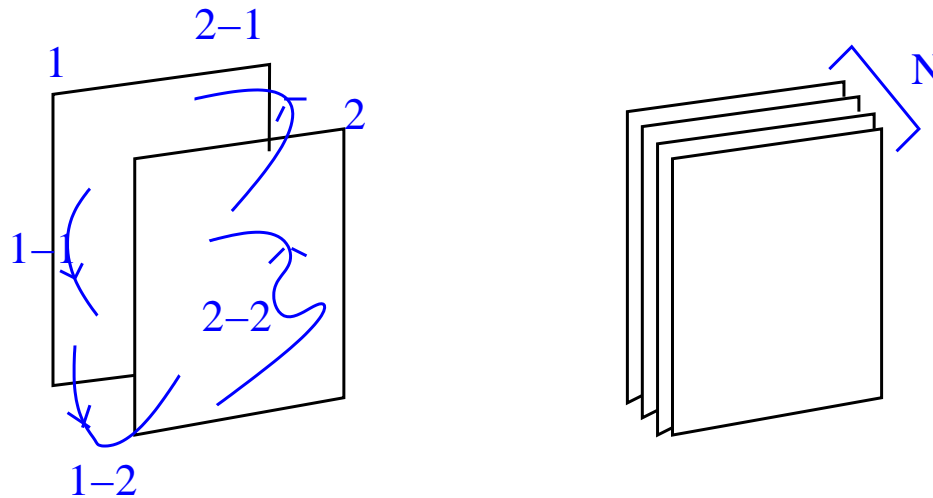
- D4+NS5 (elliptic model)

- D6+Calabi-Yau

CY \Rightarrow NS5

Brane Configurations

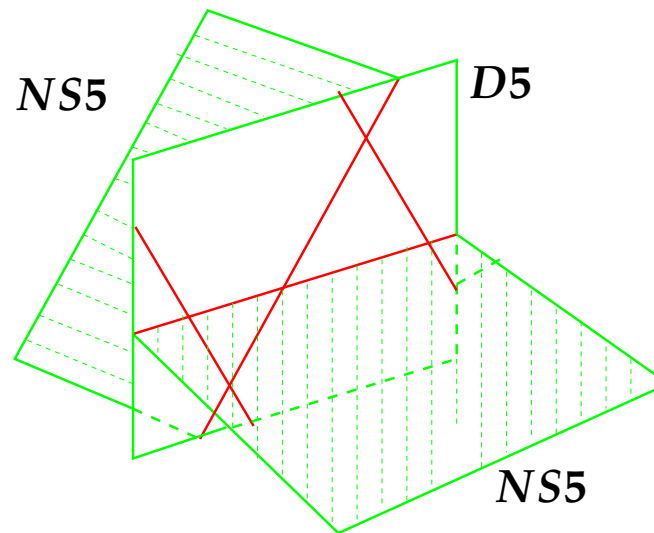
- First we prepare N D5-branes. When N D5-branes coincide, we have $U(N)$ gauge theory.



- In order to obtain $(3+1)$ -dim. theory, we compactify two directions of D5-brane on a torus. Then we have 4d $U(N)$ SYM on D5-brane.
- We still want obtain multiple gauge groups!

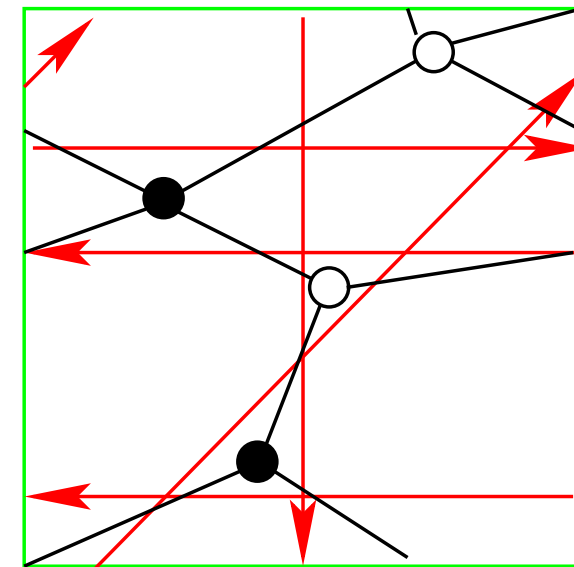
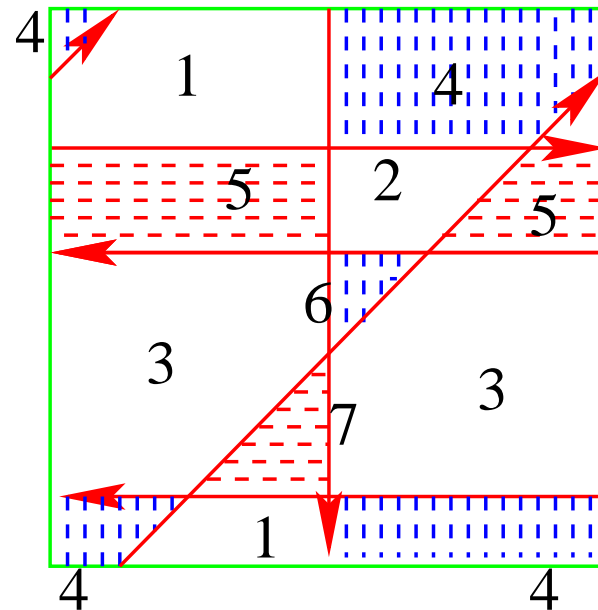
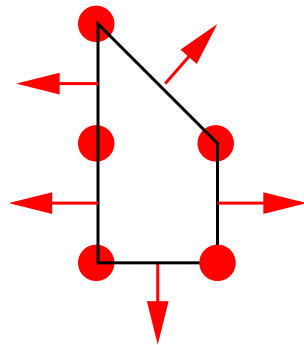
Brane Configurations

- Stack of N D5-branes are divided by NS5-branes into several regions, then SUSY is broken to $\mathcal{N} = 1$ and we have multiple gauge groups.



Brane Configurations

- Due to conservation of NS-charge, D5-brane actually becomes (N, k) -branes. ($k = 1, 0, -1$ in this talk)

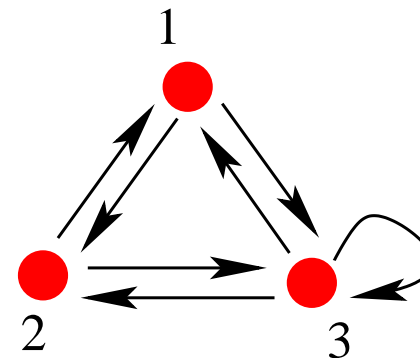
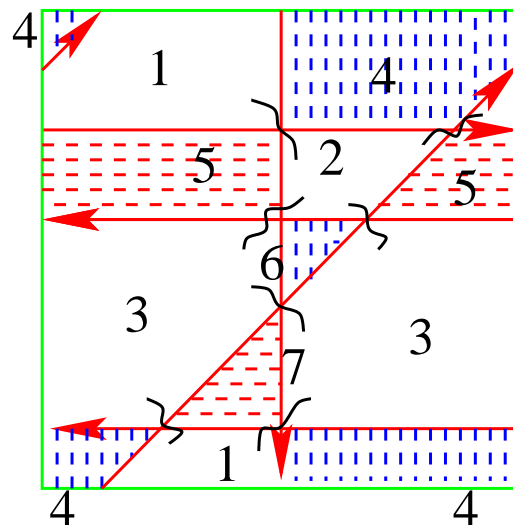


Blue Region: $(N, 1)$ -brane
 Red Region: $(N, -1)$ -brane
 White Region: $(N, 0)$ -brane

Dimer Model

Brane Configurations

- $U(N)$ gauge groups lives only on $(N, 0)$ -branes.
- We have a bifundamental for each intersection pt of $(N, 0)$ -branes.
- From this we can read off quiver!



Quiver

Blue Region: $(N, 1)$ -brane
Red Region: $(N, -1)$ -brane
White Region: $(N, 0)$ -brane

D5/NS5-System

- Summarizing, the brane configuration is:

	0	1	2	3	4	5	6	7	8	9	
D5	○	○	○	○		○		○			
NS5	○	○	○	○	Σ (2-dim surface)						

- ◆ D5-brane worldvolume: $\mathbf{R}^4 \times T^2$
- ◆ NS5-brane worldvolume: $\mathbf{R}^4 \times \Sigma$

- ◆ Projection of NS5 to 5 and 7 directions: brane tiling (coamoeba)
- ◆ Projection of NS5 to 4 and 6 directions: (p, q) -webs (amoeba)

Brane Tilings are Really Branes!

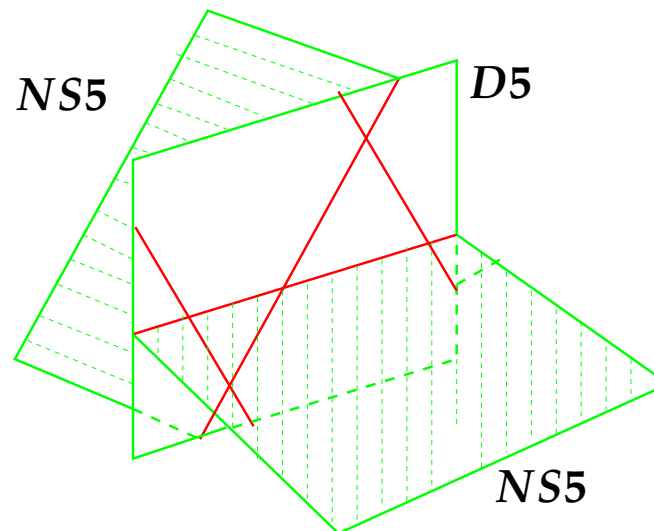
Answer 2 (physical answer)

Brane tilings represent (projection of) configurations of **physical** D5-branes and NS5-branes, which realizes quiver gauge theories in string theory.

This is important not only conceptually, but also for practical applications!

A Note on String Coupling

- Actually, we have so far talked about the limit $g_{str} \rightarrow \infty$.
- real shape of branes: difficult to determine in general (we need to solve EOM), but can be analyzed when $g_s \rightarrow 0$ and $g_s \rightarrow \infty$.
- when $g_s \rightarrow \infty$, $T_{D5} \gg T_{NS5}$, thus D5-branes become flat and NS5-branes are orthogonal to D5-branes.



Weak Coupling

Consider the weak coupling limit $g_s \rightarrow 0$. Then

$$T_{NS5} \gg T_{D5}$$

Then NS5-brane worldvolume Σ is a holomorphic curve $W(x, y) = 0$ in $(\mathbb{C}^\times)^2$, where

- $x = \exp(x_4 + ix_5), y = \exp(x_6 + ix_7)$
- $W(x, y)$ is a Newton Polynomial of the toric diagram

$$W(x, y) = \sum_{(i,j) \in \Delta} c_{(i,j)} x^i y^j$$

where $\Delta \in \mathbb{Z}^2$ is the toric diagram.

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Applications of Brane Tiling

Brane tiling has many applications.

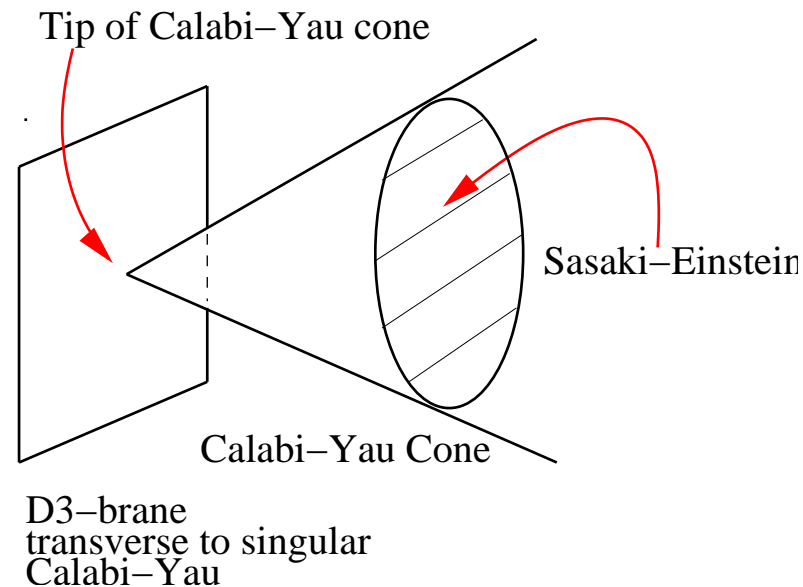
- Brane tilings clarifies the relation with Calabi-Yau geometry.
- Brane tiling gives superpotential of $\mathcal{N} = 1$ theory.
- AdS/CFT correspondence ($\mathcal{N} = 1$ version).
- Mirror symmetry
- Anomalies and exactly marginal deformations of quiver gauge theories

⋮

Relation with Calabi-Yau

Taking T-duality along 2-torus of D5/NS5-system, we have D3+Calabi-Yau.

We probe cone-type non-compact singular Calabi-Yau by D3-branes, then we have 4d $\mathcal{N} = 1$ quiver gauge theory on D3-branes.



D3-brane is transverse to CY and placed at the apex of

$$\text{CY cone: } \underbrace{\mathbb{R}^4}_{\text{D3-brane}} \times \underbrace{C(X_5)}_{\text{CY}}$$

Application to AdS/CFT

Most studied case of AdS/CFT:

$$\text{IIB on } AdS_5 \times S^5 \Leftrightarrow \mathcal{N} = 4 \text{ SYM}$$

We want to reduce SUSY to $\mathcal{N} = 1$! We replace S^5 by Sasaki-Einstein mfd X_5 .

Locally

$$AdS_5 \times X_5 \sim \mathbb{R}^4 \times C(X_5)$$

$$\underbrace{\overbrace{\left(\frac{du^2}{u^2} + u^2 ds_4^2 \right)}^{AdS_5}}_{\text{direct product}} + ds_{X_5}^2 = \underbrace{u^2 ds_4^2 + \frac{1}{u^2} (du^2 + u^2 ds_{X_5}^2)}_{\text{warped product}}$$

AdS/CFT ($\mathcal{N} = 1$ case)

AdS/CFT ($\mathcal{N} = 1$ version)

IIB on $AdS_5 \times X_5$ (X_5 : Sasaki-Einstein, $C(X)$: toric CY)
is dual to $\mathcal{N} = 1$ quiver gauge theory

Brane tiling gives gauge theory dual for each toric CY!

Simplest prediction:

$$\text{Vol}(X_5) = \frac{\pi^3}{4} \frac{1}{a}$$

where a is the central charge. Now checked for all toric CYs
(under certain assumptions)

Mirror Symmetry from Brane Tilings

Mirror Symmetry:

B-model on CY X = A-model on CY \hat{X}

Homological Mirror Symmetry:

$$\underbrace{D^b(\text{coh } X)}_{B\text{-branes}} \cong \underbrace{D^b(\mathcal{F}ut\hat{X})}_{A\text{-branes}},$$

Brane Tiling is useful for mirror symmetry [Ueda-M.Y]!

This method gives intuitive understanding from D-branes, and gives mathematically rigorous result at the same time.

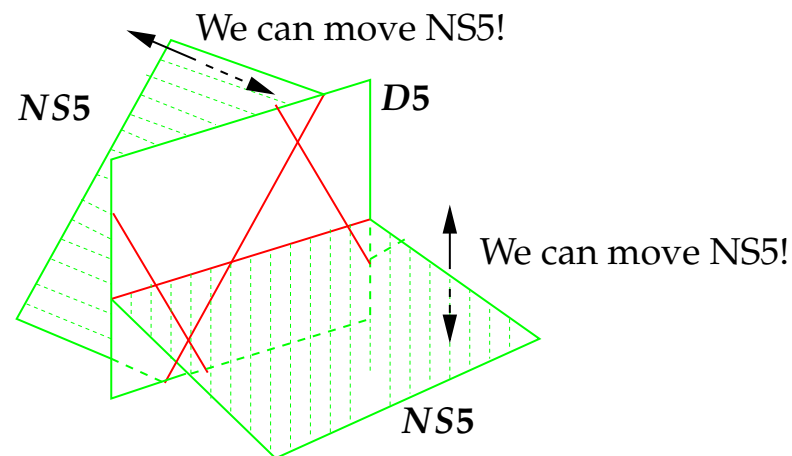
Exactly Marginal Deformations

Conformal manifold of quiver gauge theories

$\{\beta_a = \beta_k = 0\} \subset \{g_a, h_k\}$ g_a : gauge coupling, h_k : superpotential coupling

\Updownarrow [Imamura-Isono-Kimura-M.Y.]

Deformation of branes in brane tiling



Summary

- Brane tilings are simply graphs for some time, but we now know that they represent information of physical D5-branes and NS5-branes.
- They are powerful techniques to study quiver gauge theories, and have many applications!

Outlook: More to Come!

- DSB, metastable vacuum, gaugino condensation and χ -SB from brane tiling and D-branes?
- Relation with a-maximization?
- Higher dimensional generalization? (3d brane tiling)
- Inclusion of other dimension of D-branes? Solitonic states in quiver gauge theories?
- Relation with BPS state counting as in topological vertex, instanton counting in SYM, black holes?
"Plethystics" program
- Relation with tachyon condensation?
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