Heterotic—F Theory Duality Revisited

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Based on arXiv:arXiv:0805.1057[hep-th]
With H. Hayashi, R. Tatar, Y. Toda and T. Watari

Closely related to:

Donagi-Wijnholt ('08 Feb)

Beasley-Heckman-Vafa I ('08 Feb)

Some technical details relies on :

Donagi-Ovrut-He-Reinbacher ('04 May)

Blumenhagen-Moster-Reinbacher-Weigand ('06 Dec)

Today’s Topic: Het/F duality

- Want to understand **chiral matters in F-theory** by using duality with Heterotic theory

Our strategy

Cf. strategy taken by
Beaseley-Heckman-Vafa

What is F-theory?

- 12-dimensional reformulation of Type IIB with nonconstant axio-dilaton

$$\tau = C_0 + ie^{-i\phi} = \tau(z)$$

0	1	2	3	4	5	6	7	8	9	10	11
$\mathbb{R}^{3,1}$				7-brane				$z, \bar{z}$		fiber	

7-brane

Calabi-Yau four-fold

\`Fictious' dimensions

$$\tau(z) \sim \frac{i}{2\pi}\log(z - z_0) \quad \text{around 7-brane at } z = z_0$$

$$\tau \to \tau + 1 \quad \text{if we go around } z = z_0$$

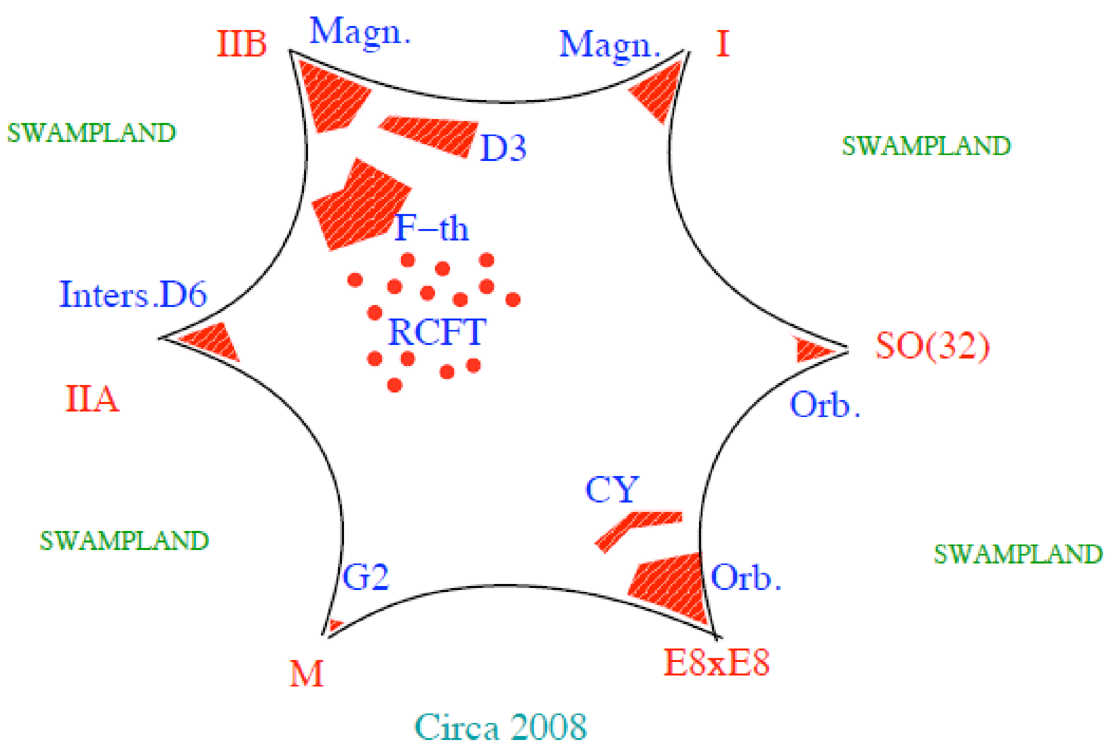
Dilaton can transform under SL(2, Z)-transformation under monodromy

Intrinsically non-perturbative (except for orientifold limit)

- Another definition: the limit of M-theory on T^2 with the size of torus T^2 -> 0
- Basic degrees of freedom (p,q)-strings (M2-branes wrapping cycles)
- (p,q) 7-branes encoded in geometry of CY4

Q. Why F-theory?

There are many other possibilities….



Q. W

ugh?

L. Ibanez @ strings 2008

Why F-theory?

- want to construct **GUT** (SU(5), SO(10), E_6...)

✓ SO(10) GUT **Spinor representation** (16) of SO(10) needed

✓ SU(5) GUT

generated perturbatively

Down-type Yukawa coupling

$$10^{ab}\bar{5}_a\bar{5}_{H\,b}$$

Up-type Yukawa coupling

$$10^{ab}10^{cd}5^e_H\epsilon_{abcde}$$

We need an epsilon tensor to construct a up-type Yukawa

We need SU(5), not U(5)

We need to go to the strong coupling regime

F/CY4, M/G2, Het-M…

More Excuses for studying F-theory

- Moduli stabilization good in F-theory
- More freedom in model building

  (e.g. gauge mediation)
- New mechanism available in F-theory without Heterotic dual

  (e.g. GUT breaking by $U(1)_Y$)
- Local model building/ bottom-up approach

  (we can forget about global issues for the moment…)

Today's Contents

1. Introduction
2. Chiral matters in F-theory: our result
3. Het/F duality and derivation of our result
4. Summary

Chiral matter in F-theory

- Today's problem: **to describe chiral matters in F-theory** (e.g. 10's and 5bar's in SU(5) GUT)

(How are quarks and leptons described in F-theory?)

Why so many developments in 2008?
After all, F-theory has 10 years of history….

The difficulties

- `F-theory' has no intrinsic formulation

Het/F dual useful

- Matters takes values in sheaf-valued cohomology; the sheaf is in fact not a line bundle

Mathematically complicated

- Want to understand codimension 3 singularities in F-theory

(almost impossible from D7+O7 viewpoint)

The advantage of our approach:
Codimension 3 singularities are under control

WHAT IS THE ANSWER?

Our Main Result

- 5bar's in SU(5) GUT

$$H^0\left(\tilde{\bar{c}}_5; \mathcal{O}(K_{B_2} + \frac{1}{2}\tilde{b}^{(c)} + \gamma)\right)$$

Four-form flux in F-theory

Covering matter curve

Codim 3 singularity

a divisor on the matter curve

Interestingly, not all, but some of the codim. 3 singularities contribute to chiral matter!

Our result: Chirality Formulae

10's: $$\chi(V) = \int_{C\times\bar{c}_V} G_F^{(4)}$$ 4-form flux

5bar's: $$\chi(\wedge^2 V) = \int_{C\times\tilde{\bar{c}}_{\wedge^2 V}} G_F^{(4)}$$

a vanishing cycle

Blow-up of matter curve (covering matter curve)

- Highly non-trivial point:

Contribution from codimension 3 singularity cancels out In the chirality formula!

HOW WE DERIVED THIS?
LET US BEGIN BY RECALLING HET/F DUALITY…

Het/F duality

- 8d Het/F duality

$$\boxed{\mathrm{Het}/T^2} \Longleftrightarrow \boxed{F/(T^2 \to K3)}$$

<u>check</u>

Narain moduli of Het theory matches with the moduli space of elliptic CY

$$SO(18,2;\mathbb{Z})\backslash SO(18,2)/SO(18)\times SO(2)$$

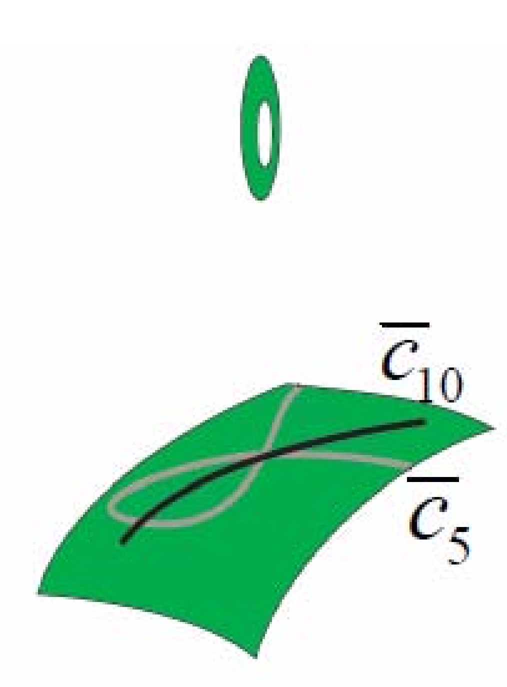
Het: 16 Wilson lines + 2 complex/Kahler moduli of torus+ 1 (real) string coupling

F-theory: 9+13-3-1=18 complex parameters and one real parameter (size of P^1)

$$y^2 = x^3 + \underbrace{g_2(z)x}_{\text{Degree 8}} + \underbrace{g_3(z)}_{\text{Degree 12}}$$

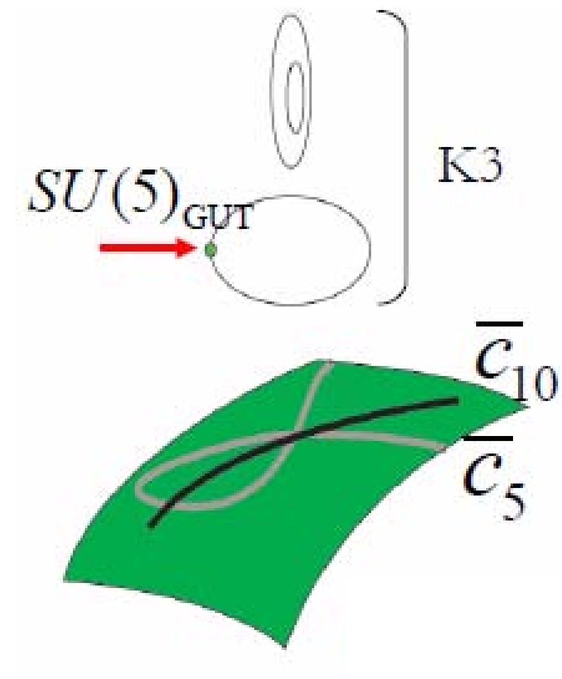
- 4d Het/F duality (Fiberwise application of 8d Het/F duality)

Het on elliptically fibered CY3 ⟺ **F on K3-fibered CY4, where K3-fiber is a elliptic fibration**



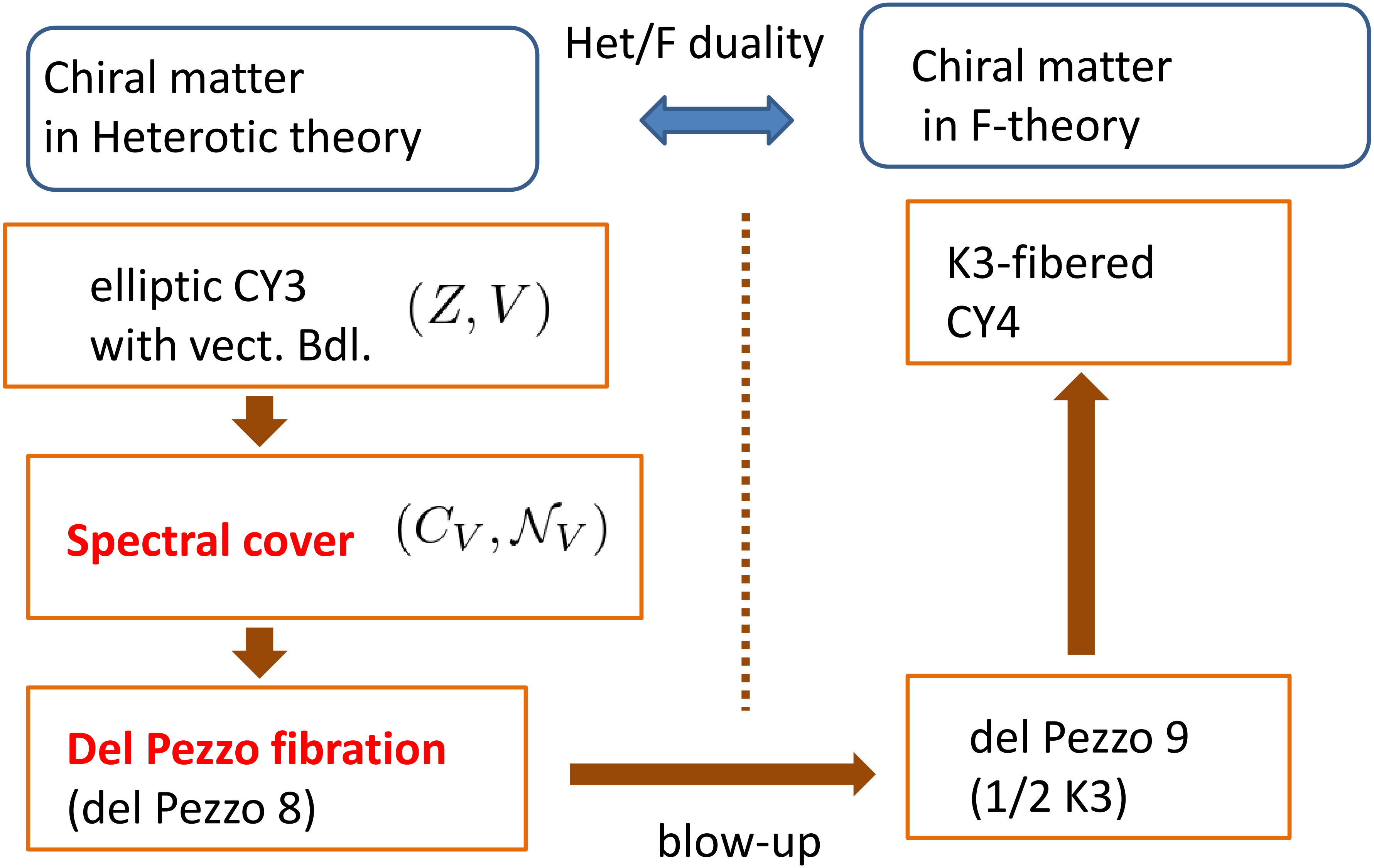
ell. fiber

dim$_C$=2



DERIVATION OF OUR RESULT

Roadmap


Chiral matter
in Heterotic theory
Het/F duality
Chiral matter
in F-theory
elliptic CY3
with vect. Bdl.
(Z, V)
K3-fibered
CY4
Spectral cover
$(C_V, \mathcal{N}_V)$
Del Pezzo fibration
(del Pezzo 8)
blow-up
del Pezzo 9
(1/2 K3)

Data in Heterotic Theory

- Consider $E_8 \times E_8$ Het on elliptically fibered Calabi-Yau Z_3 $\quad T^2 \to Z_3 \to B_2$

- Consider a SU(N) vector bundle on Z_3

(let's concentrate on `our' E_8)

⇒ Gauge symmetry is broken to commutant of SU(N)

(e.g. SU(5) bundle -> SU(5) GUT
SU(4) bundle -> SO(10) GUT)

Chiral matter in Heterotic Theory

- Matters come from decomposition of

  adj. repr. of E_8

e.g. SU(5) bundle, SU(5) GUT

V $\Lambda^2 V$

$$\mathbf{248} \to (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus [(\mathbf{10},\mathbf{5}) \oplus (\bar{\mathbf{5}},\mathbf{10})] + \mathrm{h.c.};$$

adj. 10 repr. 5bar repr.

In general, chical matters take values in

$$H^1(Z;\rho(V))$$

$$\rho(V) = V, V^\times, \wedge^2 V, \wedge^2 V^\times, \ldots, \pi_Z^* E$$

structure group of V	SU(2)	SU(3)	SU(4)	SU(5)	SU(6)
unbroken symmetry H	E_7	E_6	SO(10)	SU(5)	SU(3) × SU(2)
from V	$\mathbf{56}$	$\mathbf{27}$	$\mathbf{16}$	$\mathbf{10}$	$(\bar{\mathbf{3}}, \mathbf{2})$
from $V^\times$	(vct.-like)	$\overline{\mathbf{27}}$	$\overline{\mathbf{16}}$	$\overline{\mathbf{10}}$	$(\mathbf{3}, \mathbf{2})$
from $\wedge^2 V$	—	—	$\mathbf{10}$	$\bar{\mathbf{5}}$	$(\mathbf{3}, \mathbf{1})$
from $\wedge^2 V^\times$	—	—	(vct.-like)	$\mathbf{5}$	$(\bar{\mathbf{3}}, \mathbf{1})$
from $\wedge^3 V$	—	—	—	—	$(\mathbf{1}, \mathbf{2})$
from $\pi_Z^* E$	**adj.**	**adj.**	**adj.**	**adj.**	**adj.**

Matter localizes to “matter curves”

Matters localize to “matter curves”

[Curio, Diaconescu-Ionessei ’98]

$$\boxed{H^1(Z;\rho(V)) \simeq H^0(\bar{c}_{\rho(V)};\mathcal{F}_{\rho(V)})}$$

a curve in base B_2

a sheaf on the matter curve

Matter curve

In Heterotic theory: vanishing locus of Wilson lines

In F-theory: the intersection of 7-branes

Sheaf on the matter curve

$\mathcal{F}_{\rho(V)}$ Is a sheaf, not a line bundle in general

Our Main Result (shown again)

We have done a detailed study of the geometry to determine the sheaf completely

- 5bars's in SU(5) GUT

$$H^0\left(\tilde{\tilde{c}}_5; \underbrace{\mathcal{O}(K_{B_2} + \frac{1}{2}\tilde{b}^{(c)} + \gamma)}_{\text{The sheaf } \mathcal{F}_{\rho(V)}}\right)$$

The sheaf $\mathcal{F}_{\rho(V)}$

Four-form flux in F-theory

Covering matter curve

WHAT IS G (FOUR-FORM FLUX) AND γ?
WHAT IS THE \`COVERING' MATTER CURVE?

What is G and γ?

- 5bars's in SU(5) GUT

$$H^0\left(\tilde{\bar{c}}_5; \mathcal{O}(K_{B_2} + \frac{1}{2}\tilde{b}^{(c)} + \gamma)\right)$$

Specifies the `twist' of vector bundle over B_2 in Het

- Chirality formua

$$\chi(\wedge^2 V) = \int_{C\times\tilde{\bar{c}}_{\wedge^2 V}} G_F^{(4)}$$

Four-form flux in F-theory

Four-form flux (G-flux)

- In F-theory, we have a **four-form flux** $G_4 = dC_3$

C_3 has one index in T^2, and two indices in other directions B_2

Divided into two parts:

- NS-NS/R-R flux in type IIB

$$C_3^9 = (B_{RR} - \tau B_{NS}) \wedge (dx - \tau dy)$$

- Gauge fields on 7-branes

$$C_3^\gamma = \sum_I A_I \wedge \omega^I$$

Unified in F-theory

Relation between G and γ

$$G_4 = G_4^{\gamma} + {G_4}^9$$

Gauge fields on 7-branes
Corresponds to γ
(\`twist' of vector bundle)

$$\pi_*(G_4^{\gamma}) = G_4^{Het}$$

$$\gamma = \int_C G_4^{Het}$$

Type IIB NS-NS/R-R flux
Appears when we blow-up the del Pezzo 8 to del Pezzo 9

$$\pi : dP_9 \to dP_8$$

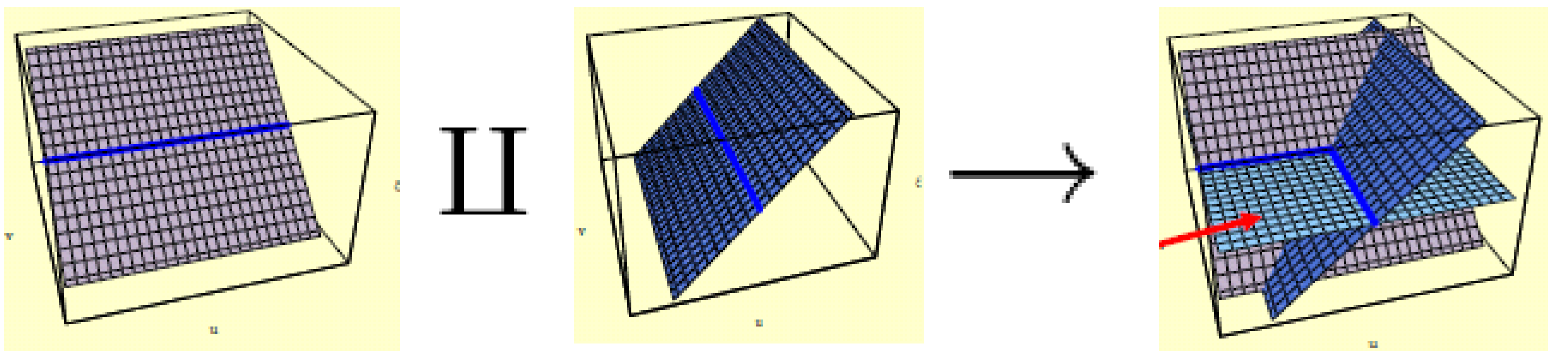
No Heterotic Analogue

Vanishing cycles
wrapped by M2-branes

`Covering' Matter Curve

If we resolve the double curve singularity, the sheaf becomes a line bundle!

$$\pi : \tilde{\bar{c}}_{\wedge^2 V} \longrightarrow \bar{c}_{\wedge^2 V}$$



$$\tilde{\mathcal{F}} \to \mathcal{F} = \pi_* \tilde{\mathcal{F}}$$

a line bundle　　　sheaf, not a line bundle

SO, WHAT HAVE WE LEARNED AFTER ALL?

Summary

- Using Heterotic—F theory duality, we have obtained detailed description of chiral matters in F-theory
- Our results are consistent with DW/BHV, but we have in addition clarified codimension 3 singularities
- New ingredients, such as chirality formulae and covering matter curves

More to come!

- Understanding our formula from purely F-theory viewpoint (translation is not enough)….
- Flavor structure/Yukawa coupling (To what extent can we learn about model-independent predictions?)
- Model building (e.g. SUSY breaking, gauge mediation etc.
  many works along this line….)

Thank you!