Heterotic—F Theory Duality Revisited

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Closely related to:

- Donagi-Wijnholt (‘08 Feb)
- Beasley-Heckman-Vafa I (‘08 Feb)

Some technical details relies on:

- Donagi-Ovrut-He-Reinbacher (‘04 May)
- Blumenhagen-Mostre-Reinbacher-Weigand (‘06 Dec)
Today’s Topic: Het/F duality

- Want to understand **chiral matters in F-theory** by using duality with Heterotic theory

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**Our strategy**

1. Heterotic

2. Het/F duality

3. F-theory

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Cf. strategy taken by Beasley-Heckman-Vafa
What is F-theory?

- 12-dimensional reformulation of Type II B with nonconstant axio-dilaton

\[ \tau = C_0 + ie^{-i\phi} = \tau(z) \]

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
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<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbb{R}^{3,1} )</td>
<td>7-brane</td>
<td>( z, \bar{z} )</td>
<td>fiber</td>
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\[ \tau(z) \sim \frac{i}{2\pi} \log(z - z_0) \quad \text{around 7-brane at } z = z_0 \]

\[ \tau \rightarrow \tau + 1 \quad \text{if we go around } z = z_0 \]
Dilaton can transform under $\text{SL}(2, \mathbb{Z})$-transformation under monodromy

Intrinsically non-perturbative (except for orientifold limit)

- Another definition: the limit of M-theory on $T^2$ with the size of torus $T^2 \to 0$

- Basic degrees of freedom $(p,q)$-strings (M2-branes wrapping cycles)

- $(p,q)$ 7-branes encoded in geometry of CY4
Q. Why F-theory?

There are many other possibilities....

Q. Why Beasley-Heckman-Vafa is not enough?

L. Ibanez @ strings 2008
Why F-theory?

- want to construct **GUT** (SU(5), SO(10), E\(_6\)…)

<table>
<thead>
<tr>
<th>✓ SO(10) GUT</th>
<th><strong>Spinor representation</strong> (16) of SO(10) needed</th>
</tr>
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<tbody>
<tr>
<td>✓ SU(5) GUT</td>
<td>generated perturbatively</td>
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- **Down-type Yukawa coupling**

\[ 10^{ab} \bar{5}_a \bar{5}_b H_b \]

- **Up-type Yukawa coupling**

\[ 10^{ab} 10^{cd} 5^e_H \epsilon_{abcde} \]

We need an epsilon tensor to construct a up-type Yukawa

We need SU(5), not U(5)

We need to go to the **strong coupling** regime

F/CY4, M/G2, Het-M…
More Excuses for studying F-theory

- Moduli stabilization good in F-theory
- More freedom in model building
  (e.g. gauge mediation)
- New mechanism available in F-theory without Heterotic dual
  (e.g. GUT breaking by U(1)_{\gamma})
- Local model building/ bottom-up approach
  (we can forget about global issues for the moment...)
Today’s Contents

1. Introduction
2. Chiral matters in F-theory: our result
3. Het/F duality and derivation of our result
4. Summary
Today’s problem: to describe chiral matters in F-theory (e.g. 10’s and 5bar’s in SU(5) GUT)

(How are quarks and leptons described in F-theory?)

Why so many developments in 2008?
After all, F-theory has 10 years of history....
The difficulties

- "F-theory" has no intrinsic formulation
- Matters takes values in sheaf-valued cohomology; the sheaf is in fact not a line bundle
- Want to understand codimension 3 singularities in F-theory (almost impossible from D7+O7 viewpoint)

The advantage of our approach: Codimension 3 singularities are under control
WHAT IS THE ANSWER?
Our Main Result

• 5bar’s in SU(5) GUT

\[ H^0 \left( \tilde{c}_5; \mathcal{O}(K_{B_2} + \frac{1}{2} \tilde{b}(c) + \gamma) \right) \]

- Covering matter curve
- Codim 3 singularity
- A divisor on the matter curve

Interestingly, not all, but some of the codim. 3 singularities contribute to chiral matter!
Our result: Chirality Formulae

10’s: $$\chi(V) = \int_{C \times \bar{c}_V} G_F^{(4)}$$

5bar’s: $$\chi(\wedge^2 V) = \int_{C \times \tilde{c} \wedge^2 V} G_F^{(4)}$$

- Highly non-trivial point: Contribution from codimension 3 singularity cancels out in the chirality formula!
HOW WE DERIVED THIS?
LET US BEGIN BY RECALLING HET/F DUALITY...
Het/F duality

- 8d Het/F duality

\[ \text{Het/T}^2 \leftrightarrow \text{F/(T}^2\rightarrow\text{K3)} \]

check

Narain moduli of Het theory matches with the moduli space of elliptic CY

\[ SO(18, 2; \mathbb{Z}) \backslash SO(18, 2)/SO(18) \times SO(2) \]

Het: 16 Wilson lines + 2 complex/Kahler moduli of torus + 1 (real) string coupling

F-theory: 9 + 13 - 3 - 1 = 18 complex parameters and one real parameter (size of \( \mathbb{P}^1 \))

\[ y^2 = x^3 + g_2(z)x + g_3(z) \]

Degree 8  Degree 12
• 4d Het/F duality (Fiberwise application of 8d Het/F duality)

Het on elliptically fibered CY3

F on K3-fibered CY4, where K3-fiber is a elliptic fibration

ell. fiber

dim$_C$=2
DERIVATION OF OUR RESULT
Roadmap

Chiral matter in Heterotic theory

Het/F duality

Chiral matter in F-theory

elliptic CY3 with vect. Bdl. $(Z, V)$

Spectral cover $(\mathcal{C}_V, \mathcal{N}_V)$

Del Pezzo fibration (del Pezzo 8)

K3-fibered CY4

del Pezzo 9 (1/2 K3)

blow-up
Data in Heterotic Theory

• Consider $E_8 \times E_8$ Het on elliptically fibered Calabi-Yau $Z_3$ $T^2 \rightarrow Z_3 \rightarrow B_2$

• Consider a SU($N$) vector bundle on $Z_3$
  (let’s concentrate on ‘our’ $E_8$)

Gauge symmetry is broken to commutant of SU($N$)

(e.g. SU(5) bundle $\rightarrow$ SU(5) GUT
SU(4) bundle $\rightarrow$ SO(10) GUT)
Chiral matter in Heterotic Theory

- Matters come from decomposition of adj. repr. of $E_8$

  e.g. SU(5) bundle, SU(5) GUT

$$248 \rightarrow (24, 1) \oplus (1, 24) \oplus [(10, 5) \oplus (\bar{5}, 10)] + \text{h.c.}$$
In general, chical matters take values in

\[ H^1(Z; \rho(V)) \]

\[ \rho(V) = V, V^\times, \wedge^2 V, \wedge^2 V^\times, \ldots, \pi^*_Z E \]

<table>
<thead>
<tr>
<th>structure group of V</th>
<th>SU(2)</th>
<th>SU(3)</th>
<th>SU(4)</th>
<th>SU(5)</th>
<th>SU(6)</th>
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<tr>
<td>unbroken symmetry ( H )</td>
<td>( E_7 )</td>
<td>( E_6 )</td>
<td>SO(10)</td>
<td>SU(5)</td>
<td>SU(3) ( \times ) SU(2)</td>
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<tr>
<td>from ( V )</td>
<td>56</td>
<td>27</td>
<td>16</td>
<td>10</td>
<td>(3, 2)</td>
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<tr>
<td>from ( V^\times ) (vct.-like)</td>
<td>27</td>
<td>16</td>
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<td>from ( \wedge^2 V )</td>
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<tr>
<td>from ( \wedge^3 V )</td>
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<td>adj.</td>
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<tr>
<td>from ( \pi^*_Z E )</td>
<td>adj.</td>
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Matter localizes to “matter curves”

Matters localize to “matter curves”
[Curio, Diaconescu-Ionessei ’98]

\[
H^1(Z; \rho(V)) \cong H^0(\tilde{c}_\rho(V); \mathcal{F}_\rho(V))
\]

Matter curve
In Heterotic theory: vanishing locus of Wilson lines
In F-theory: the intersection of 7-branes

Sheaf on the matter curve
\[ \mathcal{F}_\rho(V) \] Is a sheaf, not a line bundle in general
Our Main Result (shown again)

We have done a detailed study of the geometry to determine the sheaf completely

- 5bars’s in SU(5) GUT

\[ H^0 \left( \tilde{c}_5; \mathcal{O}(K_{B2} + \frac{1}{2} \tilde{b}(c) + \gamma) \right) \]

The sheaf \( \mathcal{F}_\rho(V) \)

Four-form flux in F-theory

Covering matter curve
WHAT IS G (FOUR-FORM FLUX) AND $\gamma$?
WHAT IS THE ‘COVERING’ MATTER CURVE?
What is G and γ?

• 5bars’s in SU(5) GUT

\[ H^0 \left( \tilde{c}_5; \mathcal{O}(K_{B_2} + \frac{1}{2} \tilde{b}(c) + \gamma) \right) \]

• Chirality formua

\[ \chi(\wedge^2 V) = \int_{C \times \tilde{c} \wedge^2 V} G_F^{(4)} \]

Specifies the `twist' of vector bundle over B_2 in Het

Four-form flux in F-theory
Four-form flux (G-flux)

- In F-theory, we have a **four-form flux** \( G_4 = dC_3 \)

  \( C_3 \) has one index in \( T^2 \), and two indices in other directions \( B_2 \)

  Divided into two parts:

  - **NS-NS/R-R flux** in type IIB
    \[
    C_3^9 = (B_{RR} - \tau B_{NS}) \wedge (dx - \tau dy)
    \]

  - **Gauge fields on 7-branes**
    \[
    C_3^\gamma = \sum_I A_I \wedge \omega^I
    \]  

Unified in F-theory
Relation between $G$ and $\gamma$

$$G_4 = G_4^\gamma + G_4^9$$

Gauge fields on 7-branes
Corresponds to $\gamma$ (‘twist’ of vector bundle)

Type IIB NS-NS/R-R flux
Appears when we blow-up the del Pezzo 8 to del Pezzo 9

$$\pi_*(G_4^\gamma) = G_4^{Het}$$

$$\gamma = \int_C G_4^{Het}$$

$$\pi : dP_9 \rightarrow dP_8$$

Vanishing cycles wrapped by M2-branes
No Heterotic Analogue
`Covering’ Matter Curve

If we resolve the double curve singularity, the sheaf becomes a line bundle!

\[ \pi : \widetilde{C} \wedge^2 V \rightarrow \overline{C} \wedge^2 V \]

\[ \widetilde{\mathcal{F}} \rightarrow \mathcal{F} = \pi_* \widetilde{\mathcal{F}} \]

a line bundle \hspace{1cm} \text{sheaf, not a line bundle}
SO, WHAT HAVE WE LEARNED AFTER ALL?
Summary

- Using Heterotic—F theory duality, we have obtained detailed description of chiral matters in F-theory.
- Our results are consistent with DW/BHV, but we have in addition clarified codimension 3 singularities.
- New ingredients, such as chirality formulae and covering matter curves.
More to come!

• Understanding our formula from purely F-theory viewpoint (translation is not enough)....
• Flavor structure/Yukawa coupling (To what extent can we learn about model-independent predictions?)
• Model building (e.g. SUSY breaking, gauge mediation etc. many works along this line....)
Thank you!