Gauge/YBE Correspondence

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(Jun ~) (Sep ~)

Jun/18/2013, Pre-Strings, KIAS

MY, to appear

See also

(F. Benini - T. Nishioka - M. Y. 1109.0283 PRD

M. Y. 1203.5784 JHEP

Y. Terashima + M. Y. 1203.5792 PRL

D. Xie + M. Y. 1207.0811 JHEP

V. V. Bazhanov - S. M. Sergeev

1006.0651, 1106.5874 NPB

V. V. Bazhanov, A. P. Kels, S. M. Sergeev 1301.5775

Motivation

exact results in SUSY theories $\Longleftrightarrow$ integrable models

many examples

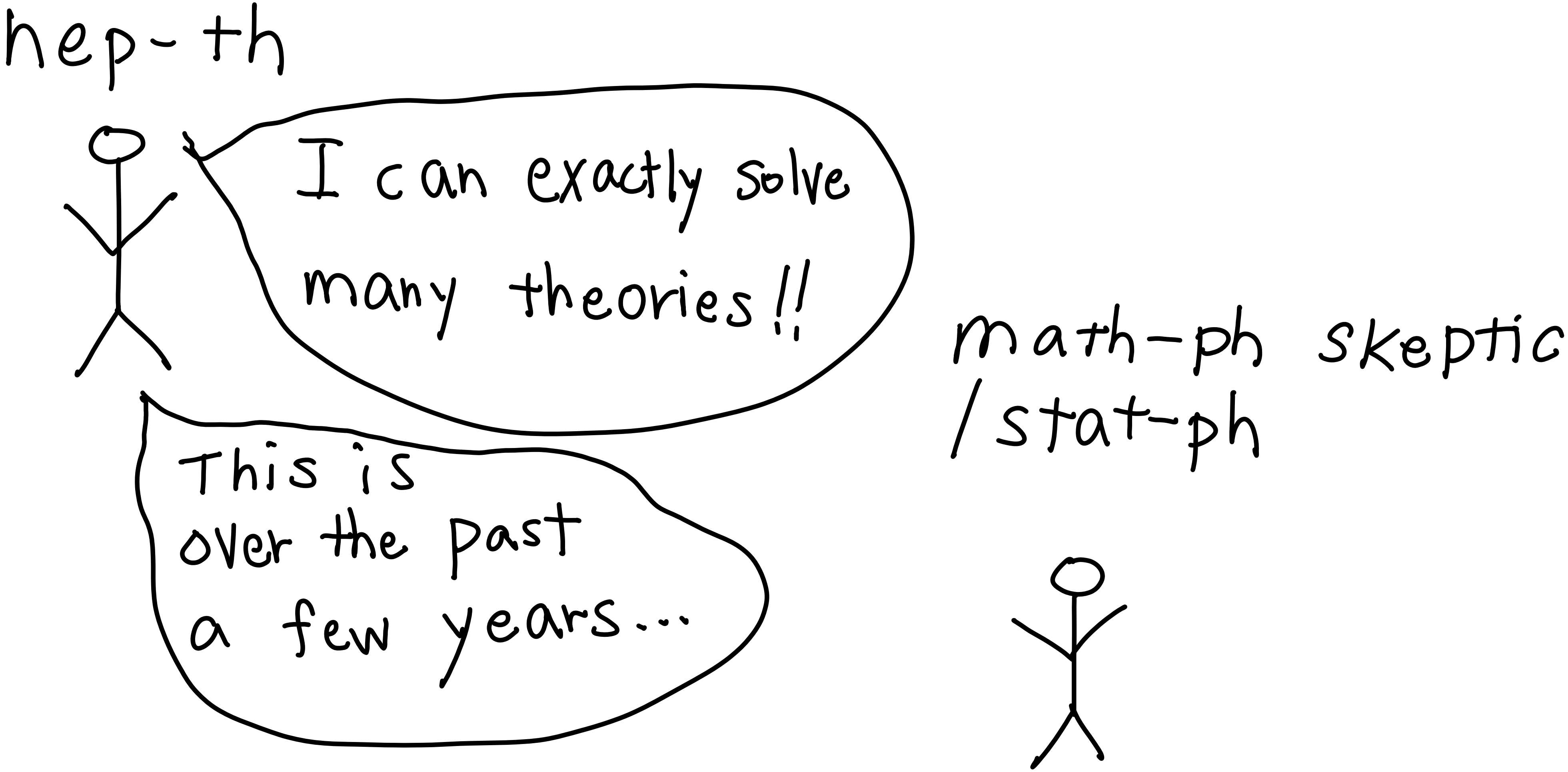
e.g. • integrability in 4d $\mathcal{N}=2$ SW theories

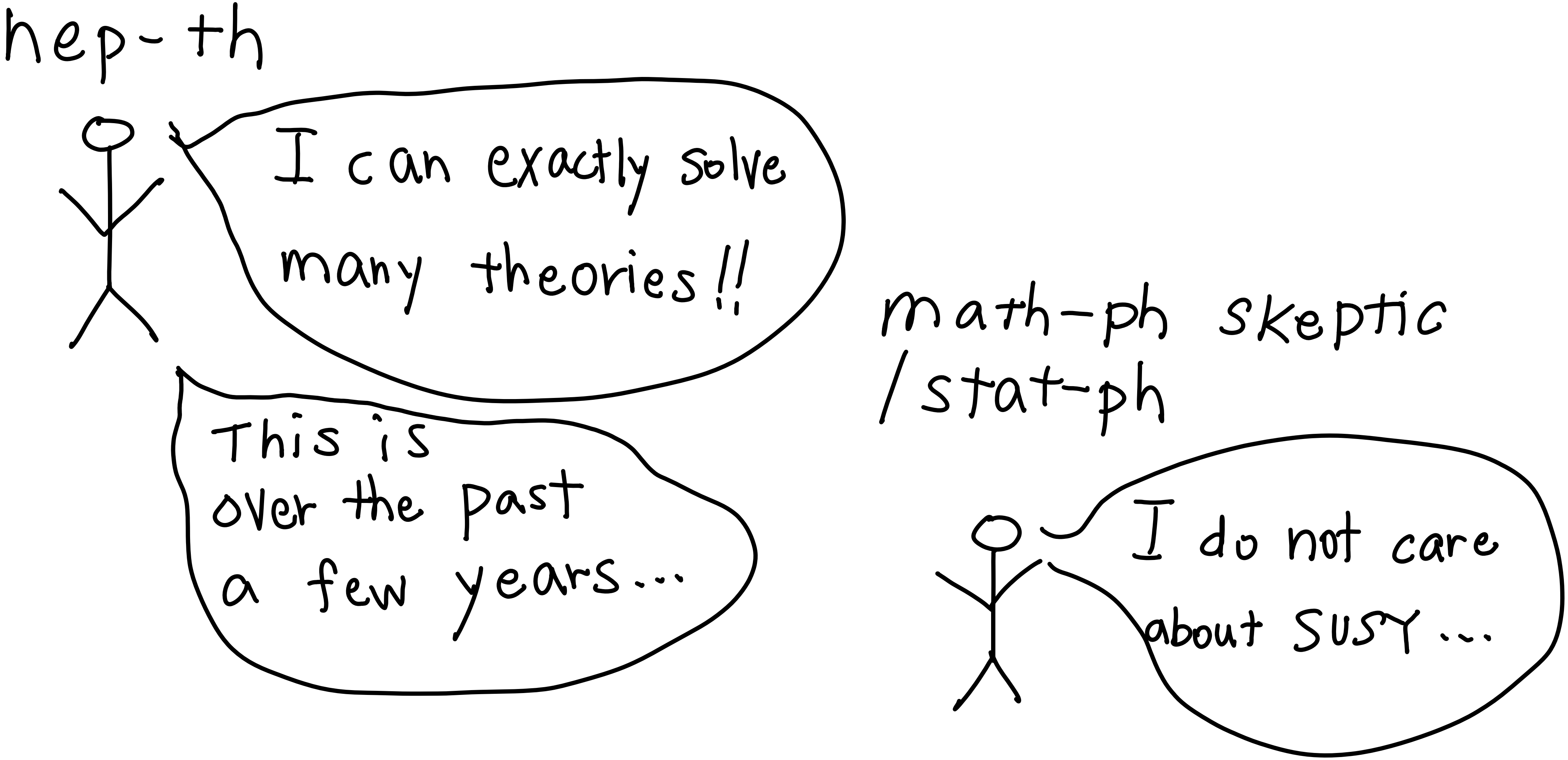
[GKMMM, Donagi-Witten ('95-)]

• Gauge / Bethe

[Nekrasov-Shatashvili ('09-);

also Moore-N-S ('97), Gerasimov-S ('06, '07)]


hep-th
I can exactly solve many theories!!
This is over the past a few years...
math-ph skeptic /stat-ph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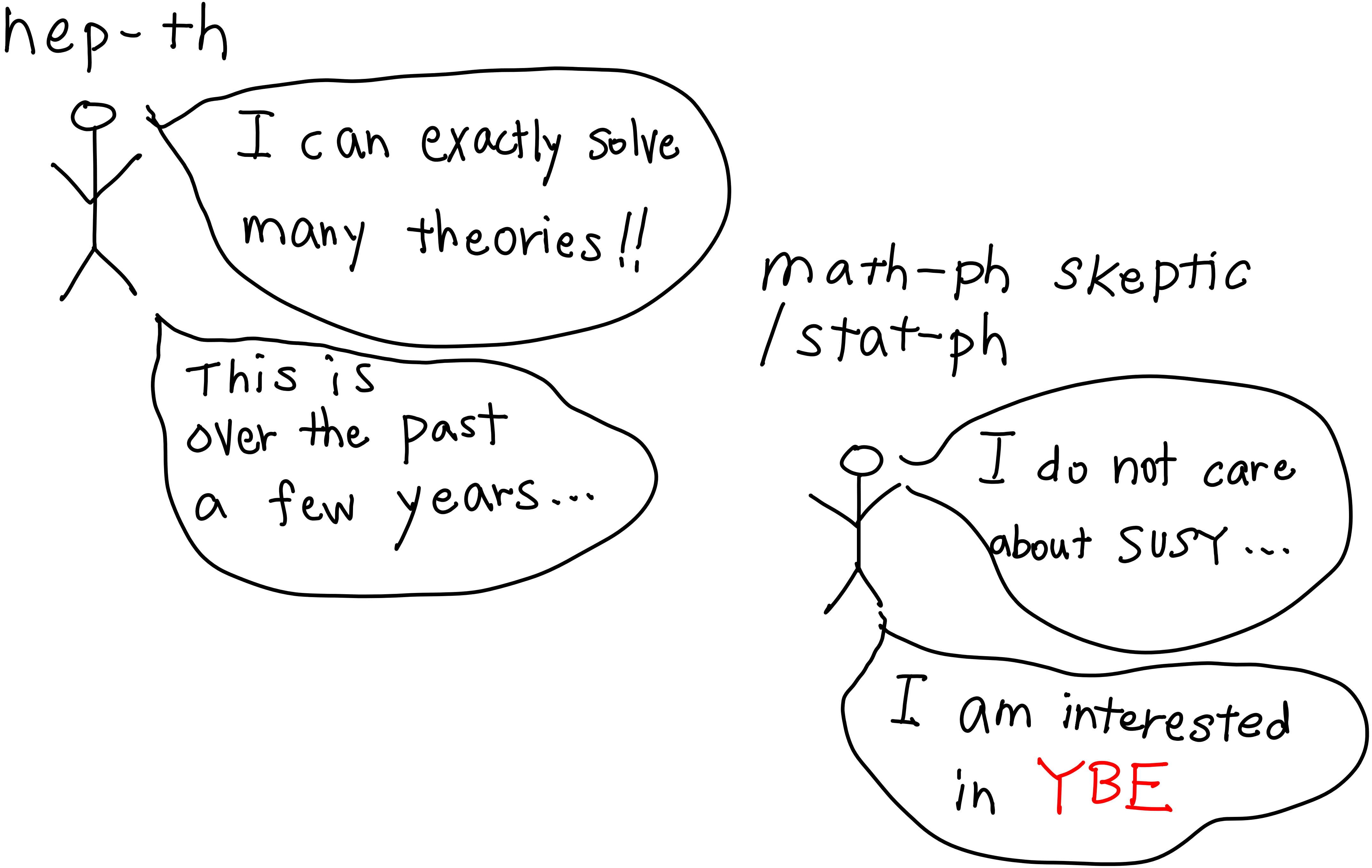
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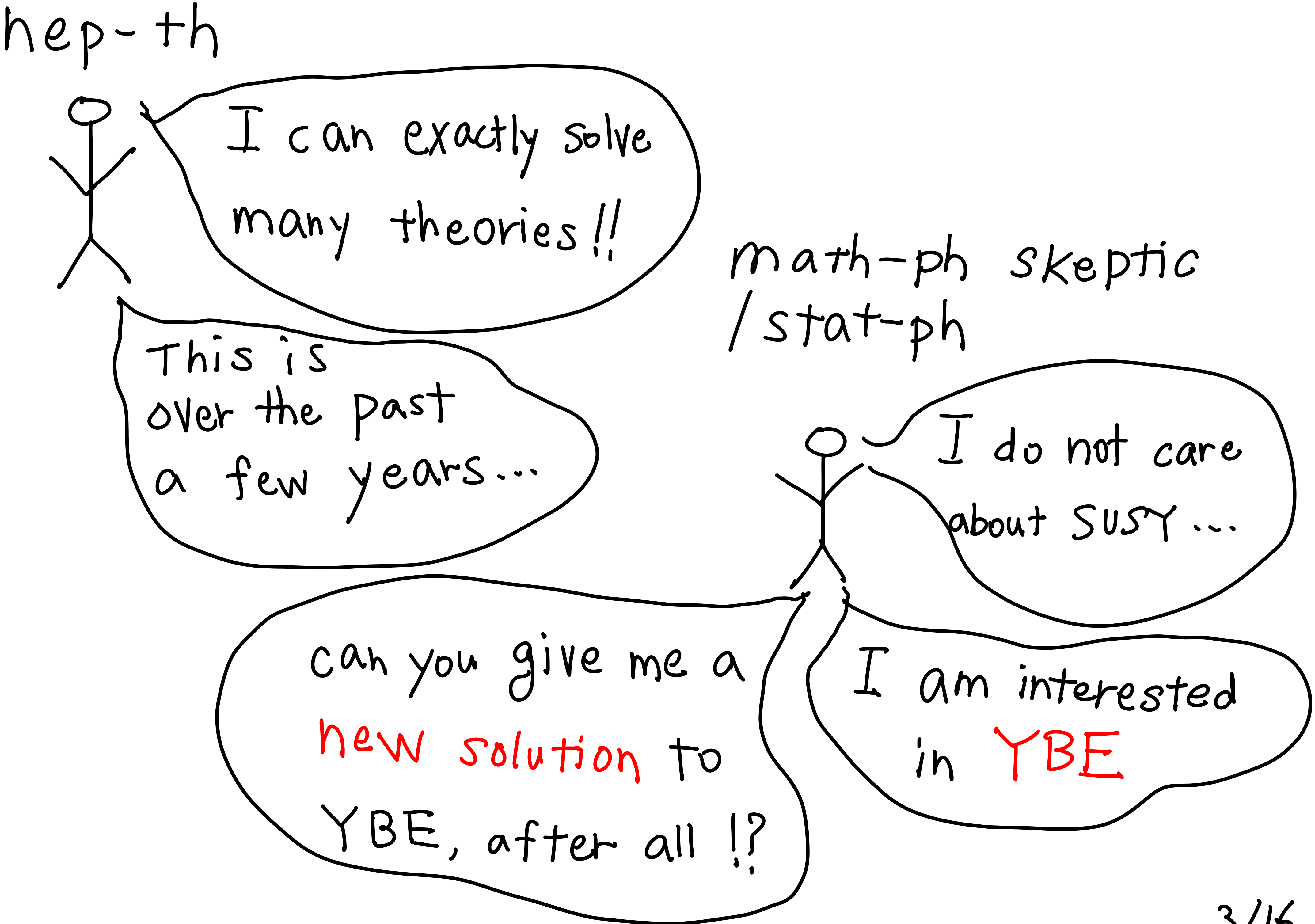
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can you give me a new solution to YBE, after all !?
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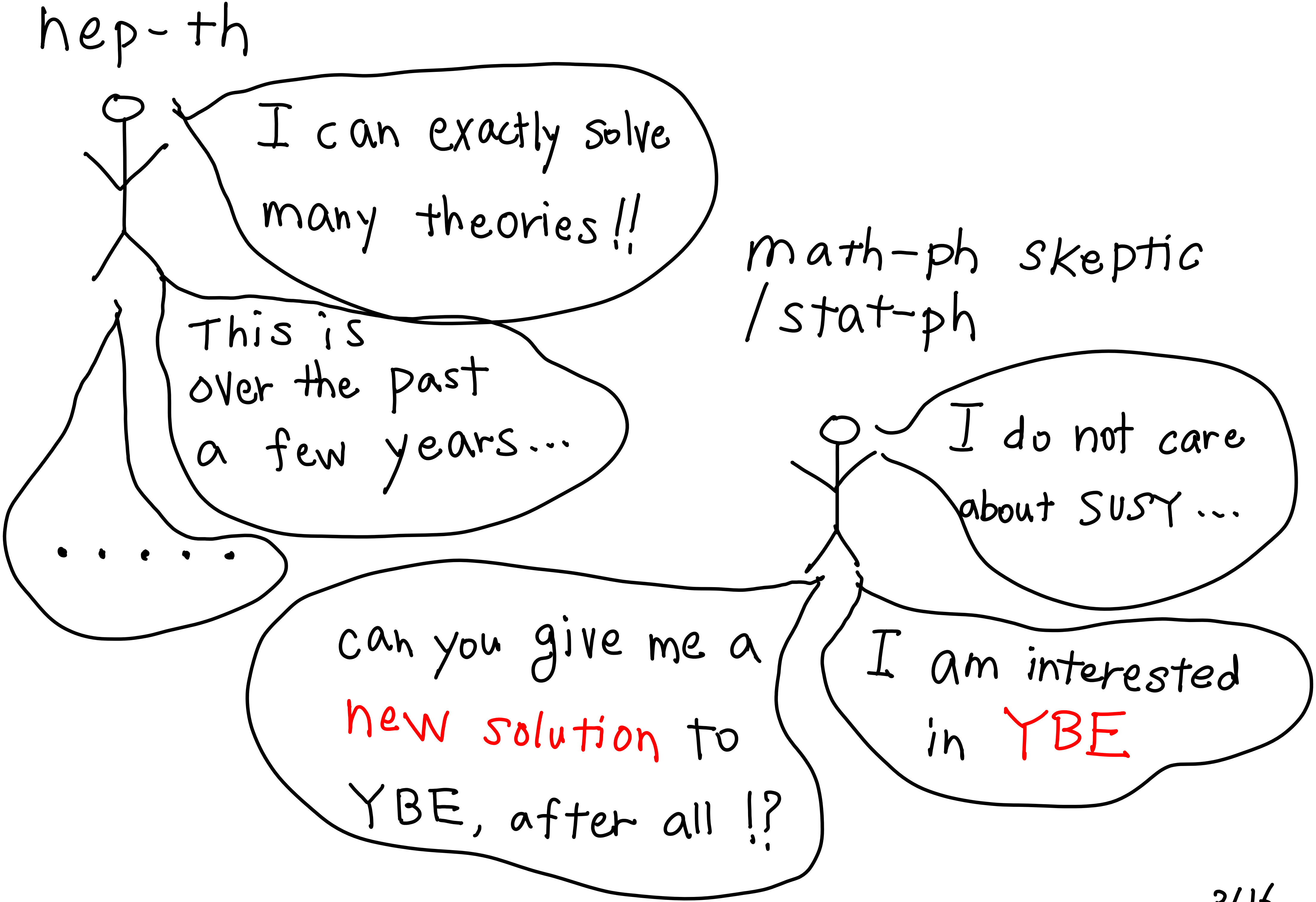
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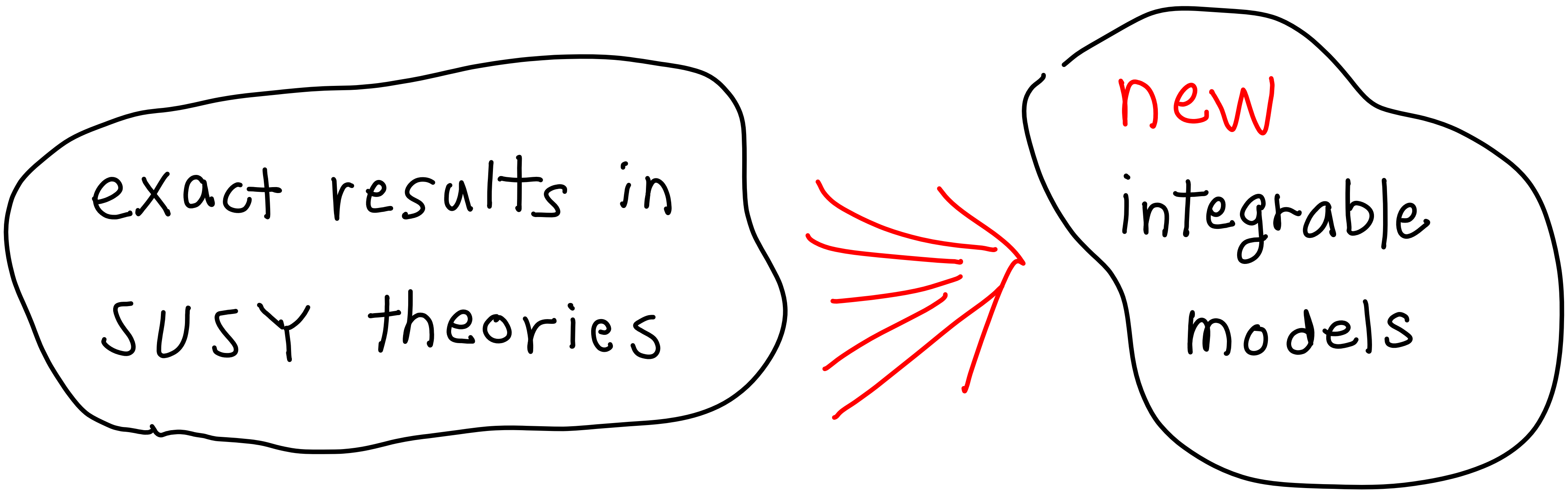
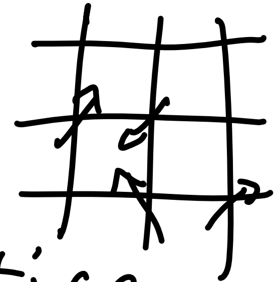
Today



Gauge/YBE

[Y (to appear)
Y (+ Terashima) ('12)]



quiver diagrams $=$ spin lattice

Boltzmann weight

$$I\left[\text{a class of 4d } \mathcal{N}=1 \text{ theories}\right] = Z\left[\text{exactly solvable model}\right]$$

$(\underbrace{p,\ q,}_{\text{fugacity}}\ \vec{u})$ R-charge

$(\underbrace{p,\ q,\ \vec{u}}_{\text{spectral parameters}})$

$$I\left[\text{a class of 4d } \mathcal{N}=1 \text{ theories}\right] = Z\left[\text{exactly solvable model}\right]$$

- I : $S^1 \times S^3$ index

  $\longleftrightarrow$ Bazhanov-Sergeev model

  ('10, '11)

- I : $S^1 \times S^3/\mathbb{Z}_n$ index

  $\longleftrightarrow$ new solution to YBE

  (to appear)

Comments

Gauge / YBE

$$I_{S^1 \times S^3/\mathbb{Z}_n}\left[\begin{matrix} 4d \\ \mathcal{N}=1 \end{matrix}\right] = Z\left[\text{integrable model}\right]$$

Gauge / Bethe [Nekrasov Shatashvili]

Yang-Yang function

$$W_{\text{twisted}}\left[\begin{matrix} 2d \\ \mathcal{N}=(2,2) \end{matrix}\right] = Y\left[\begin{matrix} \text{integrable} \\ \text{model} \end{matrix}\right]$$

$$\Downarrow$$

$$(\text{2d vacuum eqn}) = (\text{Bethe Ansatz eqn})$$

Gauge / YBE

directly gives integrable model

$$I_{S^1\times S^3/\mathbb{Z}_n}\left[\begin{matrix}4d\\ \mathcal{N}=1\end{matrix}\right] = Z\left[\text{ integrable model }\right]$$

Gauge / Bethe [Nekrasov Shatashvili]

Yang-Yang function

$$\mathcal{W}_{\text{twisted}}\left[\begin{matrix}2d\\ \mathcal{N}=(2,2)\end{matrix}\right] = Y\left[\begin{matrix}\text{integrable}\\ \text{model}\end{matrix}\right]$$

(2d vacuum eqn) = (Bethe Ansatz eqn)

hard to recover the model

Our solution to YBE is
one of the most general in the literature

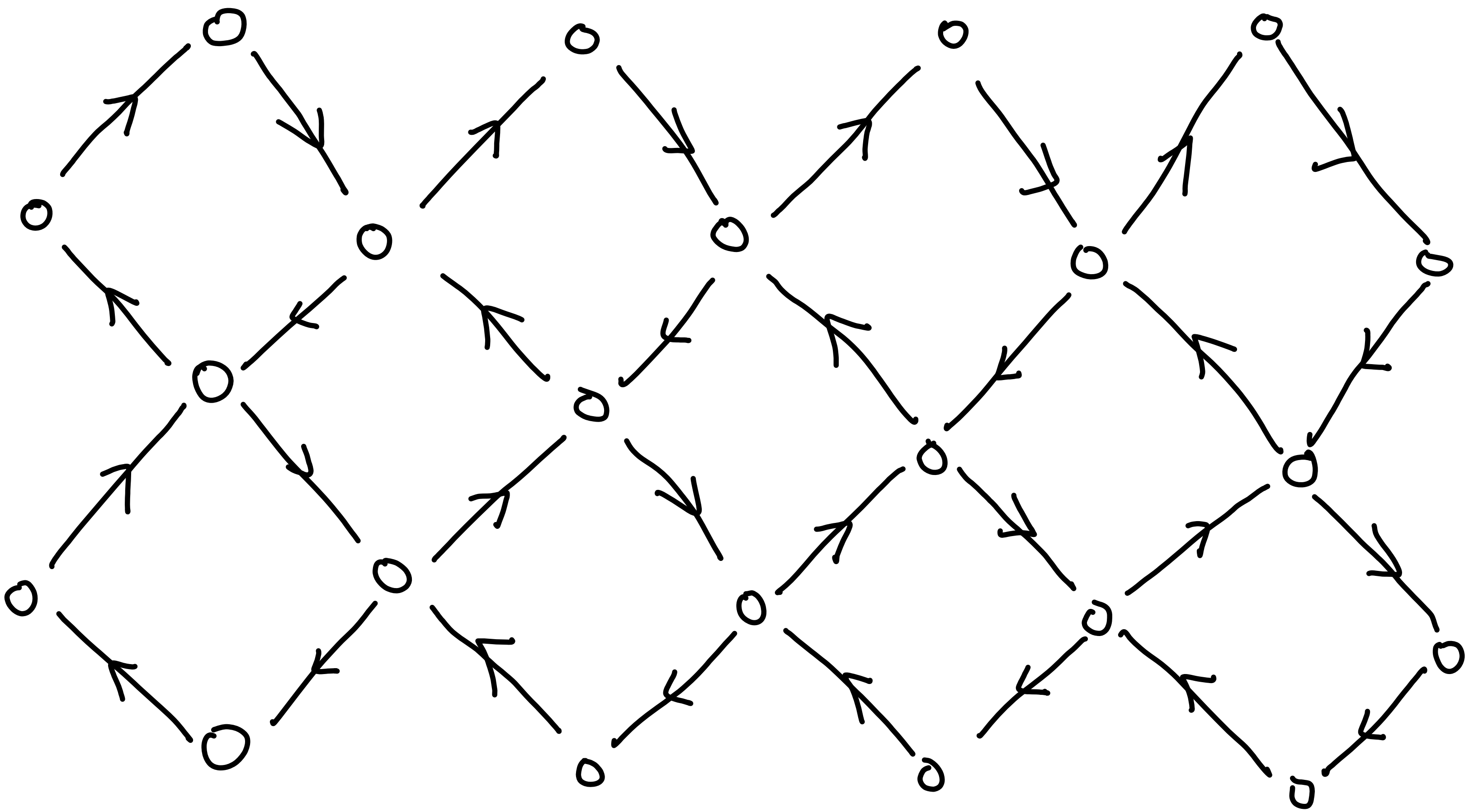
"master solution"

Our solution $\Longrightarrow$ limit & specialization

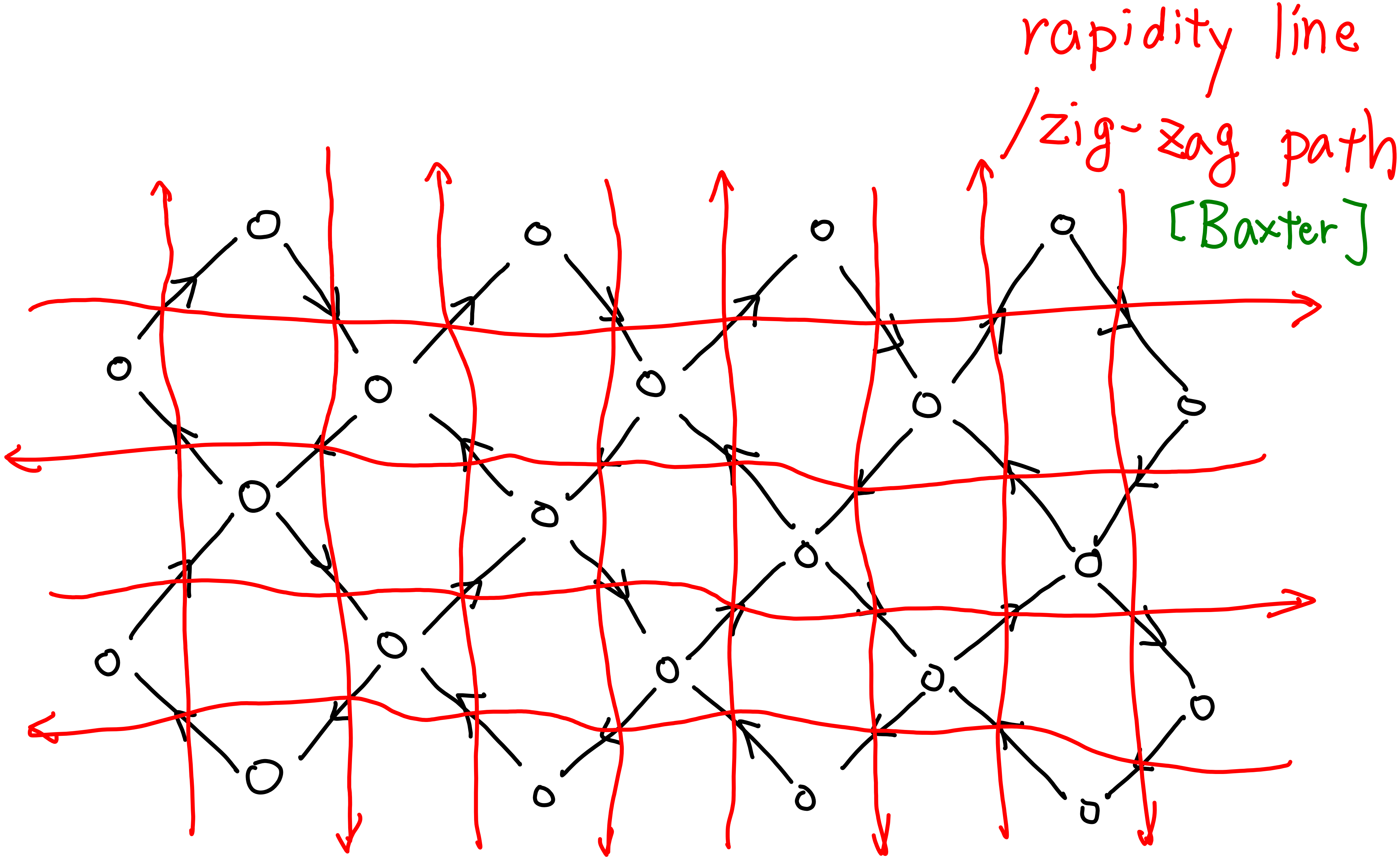
Bazhanov-Sergeev
Fadeev-Volkov
Fateev-Zamolodchikov
Kashiwara-Miwa
chiral Potts
Ashkin-Teller
⋮

Basic Idea

an oriented lattice = a quiver diagram



an oriented lattice = a quiver diagram



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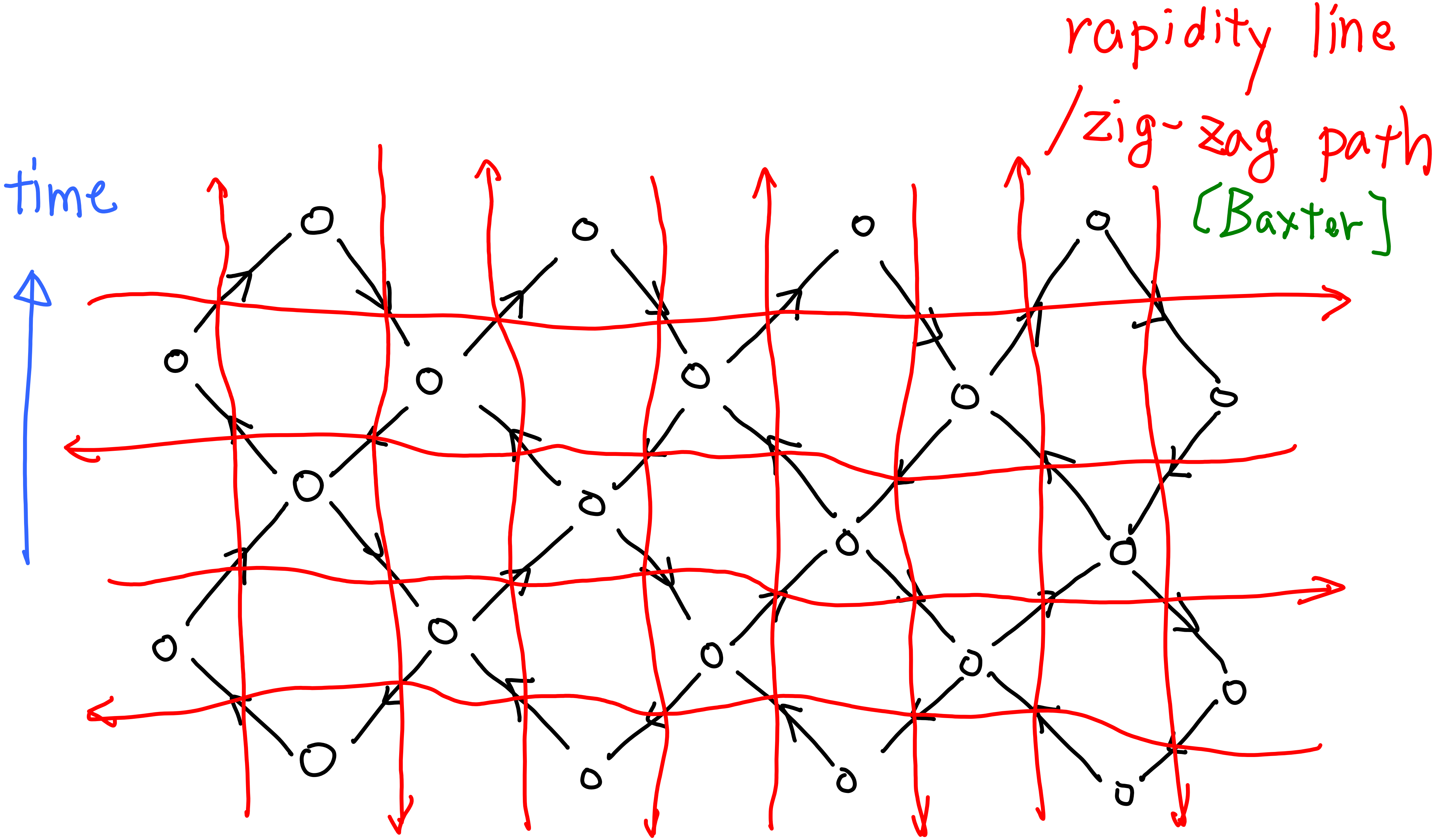
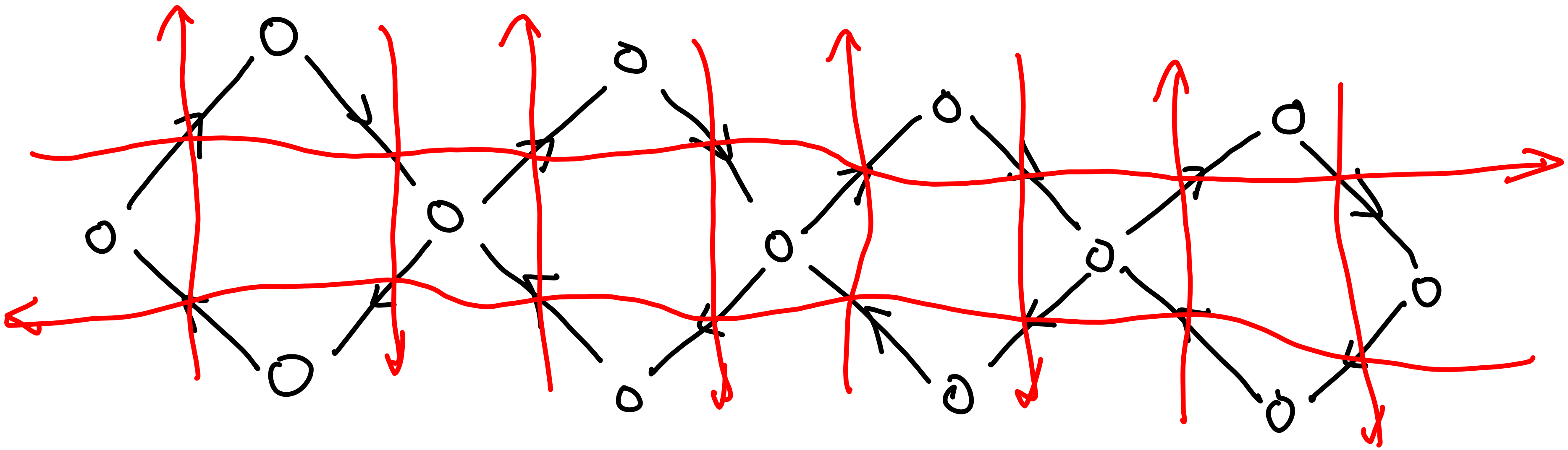
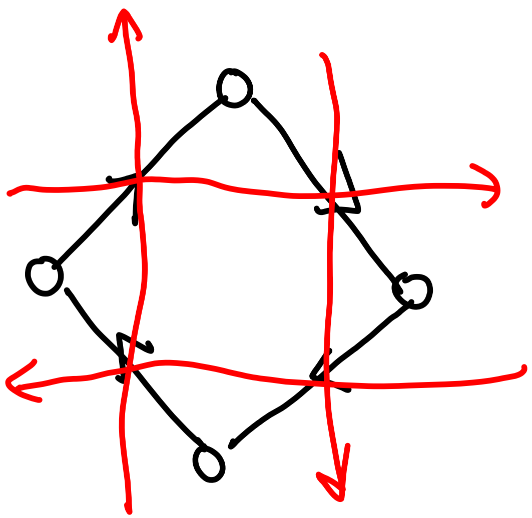


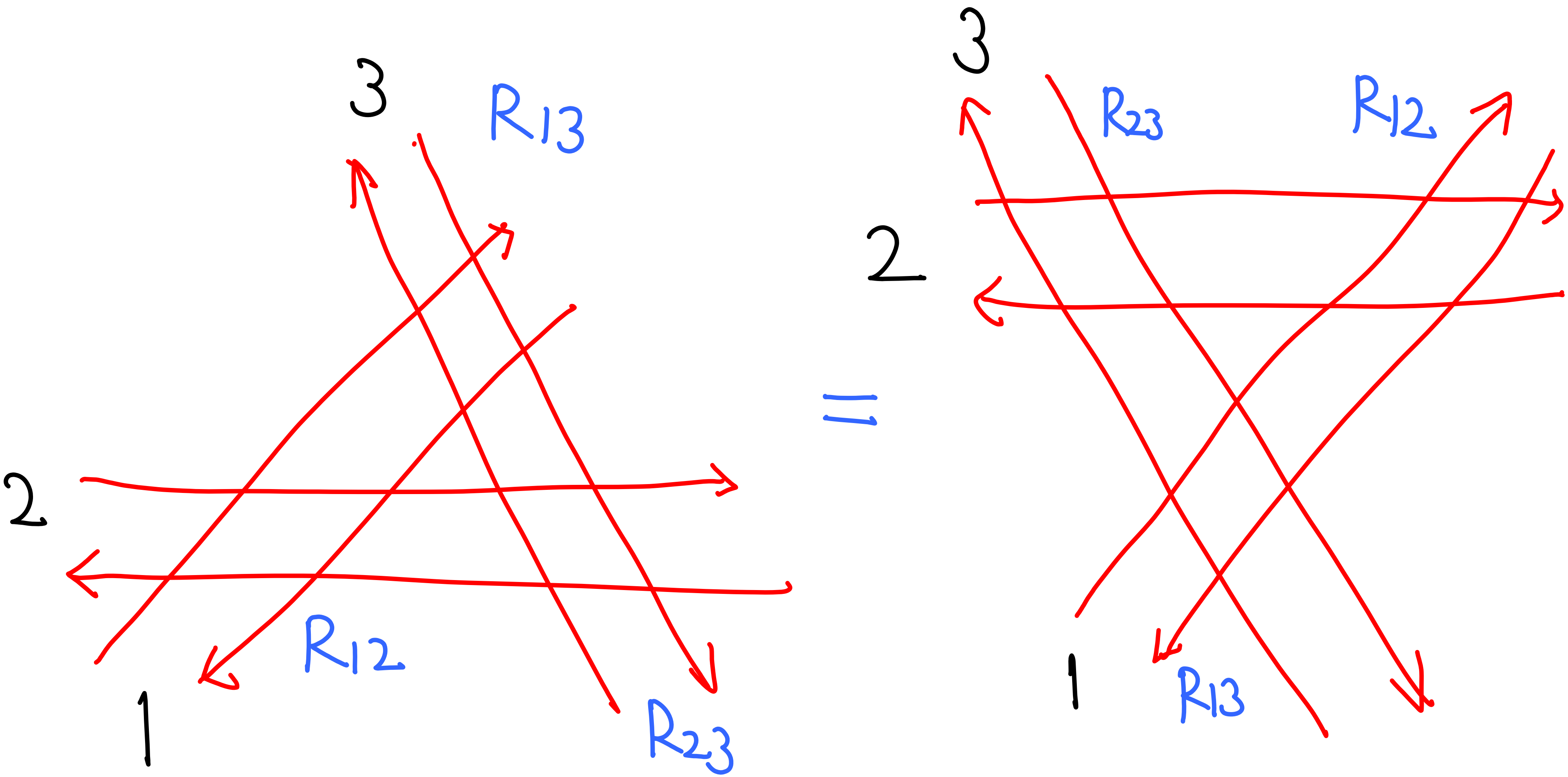
transfer matrix

$T =$ 

R-matrix

$R =$ 

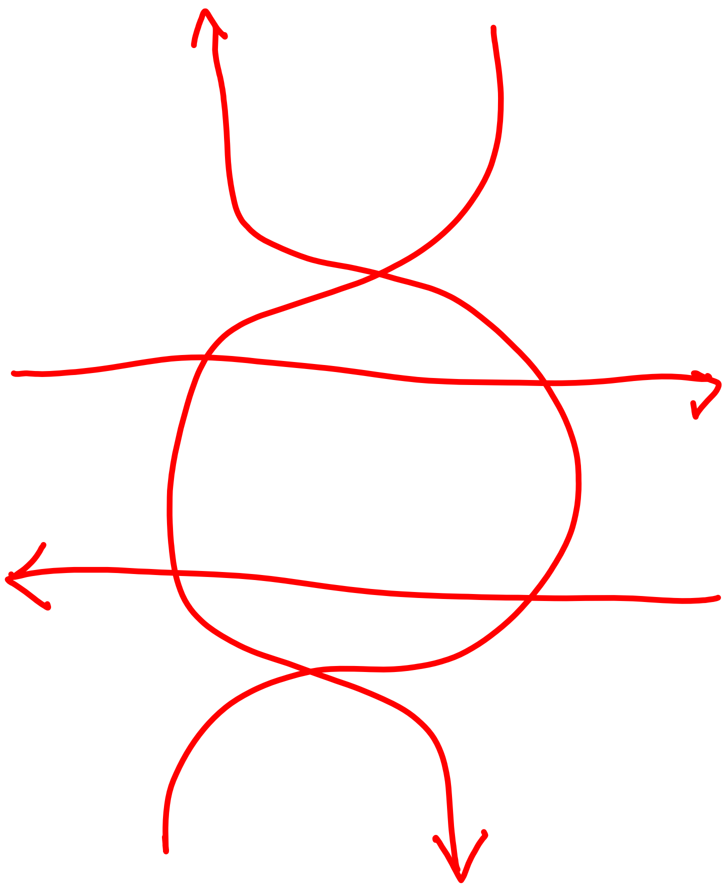
Yang-Baxter equation (YBE)



$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$$

YBE follows from simpler

"star-star relation" [Baxter ('86)]



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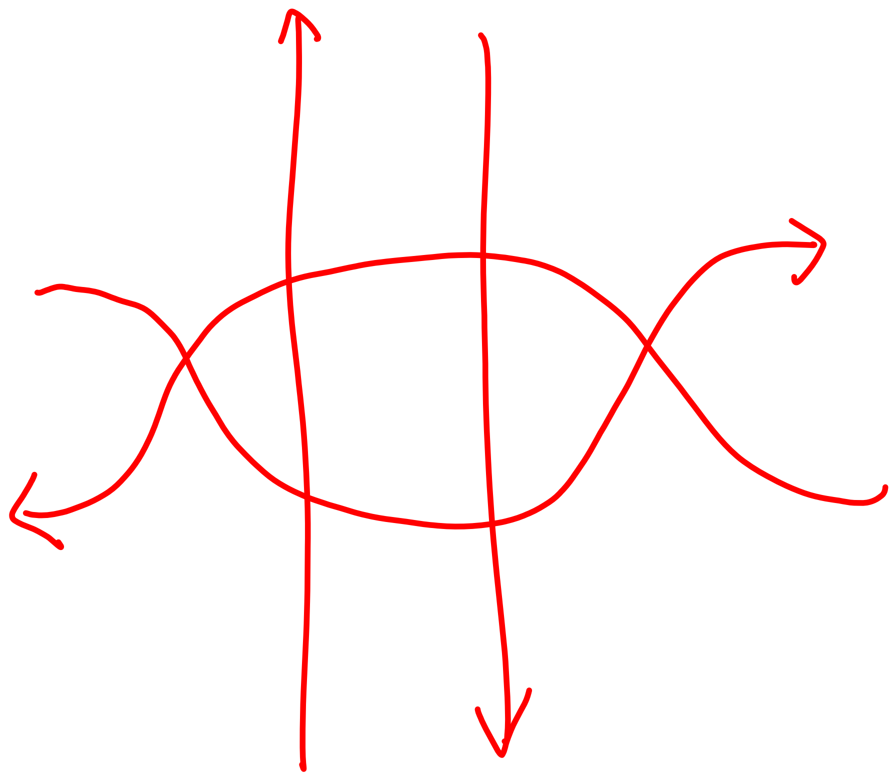

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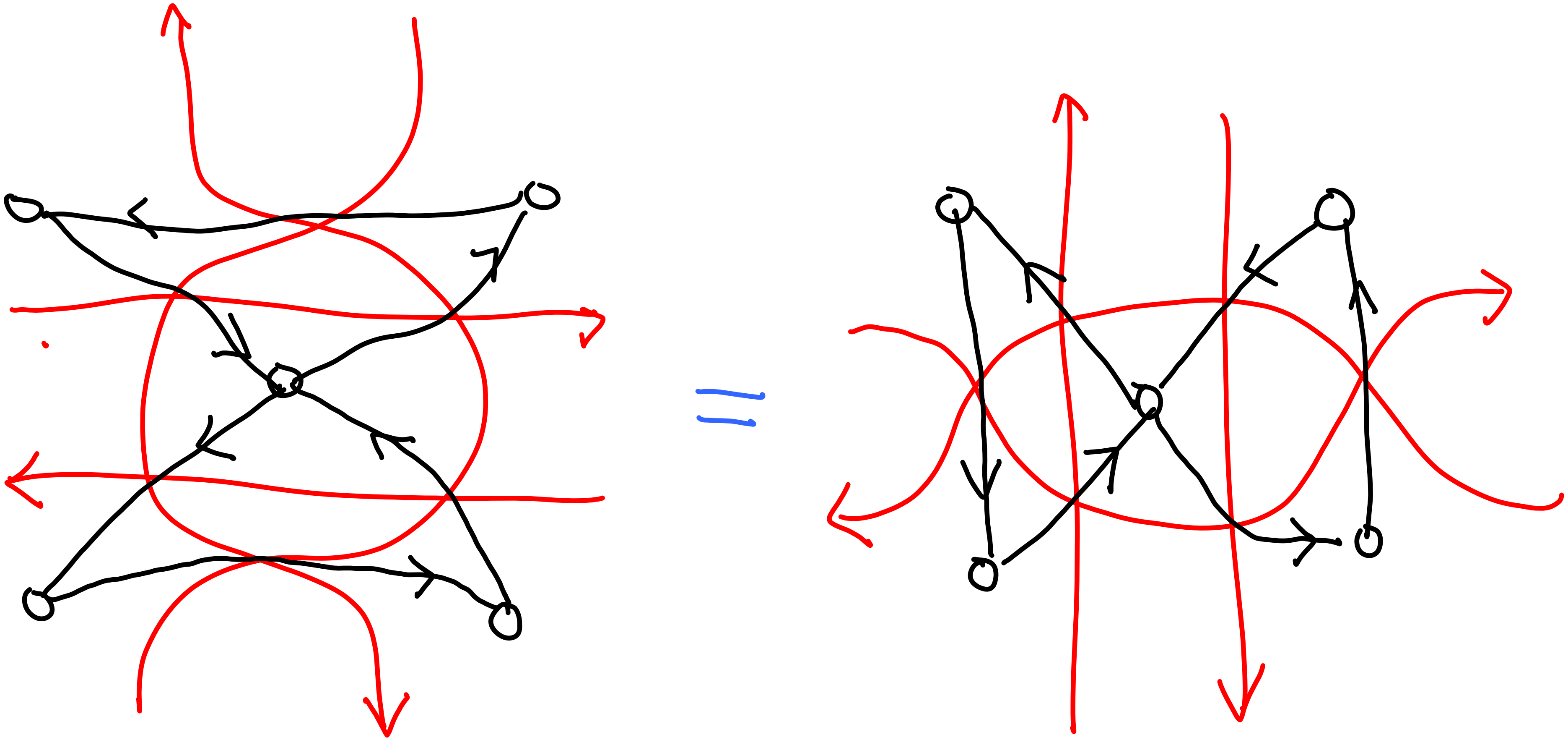

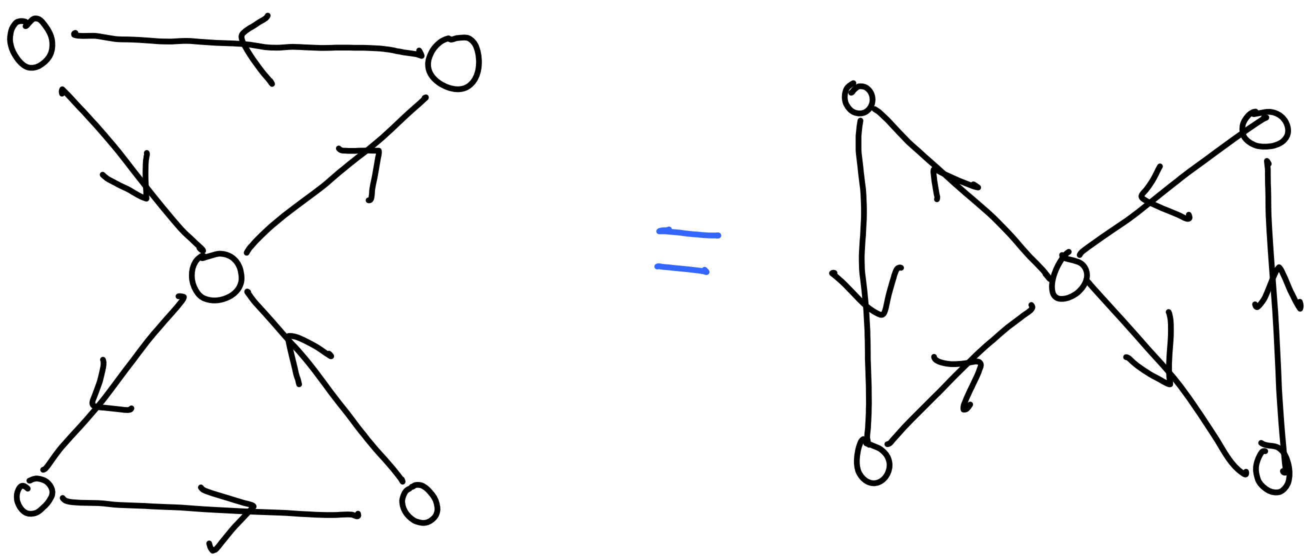

star-star relation

= 4d Seiberg duality / quiver mutation

$(N_f = 2N_c)$

$\bigcirc$ = $SU(N_c)$ gauge group $\rightarrow$ = bifundamental

4d Lens Index $I_{S^1 \times S^3/\mathbb{Z}_n}(p, q, \vec{u})$

[Benini–Nishioka–Y ('11)]

is invariant under Seiberg duality!

$\rightsquigarrow$ $\underline{\underline{\text{automatically}}}$ solve

star-star relation & YBE

- Moreover I could be written as

$$I = \sum_{\{\vec{m}_v\}} \int \prod_v d\vec{\sigma}_v \; I_{vector}[\vec{\sigma}_v, \vec{m}_v] \times I_{chiral}[\vec{\sigma}_v, \vec{m}_v]$$

· Moreover I could be written as

$$I = \sum_{\{\vec{m}_v\}} \int \prod_v d\vec{\sigma}_v \; I_{\text{vector}}[\vec{\sigma}_v, \vec{m}_v]$$

$$\times \; I_{\text{chiral}}[\vec{\sigma}_v, \vec{m}_v]$$

$$\vec{\sigma}_v = (\sigma_v^1 \cdots \sigma_v^N) \in \mathbb{R}^N$$

$$\vec{m}_v = (m_v^1 \cdots m_v^N) \in (\mathbb{Z}/\mathbb{Z}_n)^N$$

v : vertex

holonomies $\pi_1(S^1 \times S^3/\mathbb{Z}_n) = \mathbb{Z} \times \mathbb{Z}_n$

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self-interaction

nearest-neighbor

$\vec{\sigma}_v = (\sigma_v^1 \cdots \sigma_v^N) \in \mathbb{R}^N$

$\vec{m}_v = (m_v^1 \cdots m_v^N) \in (\mathbb{Z}/\mathbb{Z}_n)^N$

v : vertex

holonomies = spin $\quad \pi_1(S^1 \times S^3/\mathbb{Z}_n) = \mathbb{Z} \times \mathbb{Z}_n$

The Boltzmann weights are written in terms of orbifold elliptic gamma functions

$$\Gamma_{n,m}(x;p,q) = \prod_{\substack{i,j \geq 0 \\ i-j \equiv m \pmod n}} \frac{1-x^{-1}p^{i+1}q^{j+1}}{1-xp^{i}q^{j}}$$

$$\begin{pmatrix} n \in \mathbb{Z} \\ m \in \mathbb{Z}_n \end{pmatrix}$$

$S^1 \times S^3/\mathbb{Z}_n$

$$= \Gamma(xp^{(m \bmod n)}; pq, p^n) \times \Gamma(xq^{n-(m \bmod n)}; pq, q^n)$$

Summary

- new solution to YBE from
  4d $\mathcal{N}=1$ theory

$$I_{S^1\times S^3/\mathbb{Z}_n}\begin{bmatrix}4d\\ \mathcal{N}=1\end{bmatrix} = Z\begin{bmatrix}\text{2d spin}\\ \text{chain}\end{bmatrix}$$

"Gauge/YBE"

- 4d Seiberg duality = star-star relation

Questions:

1. underlying group structure?

$\mathcal{U}_{p,q}(\mathfrak{g})$ ⟲ operators of a class of 4d $\mathcal{N}=1$ theories

[sklyanin, ...]

2. discrete spin $\rightsquigarrow$ continous spin

$$Z = \sum_{\vec{\sigma}} e^{-S[\vec{\sigma}]} \rightarrow Z = \int d\vec{\sigma}\, e^{-\frac{1}{\hbar} S[\vec{\sigma}]}$$

discrete

Kashiwara-Miwa
chiral Potts

$\rightsquigarrow$

Bazhanov - Sergeev
$= I_{S^1 \times S^3}$

continous

Our Solution
$I_{S^1 \times S^3/\mathbb{Z}_n}$

continuous
+ discrete

$\rightsquigarrow$

??
5d index ?

all continuous