Gauge/YBE Correspondence

Masahito Yamazaki (KAVLI IPMU / IAS)

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IGST

2005

2015

2005: Summer Institute, ENS Paris

2006: int06 Workshop, AEI Potsdam

2007: 12th Claude Itzykson Meeting, Saclay/ENS Paris

2008: IGST08 Workshop, Universiteit Utrecht

2009: int09 Conference, AEI Potsdam

2010: IGST10 Conference, NORDITA Stockholm

2011: IGST11 Conference, Perimeter Institute for Theoretical Physics

2012: IGST12 Conference, ETH Zürich

2013: IGST13 Conference, Universiteit Utrecht

2014: IGST14 Conference, DESY Hamburg

Yang-Baxter Equation

1964- 2015

[McGuire
Yang
Baxter]

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Study of Exactly Soluble One-Dimensional N-Body Problems

J. B. McGuire
University of California, Los Angeles, California
(Received 9 August 1963)

In this paper it is shown that several cases of one-dimensional N-body problems are exactly soluble. The first case describes the motion of three one-dimensional particles of arbitrary mass which interact with one another via infinite-strength, repulsive delta-function potentials. It is found in this case that the stationary-state solution of the scattering of the three particles is analogous to an electromagnetic diffraction problem which has already been solved. The solution to this analogous electromagnetic problem is interpreted in terms of particles. Next it is shown that the problem of three particles of equal mass interacting with each other via finite- but equal-strength delta-function potentials is exactly soluble. This example exhibits rearrangement and bound-state effects, but no inelastic processes occur. Finally it is shown that the problem of N particles of equal mass all interacting with one another via finite- but equal-strength delta functions is exactly soluble. Again no inelastic processes occur, but various types of rearrangements and an N-particle bound state do occur. These rearrangements and the N-particle bound state are illustrated by means of a series of sample calculations.

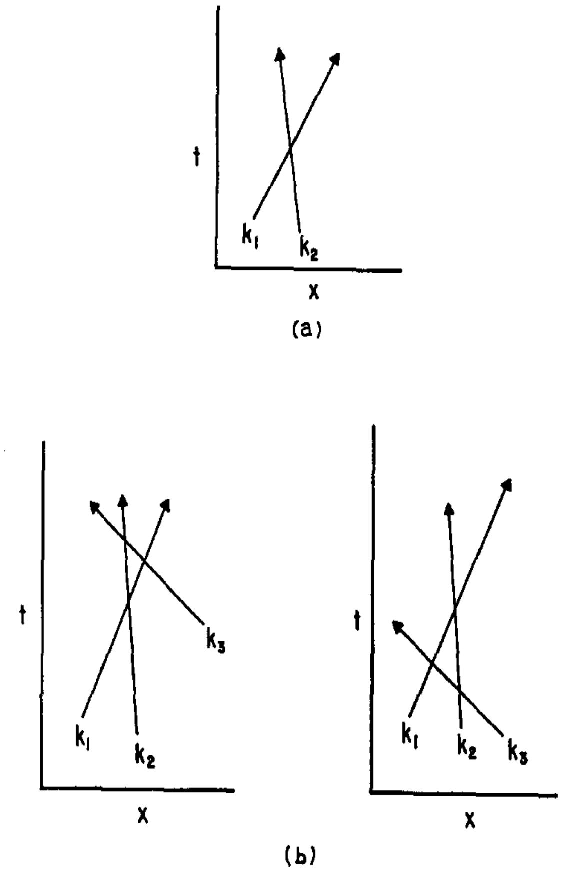
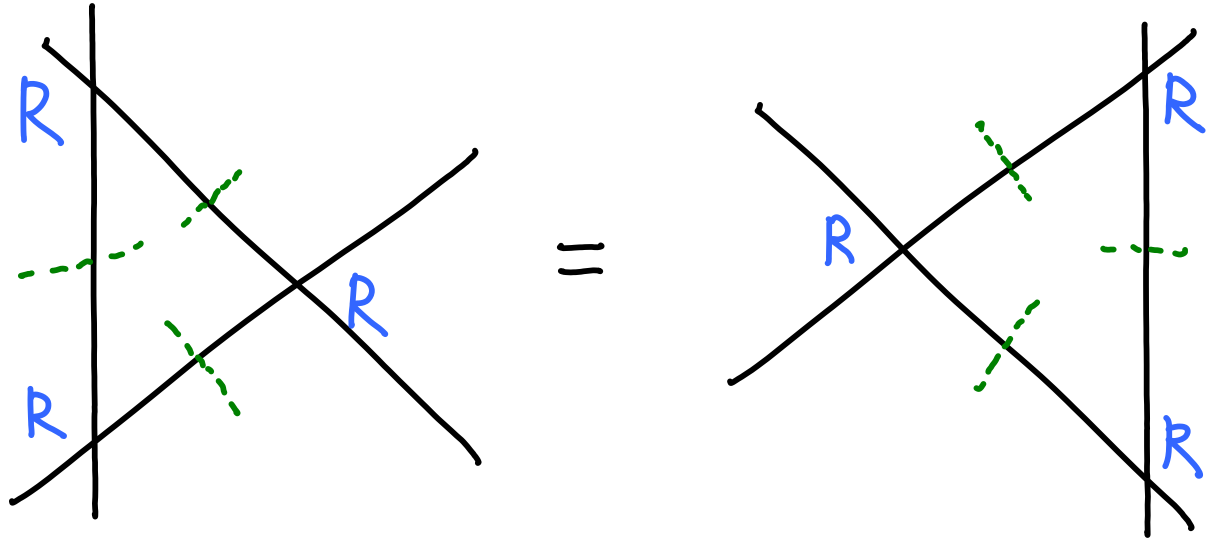
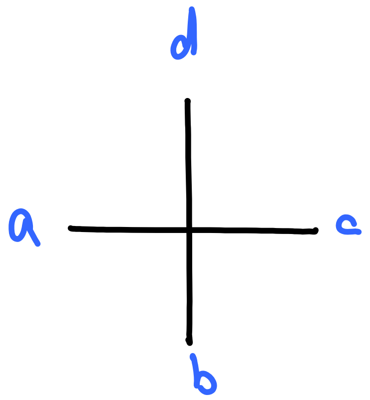


FIG. 7. Space–time plots for (a) two- and (b) three-particle problems.

YBE : highly overconstrained



$\mathcal{O}(N^6)$



$= R_{a\ b}^{\ d\ c}$ $\qquad a, b, \cdots = 1 \sim N$ $\qquad \mathcal{O}(N^4)$

Q : Why integrable models exist?

Q: Why integrable models exist?

A: Because of (SUSY) gauge-theory,

dualities

$\exists$ gauge redundancy

Lorentz sym./locality/unitarity/$\cdots$

(fundamental principles of Nature)

based on my works since 2012

M.Y. + W. Yan 1504

M.Y 1307

D. Xie + M.Y. 1207

M.Y. 1203

Y. Terashima + M.Y. 1203

+ in progress

pedagogical review

&

Some new works

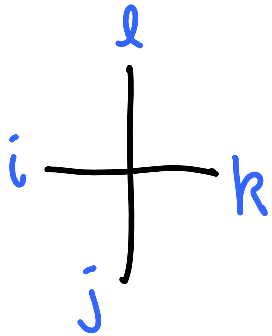
Some related recent works

A. Kels 1504

J. Yagi 1504

Basic Idea

R-matrix R

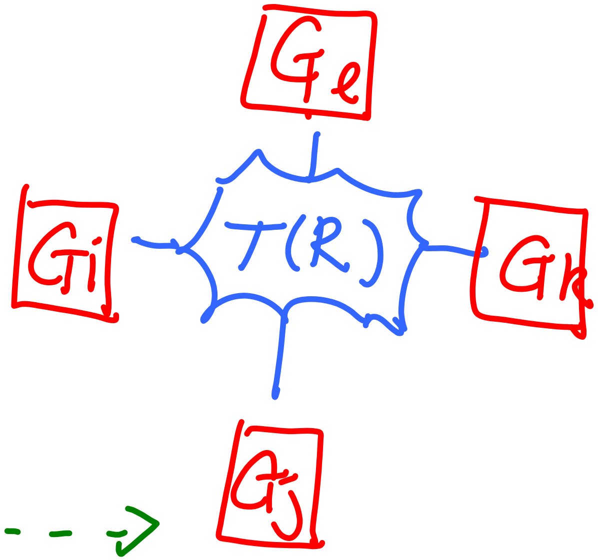
$R^{i\ \ l}_{\ j\ \ k} =$ 

$Z\left[A_\mu^i \quad A_\mu^l \quad A_\mu^j \quad A_\mu^k\right]$

bgd gauge fields

"categorification"

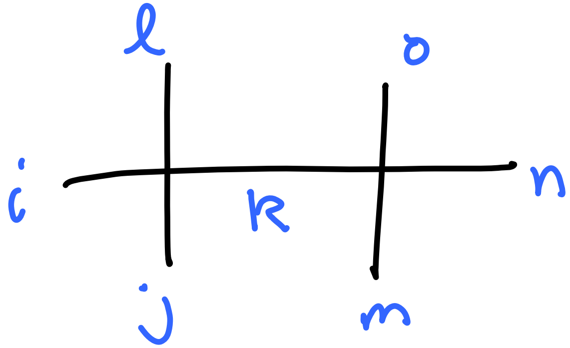
theory $T[R]$

$T[R]\begin{pmatrix} & G_l & \\ G_i & & G_k \\ & G_j & \end{pmatrix} =$ 

global sym.

Product

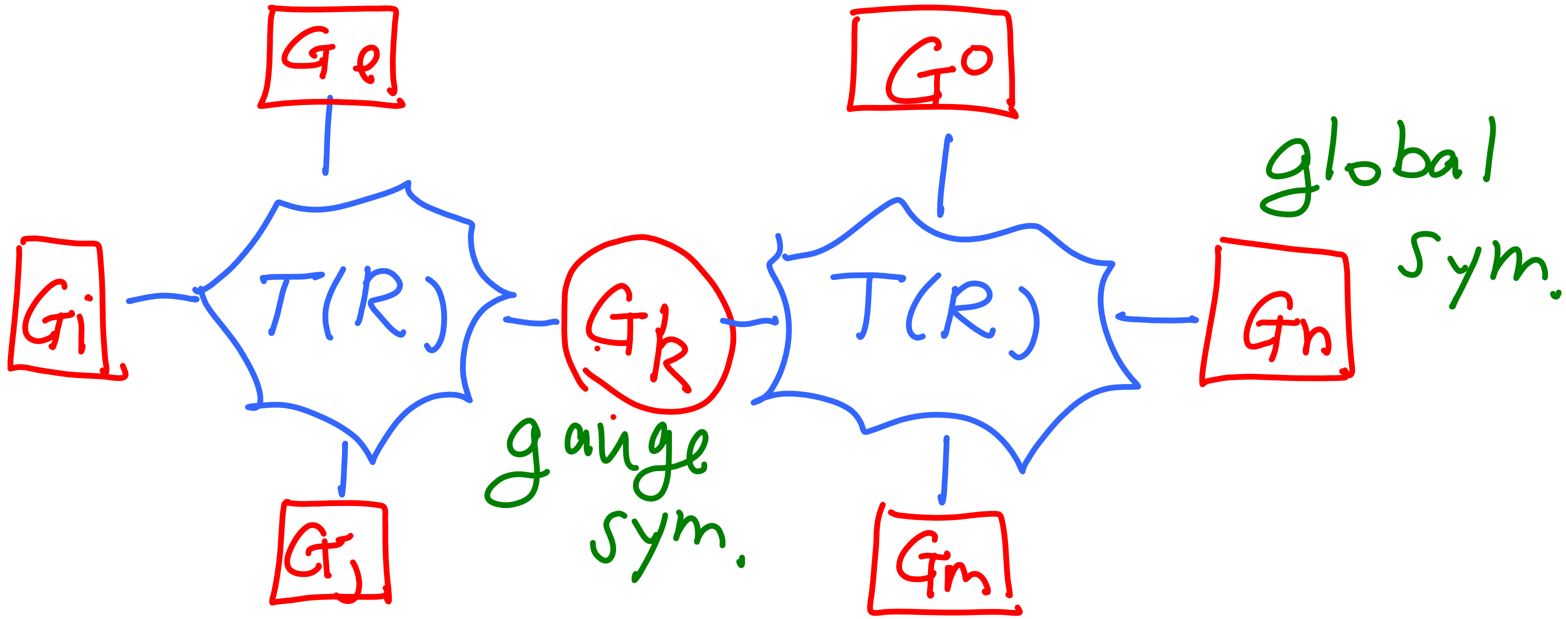
$$\sum_k R_{i\ j}^{\ l\ k}\; R_{k\ m}^{\ o\ n} =$$



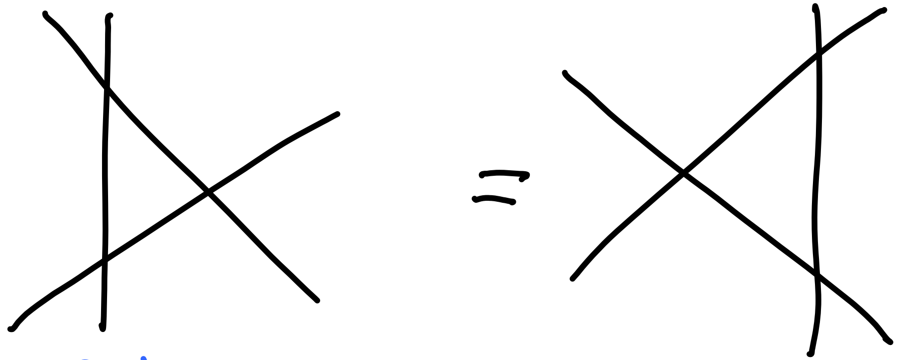
gauging

$$\int \mathcal{D}A_\mu \; Z\left[A_\mu^i \; A_\mu^l \; A_\mu^j \; A_\mu^k\right] Z\left[A_\mu^k \; A_\mu^o \; A_\mu^m \; A_\mu^n\right]$$

$$=$$



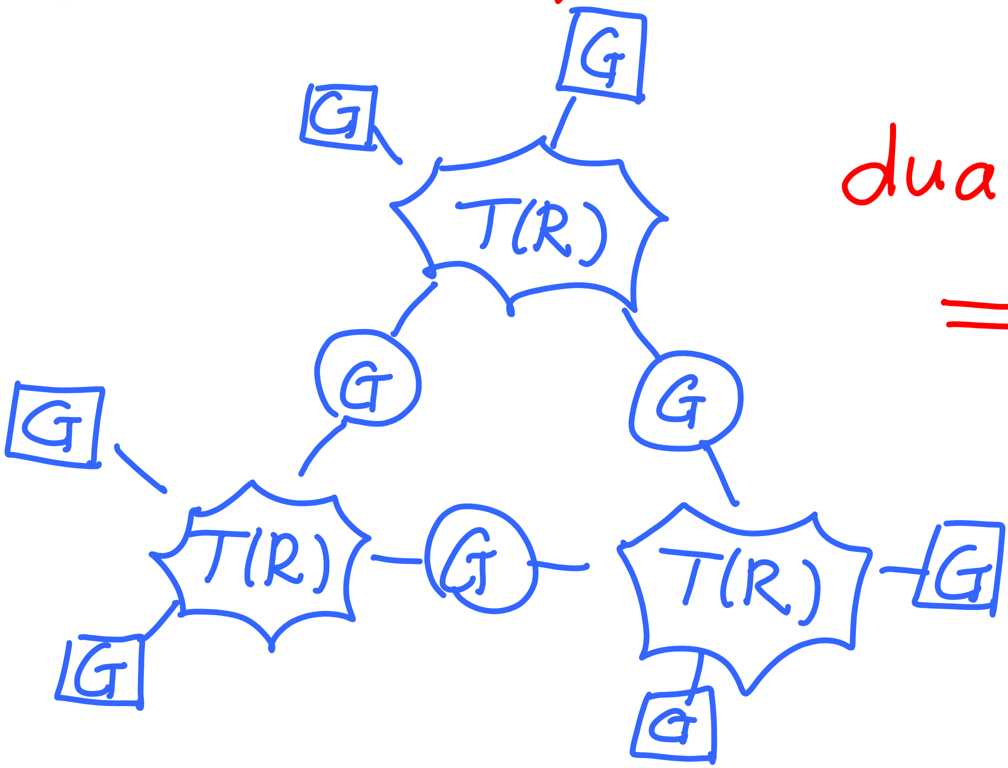
YBE



RRR = RRR

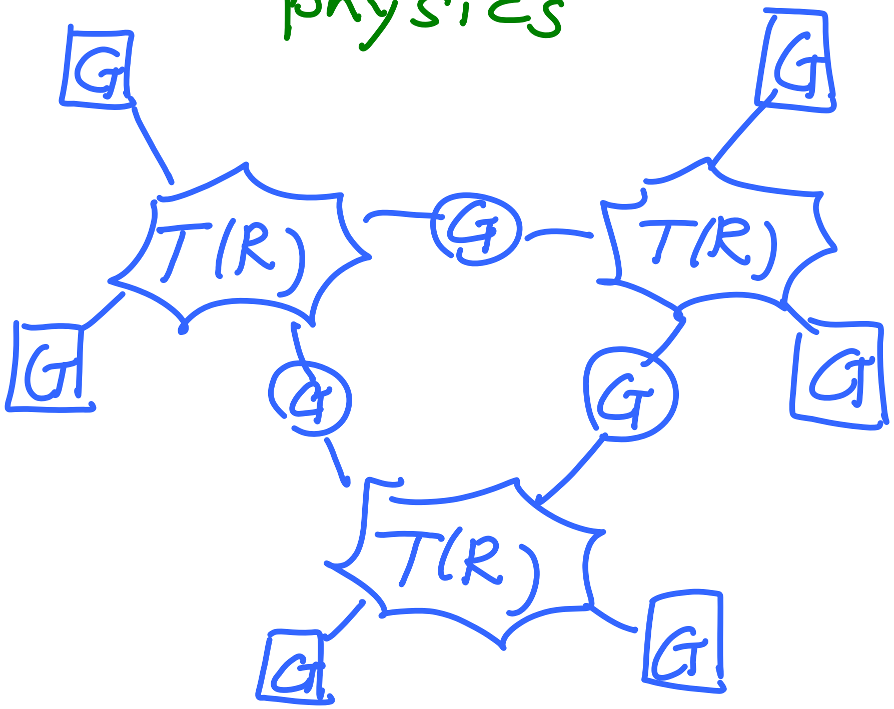
YB-duality

describe the same physics



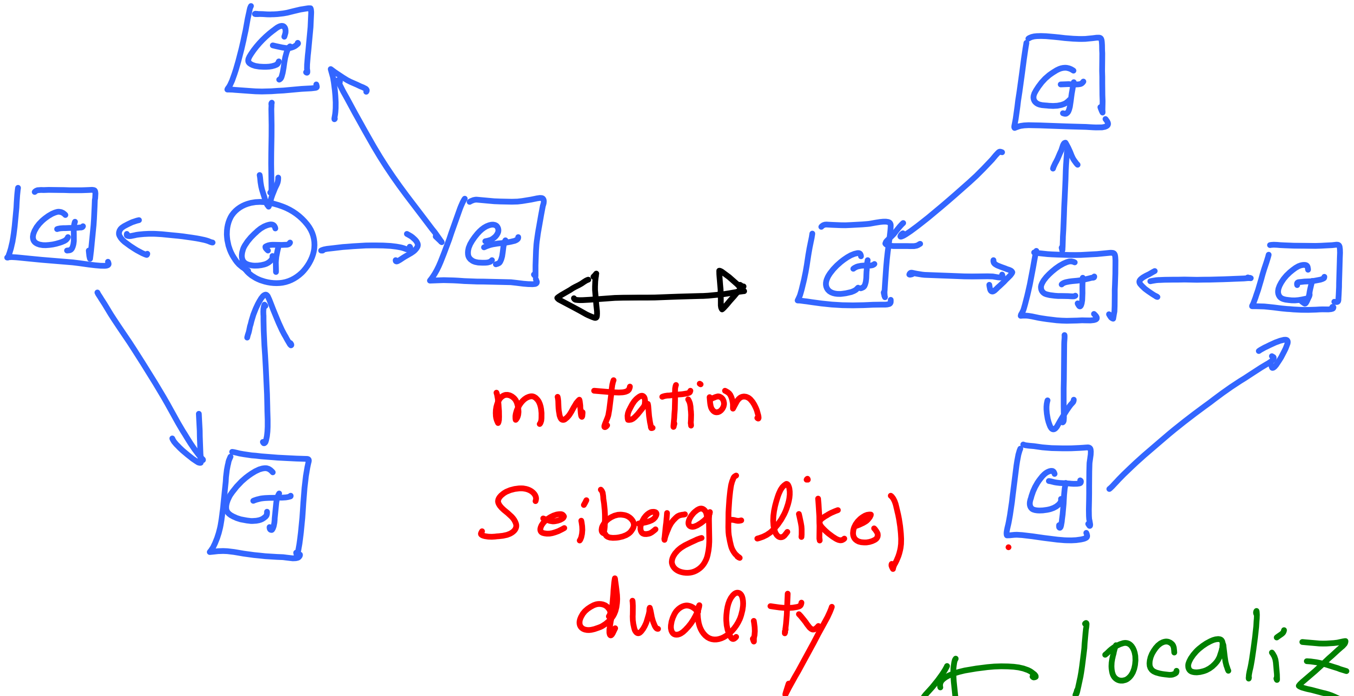
duality

=



The story works rather well for

$T[R]$ = SUSY quiver gauge theory



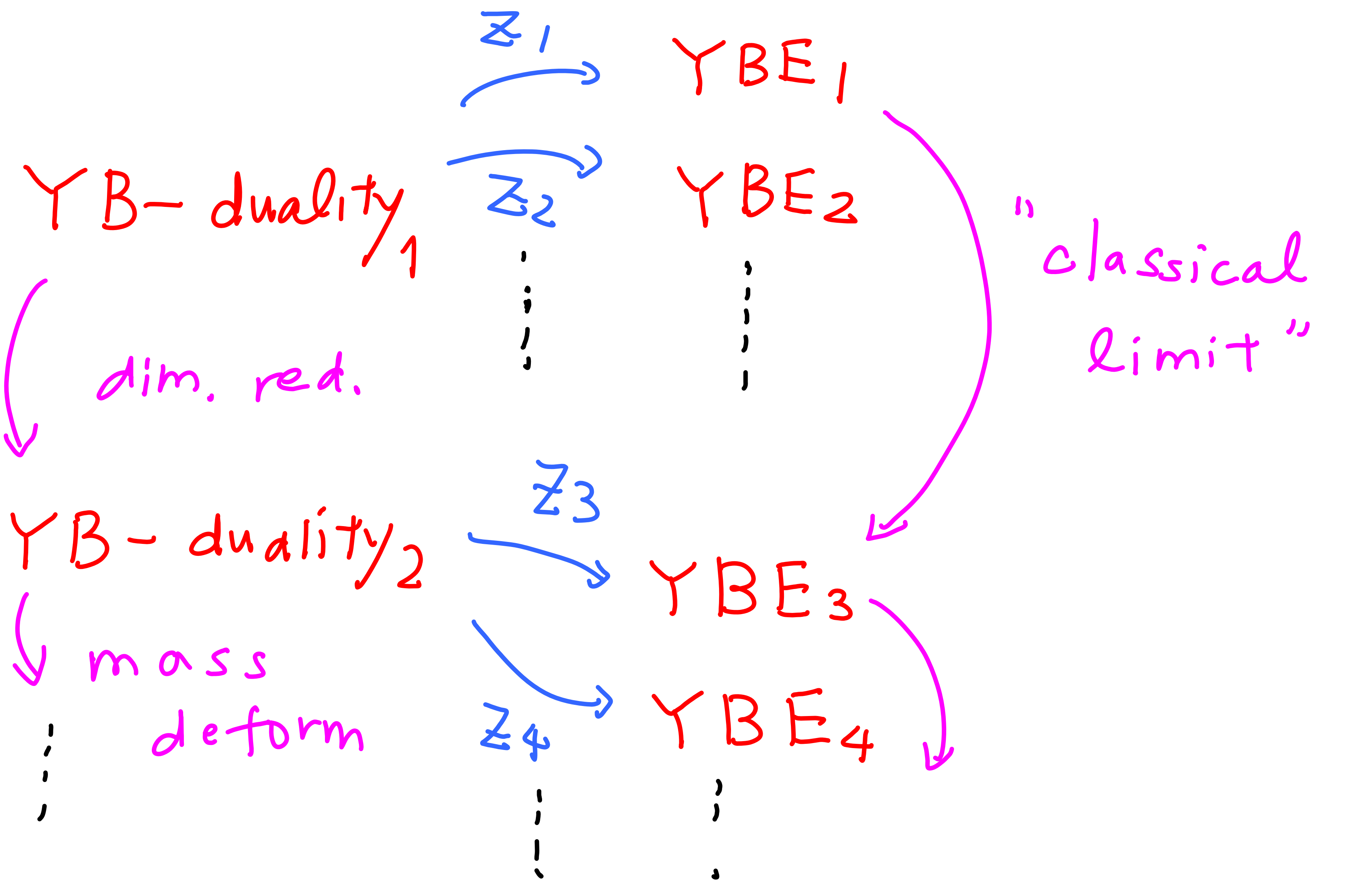
← localization

$Z[T(R)]$ = SUSY partition

spectral parameter = R-charge

YBE: automatic

We can $\begin{cases} \text{compute different } Z\text{'s} \\ \text{dimensionally reduce, mass deform} \end{cases}$



4d $\mathcal{N}=1$ Seiberg dual

$S^1 \times S^3$, $S^1 \times S^3/\mathbb{Z}_r$, $S^2 \times T^2$, ...

$\Gamma(x;p,q)$ $\Gamma_r(x;p,q)$ $\theta(x;q)$

[elliptic]

3d $\mathcal{N}=2$ Aharony dual $S^1 \times S^2$ $\to$ $(x;q)_\infty$

* 2d $\mathcal{N}=(2,2)$ Seiberg-like dual $T^2 \longrightarrow \theta(x;q)$

[rational]

* 1d $\mathcal{N}=4$ Seiberg-like dual $S^1 \longrightarrow \sin(x)$

[trigonometric]

Works in progress: 5d $\mathcal{N}=1$

2d $\mathcal{N}=(0,2)$

YBE from

2d $\mathcal{N}=(2,2)$ Dualities

[W. Yan + MY '15]

2D Seiberg-like duality

$\mathcal{N}=(2,2)$

[Benini+Cremonesi '12, Gadde+Gukov '13]

electric	magnetic
$G = U(N_c)$ w/ N_f flavors q_i $\bar{q}^i$	$G = U(N_f - N_c)$ w/ N_f flavors Q_i $\tilde{Q}^i$
	meson $M_i{}^j$
$W_{el} = 0$	$W_{mag} = \mathrm{Tr}(\tilde{Q}^i M_i{}^j Q^j)$
t_{el}	$t_{mg} = -t_{el}$

U, not SU!

$t := r + i\theta$ complexified FI

$(\mathcal{L} \supset rD + \theta F_{01})$

2D (2,2) answer from T^2:

Probably new

[W. Yan + MY '15]

spin variables

$$\mathcal{R}\begin{pmatrix} \delta & \gamma \\ \alpha & \beta \end{pmatrix}\begin{bmatrix} d & c \\ a & b \end{bmatrix}(q,y)$$

deformation parameters

spectral param

$$= \sqrt{\frac{\prod_{i,j=1}^{N}\overline{\Delta}_{2-\delta-\alpha}(d_i a_j^{-1})\overline{\Delta}_{2-\beta-\gamma}(b_i c_j^{-1})}{\prod_{i,j=1}^{N}\overline{\Delta}_{\alpha+\beta}(a_i b_j^{-1})\overline{\Delta}_{\gamma+\delta}(c_i d_j^{-1})}}\left[\frac{1}{N!}\left(\frac{\eta(q)^3}{i\theta_1(t^{-1};q)}\right)^N\right]^2$$

eta/theta function

(3.12)

$$\times\sqrt{\prod_{i\neq j}\frac{1}{\Delta(a_i a_j^{-1};q,y)}\prod_{i\neq j}\frac{1}{\Delta(c_i c_j^{-1};q,y)}}\oint\prod_i\frac{dz_i}{2\pi i z_i}\prod_{i\neq j}\frac{1}{\Delta(z_i z_j^{-1};q,y)}$$

N-comp. spin for G = U(N)

continuous spin (sum → integral)

$$\times\prod_{i,j=1}^{N}\overline{\Delta}_\alpha(a_j z_i^{-1})\overline{\Delta}_\beta(z_i b_j^{-1})\overline{\Delta}_\gamma(c_j z_i^{-1})\overline{\Delta}_\delta(z_i d_j^{-1})\,.$$

$$\Delta(a;q,y) := \frac{\theta_1(y^{-1}a;q)}{\theta_1(a;q)}$$

theta function

$$\overline{\Delta}_r\,(x;q,y) := \Delta\left(y^{\frac{r}{2}}x;q,y\right)$$

(YBE can be proven directly)

For $G=U(1)$, and after dim. red to

1D QM, simplifies to [W. Yan + MY '15]

$$\mathcal{R}^{U(1)}\begin{pmatrix}\delta & \gamma\\ \alpha & \beta\end{pmatrix}\begin{bmatrix}\boldsymbol{d} & \boldsymbol{c}\\ \boldsymbol{a} & \boldsymbol{b}\end{bmatrix}(z)$$

$$=\sqrt{\frac{\overline{\boldsymbol{\Delta}}_{\alpha+\delta}(\boldsymbol{a}-\boldsymbol{d},z)\overline{\boldsymbol{\Delta}}_{\alpha+\beta}(\boldsymbol{a}-\boldsymbol{b},z)}{\overline{\boldsymbol{\Delta}}_{\gamma+\beta}(\boldsymbol{c}-\boldsymbol{b},z)\overline{\boldsymbol{\Delta}}_{\gamma+\delta}(\boldsymbol{c}-\boldsymbol{d},z)}}\overline{\boldsymbol{\Delta}}_{\gamma-\alpha}(\boldsymbol{c}-\boldsymbol{a},z) \tag{3.31}$$

$$+\sqrt{\frac{\overline{\boldsymbol{\Delta}}_{\gamma+\beta}(\boldsymbol{c}-\boldsymbol{b},z)\overline{\boldsymbol{\Delta}}_{\gamma+\delta}(\boldsymbol{c}-\boldsymbol{d},z)}{\overline{\boldsymbol{\Delta}}_{\alpha+\beta}(\boldsymbol{a}-\boldsymbol{b},z)\overline{\boldsymbol{\Delta}}_{\alpha+\delta}(\boldsymbol{a}-\boldsymbol{d},z)}}\overline{\boldsymbol{\Delta}}_{\alpha-\gamma}(\boldsymbol{a}-\boldsymbol{c},z)\,.$$

Q: Is this new?

$$\boldsymbol{\Delta}(\boldsymbol{x},z):=\frac{\sinh(\boldsymbol{x}-z)}{\sinh(\boldsymbol{x})}\,,$$

$$\overline{\boldsymbol{\Delta}}_r(\boldsymbol{x},z):=\frac{\sinh\left(\boldsymbol{x}+(\frac{r}{2}-1)z\right)}{\sinh\left(\boldsymbol{x}+\frac{r}{2}z\right)}\,.$$

simply sine functions!

* The R-matrix is modular; $\tau \to 1/\tau$

modulus of T^2

$$\mathcal{R}\begin{pmatrix} \delta & \gamma \\ \alpha & \beta \end{pmatrix}\begin{bmatrix} \tilde{d} & \tilde{c} \\ \tilde{a} & \tilde{b} \end{bmatrix} \doteq (-\tau)^N e^{\frac{i\pi}{\tau}\frac{3N^2}{3}\zeta^2}\mathcal{R}\begin{pmatrix} \delta & \gamma \\ \alpha & \beta \end{pmatrix}\begin{bmatrix} d & c \\ a & b \end{bmatrix}.$$

$3N^2$: central charge

$$q = e^{2\pi i \tau} \to \tilde{q} = e^{2\pi i(-1/\tau)}$$

$$a = e^{2\pi i A} \to \tilde{a} = e^{2\pi i(-A/\tau)}$$

Generalization of YBE

Q: any interest?

2D $\mathcal{N}=(2,2)$ for T^2

$\rightsquigarrow Z^{\text{electric}}_{T^2} = Z^{\text{magnetic}}_{T^2}$

2D $\mathcal{N}=(2,2)$ for S^2

$\rightsquigarrow Z^{\text{electric}}_{S^2}(t_e) = Z^{\text{magnetic}}_{T^2}$ $(t_m = -t_e)$

$t := r + i\theta$ complexified FI

$(\mathcal{L} \supset rD + \theta F_{01})$

cluster y-variable [Benini Park Zhao]

$\rightsquigarrow$ "cluster-variable dependent R-matrix".

Conclusion

Gauge/YBE

$\boxed{\text{YB-duality}} \rightleftarrows \boxed{\text{YBE}}$

∴ many new solutions.

Please study them!

※ "integrability in theory space"

conserved charges map one theory to another