Pure Natural Inflation

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Refs

Y. Nomura, T. Watarai + Y 1706 hep-ph
Y. Nomura + Y 1711 hep-ph
J.-P Hong, M. Kawasaki + Y 1711 astro-ph

cf. K. Yonekura + Y 1704 hep-th
inflationary paradigm – successful 😊😊

but which inflaton potential? 😞
"Simplest" choices,

\[ V(\phi) = \phi^2, \phi^4 \] disfavored!

[Planck 2015]
* inflaton $\phi$ as axion

$$L \supset \frac{1}{32\pi^2} \frac{\phi}{f} \text{Tr} F_{\mu \nu} \tilde{F}^{\mu \nu}$$

- dynamical $\Theta$-angle
- top. term in pure Yang–Mills
- decay constant

$$\Theta = \frac{\phi}{f}$$
* inflaton as axion

\[ L = \frac{1}{32\pi^2} \frac{\phi}{f} \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} \quad [\theta = \frac{\phi}{f}] \]

\( \text{dynamical} \)
\( \Theta \)-angle

* perturbatively \( V(\phi) = 0 \) \([\phi \rightarrow \phi + c \text{ sym.}]\)

non-perturbatively

\[ V(\phi) = \Lambda^4 \left( 1 - \cos \frac{\phi}{f} \right) + \ldots \]

"dynamical scale" \( \text{1- inst.} \) \( \text{multi-inst.} \)
inflaton as axion

\[ L = \frac{1}{32\pi^2} \frac{\phi}{f} \text{Tr } F_{\mu\nu} \tilde{F}^{\mu\nu} \quad [\theta = \frac{\phi}{f}] \]

dynamical \thetabour{\text{angle}}

perturbatively \quad V(\phi) = 0 \quad [\phi \to \phi + c \text{ sym.}] 

non-perturbatively

\[ V(\phi) = \Lambda^4 \left( 1 - \cos \frac{\phi}{f} \right) + \cdots \]

"dynamical scale" \quad 1-inst.

multi-inst.

\begin{itemize}
  \item flatness \quad (\phi \to \phi + c)
  \item EFT \quad (\Lambda \ll M_{\text{UV}})
  \item rather simple
  \item 1-parameter extension of $V \sim \phi^2$
\end{itemize}
Citation counts of [Freese, Frierson, Olinto 1990] (natural inflation)
\[
V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]
\]

😞 being disfavored by observations

[Plonck 2015]

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\(n_s=50\)

\(n_s=60\)

Convex

Concave

\(f: \text{large}\)

\(f: \text{small}\)
\[ V(\phi) = \Lambda^4 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right] + \ldots \]

😊 theoretically NOT CORRECT!
(at least for pure YM @ T=0)
\[ V(\phi) = \Lambda^4 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right] + \cdots \]

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(at least for pure YM @ T=0)

※ YM theory: classically scale-invariant

IR problem: instanton gives divergent answer

known since \( N \) large

[Witten '79, '81, ..., '98]

more recently

[Dubovsky - Lawrence - Roberts '11]

[Giusti - Petroncini - Taglialatela '07]

\( \text{latticerty} \)
Expected form of $V(\phi)$

$\text{CP sym}$

$\phi \rightarrow -\phi$

$V(\phi)$

$V \sim \phi^2$

[Recall $\phi = \Theta f$]

Plateau

$\sim \Lambda^4$
periodicity in $\theta$ recovered by multiple metastable branches

\[ V(\phi) \]

\[-2\pi f \quad 0 \quad 2\pi f \quad 4\pi f \]

[a version of monodromy inflation
Silverstein - Westphal, Kaloper - Lawrence - Sorbo, ... ]

* beware tunneling in different branches
Our potential

\[ V(\phi) = M^4 \left[ 1 - \left( 1 - \left( \frac{\phi}{F} \right)^2 \right)^{-p} \right] \]

Overall scale

"Effective" decay const.

(F_{\text{nf}})

\[ \text{CP sym } \phi \rightarrow -\phi \]

\[ V \sim \phi^2 \text{ near } \phi \sim 0 \]

\[ V(\phi) \rightarrow \text{const. at } \phi \rightarrow \pm \infty \]

\[ p=3 \text{ for holographic QCD (w/ } M_{\text{KK}} \]

\[ M^4 = \frac{\lambda N^2}{3^7 \pi^2} M_{\text{KK}}^4, \quad F = \frac{8 \pi^4 N}{f} \lambda = g_{\text{YM}}^2 N \]

[ Dubovsky, Lawrence, Roberts '11]
Wow!!

Tensor/scalar ratio

$N_e = 50$

$N_e = 60$

$F = 10 \, M_{\text{pe}}$

$F = 5 \, M_{\text{pe}}$

$F = M_{\text{pe}}$

$P = 3$

(holographic)

Spectral index $n_s$

$\phi^2$

cosine

pure natural
\[ V = n^4 \left( 1 - \left( \frac{\phi}{\Phi} \right)^2 \right)^{-\frac{p}{2}} \]

- **Legend**:
  - $p = 1$
  - $p = 2$
  - $p = 3$
  - $p = 4$
  - $p = 10$
can we observe tensor modes in the near future? $r \sim 10^{-3}$, or even $10^{-4}$

$N_e = 50$

$N_e = 60$

$p = 1$
$p = 2$
$p = 3$
$p = 4$
$p = 10$

what happens here??
smaller $r$ means smaller $F$
so that $\phi / F = \Theta$ large

$V(\phi)$

$\phi$

$-2\pi f, 0, 2\pi f, 4\pi f$

$f$: small
Finite $N$ effects;

($\#$ of metastable branches) = $N$ (finite)

$V(\phi)$

$\phi$

$N$ metastable vacua

[cf. Yonekura-Y '17]

$\mathbb{Z}_N$ center sym confinement
Finite $N$ effects; bound on field value

$\phi_{\text{hor}} \leq \phi_{\text{max}}$

$\rightarrow$

lower bound on $F$

lower bound on $v$

local minimum $N$ metastable vacua another minimum

[cf. Yonekura-Y '17]
Finite $N$ effects; bound on field value $\phi_{\text{hor}} \leq \phi_{\text{max}}$ → lower bound on $F$

lower bound on $r$

local minimum

our potential troughable

new inflation - alike

another minimum
lower bound on $r$ \cite{Nomura-Y17}

(if we stay away from top of the potential)

roughly $F \gtrsim 0.1 M_{pl}$
For $F \lesssim 0\left(0.1\text{ M}_{\text{pe}}\right)$ we find spatially inhomogeneities (oscillons) $\left[\text{Hong, Kawasaki} \pm Y \ (p>0)\right]$
$\left[\text{also Amin et al. (11)} \ (p<0)\right]$

analytical/numerical
(LatticeEasy)

implications? GW? baryo/leptogenesis?
[also in progress]
Summary

* natural inflation for pure Yang-Mills, when done correctly is in complete agreement with data 😊😊

\[ V(\phi) = M^4 \left[ 1 - \left( 1 - \left( \frac{\phi}{F} \right)^2 \right)^N \right] \]

* \( F \gtrsim 0.1 M_{\text{pl}} \) → tensor modes

\( F \lesssim 0.1 M_{\text{pl}} \) → oscillons
pure natural inflation