Is Gravity the Weakest Force?

Masahito Yamazaki
(Kavli IPMU, Tokyo)

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Satoshi Shirai + MY
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Alexandra Kusenko, Volodymyr Takhistov,
Masaki Yamada + MY
1908.10930 [hep-th]
My rule today:

No complicated equations/figures
No systematic references
Loose in factors \((2 = \pi = 1)\)
My rule today:

No complicated equations/figures
No systematic references
Loose in factors (2 = \pi = 1)
OK to be naive/speculative
Attempt for big picture/
  fundamental questions

Be relaxed!
Motivation
Forces in Nature

Gravity
EM
Weak
Strong
Forces in Nature

Gravity \( g_{\mu\nu} \)

EM \( A_\mu \; U(1) \)

Weak \( A_\mu^a \; SU(2) \)

Strong \( A_\mu^a \; SU(3) \)
Forces in Nature

- Gravity $g_{\mu\nu}$ long-ranged
- EM $A_{\mu} U(1)$
- Weak $A_{\mu}^{a} SU(2)$ short-ranged (not today)
- Strong $A_{\mu}^{a} SU(3)$
Forces in Nature

- Gravity
- EM
- Weak
- Strong

\[ \text{Gravity} \quad G_{\mu\nu} \quad \text{spin 2} \]
\[ \text{EM} \quad A_{\mu} \quad U(1) \quad \text{long-ranged} \]
\[ \text{Weak} \quad A_{\mu}^{a} \quad SU(2) \quad \text{short-ranged} \]
\[ \text{Strong} \quad A_{\mu}^{a} \quad SU(3) \quad (\text{NOT today}) \]
Forces in Nature

- Gravity
- EM
- Weak
- Strong

\[ \sigma \text{ spin 0} \quad \dot{\sigma} \text{ spin 2} \]

\[ \sigma \text{ spin 1} \]

\[ \Gamma_{\mu \nu} \]

\[ A_\mu U(1) \]

\[ A_\mu \text{ SU(2)} \]

\[ A_\mu \text{ SU(3)} \]

Long-ranged

\[ \phi ? \]

Short-ranged (NOT today)
Gravity versus EM

Microscopically

\[ F_{\text{gravity}} = \frac{m^2}{8\pi M_p c^2} \frac{1}{r^2} \]

\[ F_{\text{EM}} = \frac{e^2}{4\pi} \frac{1}{r^2} \]
Gravity versus EM

Macroscopically

\[ F_{\text{gravity}} = \frac{m^2}{8\pi M_p e^2} \frac{1}{r^2} \]

\[ \downarrow \]

\[ F_{\text{EM}} = \frac{e^2}{4\pi} \frac{1}{r^2} \]

\( m: \text{huge gravity accumulates} \)

\( e \sim 0 \) (cancellation of opposite charges)
Gravity versus EM

length scale

microscopic → macroscopic

\[ F_{EM} \]

\[ \uparrow \]

\[ F_{gravity} \]
Gravity versus EM

length scale

microscopic

F_{EM}

\uparrow

F_{gravity}

everywhere?

macroscopic
Fermi small means e: gauge coupling small

e.g. electron

\[ e \lesssim \frac{m}{M_{\text{pl}}} \sim \frac{0.5 \text{ MeV}}{10^{18} \text{ GeV}} \sim 10^{-21} \]
Fermi small means $e$: gauge coupling small

e.g., electron

$$e \lesssim \frac{m}{M_{\text{pl}}} \sim \frac{0.5 \text{ MeV}}{10^{18} \text{ GeV}} \sim 10^{-21}$$

Sounds very unnatural,

but is technically natural

$$L = \int \sqrt{g} \left( \frac{1}{e^2} F_{\mu \nu} F^{\mu \nu} + R + \Lambda \right)$$
things are different if we consider quantum gravity/strings (swampland paradigm)
(Gauge) WG C
No Global Symmetry

In the extreme limit $e \to 0$
then $U(1)$ gauge sym. $\to U(1)$ global sym.
No Global Symmetry

In the extreme limit \( e \to 0 \)

then \( U(1) \) gauge sym. \( \to U(1) \) global sym.

However, \( \text{semiclassical GR/string} \)

forbids exact global sym.

\( \sim e \to 0 \) not possible

\[ "\text{known}" \text{ from long ago;} \]

\[ \text{See e.g. Banks–Seiberg (10)} \]
(Gauge) Weak Gravity Conjecture

[Arkani-Hamed, Motl, Nicolis, Vafa ('06)]

GWGC

\[ \exists \text{ a particle s.t.} \]

\[ F_{\text{gauge}} = \frac{(e q)^2}{4\pi r^2} \geq F_{\text{gravity}} = \frac{m^2}{8\pi M_p^2} \]

(\( q \): charge, \( m \): mass)

"gravity as the weakest force"
(Gauge) Weak Gravity Conjecture

[Arkani-Hamed, Motl, Nicolis, Vafa ('06)]

GWGC

\[ \exists a \text{ particle s.t. } m \leq \sqrt{2} e q \]

(\( q \) : charge, \( m \) : mass)
This bound

\[ M \leq \sqrt{2}Q \]

is opposite from BH extremality bound

\[ M \geq \sqrt{2}Q \]
Decay of extremal BH (if it happens) requires WGC particle

extremal BH

$M = \sqrt{2}Q$

$\rightarrow (M_1, Q_1)$

$\rightarrow (M_2, Q_2)$

decay

$\rightarrow (M_n, Q_n)$

$\left( \sum M_i = M, \sum Q_i = Q \right)$

$\left( M_i \leq \sqrt{2}Q_i \text{ for some } i \right)$
Gauge-Scalar \quad WGC
Forces in Nature

- Gravity, spin 2
- EM, $A_{\mu}$, $U(1)$, long-ranged
- Weak, $A_{\mu}^a$, $SU(2)$, short-ranged
- Strong, $A_{\mu}^a$, $SU(3)$
- Scalar, $\phi$, spin 0
F_{grav}, F_{gauge}, F_{scalar}

\downarrow

however, typically not "long-range"

since scalar \( \phi \) is massive
\( F_{\text{grav.}}, F_{\text{gauge}}, F_{\text{scalar}} \)

\[ \downarrow \]

However, typically not "long-range"

Since scalar \( \phi \) is massive

\( a) \) SUSY

\( b) \) Very light \((m \lesssim H^{-1})\)

\( c) \) "IR/UV"
Let's assume

a) SUSY

for the moment
BPS extremal BH in 4d \( N=2 \) SUGRA

\[ Q^2 \geq |Z|^2 + g^{i\bar{j}} D_i Z \bar{D}_{\bar{j}} \bar{Z} \]

\( \text{gauge} \quad \overset{\text{gravity}}{=} \quad \overset{\text{scalar}}{=\ M^2 + g^{i\bar{j}} (\partial_i M) (\bar{\partial}_{\bar{j}} M) \quad |Z| = M} \]

[Ceresole, d'Auria, Ferrara (95)]
Scalar Force

\[ L \geq m(\Phi) \bar{\chi} \chi \]
Scalar Force

\[ L = m(\phi) \bar{\chi}\chi = m\langle\phi\rangle \bar{\chi}\chi + (\partial_{\phi} m) \delta\phi \bar{\chi}\chi + \ldots \]

\[ \phi = \langle\phi\rangle + \delta\phi \]
$L = m(\phi) \bar{\chi}\chi = m(\langle \phi \rangle) \bar{\chi}\chi + (Q_\phi m^2) \delta \phi \bar{\chi}\chi$

$\phi = \langle \phi \rangle + \delta \phi$

\[ \text{Scalar Force} \]
$L = m(\phi) \bar{\chi} \chi = m(\langle \phi \rangle) \bar{\chi} \chi + (\partial \phi m) \delta \phi \bar{\chi} \chi$

$\phi = \langle \phi \rangle + \delta \phi$

$F \sim \frac{(\partial \phi m)^2}{r^2}$

"$\partial \phi m$: scalar charge"
GSGWC_1 (repulsive force conjecture)

\[ F_{\text{gauge}} \geq F_{\text{gravity}} + F_{\text{scalar}} \]

GSGWC_2

\[ \frac{Q}{M} \geq \left( \frac{Q}{M} \right)_{\text{extremal BH}} \]

※ GSGWC_1 + GSGWC_2 but closely related

[Heidenreich, Reece, Rudelius (19)]
Scalar WGC
In many situations, Coulomb repulsions const energy and charges tends to neutralize

\[ F_{\text{gauge}} \quad F_{\text{gravity}} \quad F_{\text{scalar}} \]

competition?
We can again start w/ a) SUSY

Previously for extremal BH

\[ Q^2 = M^2 + g i \bar{\delta} D_i M D_j \bar{M} \]

There is a similar relation

[Ceresole, d'Auria, Ferrara (95)]

\[
\left( Q \left|_{N_{ij} \rightarrow f_{ij}} \right. \right)^2 = M^2 - g i \bar{\delta} D_i M D_j \bar{M}
\]
\[ F_{\text{scalar}} \geq F_{\text{gravity}} \]

\[ \frac{|2 \phi m|^2}{4 \pi r^2} \geq \frac{m^2}{8 \pi M_{\text{pl}} r^2} \]

\[ |2 \phi m| \geq \frac{m}{M_{\text{pl}}} \]
\[
\text{A (GWGC) particle}
\]

F_{\text{scalar}} \geq F_{\text{gravity}}
Let's also go beyond a): SUSY
Let's also go beyond SUSY

Hard to expect exactly massless scalar

\[ \mathcal{L} \supset \phi \overline{\chi} \chi \text{ generates mass} \]
but can expect very light scalar $\phi$

(e.g. ALP, quintessence ----)

cf. ds conjecture
[Obied-Ooguri-Spydeveiko]
-Vafa (18)

b): very light $m \lesssim H^{-1}$

[cf. Shirai-Y (19)]
A (GWGC) particle

\[ F_{\text{scalar}} \geq F_{\text{gravity}} \]

This version is too strong

in tension w/ fifth force searches

[Shirai-Y (19)]

(\textit{Loophole by screening mechanism}
  
  e.g. chameleon)
different version?
Previously we considered

\[ \mathcal{L} \supset m(\phi) \bar{\chi} \chi \]

\( \phi \): very light

long-range forces

general

What if we use \textit{self-interaction}?

\[ \mathcal{L} \supset m(\phi) \phi^2 \]

\[ = m\phi^2 - A\phi^3 + \lambda\phi^4 + \ldots \]
\[(\mathcal{M} m)^2 \geq \frac{m^2}{M_{Pl}^2}\]

\[m = V'' = V_{\phi \phi}\]

\[(V'')^2 \geq \frac{(V'')^2}{M_{Pl}^2}\]
$SWGC_1$

\[(\partial \phi \, m)^2 \geq \frac{m^2}{M_{pl}^2}\]

\[m = V'' = V_{\phi \phi}\]

\[(V'')^2 \geq \left(\frac{V''}{M_{pl}^2}\right)^2\]

$SWGC_2$

[\text{Gonzalo, Ibanez (19)}]

\[2 (V'')^2 - V'' V''' \geq \frac{(V'')^2}{M_{pl}^2}\]
\[ 2 (V'')^2 - V'' V''' \geq \frac{(V'')^2}{M_p l^2} \]

if \( V'' > 0 \)

\[ 2 \left( \frac{V'''}{V''} \right)^2 - V''' \geq \frac{V''}{M_p l^2} \]

\text{gravity}

repulsion

attraction
\[ 2 (V'')^2 - V'' V'''' \geq \frac{(V'')^2}{M_{pl}^2} \]

\[ \text{if } V'' > 0 \]

\[ 2 \frac{(V''')^2}{V''} - V'''' \geq \frac{(V'')^2}{M_{pl}^2} \]

i.e. schematically

\[ F_{\text{net scalar}} = F_{\text{att. scalar}} - F_{\text{rep. scalar}} \geq F_{\text{grav}}. \]
e.g. Higgs [Shirai - Y (’19)]

\[
\left( \chi - V^{\prime \prime} / M_P^2 \right) / h^2
\]

\[h \text{ [GeV]}\]
\[2(V'')^2 - V'' V''' \geq \frac{(V'')^2}{M p e^2}\]

- This does not require a very light scalar
  (in general no long-range scalar force)

- hence c) "UV/IR" statement
\[ 2 \left(V''\right)^2 - V'' V^{''''} \geq \frac{(V'')^2}{M_{pe}^2} \]

\[ M_{pe} \to \infty \]

\[ 2(V^{''''})^2 - V'' V^{'''''} \geq 0 \]

Still non-trivial constraint

VERY different from other swampland conjectures

(Not a swampland conjecture?)
Currently little support for SWGC2, likely not true as a general statement? Contrary to the claims of [Gonzalo (19)] Ibanez

However, the main point

\[ F_{\text{scalar}} \geq F_{\text{gravity}} \]

could be true in many cases?
String moduli

In String Theory, "generically"

$O(100)$ scalar moduli

Often assumed: gravitational interaction
(c.f. moduli problem)
In String Theory, "generically"

$O(100)$ scalar moduli

Often assumed: gravitational interaction
(cf. moduli problem)

Scenario changed if $F_{\text{scalar}} \geq F_{\text{grav}}$. 
If \( F \) is scalar, \( N \) is \( F \) grav.

\[
V(\Phi) = -\frac{1}{2} m_2^2 \phi_2^2 - \frac{1}{3} A \phi_3^3 + \frac{1}{4} \phi_4^4
\]
If \( F_{\text{scalar}} \geq F_{\text{grav}} \),

for \( V(\phi) = \frac{1}{2} m^2 \phi^2 - \frac{A}{3} \phi^3 + \frac{2}{4} \phi^4 \)

\[ S W G C \rightarrow A^2 - \lambda m^2 \geq \left( \frac{m^2}{M_{pl}} \right)^2 \]

F\text{att..}\ F_{\text{ scalar}} \ F_{\text{ rep..}} \ F_{\text{ scalar}} \ F_{\text{ grav.}}
If \( F_{\text{scalar}} \geq F_{\text{grav}} \).

For \( V(\phi) = \frac{1}{2} m^2 \phi^2 - \frac{A}{3} \phi^3 + \frac{\lambda}{4} \phi^4 \).

\[
SWGC \quad \Rightarrow \quad A^2 - \lambda m^2 \gtrsim \left( \frac{m^2}{M_{\text{pl}}} \right)^2
\]

\( A \gtrsim \frac{m^2}{M_{\text{pl}}} \) \quad \text{oscillon formation} \quad \text{[Kusenko-Takistov-Yamada-Y]}(19)

\[A \gtrsim \lambda m^2\]

\[\lambda: \text{small} \quad \text{gravity small}\]

\[A^2 \geq \frac{m^2}{M_{\text{pl}}} \quad \text{Q-ball formation} \quad \text{(Floquet exponent} \geq H)\]
If \( F_{\text{scalar}} \geq F_{\text{grav}} \), scalar fields fragment into localized lumps after inflation.

\[ \text{[Kusenko-Takhistov-Yamada-Y (19)]} \]

\( Q \)-ball / oscillon \[ \text{[Kusenko-Shaposhnikov (97), …]} \]

PBH \[ \text{[Alex’s talk]} \]

\( \vdots \text{ as DM?} \)
Summary

GWGC

\[ F_{\text{gauge}} \geq F_{\text{gravity}} \]

Swampland

QG/string ?
Summary

GWGC

\[ F_{\text{gauge}} \geq F_{\text{gravity}} \]

GSWGC

\[ F_{\text{gauge}} \geq F_{\text{scalar}} + F_{\text{gravity}} \]

SWGC

\[ F_{\text{scalar}} \geq F_{\text{gravity}} \]

most controversial but most powerful

Swampland

QG/string ?
Q: Is gravity the weakest force?

Q: If so, in what sense? Why?