EFT & Beyond

Masahito Yamazaki
(IPMU, Tokyo)

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原子核三者夏の学校

Lec II

Decoupling theorem

effect of massive mode w/ mass suppressed by powers of M in pert. theory

"decoupling of UV from IR"

matter content

sym.

SMEFT

$$\mathcal{L} = \mathcal{L}_0 + \sum_n \frac{C_n \, \mathcal{O}_n}{\Lambda_{UV}^{|\dim \mathcal{O}_n| - D}}$$

Higgs lepton

$$\mathcal{L}_5 \supset \frac{HHLL}{\Lambda_{UV}}$$

$$\mathcal{L}_6 \supset \frac{1}{\Lambda_{UV}^2} QQQL$$

$m^2 H^2$

hierarchy problem

Λ cosmological const.

$$\Lambda^4 \sim 10^{-120} M_{pl}^4 \quad \Lambda_{UV}^4$$

※ one loophole of decoupling theorem:

$$\frac{1}{\frac{k^2}{\alpha} - \Pi(k^2)}$$

$k^2 \tilde{\Pi}(k^2)$

$$\underbrace{k^2 \tilde{\Pi}(k^2)}_{\displaystyle \underbrace{\tilde{\Pi}(0)} + k^2 \tilde{\tilde{\Pi}}(0)}$$

1PI

$\tilde{\tilde{\Pi}}(0) \sim \dfrac{1}{M^2}$

α vs $\tilde{\Pi}(0)$

SSB of gauge sym.

$$|D_\mu H|^2 \supset e^2 A_\mu^2 \underbrace{H^2}_{\displaystyle \Downarrow\ v^2}$$

$m_A \sim e v$

$m_A^2 \sim \alpha v^2$

$$\frac{1}{\underbrace{\frac{k^2 - \alpha v^2}{\alpha}} - \underbrace{\Pi(k^2)}}$$

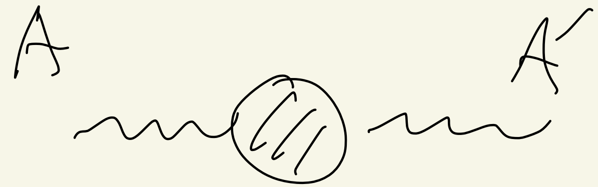
$\dfrac{k^2}{\alpha} \sim \underbrace{v^2}$

$$\Pi(k^2) = \boxed{\Pi(0)} + k^2 \boxed{\tilde{\Pi}(0)} + \underbrace{k^4 \tilde{\tilde{\Pi}}(0)}_{} + \cdots$$

$\tilde{\tilde{\Pi}}(0) \sim 1/M^2$

$\begin{pmatrix} \alpha \\ v \end{pmatrix}$ vs. $\begin{pmatrix} \Pi(0) \\ \tilde{\Pi}(0) \end{pmatrix}$

in EW in SM



$$SU(2)_W \times U(1)_Y \to U(1)_{EM}$$

$W^\pm, Z, \gamma$

$$\Pi_{WW}, \quad \Pi_{ZZ}, \quad \Pi_{Z\gamma}, \quad \Pi_{\gamma\gamma}$$

$$\begin{array}{cc} \begin{pmatrix} \circ & \circ & \times & \times \\ \circ & \circ & \circ & \circ \end{pmatrix} & \begin{array}{l} \Pi(0) \\ \Pi'(0) \end{array} \end{array}$$

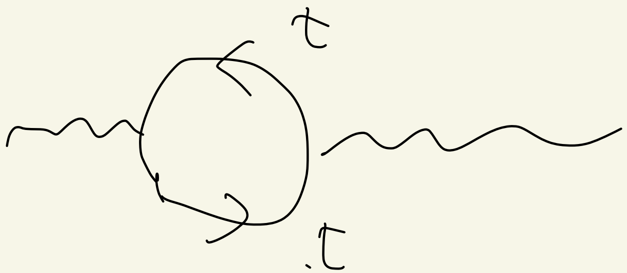
6 parameters

3 parameter

$\alpha_{SU(2)_W}, \ \alpha_Y, \ v_{Higgs}$

3 parameters

S, T, U



UV/IR non-decoupling

- Non-commutative F.T.
- "X-cube"
- gravity

Non-commutative Field Theory

$[x_i, p_j] = i\hbar\delta_{ij}$

$\Delta x \Delta p \gtrsim \hbar$

- Non-commutative Spacetime

$$[x^\mu, x^\nu] = i \underbrace{\Theta^{\mu\nu}}_{\text{anti-sym.}} = i \begin{pmatrix} \begin{matrix} 0 & \theta_1 \\ -\theta_1 & 0 \end{matrix} & & \\ & \begin{matrix} 0 & \theta_2 \\ -\theta_2 & 0 \end{matrix} & \\ & & \ddots \end{pmatrix}$$

(∴ Lorentz sym broken)

$\Downarrow$

uncertainty relation

$$\Delta x^\mu \Delta x^\nu \gtrsim |\Theta_{\mu\nu}|$$

"UV-IR mixing"

$\Delta x \to 0$ $\Delta x \to \infty$

$$x^\mu \to \sum \underbrace{\Lambda_\mu{}^\nu}\, x^\nu$$

* noncommutative star product $\quad \phi_1(x)\,\phi_2(x)$

$$(\phi_1 * \phi_2)(x) := e^{\frac{i\Theta_{\mu\nu}}{2}\partial^\mu_y \partial^\nu_z}\,\phi_1(y)\,\phi_2(z)\Big|_{y=z=x}$$

$\phi_1(x+\xi)\,\phi_2(x-\xi)$

$$= \phi_1\phi_2 + i\Theta_{\mu\nu}\,\partial^\mu\phi_1\,\partial^\nu\phi_2 + \cdots$$

Non-locality

$\phi_1 * (\phi_2 * \phi_3) = (\phi_1 * \phi_2) * \phi_3$ associative

$\mathcal{L} \ni \phi_1(x)\,\phi_2(x) \longrightarrow \mathcal{L}_{NC} \ni (\phi_1 * \phi_2)(x)$

$[x^\mu, x^\nu]_* = x^\mu * x^\nu - x^\nu * x^\mu = i\Theta_{\mu\nu}$

replace $\underset{\text{ordinary}}{\wedge}$ product by $*$-product

* ϕ^4 - theory 4D

$\phi_1 \star \phi_2$

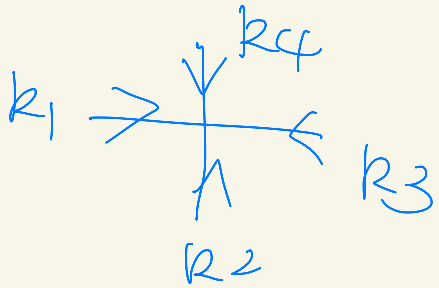
$$\mathcal{L}_{NC} = \int d^4x \left(\partial_\mu \phi \cancel{\star} \partial^\mu \phi - \frac{m^2}{2}\phi \cancel{\star} \phi - \frac{\lambda}{4} \phi\star\phi\star\phi\star\phi \right)$$

$\sum_{i<j} k_i \times k_j$

$k_1 \quad k_2 \quad k_3 \quad k_4$

$= (p, -p, k, -k)$

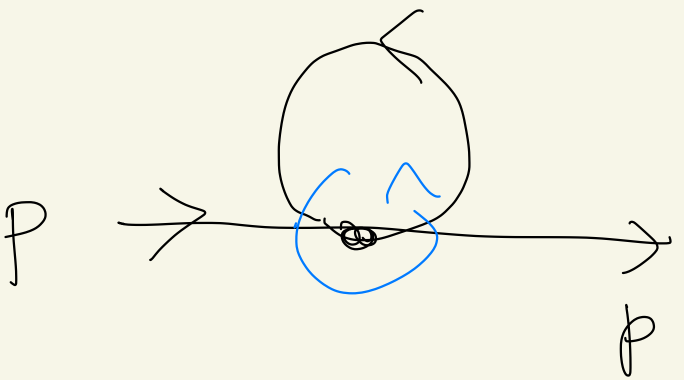
$\lambda \, \delta(k_1 + \cdots + k_4)$



Fourier space

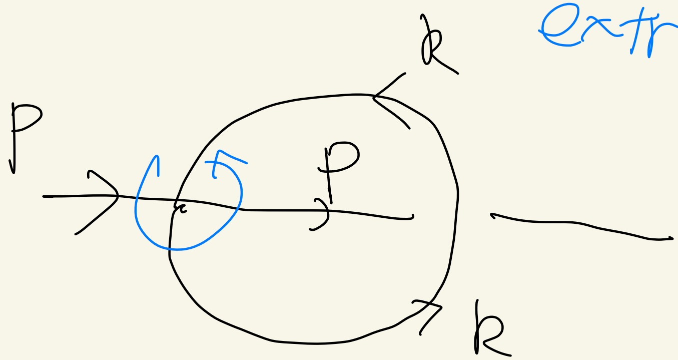
$$e^{\frac{i}{2}\sum_{i<j} \underbrace{k_i \times k_j}_{= k_i^\mu \Theta_{\mu\nu} k_j^\nu}}$$

extra phase $e^{i\delta}$



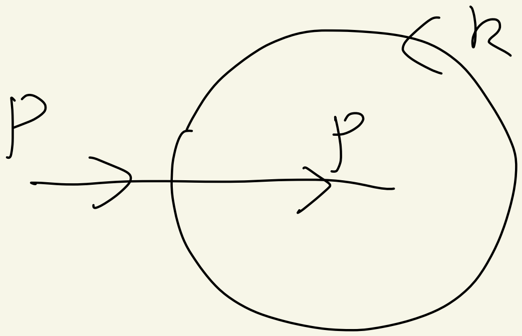
$\delta = 0$

planar



$(p, -k, -p, k)$

$\delta \neq 0$ non-planar



$$\int \frac{d^4k}{k^2+m^2}\, e^{ik\times p}$$

$$\frac{1}{k^2+m^2} = \int d\alpha\, e^{-\alpha(k^2+m^2)}$$

$$\int \frac{d\alpha}{\alpha^2}\, e^{-\alpha m^2 - \frac{p\circ p}{\alpha}}$$

$$|p_\mu \Theta^2_{\mu\nu} p_\nu|$$

$$e^{-\alpha m^2 - \frac{p\circ p + 1/\Lambda_{UV}^2}{\alpha}}$$

$$1/\Lambda_{eff}^2 = \overbrace{p\circ p}^{NC} + \frac{1}{\Lambda_{UV}^2}$$

$$\left(\Lambda_{eff}^2 = \frac{1}{p\circ p + \frac{1}{\Lambda_{UV}^2}} \right)$$

$$\Lambda_{\text{eff}}^2 = \frac{1}{1/\Lambda_{UV}^2 + \text{pop}}$$

$\text{pop} \to 0$ IR limit

$1/\Lambda_{UV}^2 \to 0$ UV limit

* when $\text{pop} \ll \frac{1}{\Lambda^2}$ $\leadsto$ $\Lambda_{\text{eff}} = \Lambda_{UV}$

"IR first"

(NC gone
usual QFT)

* when $\text{pop} \gg \frac{1}{\Lambda^2}$ $\qquad \Lambda_{\text{eff}}^2 = \frac{1}{\text{pop}}$ (NC remains)

"UV first"

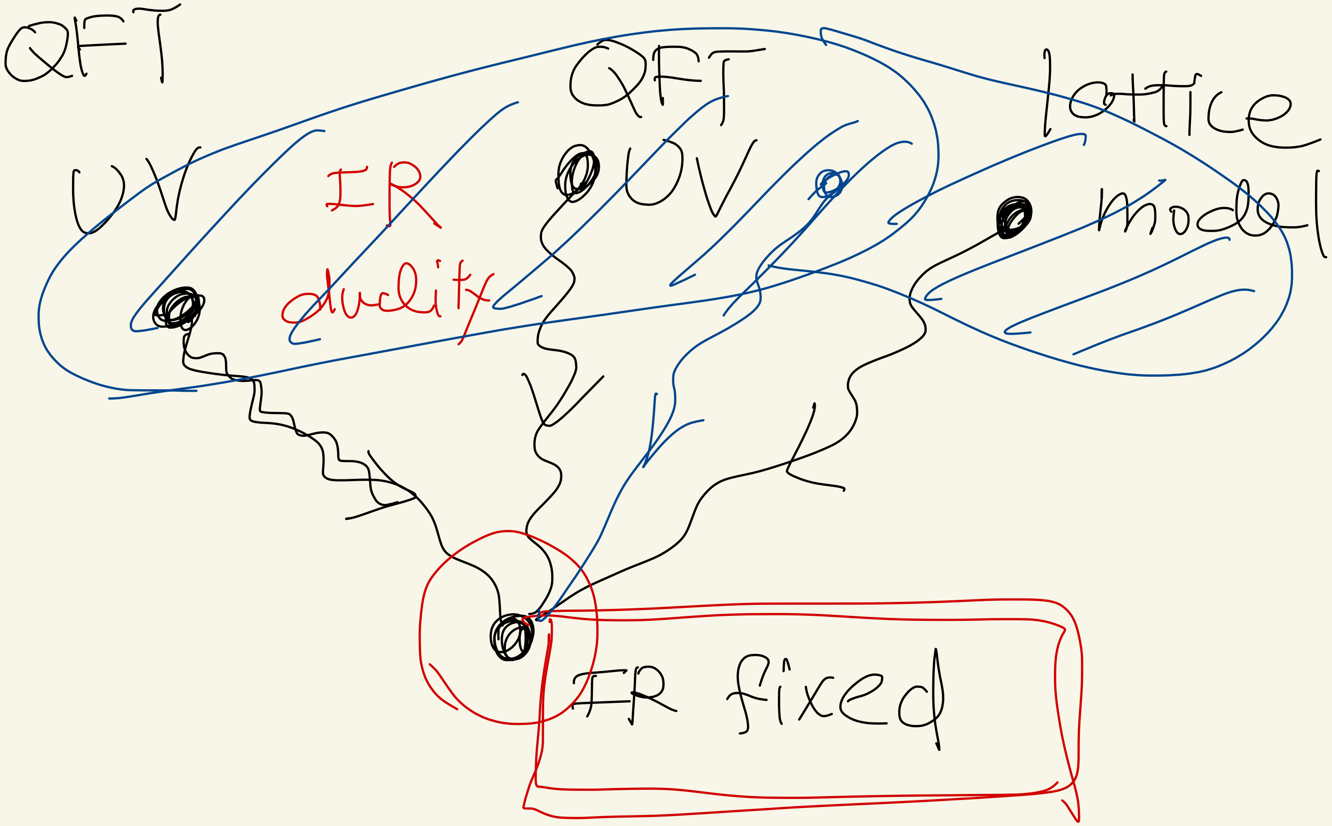
IR limit
UV limit
} do not commute!

"UV/IR mixing"

Fracton / X-cube

Renormalization group flow

Space of QFT (+α)



Universality

IR behavior?

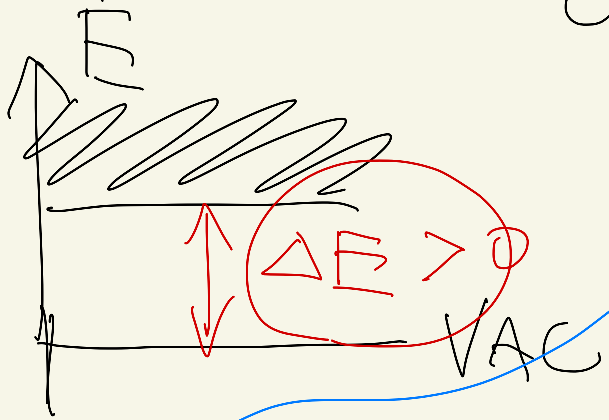
e.g. 4D QCD $\Delta E > 0$: mass gap

is

Λ : dynamical scale

gapped

$\Delta E > 0$



VAC: trivial

topological

ground state degeneracy

GSD

=

1

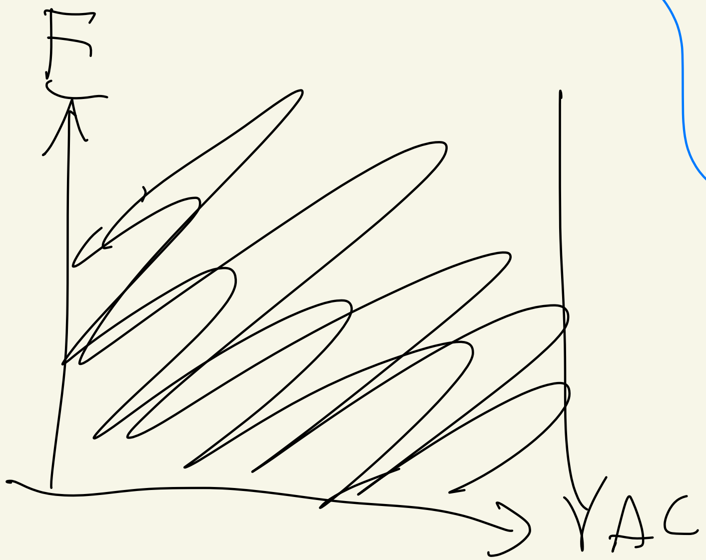
topological order ≒ TQFT

GSD > 1

topology-dependent

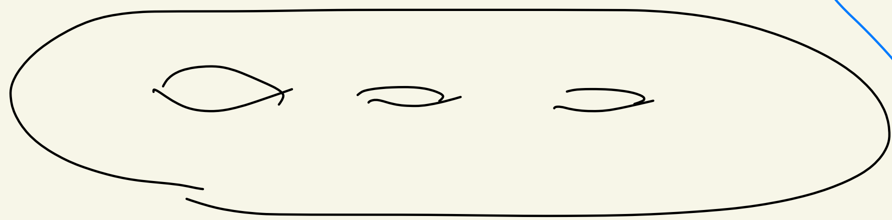
gapless

$\Delta E = 0$



e.g. (1+1)D "toric code" (lattice model)

$\rightsquigarrow$ genus g Riemann surface



ground state deg.

2^{2g}

(e.g. $g=3$)

$\mathbb{Z}_2$-gauge theory

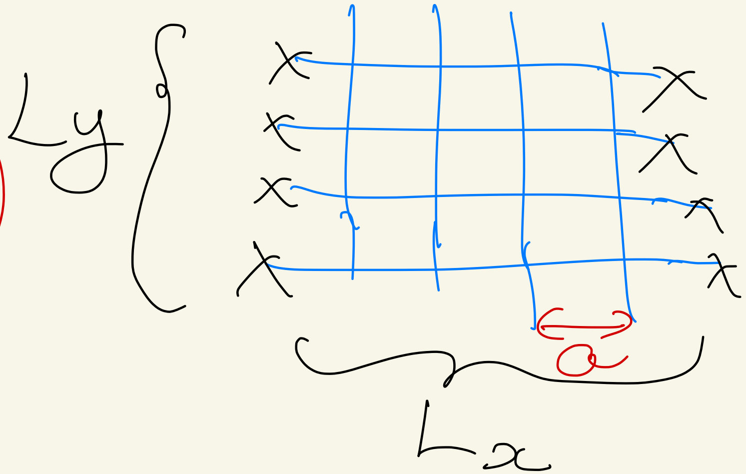
topology-dependent : topological order

Fracton (e.g. X-cube):

- lattice model w/ H.
- (ground state deg) $\sim 2^{O(L)}$

lattice size

New phase of matter?



- gapped

"UV/IR mixing"

a: lattice spacing

$L \to \infty$

$a \to 0$ $\quad$ La: fixed

$\downarrow$

continuum

gapped

$$\langle O(x)\, O(y) \rangle \sim e^{-\Delta E |x-y|}$$

exponential

gapless / CFT

$$\langle O(x)\, O(y) \rangle \sim \frac{1}{|x-y|^{2\Delta_O}}$$

power-law

TQFT

$$\langle O(x)\, O(y) \rangle = \text{const,}$$

$$(x \neq y)$$