θ-vacua in 4d Yang-Mills Theories

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2010.08810 & 2102.08784 [hep-lat])

Today :

4D Pure Yang-Mills Theory

w/ θ-angle w/ $G = SU(2)$

Q : Free Energy $F(\theta) = -\frac{1}{V} \ln \frac{Z(\theta)}{Z(0)}$

as a function of θ ?

Q : Fate of CP-sym. @ $\theta = \pi$?

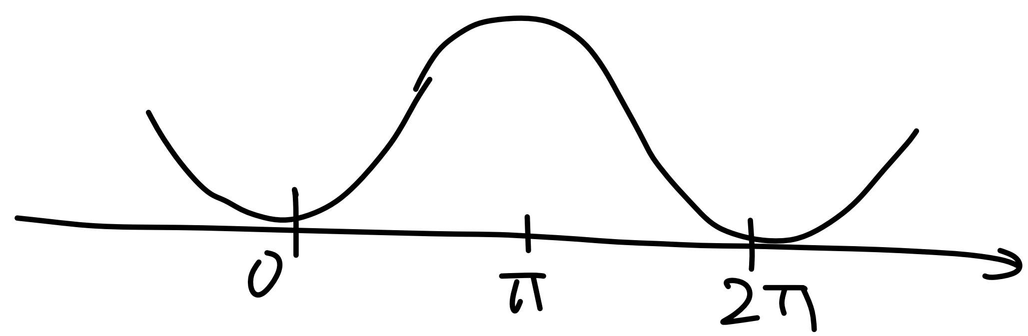
Q : gapped (confinement) ? gapless ?

Instanton (DIGA) ['t Hooft]

$$F(\theta) \sim \int^{\rho\to\infty:\,\mathrm{IR}} \frac{d\rho}{\rho^5}\, e^{-\frac{8\pi^2}{g^2(\mu)}} (\mu\rho)^{b_1 = \frac{11}{3}N} (1-\cos\theta)$$

$$+ (2\text{-instanton}) + \cdots\cdots$$

※ 2π- periodic



※ Works well at $T \gtrsim T_c$

※ However, not correct for $T \ll T_c$

($\rho \to \infty$ divergent : IR problem)

Large N [Witten]

$$\mathcal{L} \sim \frac{1}{N^{-1}} \left(\frac{1}{g^2 N} \mathrm{Tr}\, F \wedge * F + \frac{\theta}{N} \mathrm{Tr}\, F \wedge F \right)$$

topological susceptibility

$$E(\theta) = N^2 f\left(\frac{\theta}{N}\right) = \frac{1}{2} \chi \theta^2 \left(1 + b_2 \theta^2 + \cdots \right)$$

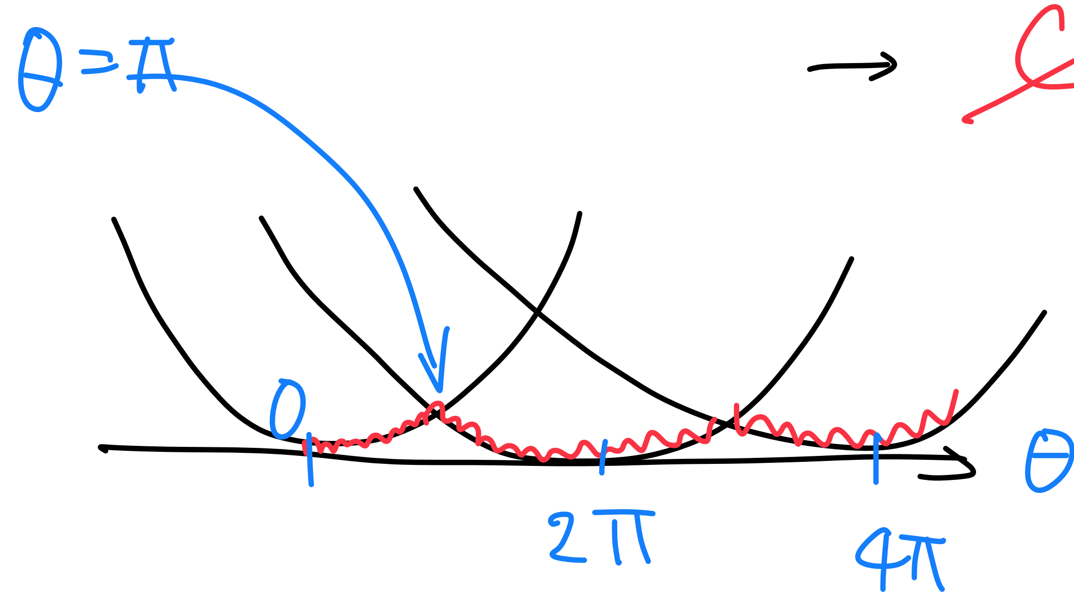
dimensionless coefficient

$$\chi = \chi^{(0)} + O\left(\frac{1}{N^2}\right) \to \chi^0$$

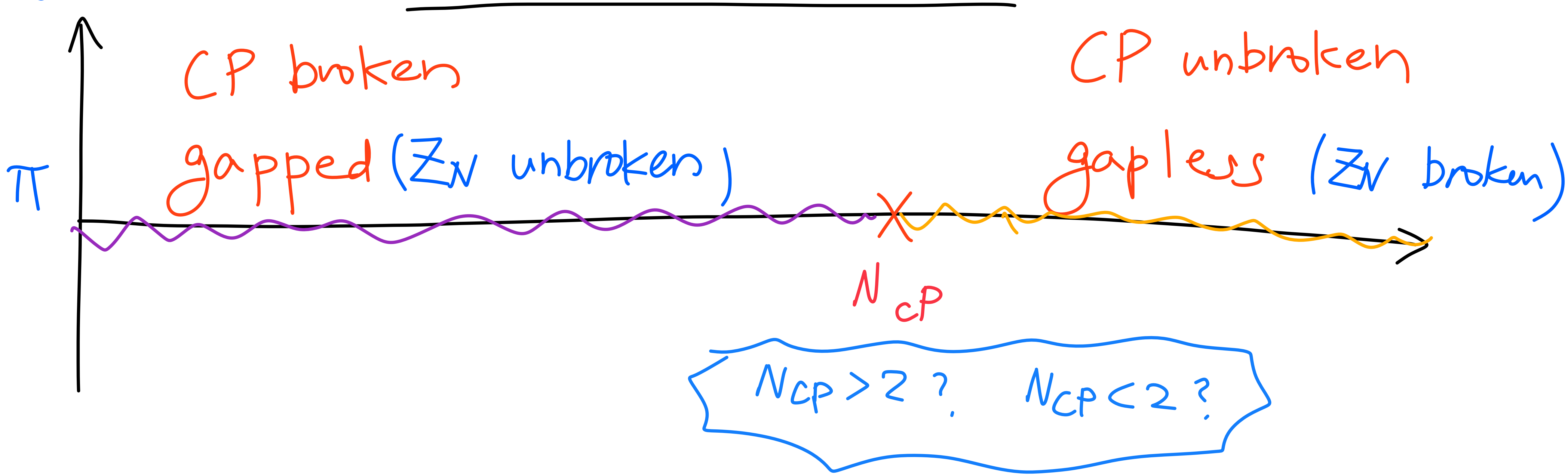
$$b_{2n} = \frac{b_{2n}^{(0)}}{N^{2n}} + O\left(\frac{1}{N^{2n+2}}\right) \to 0$$

※ NOT 2π-periodic $\to$ multiple branches

$\to$ ~~CP~~ @ $\theta = \pi$ [Daschen]



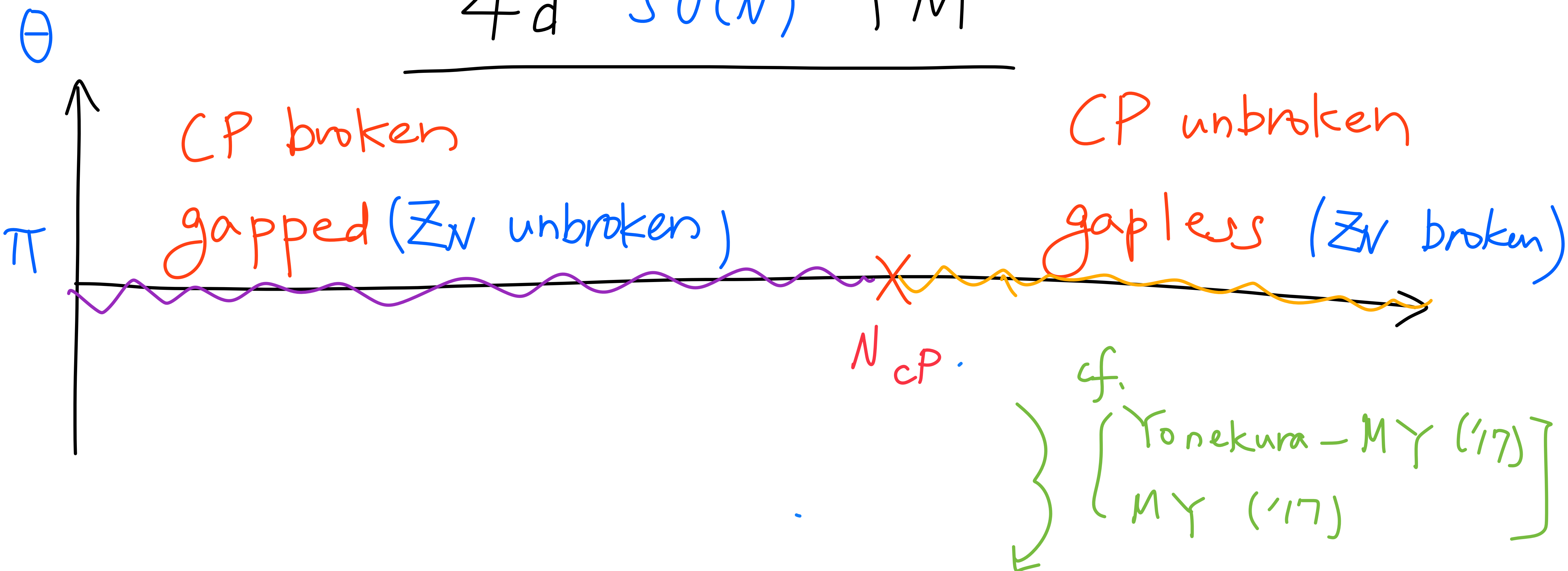
4d $SU(N)$ YM



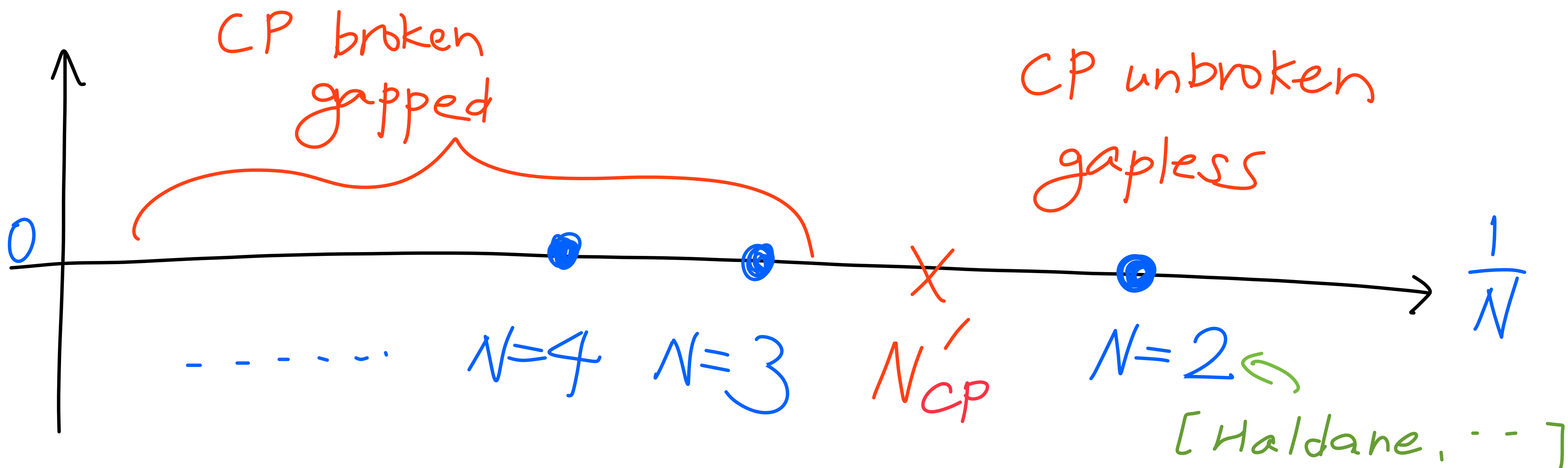
(※ mixed $CP - \mathbb{Z}_N$ anomaly @ $\theta = \pi$

[Gaiotto - Kapustin - Seiberg - Willet])

4d $SU(N)$ YM



2d CP^{N-1} model



Lattice

"Just do it" on lattice?

However

① definition of Q : discretized Q not quantized

(due to short-distance modes)

smearing

② Sign Problem $e^{-S_g + i\theta Q}$

widely fluctuating @ $\theta \sim \pi$

[Kitano-Yamada-MY ('20)]

- expansion around $\theta \sim 0$

$$F(\theta) = \frac{1}{2}\chi\theta^2(1+b_2\theta^2+b_4\theta^2+\cdots)$$

- "subvolume method"

[Kitano-Matsudo-Yamada-MY ('21)]

both uses confs @ $\theta=0$

Expansion around $\theta = 0$.

2010.08810

(many refs, e.g. Bhanot - Rabinovici - Seiberg - Woit ('84)

⋮

Del Debbio - Panagopoulos - Vicari ('02)

Bonati - D'Elia - Rossi - Vicari ('13, '16)) generate gauge conf. at $\theta=0$ ← no sign problem

↓

measure top. charge Q

↓

$$\chi = \frac{\langle Q^2 \rangle_{\theta=0}}{V} \,,$$

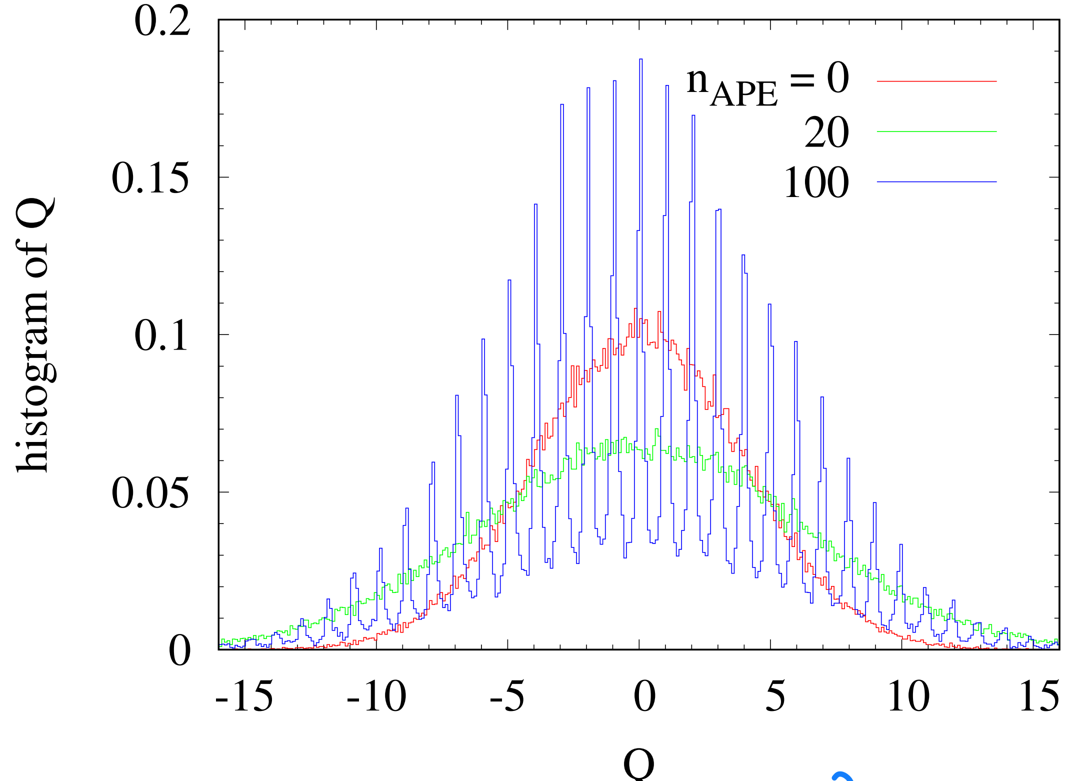
$$b_2 = -\frac{\langle Q^4 \rangle_{\theta=0} - 3 \langle Q^2 \rangle_{\theta=0}^2}{12 \langle Q^2 \rangle_{\theta=0}} \,,$$

$$b_4 = \frac{\langle Q^6 \rangle_{\theta=0} - 15 \langle Q^2 \rangle_{\theta=0} \langle Q^4 \rangle_{\theta=0} + 30 \langle Q^2 \rangle_{\theta=0}^3}{360 \langle Q^2 \rangle_{\theta=0}} \,,$$

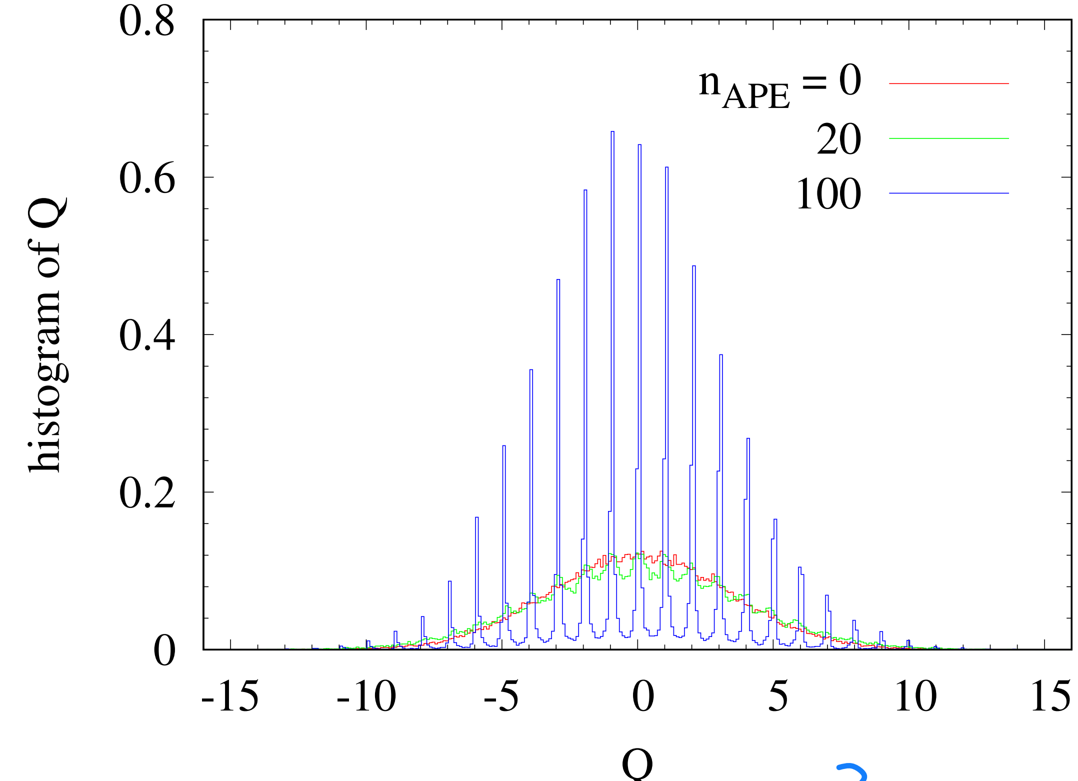
$$F(\theta) = \frac{1}{2} \chi \theta^2 \left(1 + b_2 \theta^2 + b_4 \theta^4 + \cdots \right)$$

cf. pure natural inflation [Nomura - Watari - MY ('17)
Nomura - MY ('17)]

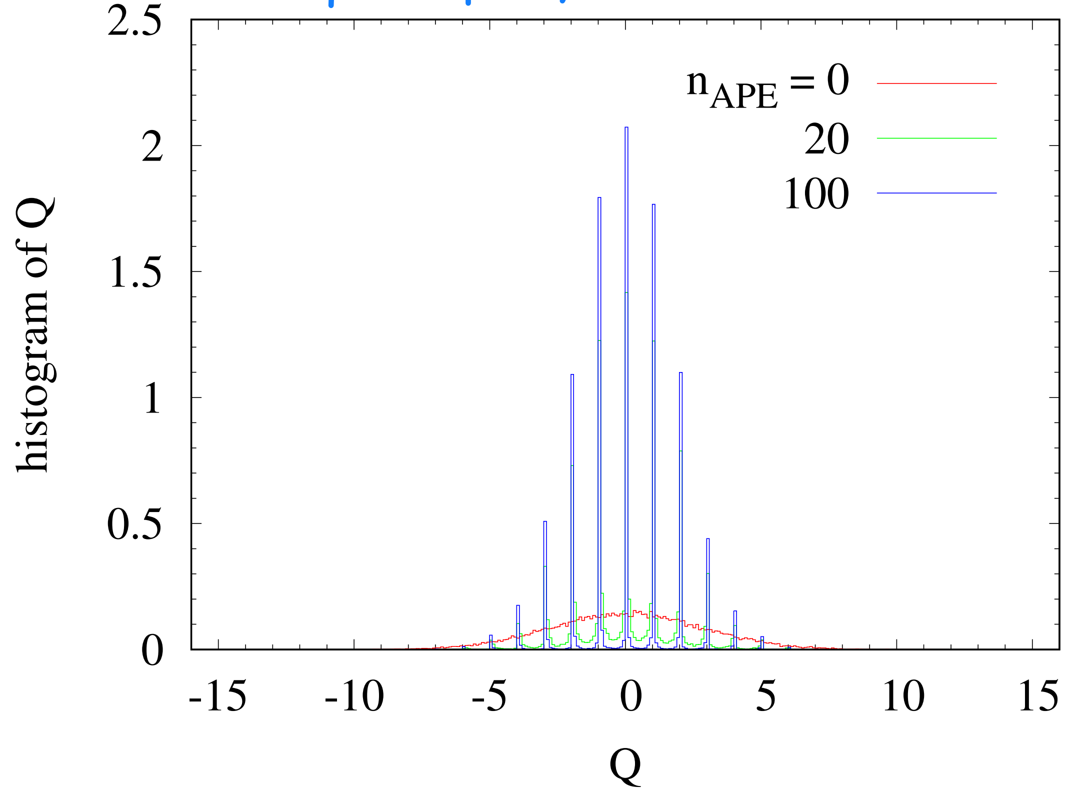
$\beta = 1,75 \quad 16^3 \times 32$



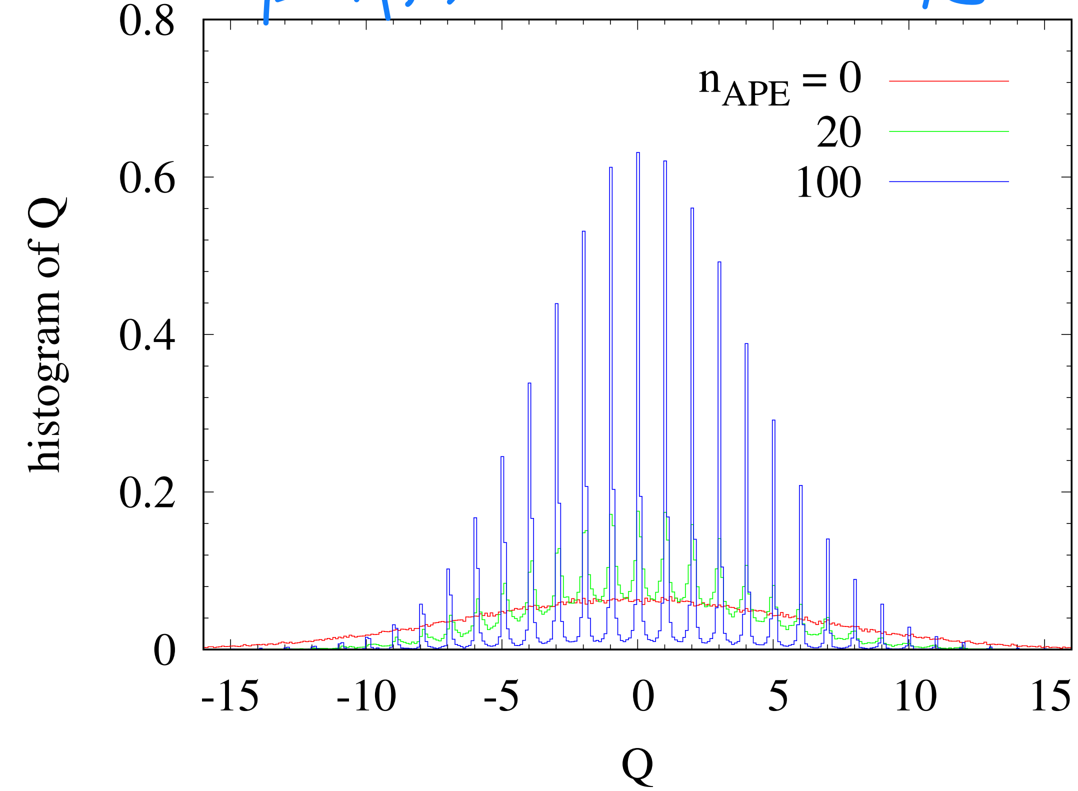
$\beta = 1,85 \quad 16^3 \times 32$

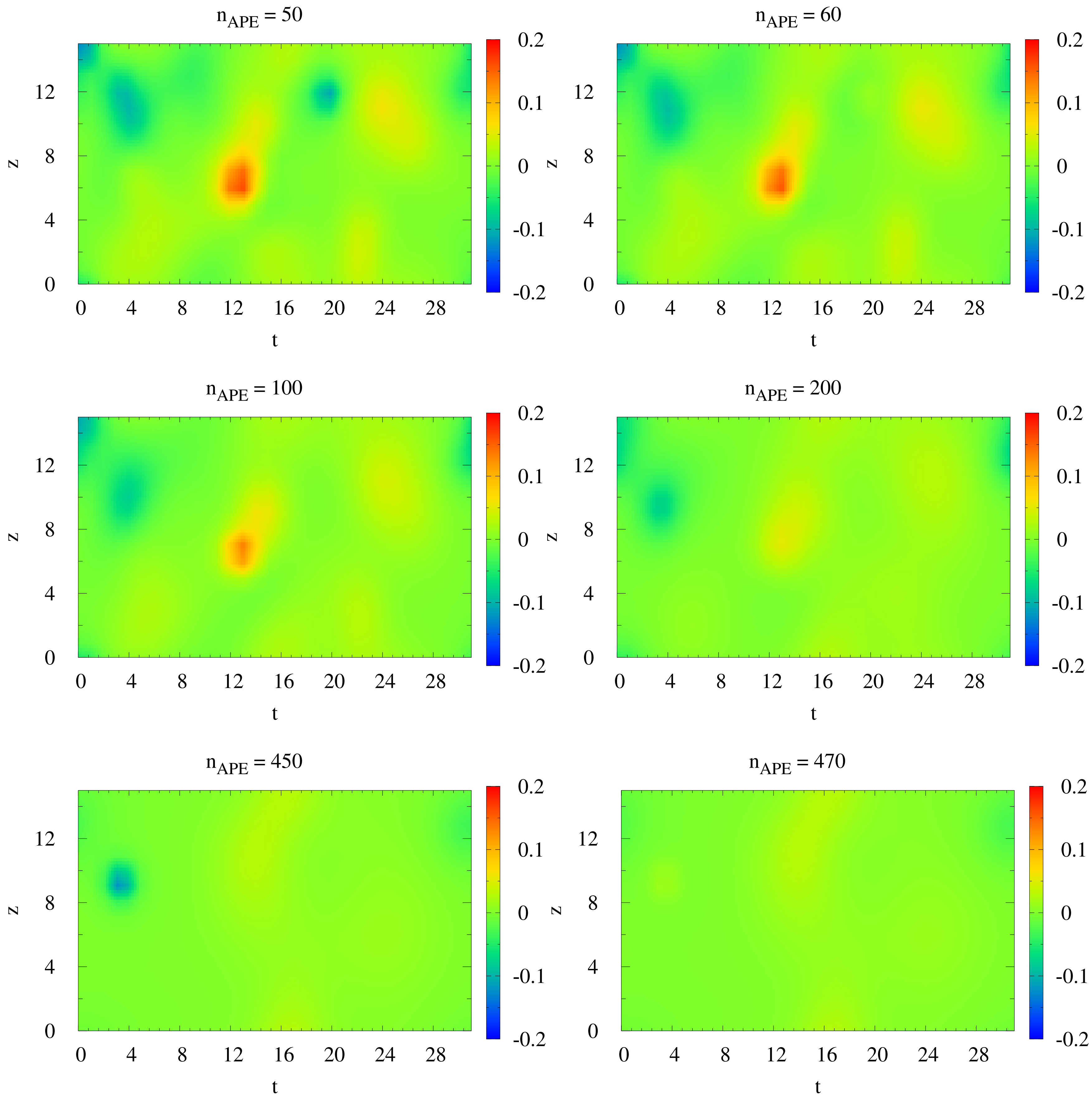


$\beta = 1,975 \quad 16^3 \times 32$



$\beta = 1,975 \quad 24^3 \times 48$



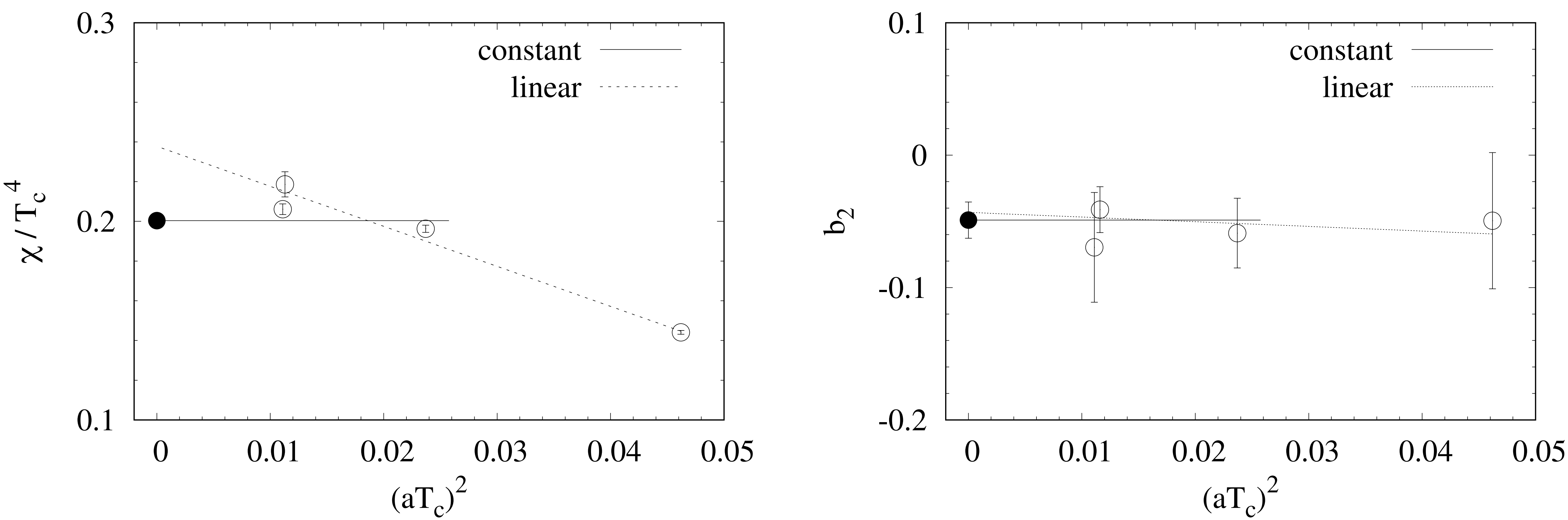


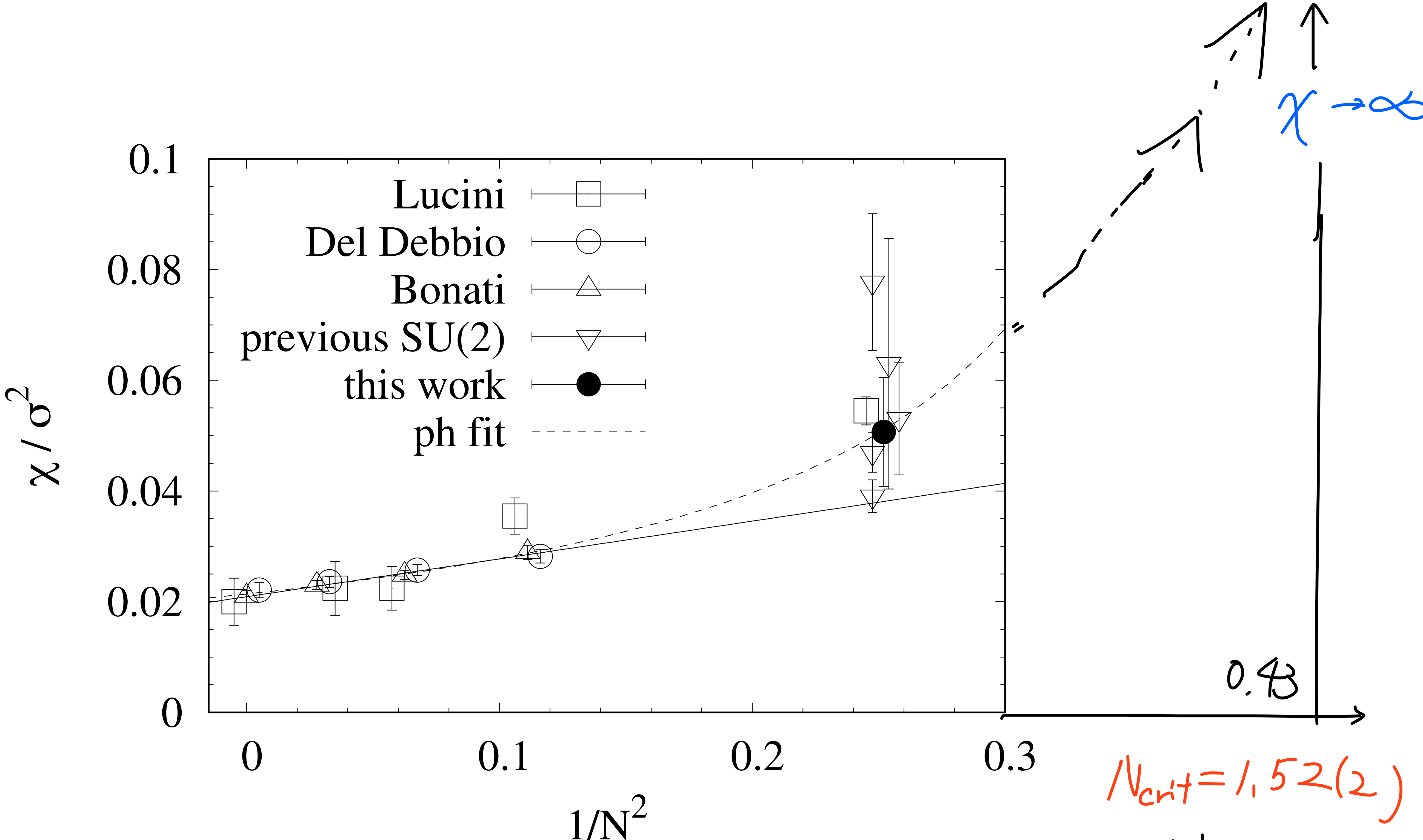
cf. [Bilson-Thompson, Leinweber, Willams, Dunne ('03)]

$$\frac{\chi}{T_c^4} = 0.200(39) \ , \qquad \frac{\chi^{1/4}}{T_c} = 0.674(31) \ , \qquad b_2 = -0.049(20) \ ,$$

seems to be the first determination of b_2!

[cf. Bonanno, Bonati, D'Elía ('18) $b_4 = 6(2) \cdot 10^{-4}$]



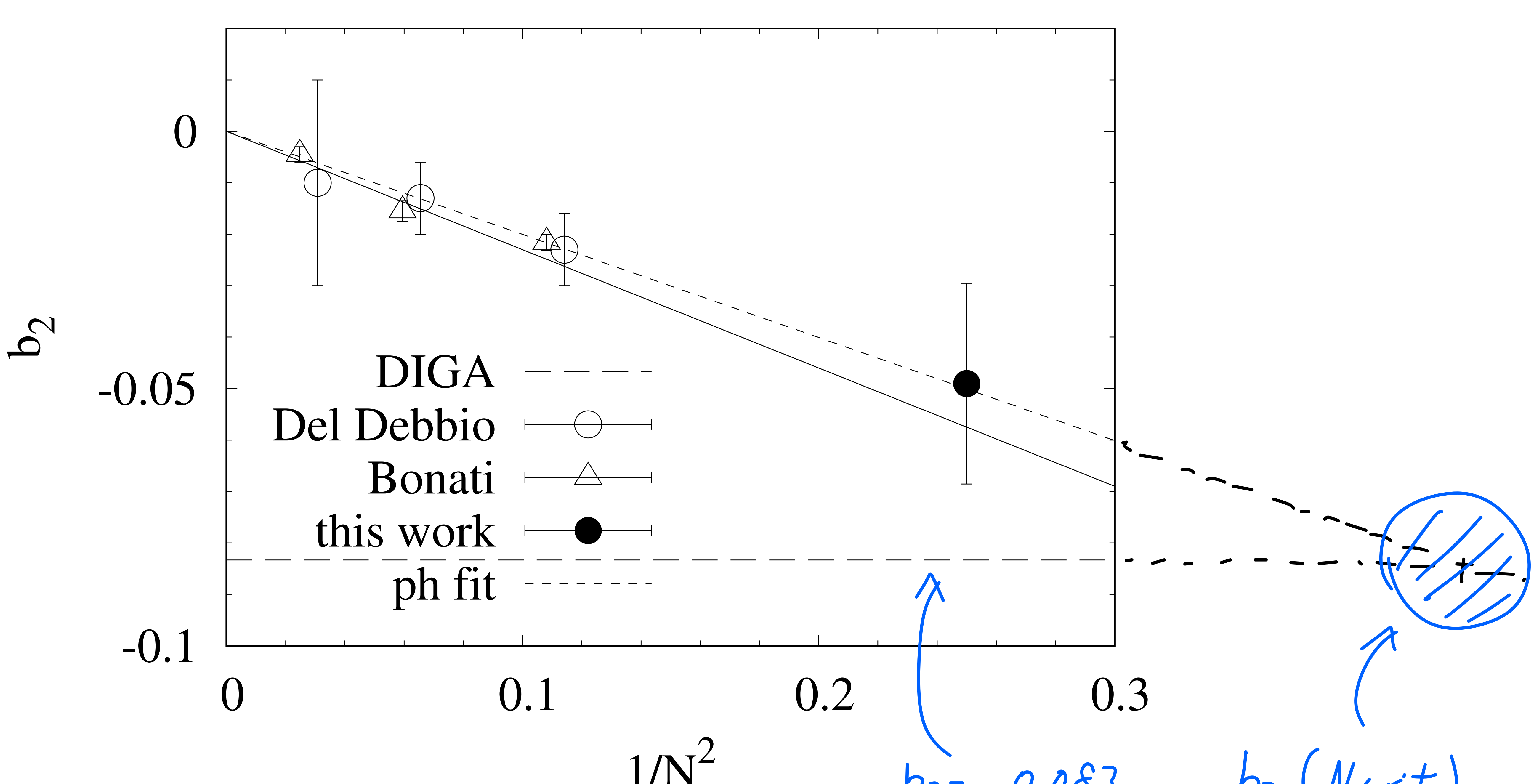


$N_{crit} = 1.52(2)$

when fitted with

[cf. Lüscher ('82) for 2d CP^{N-1} model]

$\left(\begin{array}{l} \chi \to \infty \\ \text{at} \\ N = N_{crit} \end{array} \right) \rightsquigarrow \dfrac{\chi}{\sigma} = \left(\dfrac{\chi}{\sigma}\right)_{N=\infty} \dfrac{N^2}{N^2 - N_{crit}^2}$



$b_2 = -0.083$
(instanton)

$b_2(N_{crit})$
$\parallel$
$-0.087(5)$

$N_{crit} \sim 1.5$

Subvolume Method

2102.08784

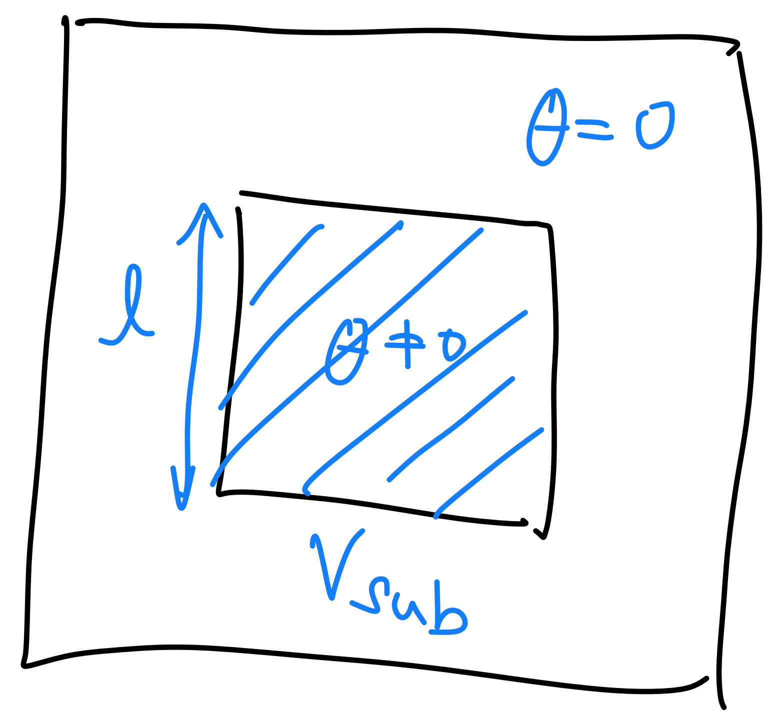
Subvolume Method

[Kitano, Matsudo, Yamada, MY ('21)]

(cf. [Keith-Hynes, Thacker ('08)] for 2d $\mathbb{CP}^1$-model)

$$e^{-\overbrace{V_{sub}}^{\ell^4} F_{sub}(\theta)} = \frac{1}{Z(0)} \int \mathcal{D}U \, e^{-S_g + i\theta Q_{sub}}$$

$$= \left\langle e^{i\theta Q_{sub}} \right\rangle = \left\langle \cos(\theta Q_{sub}) \right\rangle$$



$Q_{sub} \notin \mathbb{Z}$

better w/ sign problem !

Fit

$$F_{sub}(\theta) \sim F(\theta) + \frac{s(\theta)}{\ell} + \mathcal{O}\left(\frac{1}{\ell^2}\right)$$

surface tension

inside region

$$(aT_c)^{-4} \ll V_{sub} \ll V_{full}$$

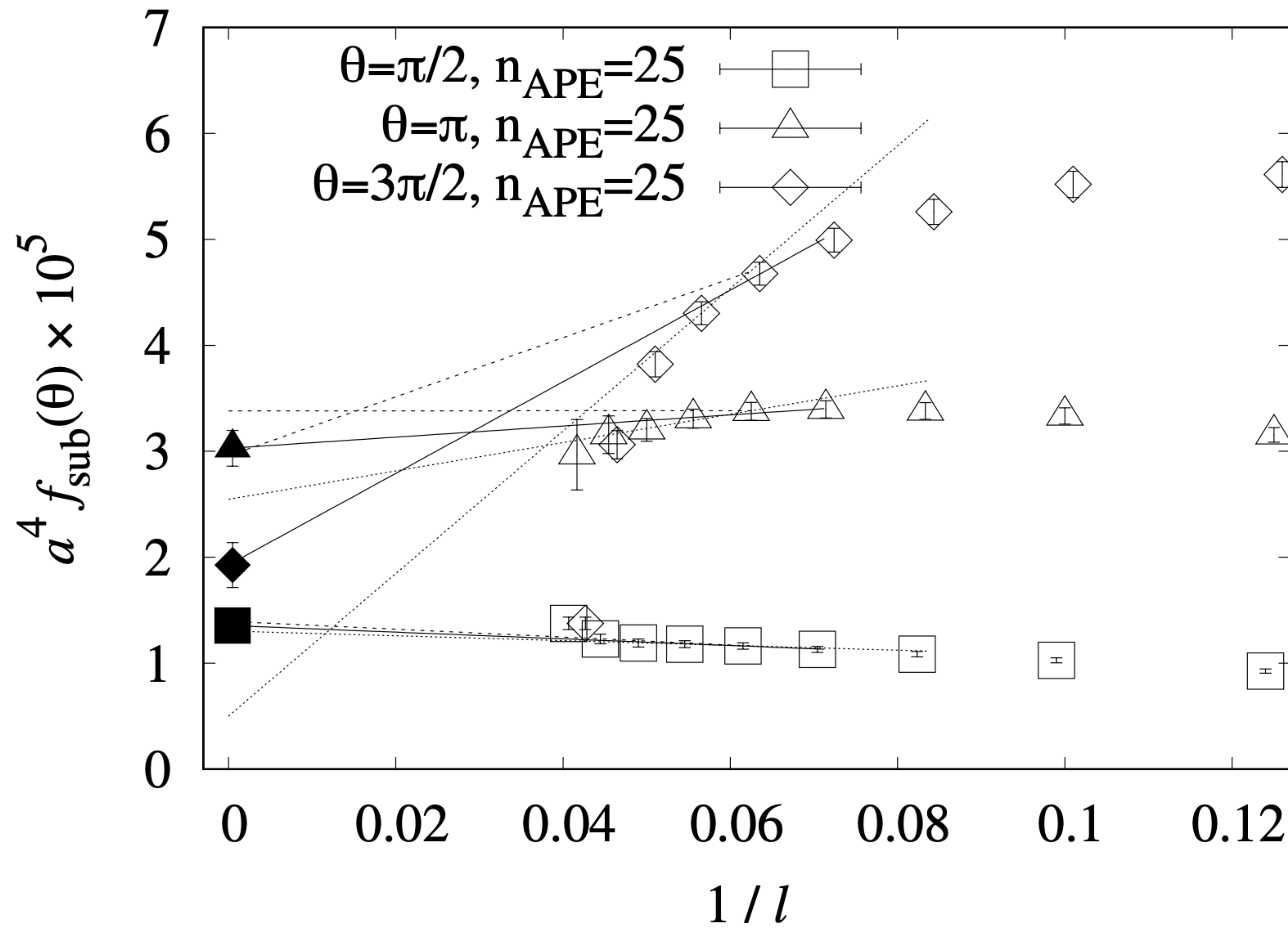
- SU(2) YM (Symanzik)
- $\beta = \frac{4}{g^2} = 1.975 \longrightarrow 1/(aT_c) = 9.50$
- $V_{full} = 24^3 \times \{48, 8, 6\}$

  T=0 T=1.2 T_c T=1.6 T_c

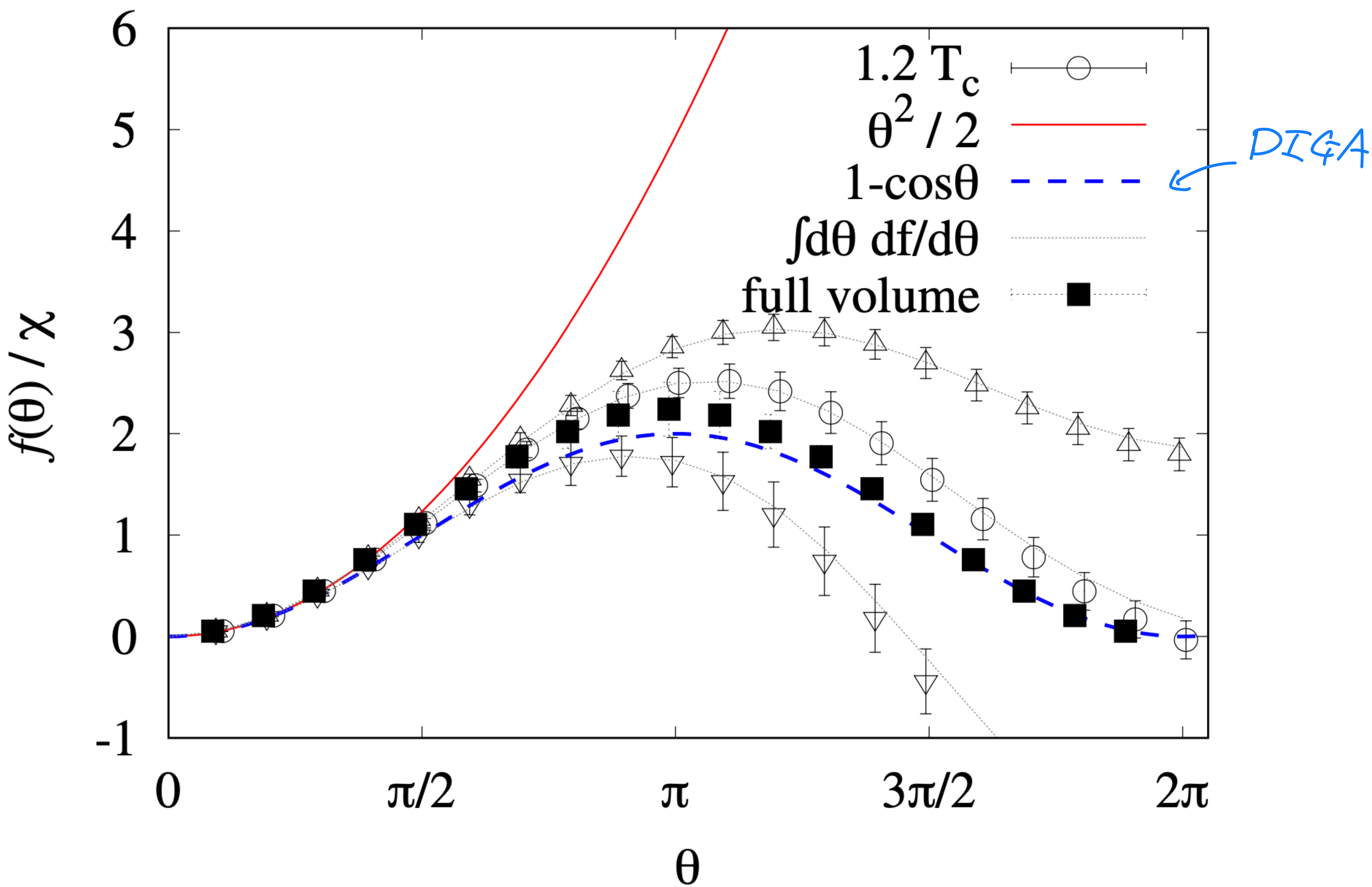
- #(config) = {68000, 10000, 10000}

$(aT_c)^{-1} < \ell < \ell_{full}$

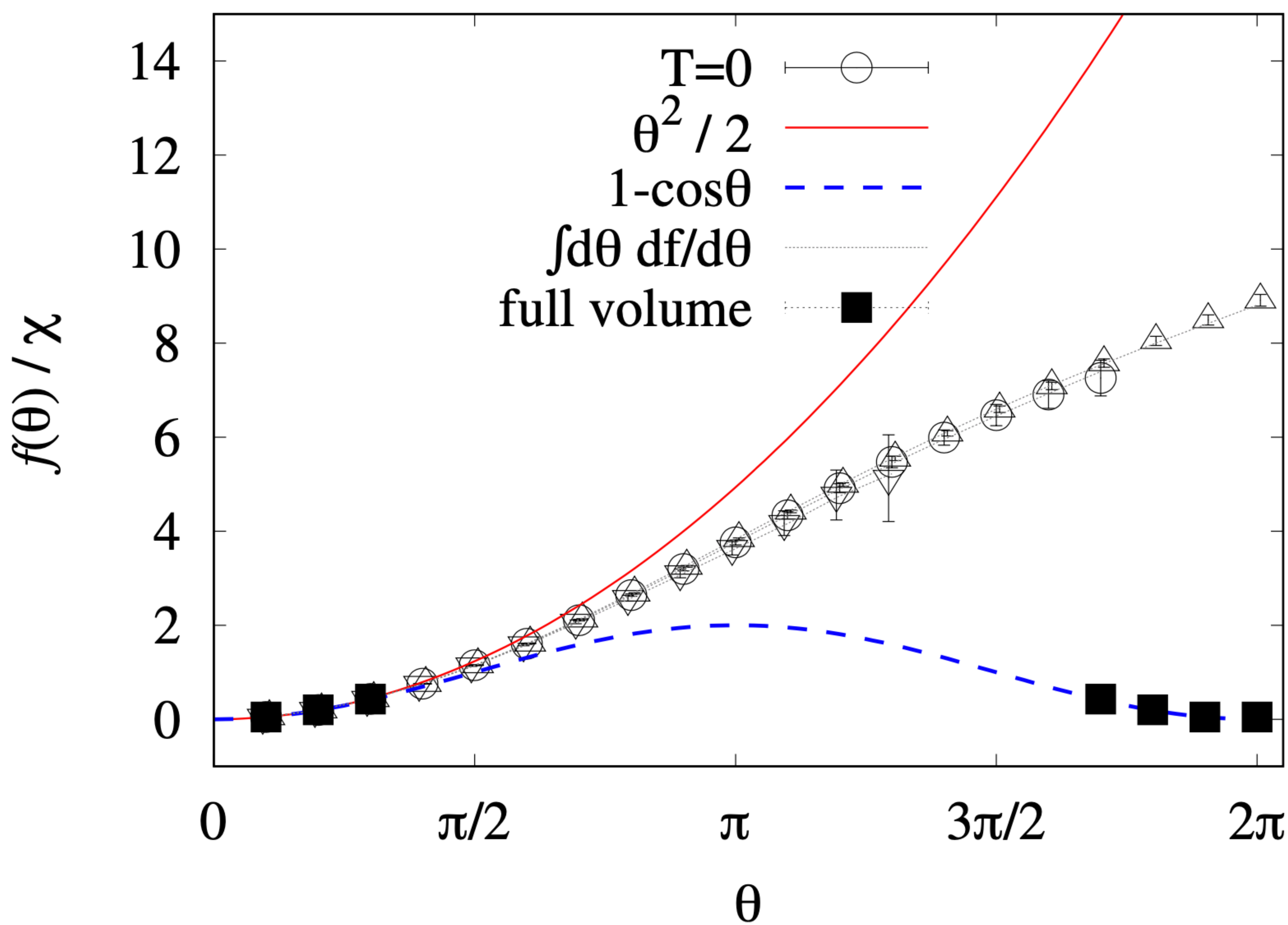
- $V_{sub} = \ell^4$ w/ $\ell \in \{10, 12, \ldots, 24\}$
- lineor extrapolation $F_{sub}(\theta) = \underbrace{F(\theta)}_{\text{result}} + \frac{S(\theta)}{\ell} + \mathcal{O}(1/\ell^2)$


θ=π/2, n_APE=25
θ=π, n_APE=25
θ=3π/2, n_APE=25
a^4 f_sub(θ) × 10^5
1 / l
0
1
2
3
4
5
6
7
0
0.02
0.04
0.06
0.08
0.1
0.12

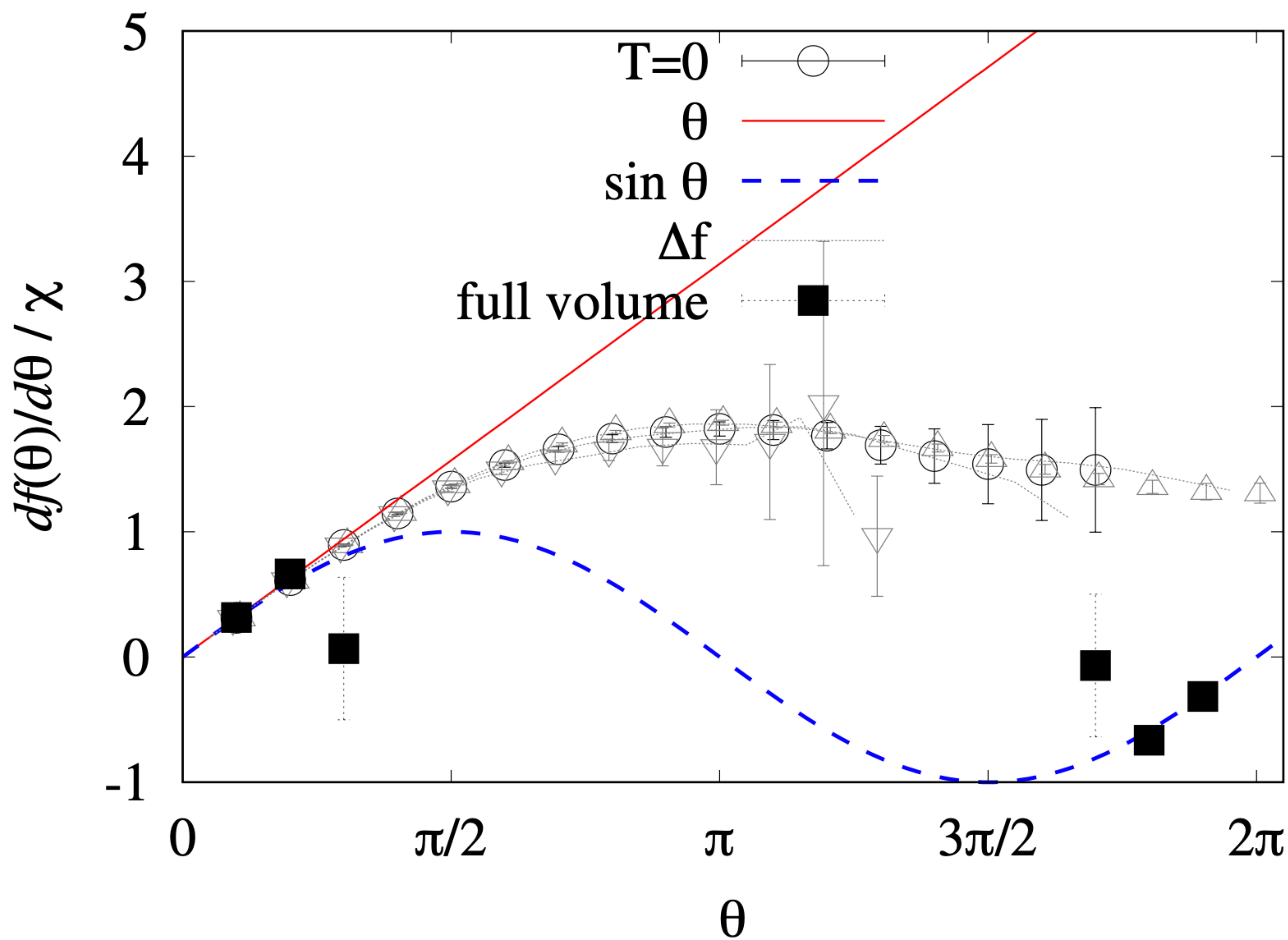
$f(\theta)$ @ $T = 1.2\,T_c > T_c$



$f(\theta)$ @ $T=0$; clearly NOT 2π-periodic



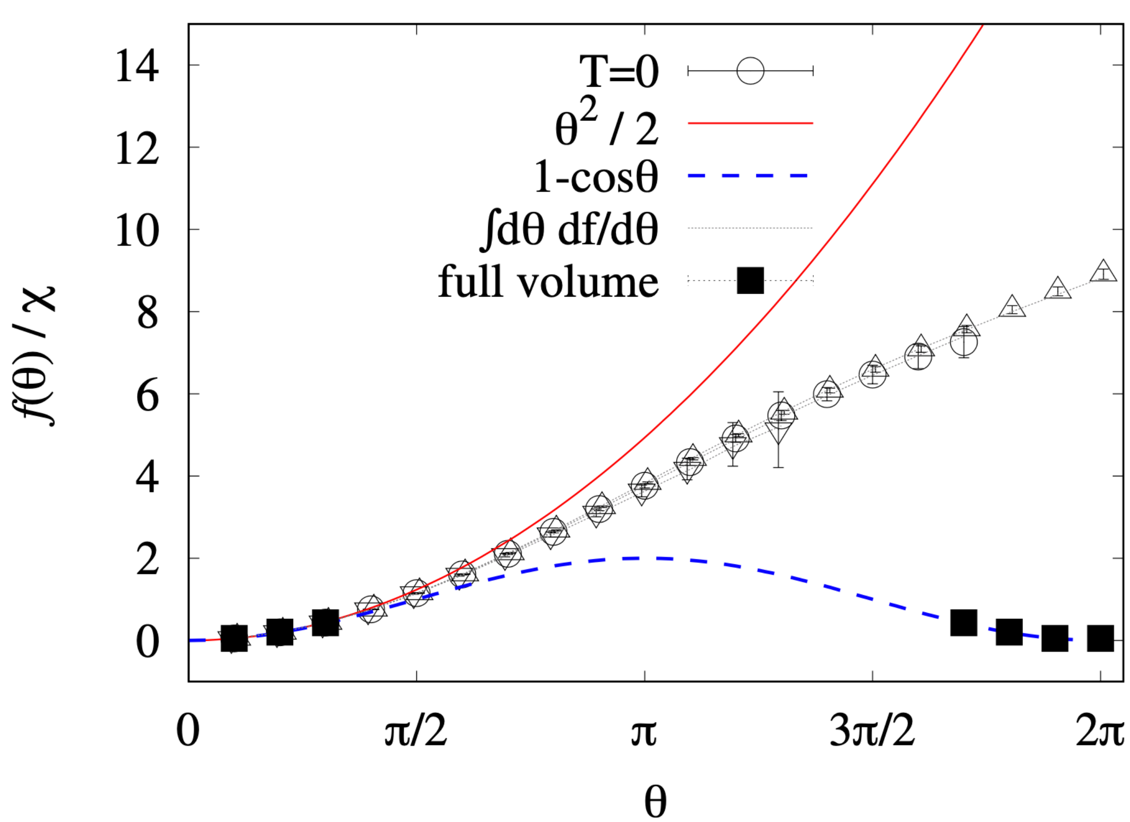
$\frac{df(\theta)}{d\theta}$ @ $\theta = 0$; clearly $\left.\frac{df(\theta)}{d\theta}\right|_{\theta=\pi} > 0$!



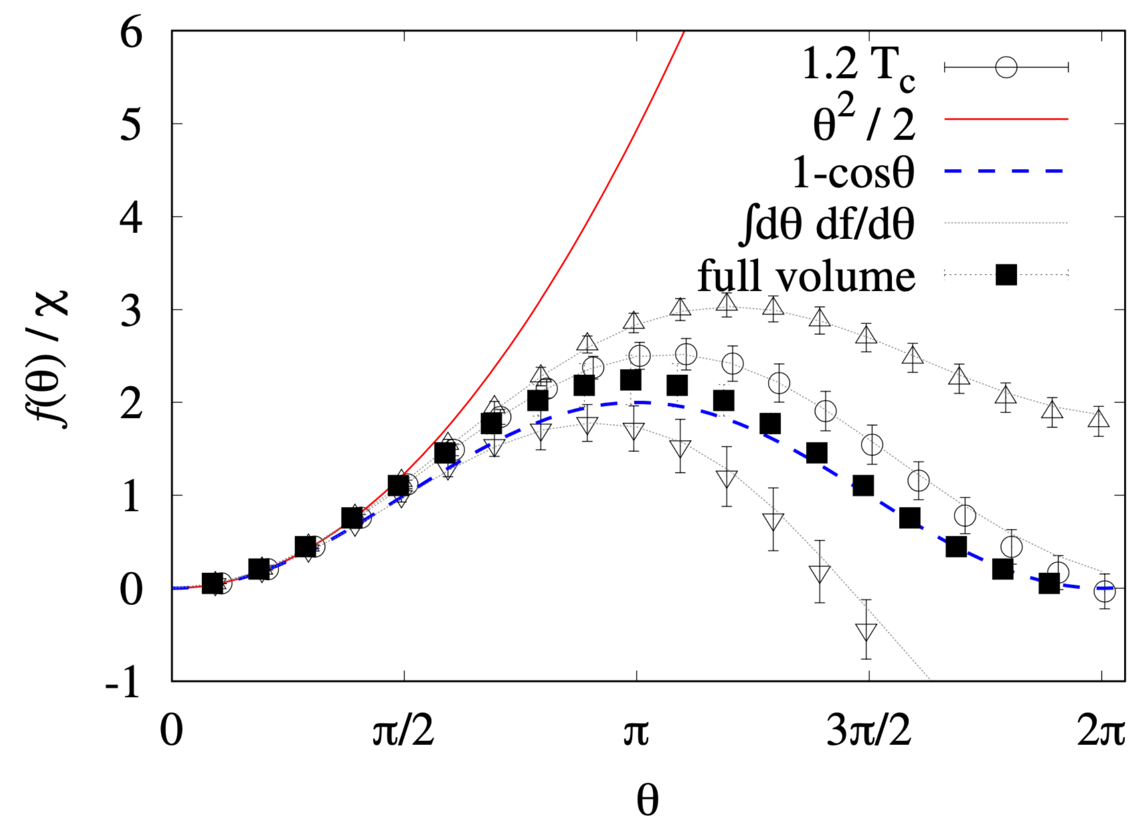
Summory

- $F(\theta)$ for 4d $SU(2)$ YM for $0 \leq \theta \lesssim \frac{3\pi}{2}$

  using subvolume method (despite sign problem!!)



$T = 0$



$T = 1.2\,T_c$

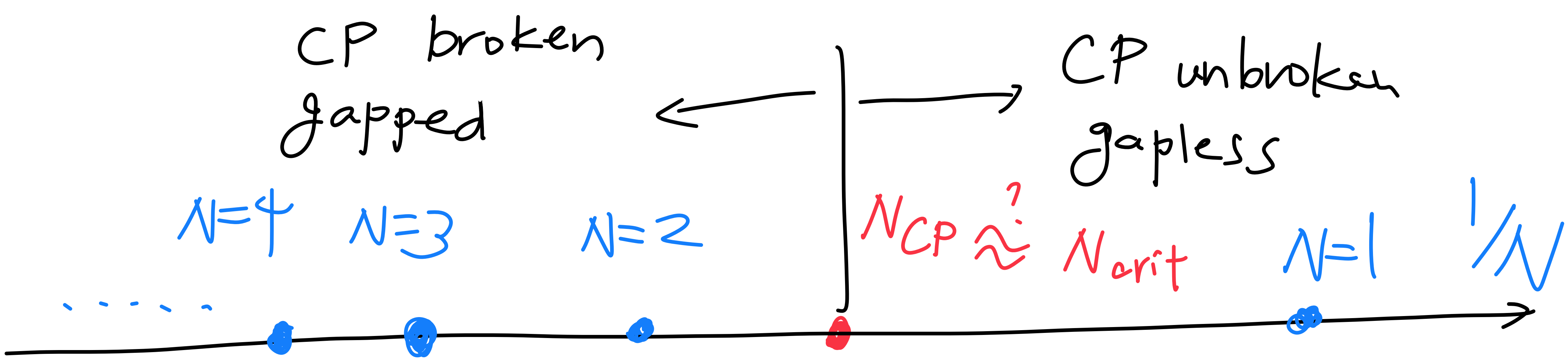
Summory

* 4d SU(2) YM : still "large N"

spontaneous CP breaking, mass gap @ $\theta = \pi$

$$\frac{\chi^{1/4}}{T_c} = 0.674(31), \quad b_2 = -0.049(20)$$

[Quantitatively different from 2d CP^{N-1} model]



Future Works

- Improve systematics
- Explore
  - (T, θ) - phase diagram [e.g. $T_c(\theta)$]
  - different theories [SU(N) YM, SU(N) YM + matter, ⋮]
- Similar story for μ?

quark-gluon plasma, (T, μ) - phase diagram?