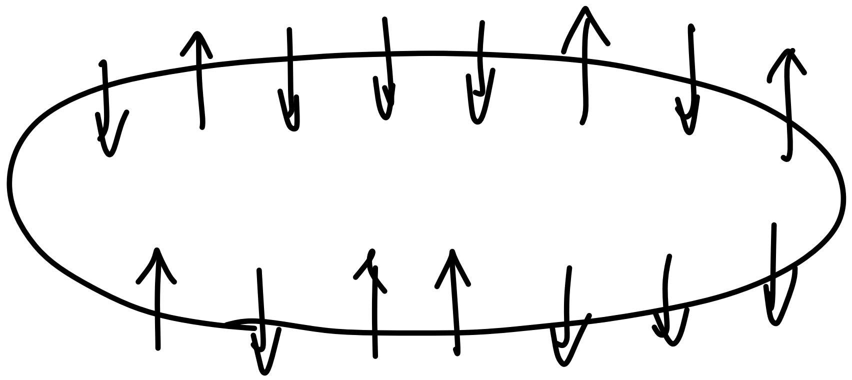

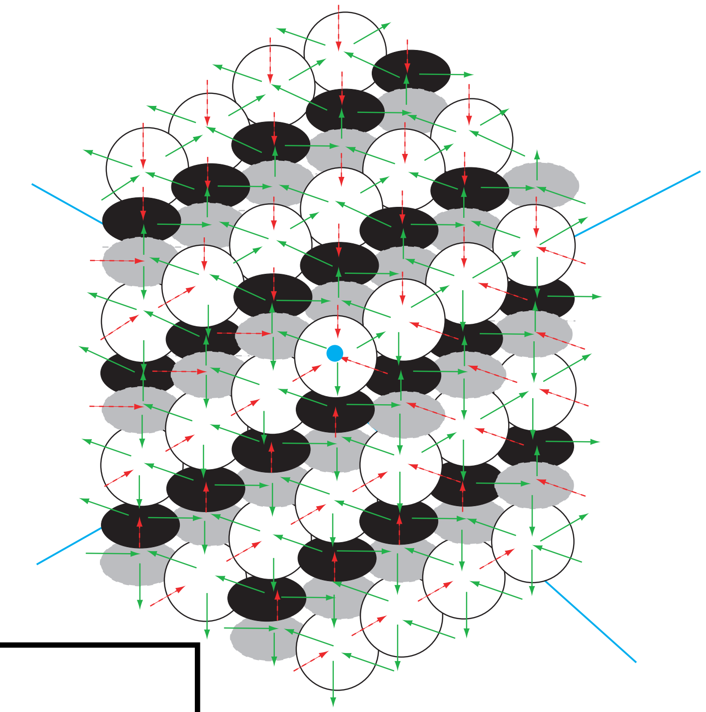

# ゲージ・ベーテ対応
# を問い直す

山崎　雅人

INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

YITP / online

Aug. 22, 2022

思い出話から…

# 基研研究会「量子場理論と弦理論の発展」

## 2008年7月28日--8月1日

## 京都大学基礎物理学研究所
## 湯川記念館 Panasonic 国際交流ホール 及び 会議室(Y206, Y306)

| | | |
|---|---|---|
| 山崎 雅人 | 東京大学理学系研究科物理学専攻 | A New N=4 Membrane Action via Orbifold [内容紹介 (ppt)] |

arXiv > hep-th > arXiv:0805.1997

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High Energy Physics – Theory

*[Submitted on 14 May 2008 (v1), last revised 21 Nov 2008 (this version, v2)]*

# A New N=4 Membrane Action via Orbifold

Hiroyuki Fuji, Seiji Terashima, Masahito Yamazaki

We propose a new Lagrangian describing N=4 superconformal field theory in three dimensions. This theory is believed to describe interacting field theory on the worldvolume of a M2-brane on an orbifold, and is obtained as a Z_2-quotient of the theory proposed by Bagger and Lambert. Despite unusual Chan-Paton structures, we can take Z_2-orbifold by using SU(2)\times SU(2) bifundamental representations. We also analyze the moduli space of

# 江口先生還暦記念研究会

Welcome

Programme Theme

Invited Speakers

Organizers

Program & Slides

Conference Proceedings

Conference Banquet

List of Participants

## 30 Years of Mathematical Methods in High Energy Physics

**March 17-19, 2008 RIMS, Kyoto, Japan**

Welcome to the home page of **30 Years of Mathematical Methods in High Energy Physics** in honor of Professor Tohru Eguchi's 60th birthday. The conference is jointly hosted by Yukawa Institute for Theoretical Physics (YITP) and Research Institute for Mathematical Sciences (RIMS); it is also supported by Japan Society for the Promotion of Science (JSPS) and Inoue Foundation for Science.

The conference venue is **the auditorium (room 420) of RIMS**, Kyoto University, Japan. **Registration starts at 8:30 a.m.** Monday March 17.

https://www.yukawa.kyoto-u.ac.jp/assets/contents/seminar/archive/2007/yitp-w-07-20/index.html

# Massive Puzzle
# Quantum Integrability
# and
# Gauge Theories

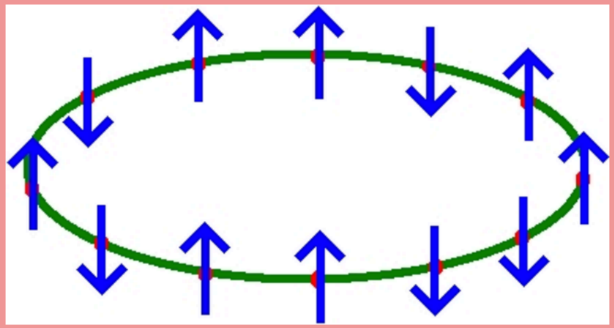
*Nikita Nekrasov*
*IHES*
*Kyoto, March 2008*

## Gauge theory -- spin chain

Identical to the Bethe ansatz equations

$$\left(\frac{\lambda_i + is}{\lambda_i - is}\right)^L = \prod_{j \neq i} \frac{\lambda_i - \lambda_j + i}{\lambda_i - \lambda_j - i}$$

$s$



arXiv > hep-th > arXiv:0901.4748

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## High Energy Physics – Theory

*[Submitted on 29 Jan 2009 (v1), last revised 4 Feb 2009 (this version, v2)]*

# Quantum integrability and supersymmetric vacua

Nikita A. Nekrasov, Samson L. Shatashvili

This is an announcement of some of the results of a longer paper where the supersymmetric vacua of two dimensional N=2 susy gauge theories with matter are shown to be in one-to-one correspondence with the eigenstates of integrable spin chain Hamiltonians. The correspondence between the Heisenberg spin chain and the two dimensional U(N) theory with fundamental hypermultiplets is reviewed in detail. We demonstrate the isomorphism of the equivariant quantum cohomology of the cotangent bundle to the Grassmanian manifold Gr(N,L) and the ring of quantum integrals of motion of the length L SU(2) XXX spin chain, in the N-particle sector.
This paper accompanies arXiv:0901.4744

Comments: 21 pp., short version II, conference in honour of T.Eguchi's 60th anniversary; v2. typos and refs corrected

## Supersymmetric vacua and Bethe ansatz

#7

Nikita A. Nekrasov (IHES, Bures-sur-Yvette), Samson L. Shatashvili (IHES, Bures-sur-Yvette and Hamilton Math. Inst., Dublin and Trinity Coll., Dublin) (Jan, 2009)

Published in: *Nucl.Phys.B Proc.Suppl.* 192-193 (2009) 91-112 • Contribution to: ESF School in High Energy Physics and Astrophysics: Theory and Particle Physics: The LHC Perspective and Beyond • e-Print: 0901.4744 [hep-th]

pdf DOI cite 261 citations

## Quantum integrability and supersymmetric vacua

#8

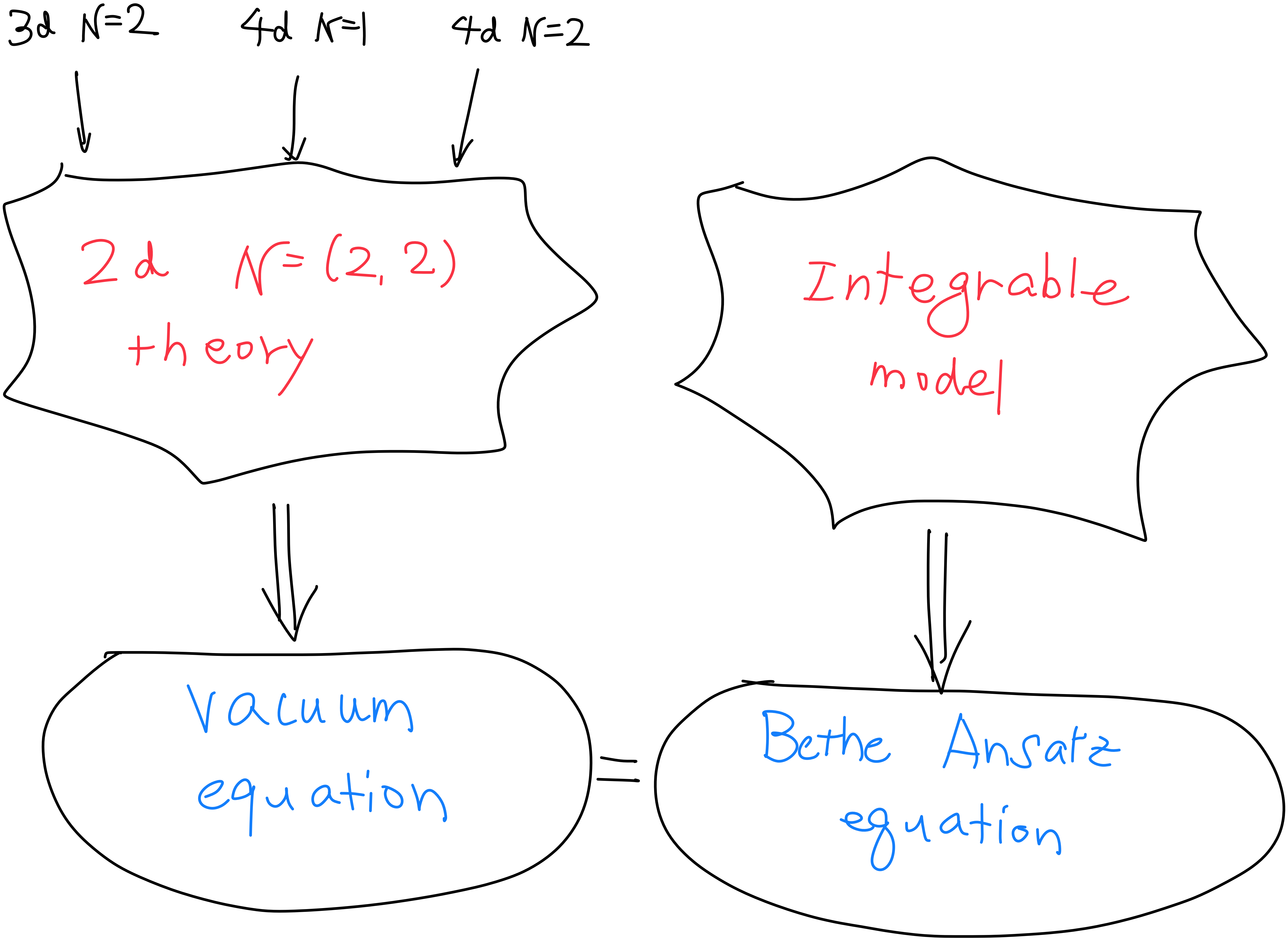
Nikita A. Nekrasov (IHES, Bures-sur-Yvette), Samson L. Shatashvili (IHES, Bures-sur-Yvette and Hamilton Math. Inst., Dublin and Trinity Coll., Dublin) (Jan, 2009)

Published in: *Prog.Theor.Phys.Suppl.* 177 (2009) 105-119 • Contribution to: 30 Years of Mathematical Methods in High Energy Physics (In honor of Professor Tohru Eguchi's 60th Birthday) • e-Print: 0901.4748 [hep-th]

pdf DOI cite 214 citations

Many precursors, e.g.
[Gorsky-Nekrasov, Minahan-Polychronakos, Douglas ('94), Gerasimov (~'93)
Losev/Moore+Nekrasov-Shatashviili ('97-'98)
Gerasimov-Shatashvili ('06-'07)]

# Gauge / Bethe 101


3d N=2
4d N=1
4d N=2
2d N=(2,2) theory
Integrable model
Vacuum equation
=
Bethe Ansatz equation

"Gonge"

e.g. 2d $\mathcal{N}=(2,2)$ $\underbrace{U(N_c)\text{ gauge}}$ + $\underbrace{N_f\text{ flavors}}$

Vector multiplet

$(A_\mu, \sigma, \lambda, \bar{\lambda})$

0,1 $\quad$ $A_2 + i A_3$

chiral multiplet

give mass

+ 1 adjoint

effective theory after integrating out matters

$\int d\theta^+ d\bar{\theta}^- \widetilde{W}(\Sigma)$ : effective twisted superpotential

twisted superfield

$$\Sigma = \sigma - i\sqrt{2}\theta^+ \bar{\lambda}_+ - i\sqrt{2}\bar{\theta}^- \lambda_- + \cdots$$

The vacuum equation

$$\exp\Big(\underbrace{\frac{\partial \widetilde{W}(\sigma)}{\partial \sigma}}_{}\Big) = 1$$

( $2\pi i\,\mathbb{Z}$ ; flux sector $\frac{1}{2\pi}\int F \in \mathbb{Z}$ )

$\rightsquigarrow$

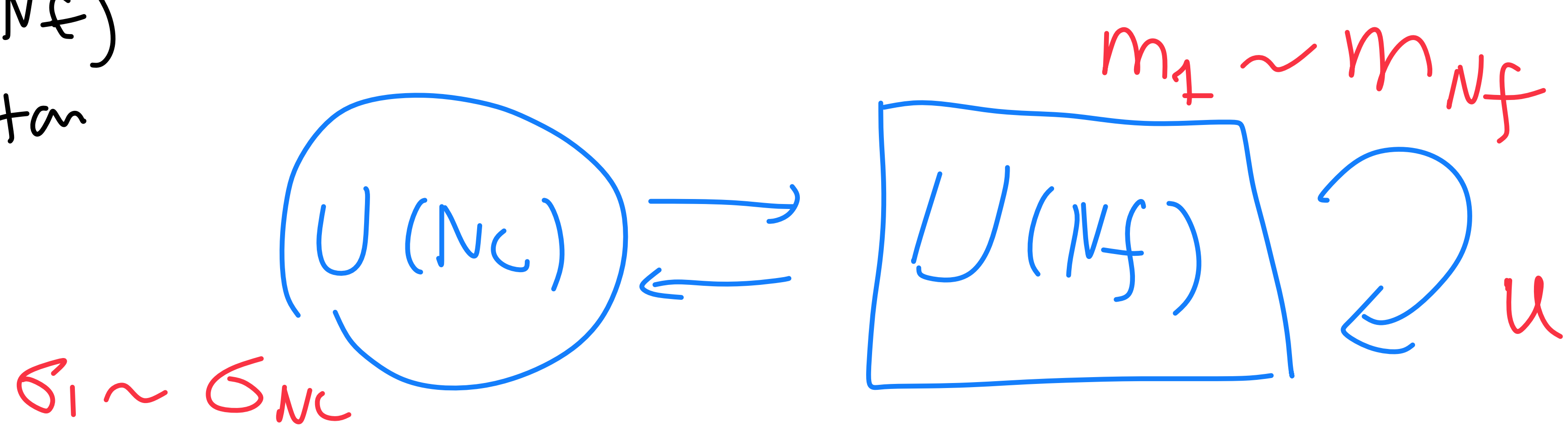
FI param.

$U(1)_R^{\mathcal{N}=4}$ on adj

$$\prod_{a=1}^{N_f} \frac{\sigma_i - m_a + u/2}{\sigma_i - m_a - u/2} = e^{2\pi i t} \prod_{\substack{j\neq i \\ 1\le j\le N_c}} \frac{\sigma_i - \sigma_j + u}{\sigma_i - \sigma_j - u}$$

$U(N_c)$ Cartan

$U(N_f)$ Cartan



"Bethe"

XXX spin chain

$$\hat{H} = -J \sum_{i=1}^{N_f} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}$$

spin - 1/2

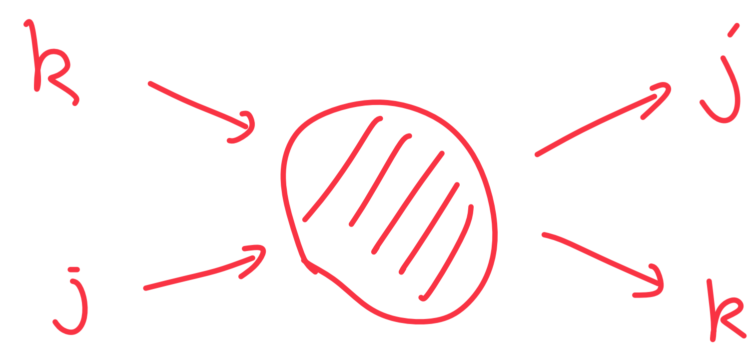
# Bethe Ansatz equation

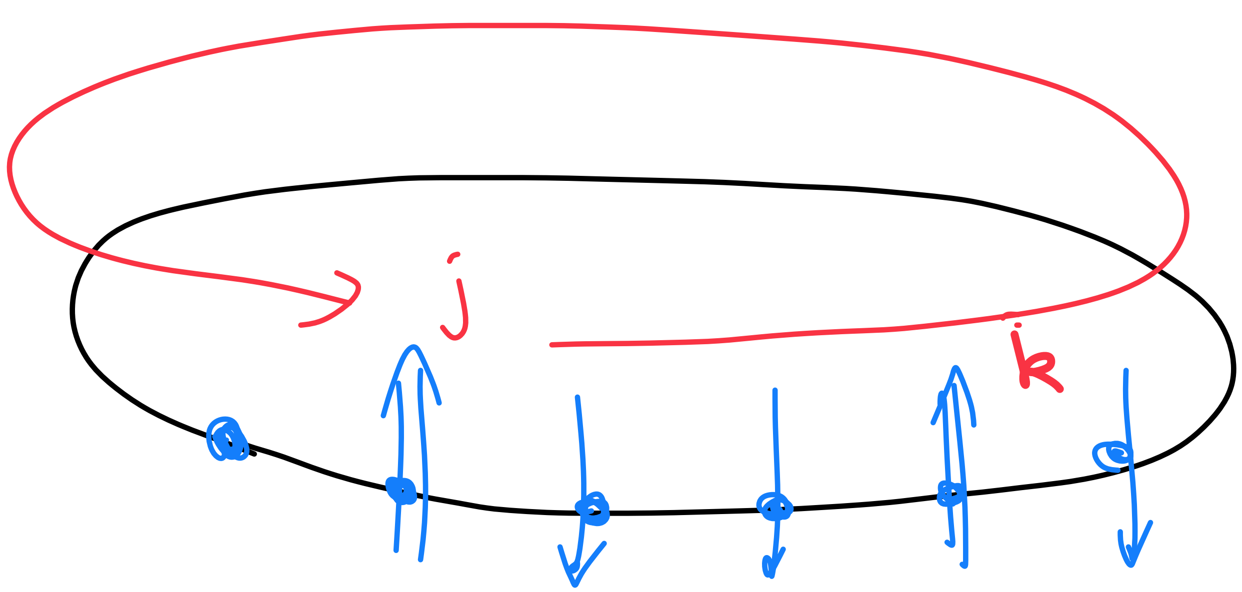
$$\left(\frac{\sigma_j + i}{\sigma_j - i}\right)^{N_f} = \prod_{\substack{k\neq j \\ 1\le k\le N_c}} \frac{\sigma_j - \sigma_k + 2i}{\sigma_j - \sigma_k - 2i}$$

$e^{i p_j N_f}$

$S_{jk}$ : S-matrix

($\sigma$ : rapidity)



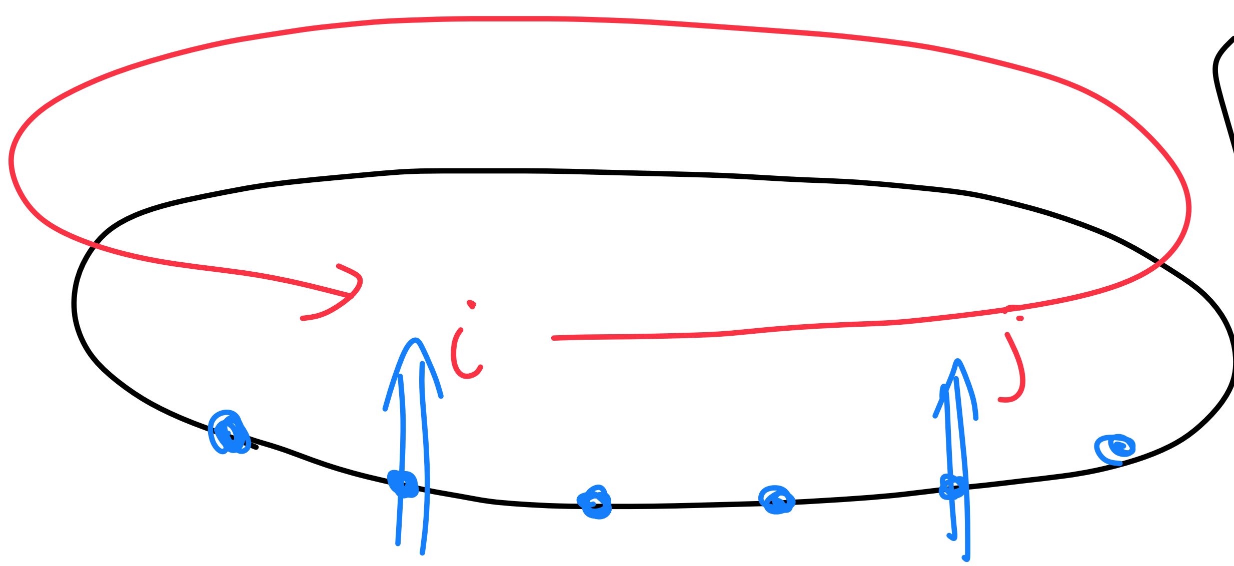

$$\begin{pmatrix} N_c & \uparrow & \text{spins} \\ N_f & \uparrow\downarrow & \text{spins} \end{pmatrix}$$

# Bethe Ansatz equation

$$\left(\frac{\sigma_j + i}{\sigma_j - i}\right)^{N_f} = \prod_{\substack{k\neq j\\ 1\le k\le N_c}} \frac{\sigma_j-\sigma_k+2i}{\sigma_j-\sigma_k-2i}$$

$\Big\downarrow$ inhomogeneity + twist

$$\prod_{a=1}^{N_f}\left(\frac{\sigma_i - m_a + i}{\sigma_i - m_a - i}\right) = e^{t}\left(\prod_{\substack{k\neq j\\ 1\le k\le N_c}} \frac{\sigma_j-\sigma_k+2i}{\sigma_j-\sigma_k-2i}\right)$$




$m_a$ : inhomogeneity
$\parallel$
twisted mass for $SU(N_f)$

$t$ : twist along chain
$\parallel$
FI param.

Generalizations :

- 2d $\mathcal{N}=(2,2)$

  $U(N_c) + N_f$ flavors + adj
  
  ||
  
  XXX chain
  
  $Y_\hbar(sl_2)$

  $\xleftarrow{S^1}$

  3d $\mathcal{N}=2$

  $U(N_c) + N_f$ + adj
  
  ||
  
  XXZ chain
  
  $\mathcal{U}_q(sl_2)$

  $\xleftarrow{S^1}$

  4d $\mathcal{N}=1$

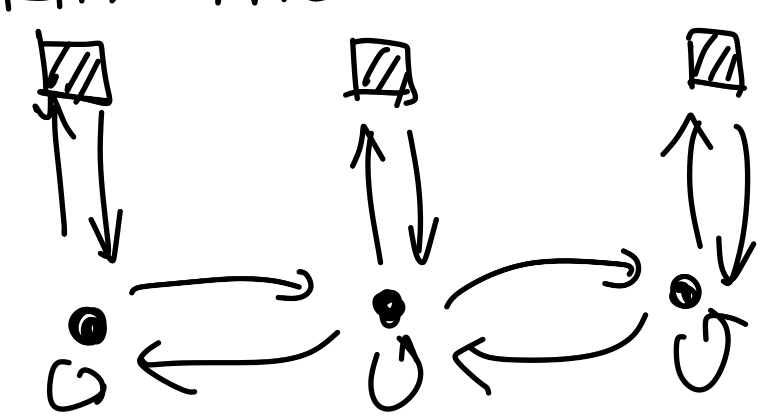
  $U(N_c) + N_f$ + adj
  
  ||
  
  XYZ chain
  
  $E_{q,\tau}(sl_2)$

- $U(N_c)$ $N_f$

  $\rightsquigarrow$

  Dynkin theories

  $Y_\hbar(\mathfrak{g})$

$\boxed{\text{6d } (2,0)\ A_N \text{ SCFT}}$

on $S_2$ ← cover

"UV"

"IR"

$\{$SW curve$\}$ = $\{$spectral curve of integrable model$\}$

$\boxed{\text{4d } \mathcal{N}=2 \text{ "class S"}}$

on $\mathbb{R}^4_{\epsilon_1,\epsilon_2}$

$$\log Z \sim \frac{1}{\epsilon_1\epsilon_2}\mathcal{F}$$

$$a_D = \partial\mathcal{F}/\partial a$$

on $\mathbb{R}^2_{\epsilon_1}$

$\boxed{\text{2d } \mathcal{N}=(2,2)}$

on $\mathbb{R}^2_{\epsilon_2}$

$$\log Z \sim \frac{1}{\epsilon_2}\widetilde{W}$$

$$\beta = \frac{\partial \widetilde{W}_{oper}}{\partial \alpha}$$

"quantize"

$$\widetilde{W} = \widetilde{W}_{oper} - \widetilde{W}^*$$

Puzzle ? (circa 2008)

2d $\mathcal{N}=(2,2)$

$Q, W$

quiver

There should exist IM

$\infty$-dim algebra

$\mathcal{Y}_{Q,W} \neq \mathcal{Y}(\mathfrak{g})$

R-matrix, Bethe Ansatz, ...

Q: $\mathcal{Y}_{Q,W}$ ???

# Candidate for $\mathcal{Y}_{Q,W}$ : Quiver Yangian

Wei Li + MY

(2003.08909 [hep-th])

Dimitry Galakhov + MY

(2008.07006 [hep-th])

Dimitry Galakhov+Wei Li + MY

(2106.01230 [hep-th])

(2108.10286 [hep-th])

(2206.13340 [hep-th])

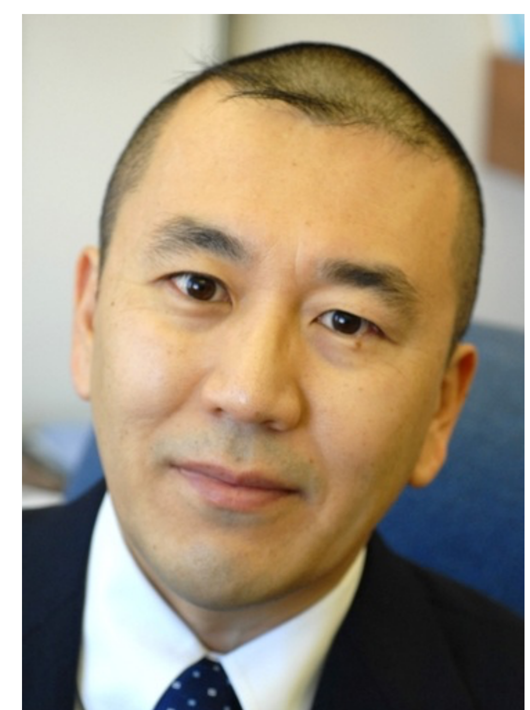
also works by Noshita, Watanabe, Bao,···

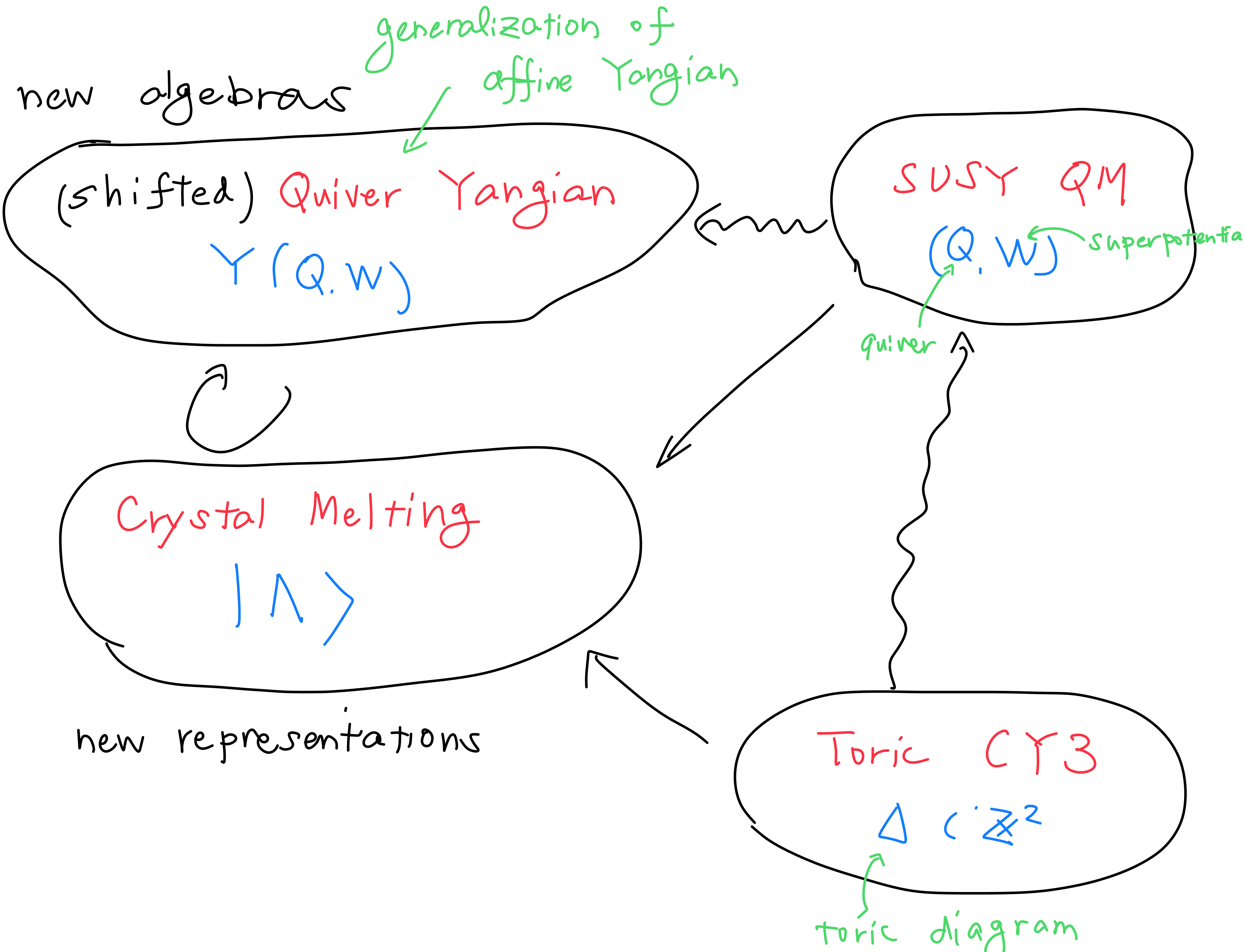
Also earlier works, e.g.

Hirosi Ooguri + MY (0811.2810 [hep-th])

MY (Ph.D. thesis, 1002.1709 [hep-th])

MY (Master thesis, 0803.4474 [hep-th])




new algebras
generalization of affine Yangian
(shifted) Quiver Yangian
Y(Q, W)
SUSY QM
(Q, W)
superpotential
quiver
Crystal Melting
|Λ⟩
new representations
Toric CY3
Δ ⊂ ℤ²
toric diagram

# Quiver Yangian

# in a nutshell

[See MY 2203.14314 for review]

Relations

$\mathcal{Y}(Q, W)$

$$\psi^{(a)}(z)\,\psi^{(b)}(w) = \psi^{(b)}(w)\,\psi^{(a)}(z)\;,$$
$$\psi^{(a)}(z)\,e^{(b)}(w) \simeq \varphi^{b\Rightarrow a}(\Delta)\,e^{(b)}(w)\,\psi^{(a)}(z)\;,$$
$$e^{(a)}(z)\,e^{(b)}(w) \sim (-1)^{|a||b|}\varphi^{b\Rightarrow a}(\Delta)\,e^{(b)}(w)\,e^{(a)}(z)\;,$$
$$\psi^{(a)}(z)\,f^{(b)}(w) \simeq \varphi^{b\Rightarrow a}(\Delta)^{-1}\,f^{(b)}(w)\,\psi^{(a)}(z)\;,$$
$$f^{(a)}(z)\,f^{(b)}(w) \sim (-1)^{|a||b|}\varphi^{b\Rightarrow a}(\Delta)^{-1}\,f^{(b)}(w)\,f^{(a)}(z)\;,$$
$$\left[e^{(a)}(z), f^{(b)}(w)\right\} \sim -\delta^{a,b}\,\frac{\psi^{(a)}(z) - \psi^{(b)}(w)}{z-w}\;, \qquad (\Delta = z - w)$$

"$\simeq$" means equality up to $z^n w^{m\geq 0}$ terms

"$\sim$" means equality up to $z^{n\geq 0} w^m$ and $z^n w^{m\geq 0}$ terms

bonding factor

$$\varphi^{a\Rightarrow b}(u) \equiv \frac{\prod_{I\in\{b\to a\}}(u + h_I)}{\prod_{I\in\{a\to b\}}(u - h_I)}$$

flavor charge of arrow = bifundamental

edge

## Generators

($z$: spectral parameter)

$$e^{(a)}(z) \equiv \sum_{n=0}^{+\infty} \frac{e_n^{(a)}}{z^{n+1}}\,, \qquad \psi^{(a)}(z) \equiv \sum_{n=-\infty}^{+\infty} \frac{\psi_n^{(a)}}{z^{n+1}}\,, \qquad f^{(a)}(z) \equiv \sum_{n=0}^{+\infty} \frac{f_n^{(a)}}{z^{n+1}}\,,$$

(can truncate to $n=-k$)

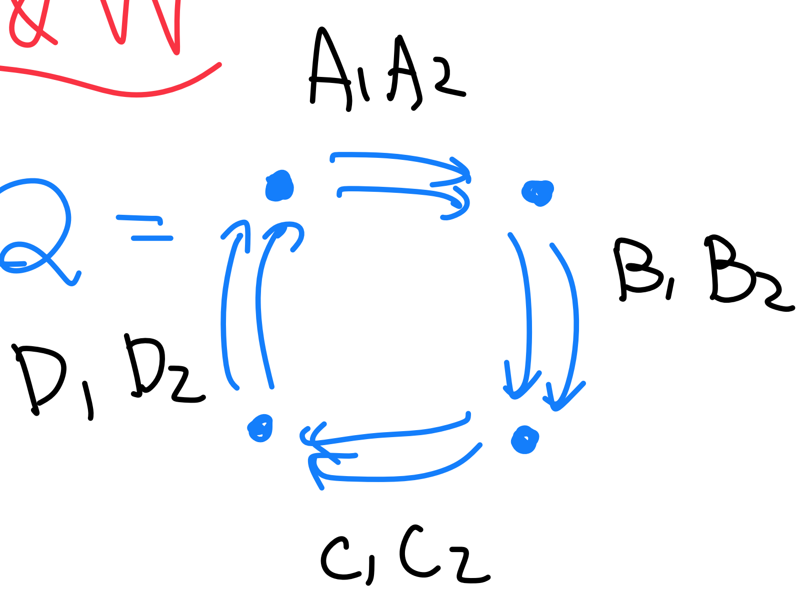
$a$ : quiver vertex

## $\mathbb{Z}_2$-grading

$$|a| = \begin{cases} 0 & (\exists \text{ edge } I \text{ s.t. } I \text{ starts and ends at } a) \\ 1 & (\text{otherwise}) \end{cases}$$

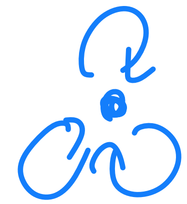
## Q & W

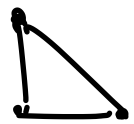
$Q =$




$$W = \mathrm{Tr}\,(A_1 B_1 C_1 D_1 - A_1 B_2 C_1 D_2 - A_2 B_1 C_2 D_1 + A_2 B_2 C_2 D_2)$$

$(CY_3 = K_{\mathbb{P}^1 \times \mathbb{P}^1})$

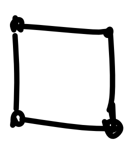
* $\mathbb{C}^3 \rightsquigarrow$ $Q=$  $\rightsquigarrow Y(\widehat{\mathfrak{gl}_1})$



$W = \mathrm{Tr}(XYZ - XZY)$

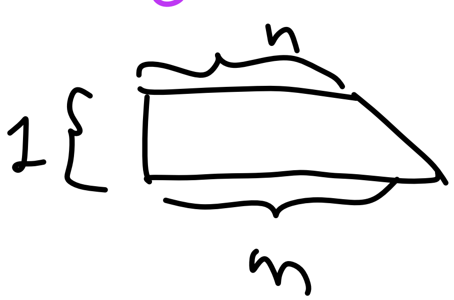
[Miki; Ding-Iohara;⋯
Tsymbaulik; Prochazka;
Gaberdiel, Gopakumar, Li, Peng,⋯]

* conifold $\rightsquigarrow$ $Q=$  $\rightsquigarrow Y(\widehat{\mathfrak{gl}_{1|1}})$

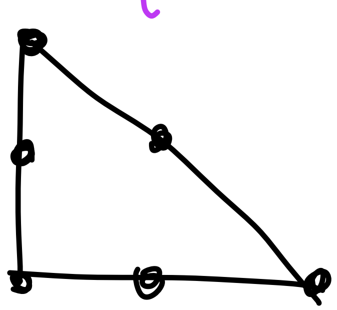


$W = \mathrm{Tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1)$

* $xy = z^n w^m \rightsquigarrow Y(\widehat{\mathfrak{gl}_{m|n}})$ [Bezerra-Mukhin ('19)]




* $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2) \rightsquigarrow Y(\widehat{D(2,1,\alpha)})$ [Noshita-Watanabe ('21)]

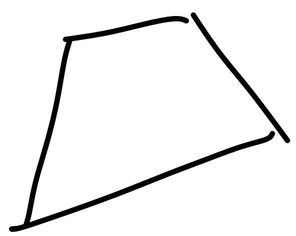


$Y(\hat{\mathfrak{g}})$ for $\left( \begin{array}{l} \text{non-chiral quiver} \\ \text{toric CY3 w.o. 4-cycle} \end{array} \right.$

$\left\{\begin{array}{l}\text{chiral quiver} \\ \text{toric } CY_3 \text{ w/ cpt 4-cycle}\end{array}\right.$

* general toric $CY_3$ $\rightsquigarrow$ $Y(Q, W)$

has no "$\mathfrak{g}$"



$\Delta \subset \mathbb{Z}^2$

new algebra

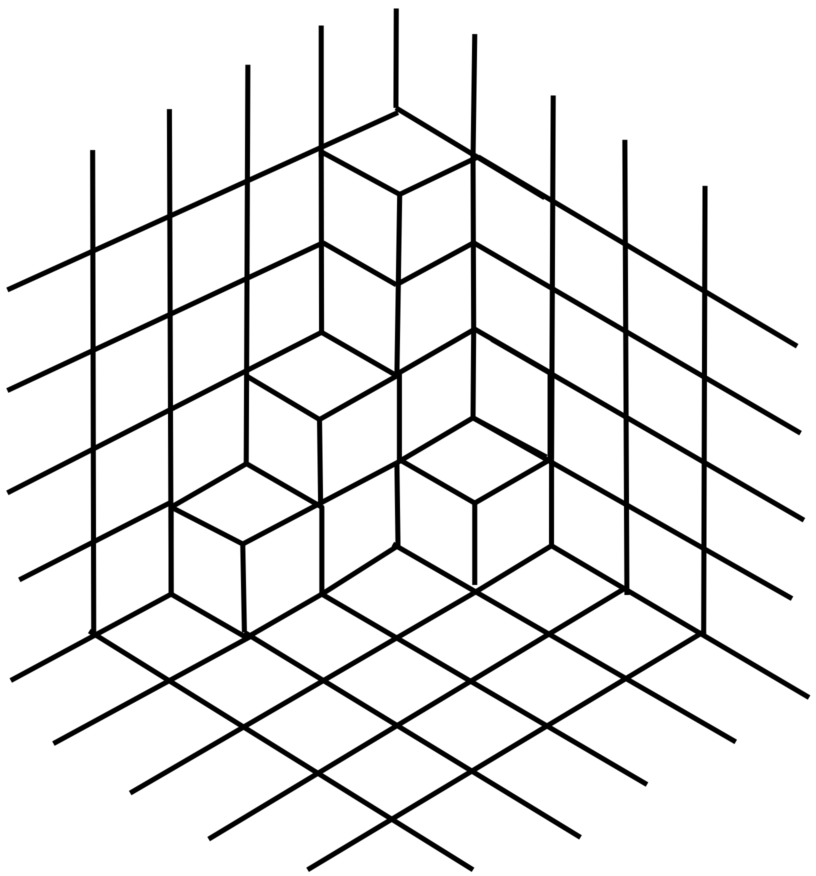
beyond $Y(\mathfrak{g})$

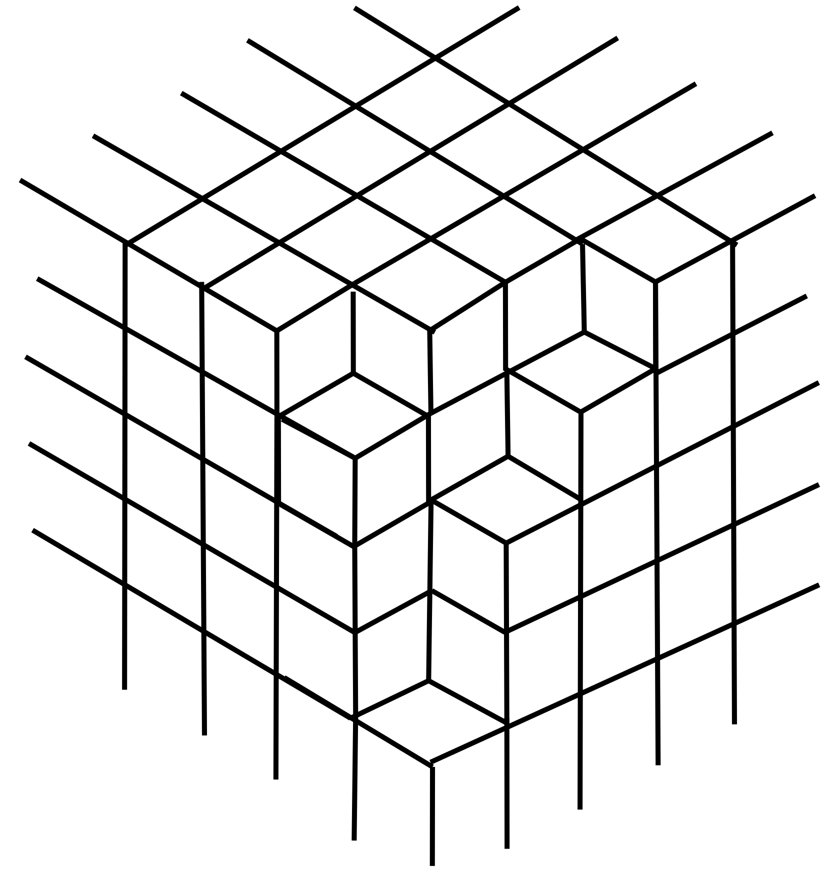
$Y(\hat{\mathfrak{g}})$ !

# Representations from Crystal Melting

cf. earlier developments on quantum toroidal algebras (Ding-Iohara-Miki) and affine Yangians by [Feigin, Jimbo, Miwa, Mukhin; Tsymbaulik; Prochazka; Rapcak; Gaberdiel, Gopakumar; Li, Peng,⋯]

$\mathbb{C}^3$: crystal melting [Okounkov-Reshetikhin-Vafa; Iqbal, Nekrasov,...]





plane partition

$$M(q) \equiv \sum_{\Lambda \in \text{ plane partition}} q^{|\Lambda|} = \prod_{k=1}^{\infty} \frac{1}{(1-q^k)^k}$$

$$= 1 + q + 3q^2 + 6q^3 + 13q^4 + 24q^5 + 48q^6 + \dots ,$$

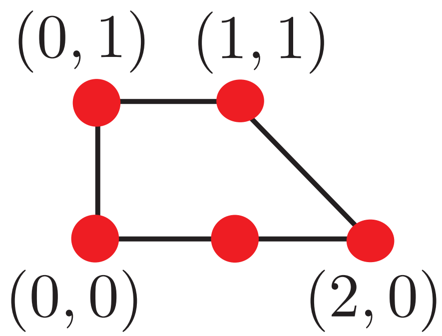
$$= Z^{\mathbb{C}^3}_{\text{top A-model}}$$

# The story generalizes to an arbitrary toric CY3

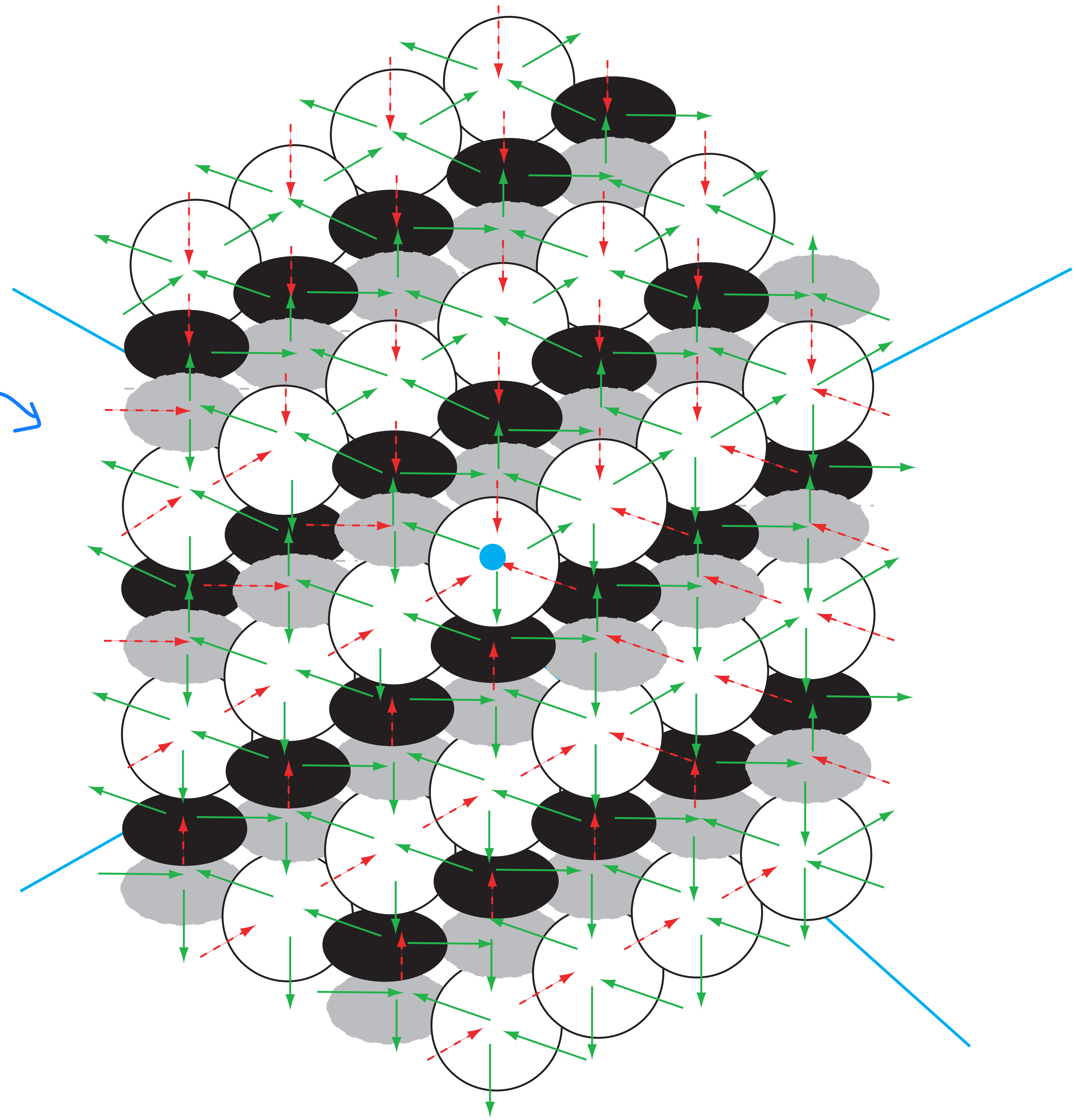
[Ooguri-MY '08'09]

See also [Szendroi; Bryant, Young; Mozgovoy, Reineke; Nagao, Nakajima; Ooguri, MY; Jafferis, Chuang, Moore; Sulkowski; Aganagic, Vafa; ⋯]

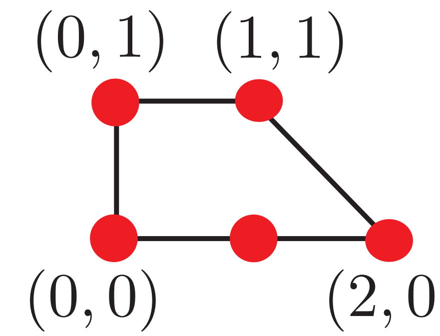
toric diagram (SPP $xy = zw^2$)



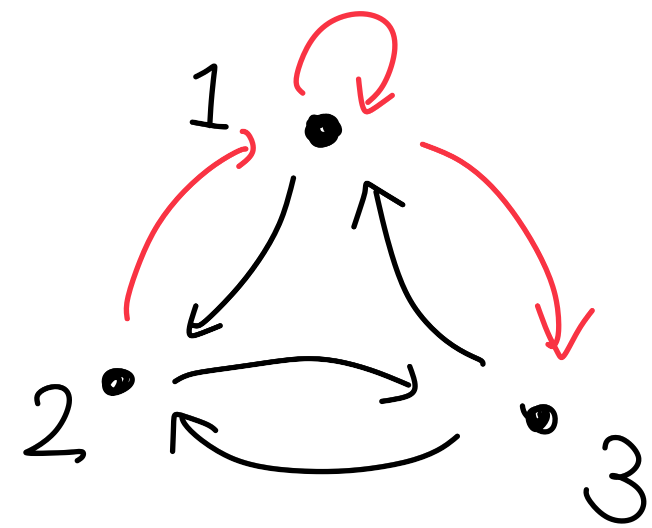

[Ooguri-MY '08]



We have an associated SQM




Q




{atom in the crystal}

= {(open path) starting at a vertex}

module F-term $(\partial W = 0)$


= {"chiral ring operator"}

We can place the atoms in 3D according to their R + flavor charges

# Representations from Crystal Melting

Representation by crystal melting [Li-MY '20], inspired by [FFJMM] and [Prochazka]

crystal

$$\psi^{(a)}(z)|\mathrm{K}\rangle = \Psi^{(a)}_{\mathrm{K}}(z)|\mathrm{K}\rangle\,,$$

$$e^{(a)}(z)|\mathrm{K}\rangle = \sum_{\boxed{a}\in \mathrm{Add}(\mathrm{K})} \frac{E^{(a)}(\mathrm{K}\to\mathrm{K}+\boxed{a})}{z-h(\boxed{a})}|\mathrm{K}+\boxed{a}\rangle\,,$$

$$f^{(a)}(z)|\mathrm{K}\rangle = \sum_{\boxed{a}\in \mathrm{Rem}(\mathrm{K})} \frac{F^{(a)}(\mathrm{K}\to\mathrm{K}-\boxed{a})}{z-h(\boxed{a})}|\mathrm{K}-\boxed{a}\rangle\,,$$

add/remove an atom

Representation by crystal melting [Li-MY '20], inspired by [FFJMM] and [Prochazka]

crystal

$$\psi^{(a)}(z)|\mathrm{K}\rangle = \Psi^{(a)}_{\mathrm{K}}(z)|\mathrm{K}\rangle \ ,$$

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$$f^{(a)}(z)|\mathrm{K}\rangle = \sum_{\boxed{a}\in \mathrm{Rem}(\mathrm{K})} \frac{F^{(a)}(\mathrm{K}\to\mathrm{K}-\boxed{a})}{z-h(\boxed{a})}|\mathrm{K}-\boxed{a}\rangle \ ,$$

poles for atom $\boxed{a}$

add/remove an atom

Representation by crystal melting [Li-MY '20], inspired by [FFJMM] and [Prochazka]

$$\psi^{(a)}(z)|\mathrm{K}\rangle = \Psi^{(a)}_{\mathrm{K}}(z)|\mathrm{K}\rangle\ ,$$

$$e^{(a)}(z)|\mathrm{K}\rangle = \sum_{\boxed{a}\in \mathrm{Add}(\mathrm{K})} \frac{E^{(a)}(\mathrm{K}\to\mathrm{K}+\boxed{a})}{z-h(\boxed{a})}|\mathrm{K}+\boxed{a}\rangle\ ,$$

$$f^{(a)}(z)|\mathrm{K}\rangle = \sum_{\boxed{a}\in \mathrm{Rem}(\mathrm{K})} \frac{F^{(a)}(\mathrm{K}\to\mathrm{K}-\boxed{a})}{z-h(\boxed{a})}|\mathrm{K}-\boxed{a}\rangle\ ,$$

poles for atom $\boxed{a}$

$$h(\boxed{a}) \equiv \sum_{I\in \mathrm{path}[\mathfrak{o}\to\boxed{a}]} h_I\ .$$

$\Psi^{(a)}_{\mathrm{K}}$ :
$$\Psi^{(a)}_{\mathrm{K}}(u) = \psi^{(a)}_0(z) \prod_{b\in Q_0} \prod_{\boxed{b}\in \mathrm{K}} \varphi^{b\Rightarrow a}(u-h(\boxed{b}))\ ,$$

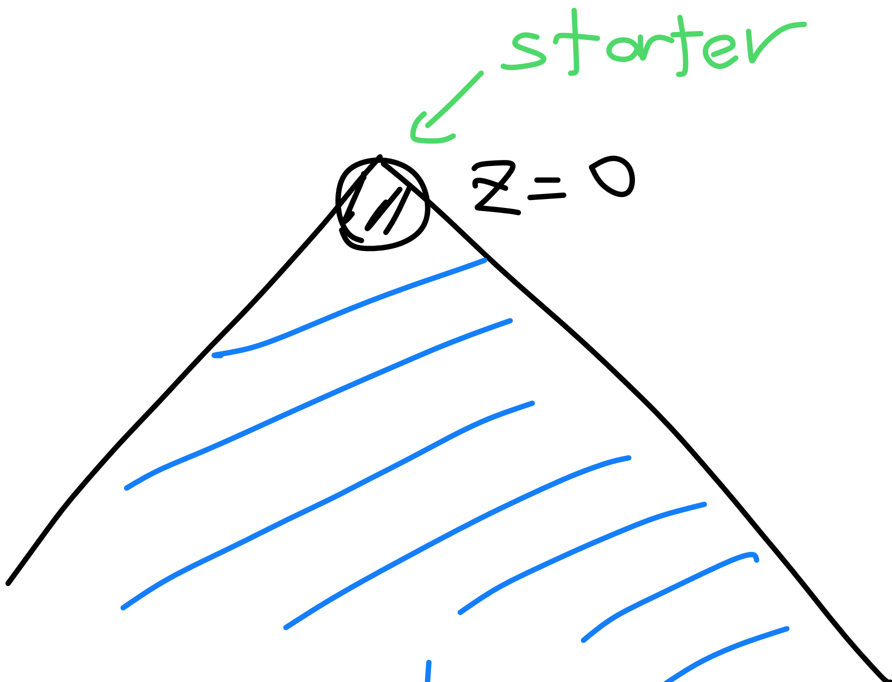
$$\varphi^{a\Rightarrow b}(u) \equiv \frac{\prod_{I\in\{b\to a\}}(u+h_I)}{\prod_{I\in\{a\to b\}}(u-h_I)}$$

$E^{(a)}/F^{(a)}$ :
$$E^{(a)}/F^{(a)} = \sqrt{\pm \operatorname*{Res}_{u=h(\boxed{a})} \Psi^{(a)}_{\mathrm{K}}(u)}$$

$$\psi^{(a)}(z)\,|\emptyset\rangle = \underbrace{\psi_0^{(a)}(z)}_{}\,|\emptyset\rangle$$

[Galakhov-Li-MY '21]

vacuum charge function $\longleftrightarrow$ representation

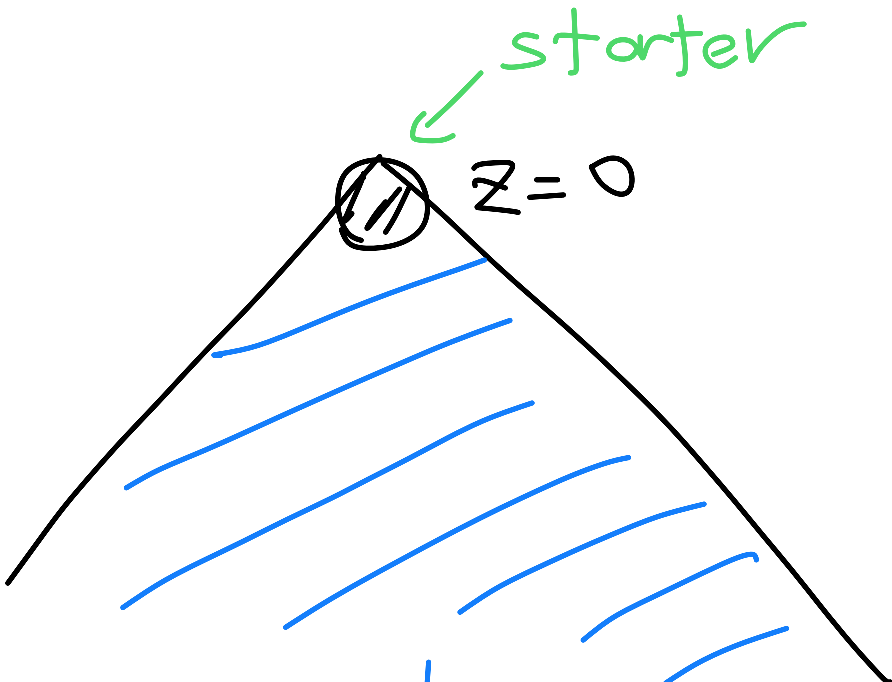



$\psi_0 \sim \frac{1}{z}$

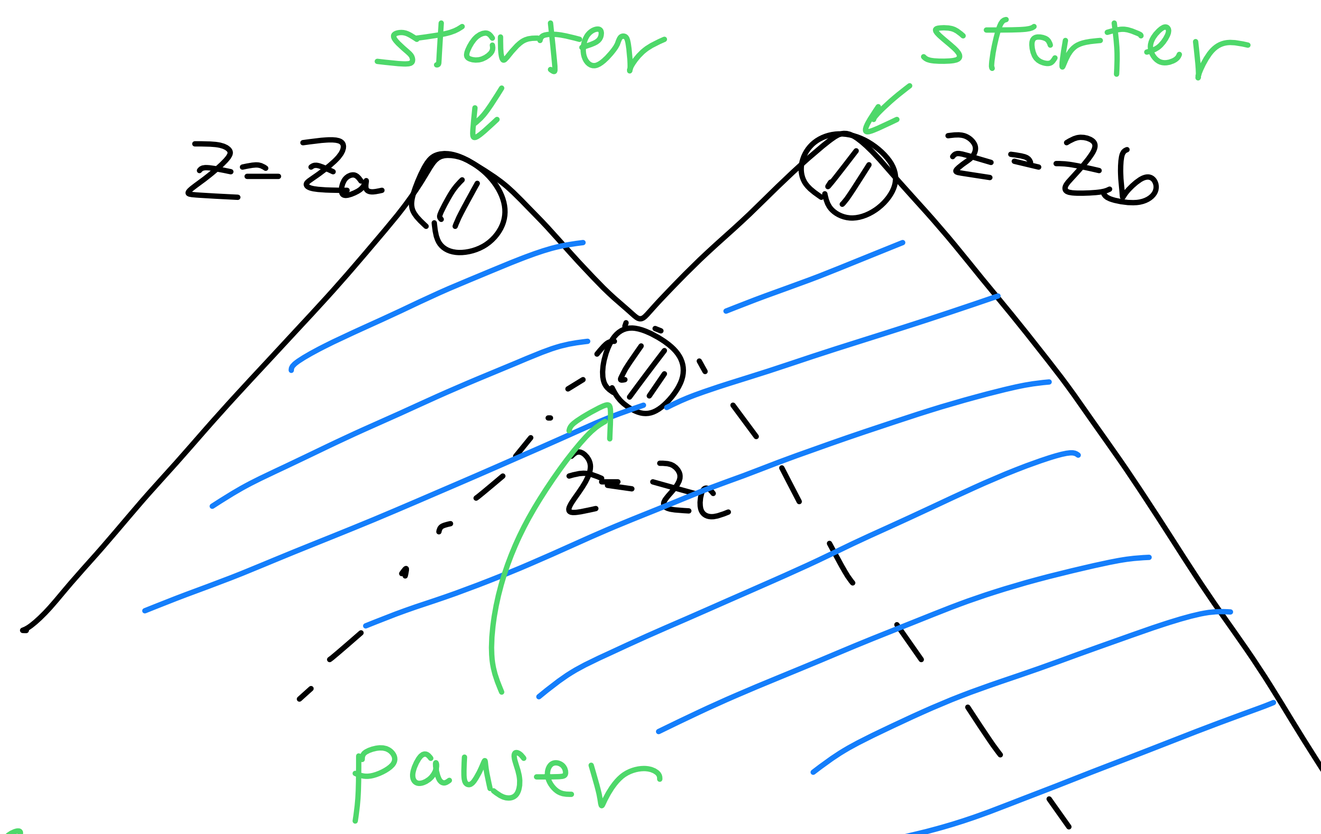
$$\psi^{(a)}(z)\,|\emptyset\rangle = \underbrace{\psi_0^{(a)}(z)}\,|\emptyset\rangle$$

[Galakhov-Li-MY '21]

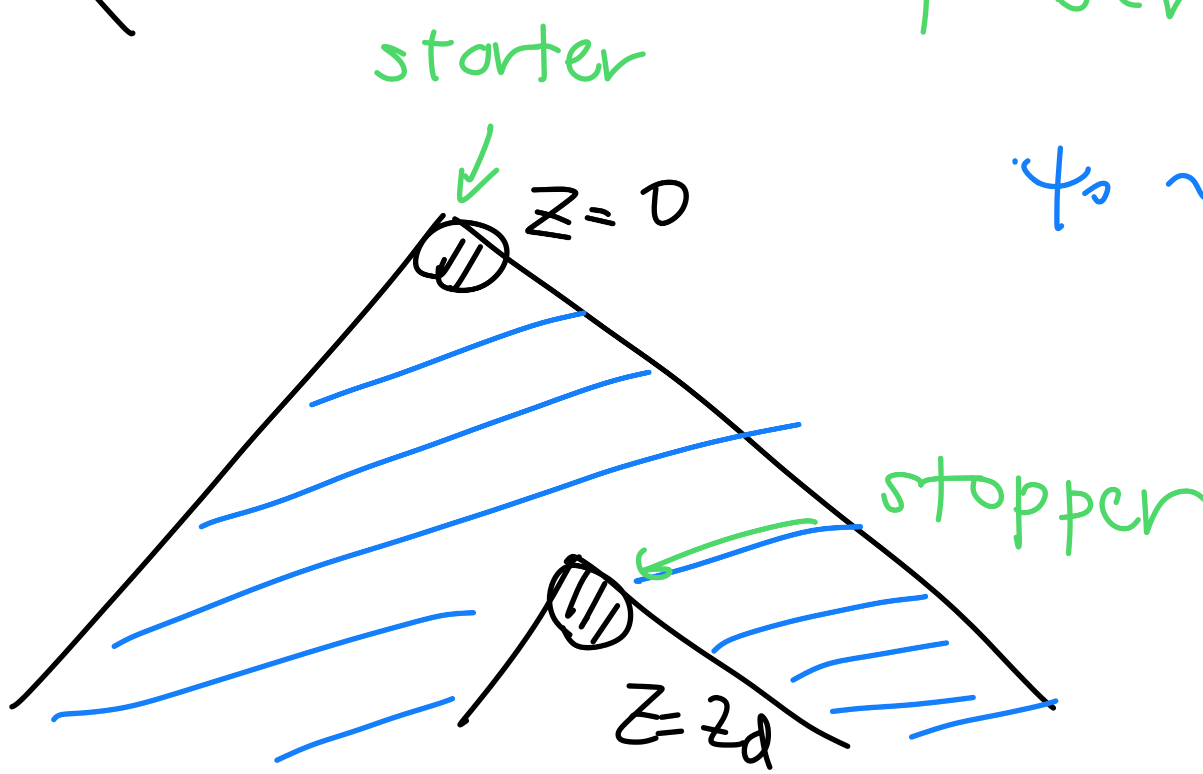
vacuum charge function $\longleftrightarrow$ representation




$\psi_0 \sim \frac{1}{z}$




$\psi_0 \sim \frac{z - z_c}{(z - z_a)(z - z_b)}$




$\psi_0 \sim \frac{z - z_d}{z}$

# Resolution of the Puzzle?

2d $\mathcal{N}=(2,2)$

$Q, W$ $\rightsquigarrow$ $\mathcal{Y}_{Q,W} \neq \mathcal{Y}(\mathfrak{g})$

toric $CY_3$

should be some $\infty$-dim. algebra

$\mathcal{Y}_{Q,W}$ = quiver Yangian??

R-matrix

Bethe Ansatz

[Galakhov-Li-Y ('22)]

Vacuum equation = "Would-be BAE"

exp(FI param.)

$$1 = \mathbf{BAE}_i^{(a)}\left(\vec{\sigma}, \vec{u}, \vec{\mathfrak{q}}\right) := \mathfrak{q}_a^{-1} \prod_{\substack{1 \le j \le N_a \\ j \ne i}} \varphi^{a \Leftarrow a}\left(\sigma_i^{(a)} - \sigma_j^{(a)}\right) \times$$

$$\times \prod_{\substack{b \in Q_0 \\ b \ne a}} \prod_{k=1}^{N_b} \varphi^{a \Leftarrow b}\left(\sigma_i^{(a)} - \sigma_k^{(b)}\right) \prod_{\mathfrak{f}} \varphi^{a \Leftarrow \mathfrak{f}}\left(\sigma_i^{(a)} - u_{\mathfrak{f}}\right)$$

net deg $\neq 0$
in general

$$\varphi^{a \Rightarrow b}(u) \equiv \frac{\prod_{I \in \{b \to a\}} (u + h_I)}{\prod_{I \in \{a \to b\}} (u - h_I)}$$

We now have an algebra $Y$

and a representation $\mathcal{C}$ from crystal

$\rightsquigarrow$

Natural to consider

"crystal chains" $\mathcal{C}^{\otimes n}$

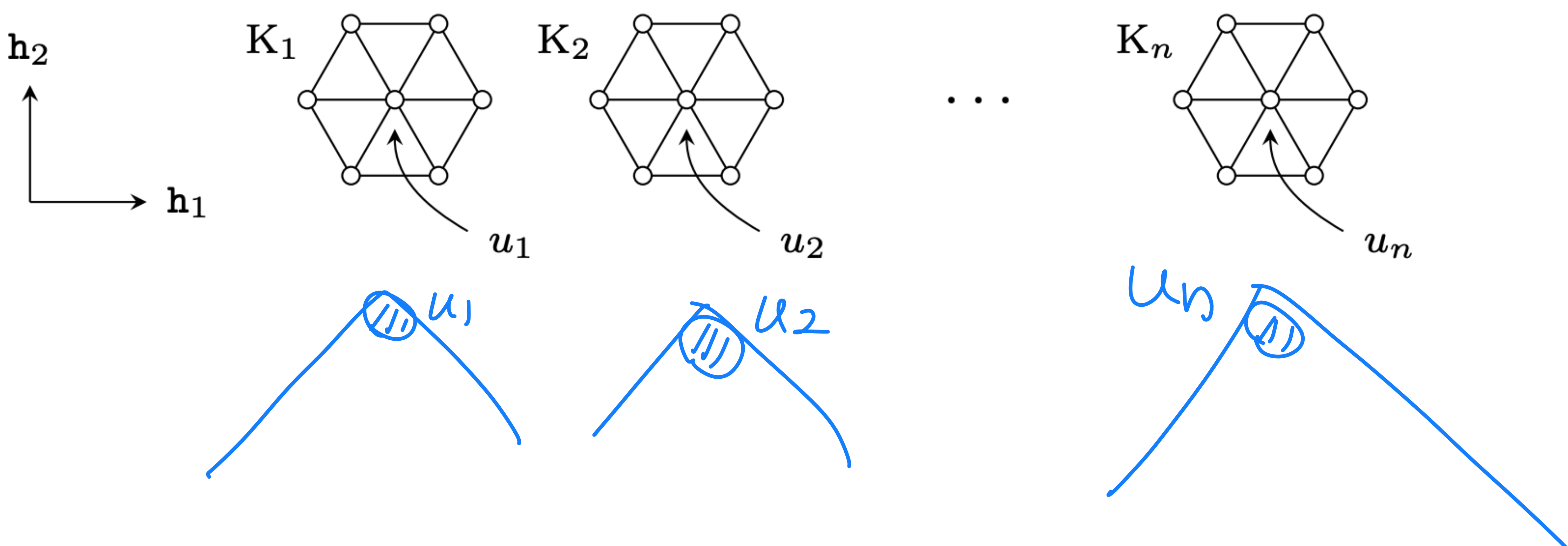
&

R-matrix, BAE, ....

We can make "crystal chains" by bringing together crystals in

Spectral-parameter plane

[Galakhov-Y, Galakhov-Li-Y ('21)]




$$|\mathrm{K}_1, {}^\sharp\mathcal{C}_1\rangle_{u_1} \otimes |\mathrm{K}_2, {}^\sharp\mathcal{C}_2\rangle_{u_2} \otimes \ldots \otimes |\mathrm{K}_n, {}^\sharp\mathcal{C}_n\rangle_{u_n} \,.$$

We can derive representations

[Galakhov-Y, Galakhov-Li-Y ('21)]

$$\mathbf{\Delta}_0^{(n)}(\psi(z))\bigotimes_{i=1}^{n}|\mathrm{K}_i\rangle_{u_i}=\prod_i \Psi_{\mathrm{K}_i}(z-u_i)\times\bigotimes_i|\mathrm{K}_i\rangle_{u_i}\,,$$

$$\mathbf{\Delta}_0^{(n)}(e(z))\bigotimes_{i=1}^{n}|\mathrm{K}_i\rangle_{u_i}=\sum_i\sum_{\square\in\mathrm{Add}(\mathrm{K}_i)}\prod_{j<i}\Psi_{\mathrm{K}_j}(u_i+h_\square-u_j)\times\frac{[\mathrm{K}_i\to\mathrm{K}_i+\square]}{z-(u_i+h_\square)}\times \bigotimes_{j<i}|\mathrm{K}_j\rangle_{u_j}\otimes|\mathrm{K}_i+\square\rangle_{u_i}\otimes\bigotimes_{k>i}|\mathrm{K}_k\rangle_{u_k}\,,$$

$$\mathbf{\Delta}_0^{(n)}(f(z))\bigotimes_{i=1}^{n}|\mathrm{K}_i\rangle_{u_i}=\sum_i\sum_{\square\in\mathrm{Rem}(\mathrm{K}_i)}\prod_{k>i}\Psi_{\mathrm{K}_k}(u_i+h_\square-u_k)\times\frac{[\mathrm{K}_i\to\mathrm{K}_i-\square]}{z-(u_i+h_\square)}\times \bigotimes_{j<i}|\mathrm{K}_j\rangle_{u_j}\otimes|\mathrm{K}_i-\square\rangle_{u_i}\otimes\bigotimes_{k>i}|\mathrm{K}_k\rangle_{u_k}\,,$$

and "standard coproduct"

※ not inv. under permutations

$$\mathbf{\Delta}_0 e=e\otimes 1+\psi\overset{\rightarrow}{\otimes}e\,,$$

$$\mathbf{\Delta}_0 f=1\otimes f+f\overset{\leftarrow}{\otimes}\psi\,,$$

$$\mathbf{\Delta}_0\psi=\psi\otimes\psi\,.$$

However,

$$\mathbf{\Delta}_0 e = e \otimes 1 + \psi \overset{\rightarrow}{\otimes} e\,,$$
$$\mathbf{\Delta}_0 f = 1 \otimes f + f \overset{\leftarrow}{\otimes} \psi\,,$$
$$\mathbf{\Delta}_0 \psi = \psi \otimes \psi\,.$$

- $\Delta_0$ does NOT reproduce R-matrix needed for (BAE) = (vacuum equation)
- For rational/Yangian case does NOT come from a coproduct $\Delta_0 : Y \to Y \otimes Y$

[Prochazka ('15)] [Galakhov-Li-Y ('22)]

We need to search "correct" $\Delta$:

cf. stable envelope of [Maulik-Okounov]

$$\Delta = \mathcal{U}^{-1} \Delta_0 \mathcal{U}$$

# "Yes-Go"

[Galakhov-Li-Y ('22)]
See also [Feigin-Jimbo-Miwa-Mukhin ('15)]
[Litvinov-Vilkovisky ('20)] [Chistyakova-Litvinov-Orlov ('21)]
[Kolyaskin, A. Litvinov, and A. Zhukov ('22)] [Bao ('22)]

We can choose shift $= 0$

* For 2d-crystal repr. (Fock module) of $Y(\hat{\mathfrak{g}})$ w/ $\mathfrak{g} = \mathfrak{gl}_{m|n}, D(2,1;\alpha)$

We can derive BAE and verify Gauge/Bethe!

# "No - Go"

[Galakhov-Li-Y ('22)]

shift $\neq 0$

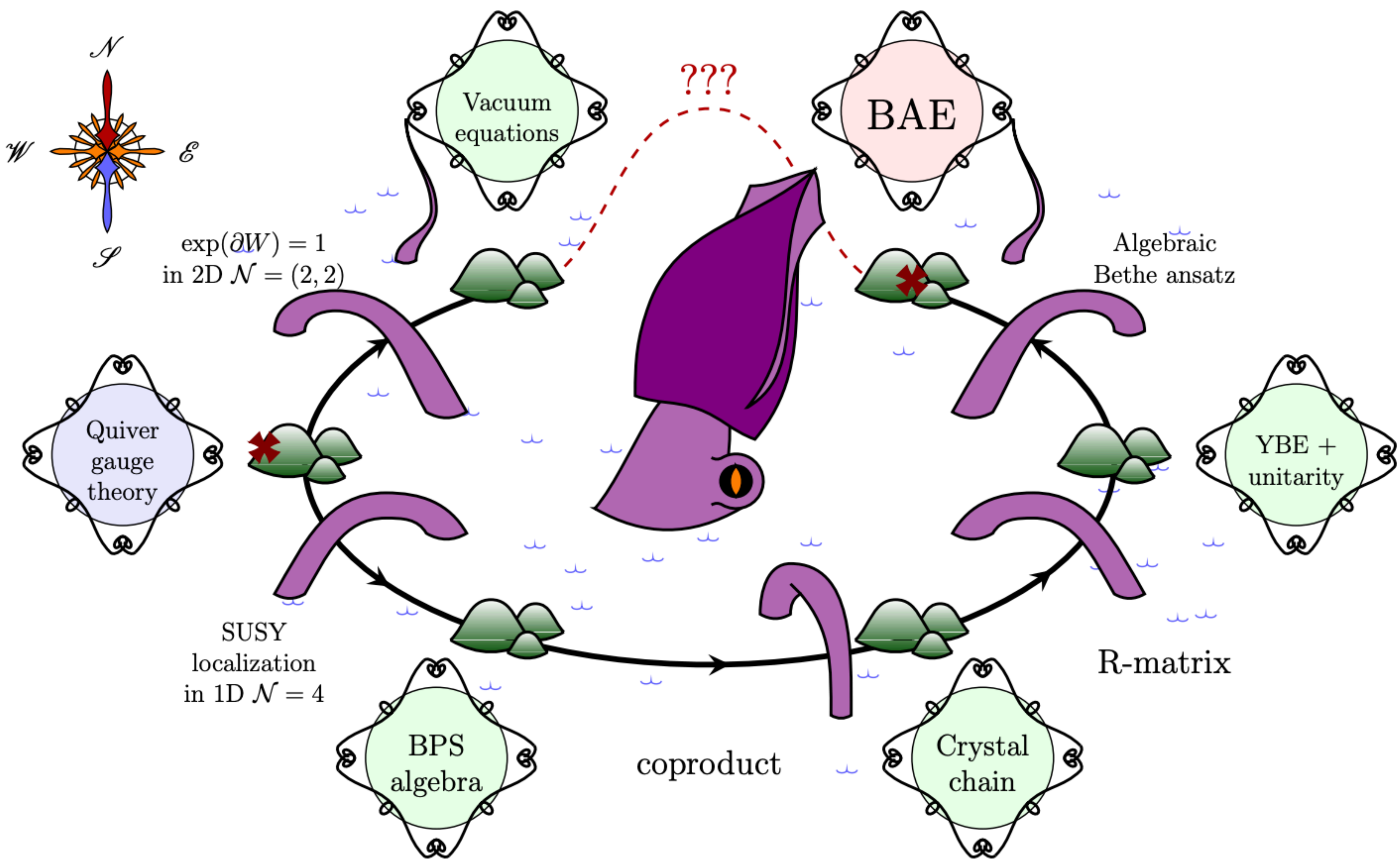
* For $Y(Q, W)$ without underlying $\mathfrak{g}$

[ chiral quiver / toric $CY_3$ with 4-cycle ]

We have obstructions (under some assumptions)

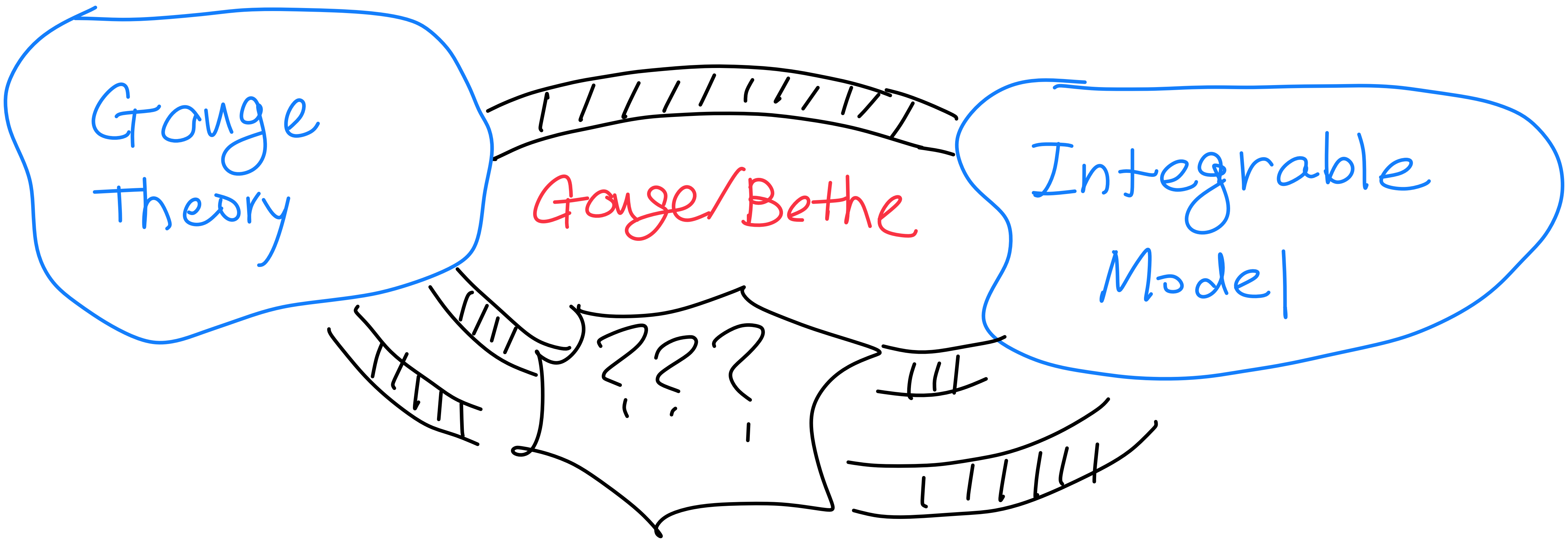
to finding consistent $\Delta$ / $R$

whose BAE matches vacuum eqn.



$\mathscr{N}$
$\mathscr{W}$
$\mathscr{E}$
$\mathscr{S}$
Vacuum equations
???
BAE
$\exp(\partial W) = 1$ in 2D $\mathcal{N} = (2,2)$
Algebraic Bethe ansatz
Quiver gauge theory
YBE + unitarity
SUSY localization in 1D $\mathcal{N} = 4$
R-matrix
BPS algebra
coproduct
Crystal chain

# Summary




- Still foundational issues to be solved
- "beyond $Y(\mathfrak{g})$"
  quiver Yangian $Y(Q,W)$ provides new clue

Mertonian Norms of Science

## Universalism

Universalism[5] finds immediate expression in the canon that truth-claims, whatever their source, are to be subjected to *preestablished impersonal criteria*: consonant with observation and with previously confirmed knowledge. The acceptance or rejection of claims entering the lists of science is not to depend on the personal or social attributes of their protagonist; his race, nationality, religion, class, and personal qualities are as such irrelevant. Objectivity precludes particularism. The circumstance that scientifi-

## Organized Skepticism

institutions. Science which asks questions of fact, including potentialities, concerning every aspect of nature and society may come into conflict with other attitudes toward these same data which have been crystallized and often ritualized by other institutions. The scientific investigator does not preserve the cleavage between the sacred and the profane, between that which requires uncritical respect and that which can be objectively analyzed.

"The Normative Structure of Science"

# 大栗先生還暦記念研究会

## HirosiFest @ Kavli IPMU, October 20-21
## HirosiFest @ Caltech, October 27-28

## HirosiFest @ Kavli IPMU

20-21 October 2022
Kavli IPMU, Kashiwa, Japan
Asia/Tokyo timezone

- Overview
- Timetable
- Accommodation
- Access
- Visa
- Contact Information

**Dates**:  October 20-21, 2022
**Venue:** Lecture Hall, Kavli IPMU, Kashiwa, Japan

**Overview:** Prof. Hirosi Ooguri is one of the founding members of the Kavli IPMU and is currently the director of Kavli IPMU. He has made tremendous contributions in physics and mathematics and is one of the most outstanding researchers of his generation. He is turning sixty this year, and we would like to take this opportunity to celebrate his remarkable contributions in physics and mathematics, bringing together experts worldwide.

Kavli IPMU イベント：on-site参加者は少数に限定，オンライン中心を予定．registration @ https://indico.ipmu.jp/event/402/

# ポスドク公募中！！（2023年4月-9月の半年雇用）

素粒子全般，宇宙論，重力理論，天体物理，物性理論，量子情報を含め
かなり広い分野を対象　（とりあえずの締切が通常よりも早いので注意）

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## University of Tokyo, Kavli Institute for the Physics and Mathematics of the Universe

**Position ID:** UTokyo-KavliIPMU-DMQG2022 [#22199]
**Position Title:** Project researcher position in Theoretical Physics
**Position Type:** Postdoctoral
**Position Location:** Kashiwa, Chiba 277-8583, Japan [map]
**Subject Areas:** Astrophysics
Condensed Matter Physics
High-Energy Theory
Cosmology
String Theory and Mathematical Physics (more...)
**Appl Deadline:** 2022/09/30 11:59PM (listed until 2023/01/31)
**Position Description:**

Applications are invited for a half-year postdoctoral research position at the Kavli IPMU, the University of Tokyo.
<**https://academicjobsonline.org/ajo/jobs/22199**>

The position is funded by KAKENHI project, "Dark Matter in Quantum Gravity" (20H05860, PI: Masahito Yamazaki), which is the sub-group C01 under Grant-in-Aid for Transformative Research Areas (A), "What is dark matter? - Comprehensive study of the huge discovery space in dark matter" (PI: Hitoshi Murayama).
The postdoctoral scholar will work on one of the following areas: cosmology, phenomenology, quantum computation and/or string theory, related to "Dark Matter in Quantum Gravity" together with Yasunori Nomura, Satoshi Shirai, Masahito Yamazaki at the Kavli IPMU and Ryo Saito at Yamaguchi University. Our research interest is very broad, and we welcome applications from researchers in diverse areas.

Applicants should have a Ph.D. in physics or related areas by the starting date of the position.
The appointment will start from April 1, 2023 through the end of September 2023.
In exceptional circumstances and if there is mutual agreement, there is also a possibility of starting the position already in early January 2023; in either case the appointment will terminate by the end of September 2023.