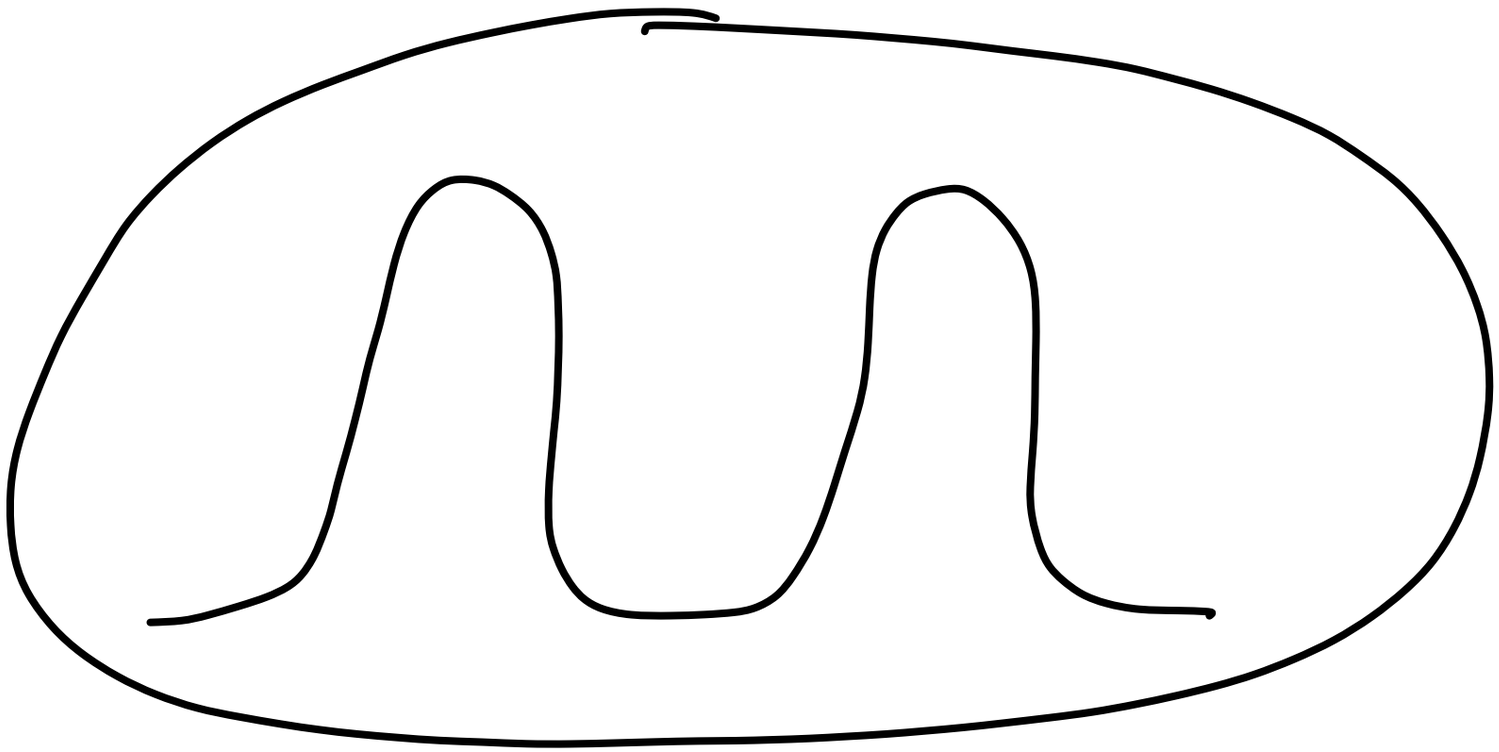
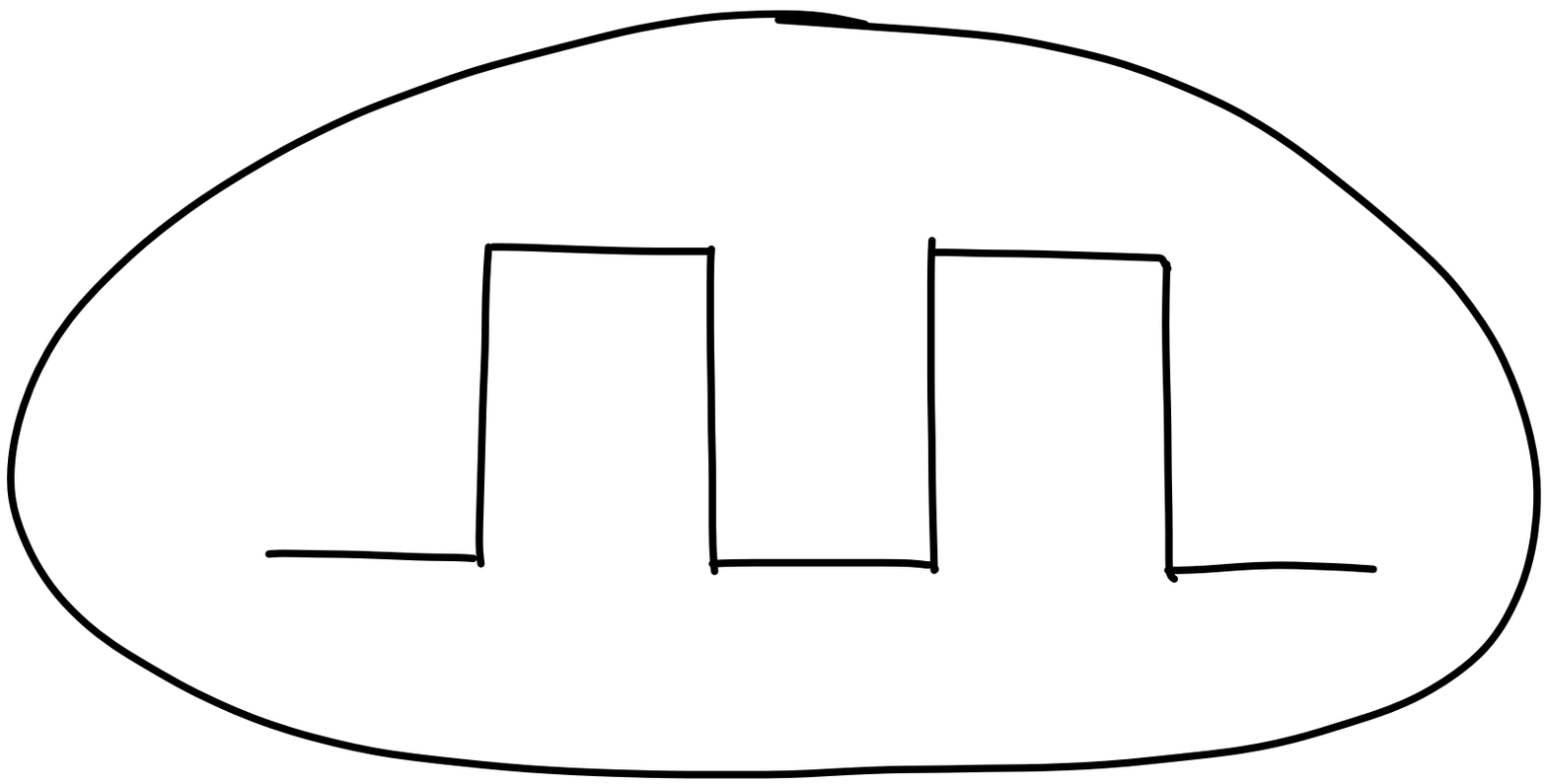
Theoretical Engineering of Integrable Models from Extra Dimensions

Masahito Yamazaki
(Kavli IPMU / TSQS, UTokyo)

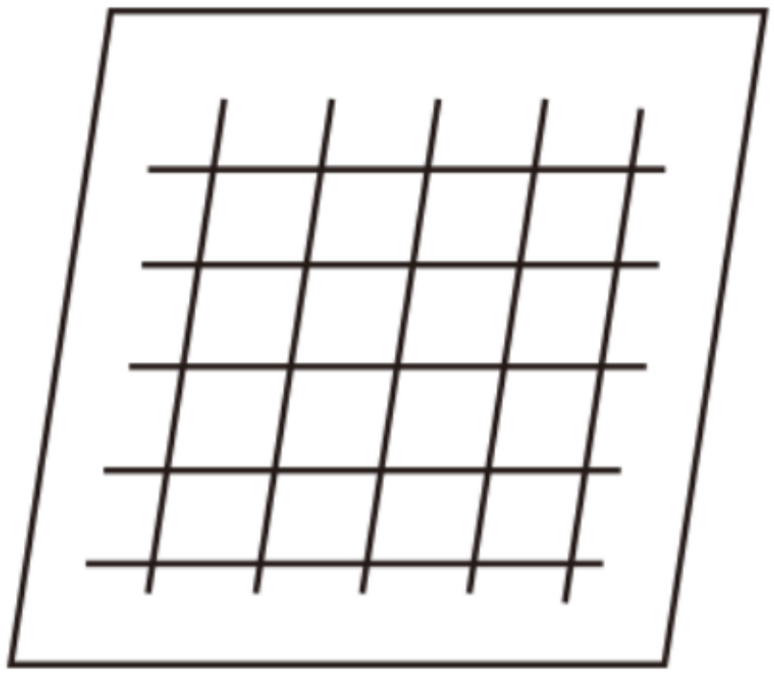
TSQS 2022, Nov. 8 @ Koshiba Hall

Is the world digital or analog?

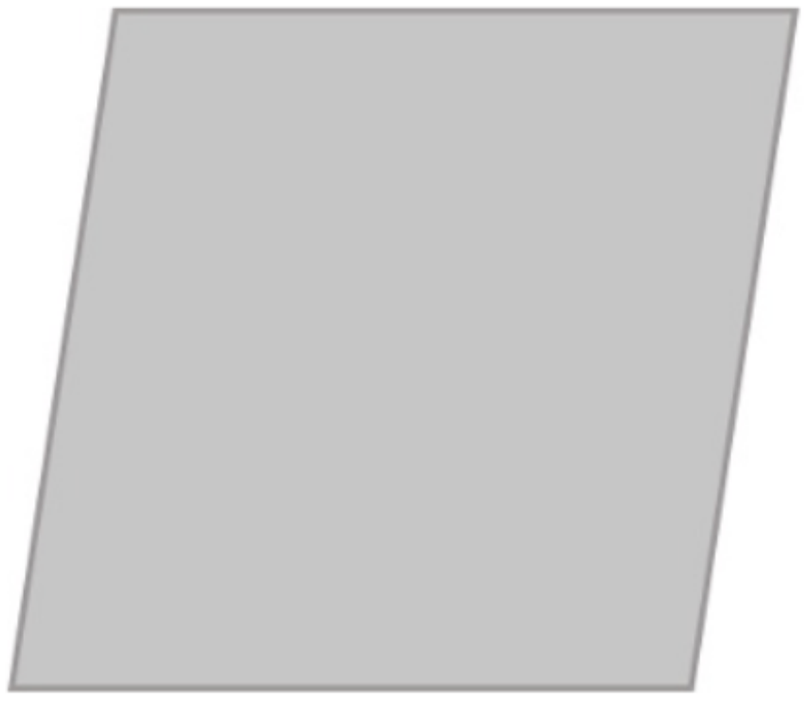




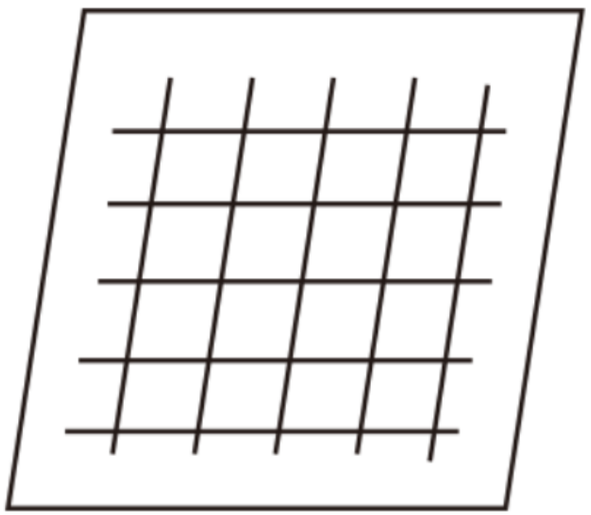
discrete lattice model

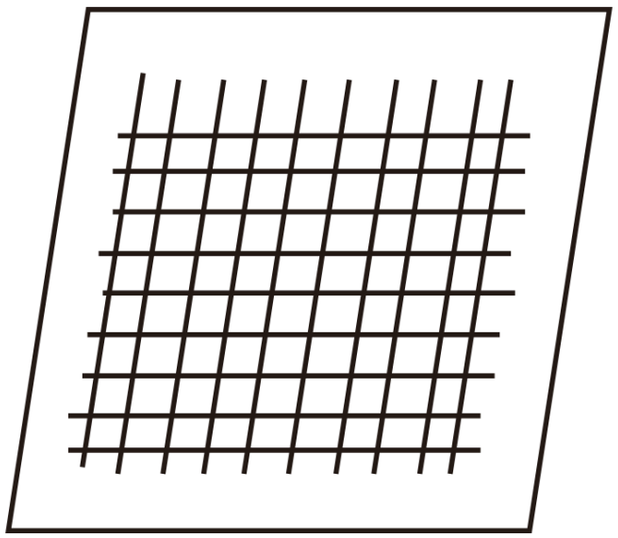


continuous field theory

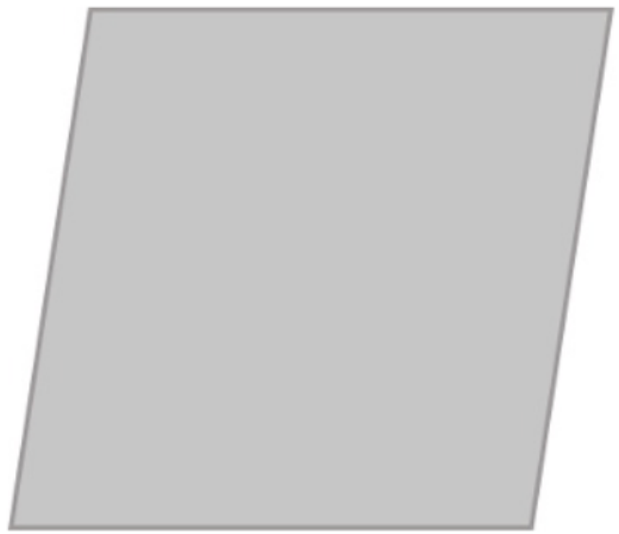


lattice model





field theory



“UV”

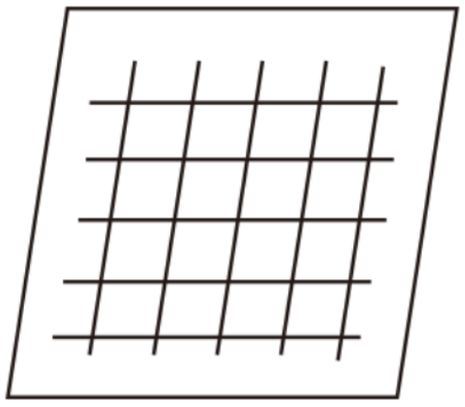
“IR”

renormalization
group flow

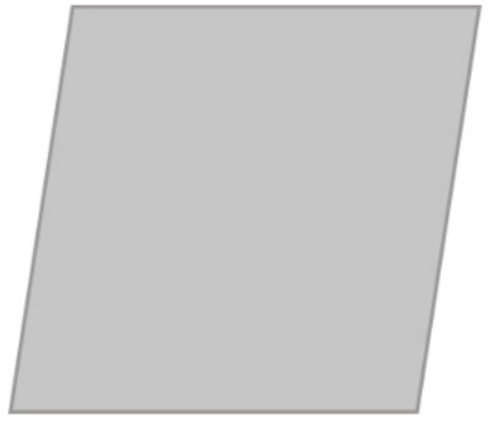
Discretizations in general break the symmetries of the field theory, but we can choose “nice” discretizations

Q: What happens if we have enough symmetries, so that the theories are solvable/integrable?

integrable
lattice model



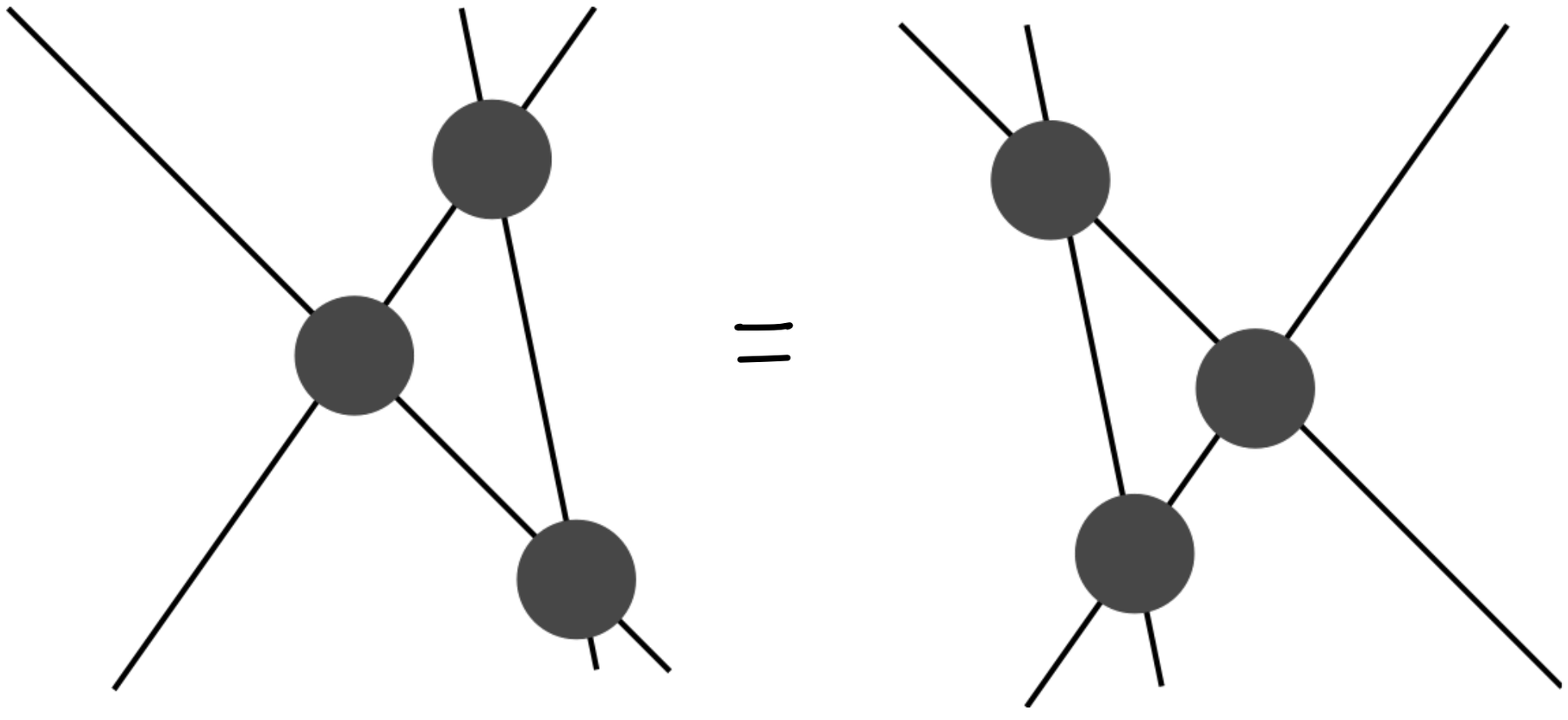
integrable
field theory



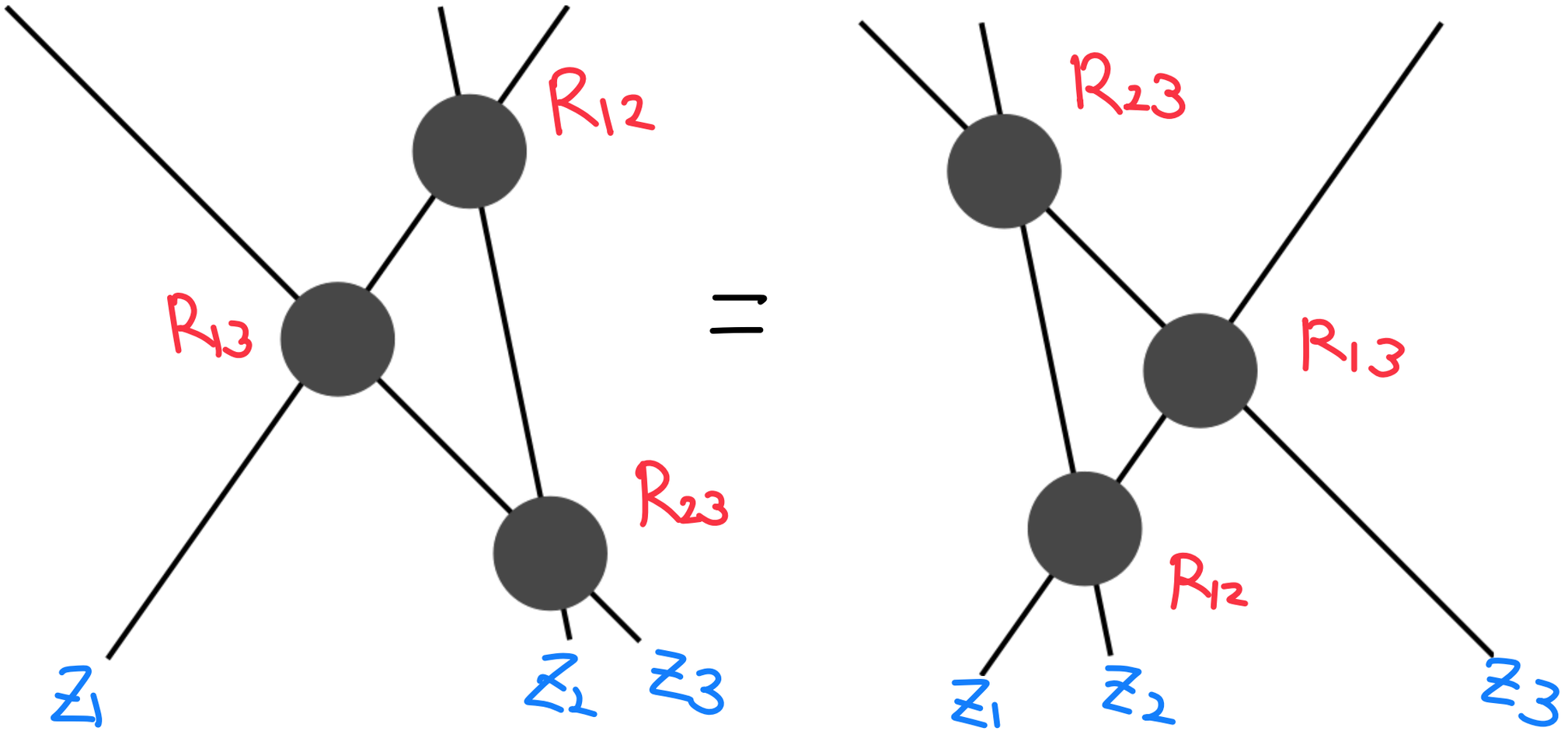
$$Q_n \sim \sum p_i^{\,n} \quad (n = 1, 2, \cdots) \quad \text{conserved}$$

[S.Parke '80] cf. [Coleman-Mandula '67]

Factorized scattering (Yang-Baxter equation)



Factorized scattering (Yang-Baxter equation)



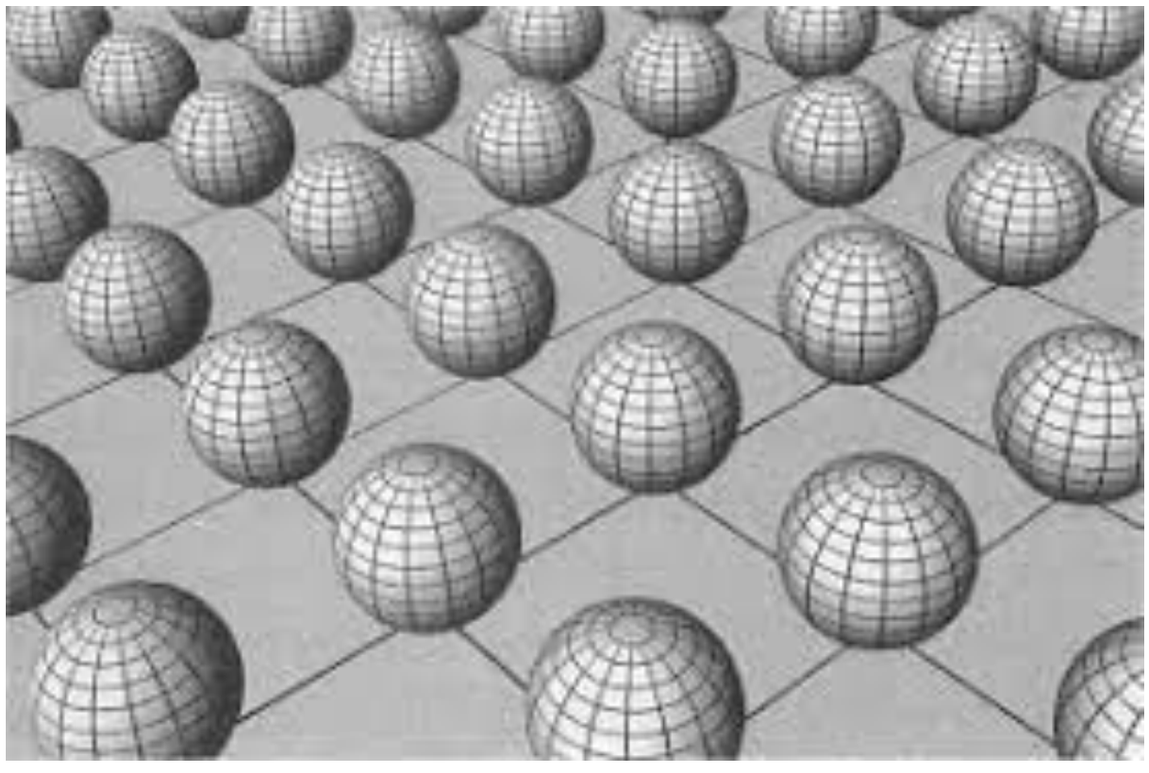
$$R_{ij} = R_{ij}(z_i - z_j)$$

R-matrix

"spectral parameter"

Extra Dimensions

come to rescue

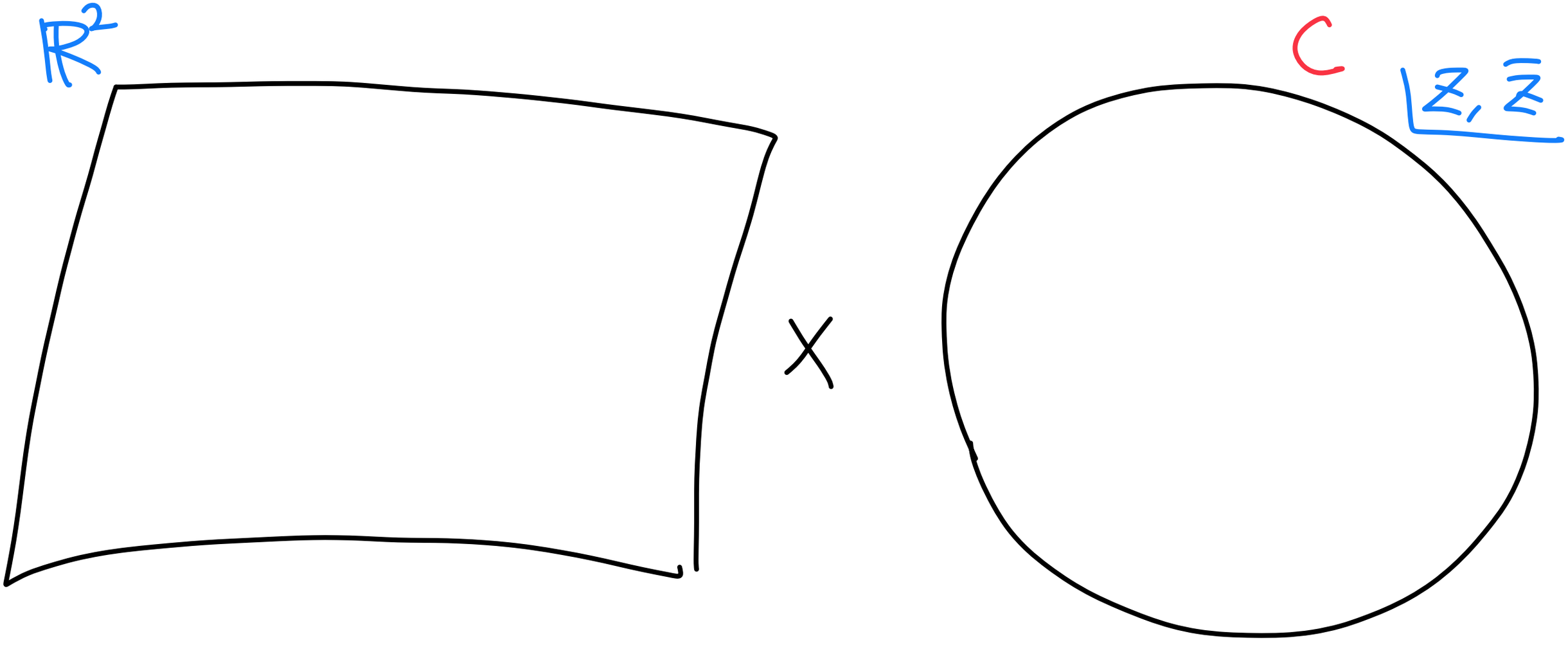


[Costello-Witten-MY ’17]
[Costello-Witten-MY ’18]
[Costello-MY ’19]
[Ashwinkumar-Sakamoto-MY ’22]

Consider “4d Chern-Simons theory”

[Costello ’13]

$$S = \frac{1}{k} \int_{\mathbb{R}^2 \times C} dz \wedge \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

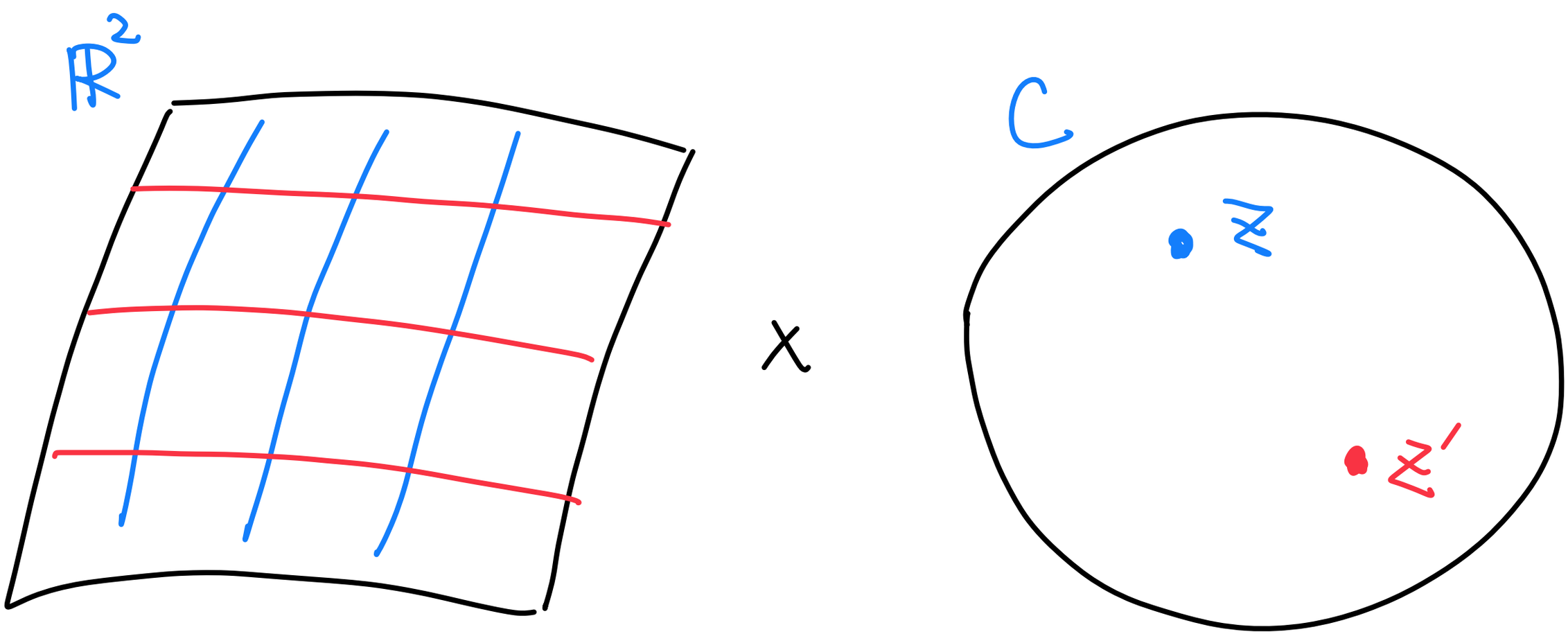


2d part: topological
(gapped)

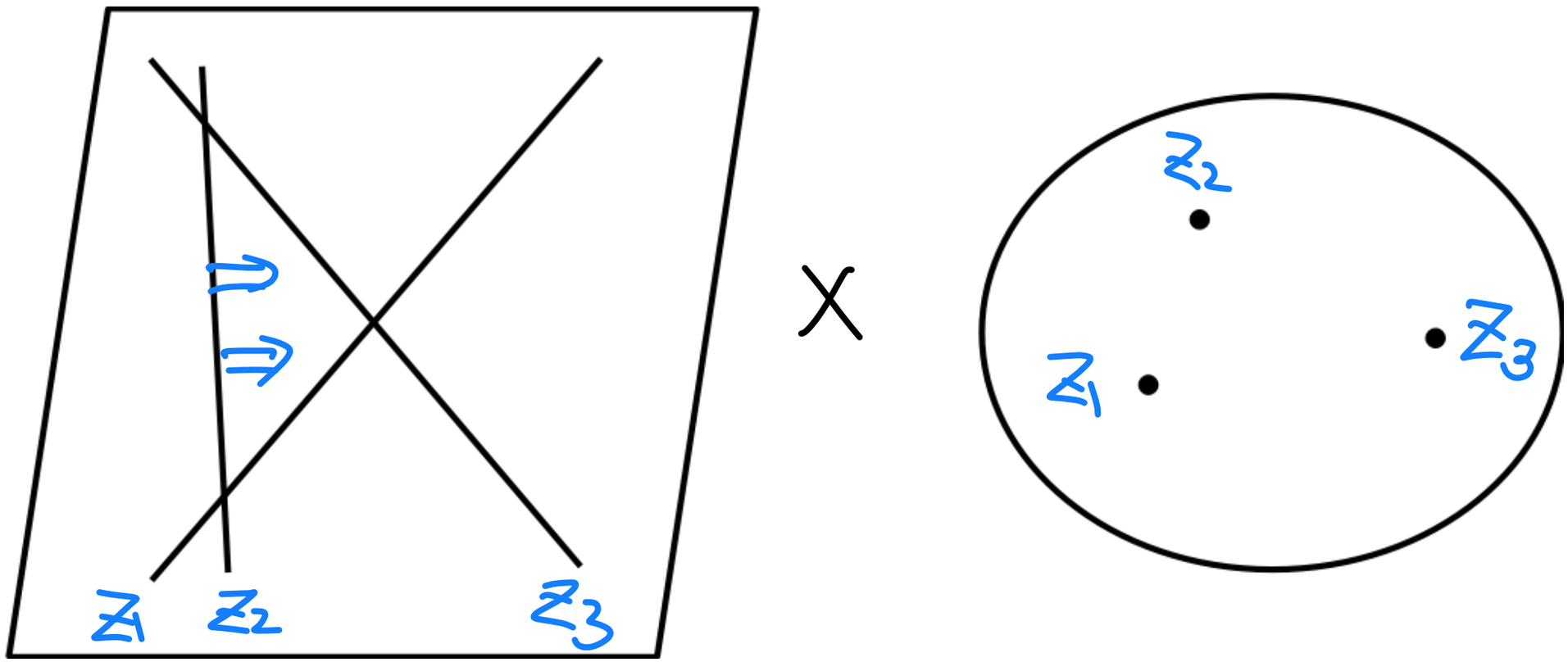
2d part: holomorphic
(gapless)

We make integrable lattice from Wilson lines

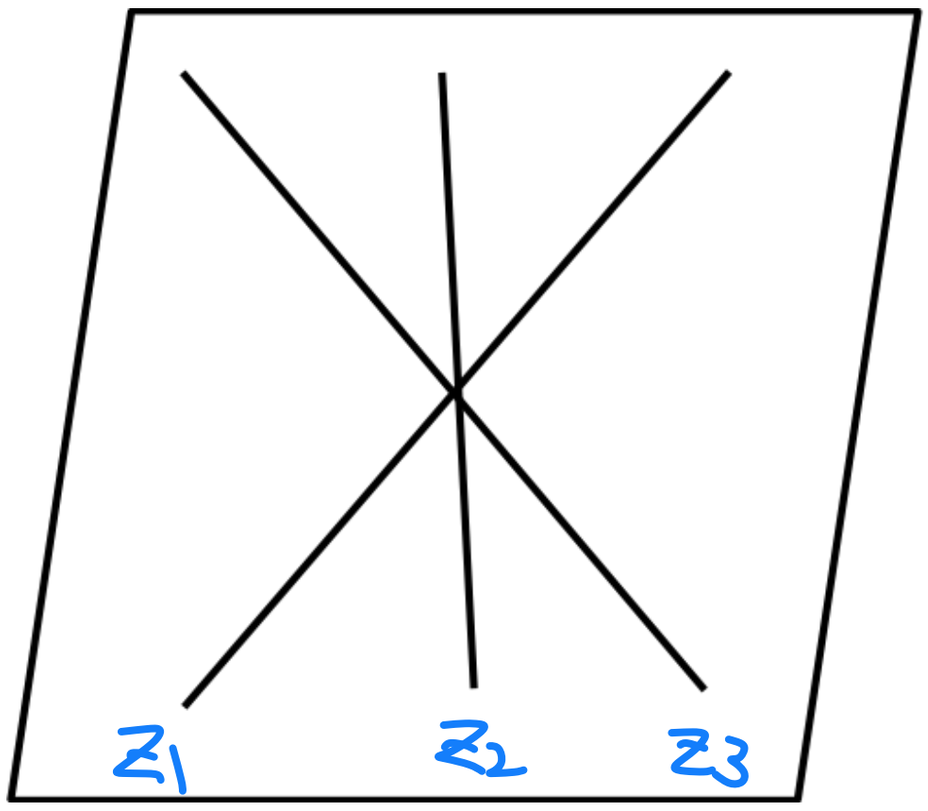
$$\langle W \rangle = \left\langle P \exp \int A \right\rangle$$

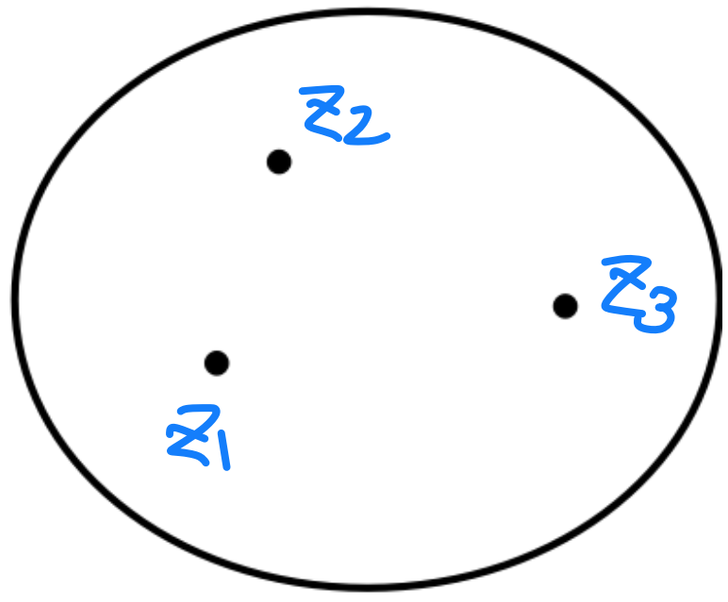


Yang-Baxter equation automatic
from topological invariance



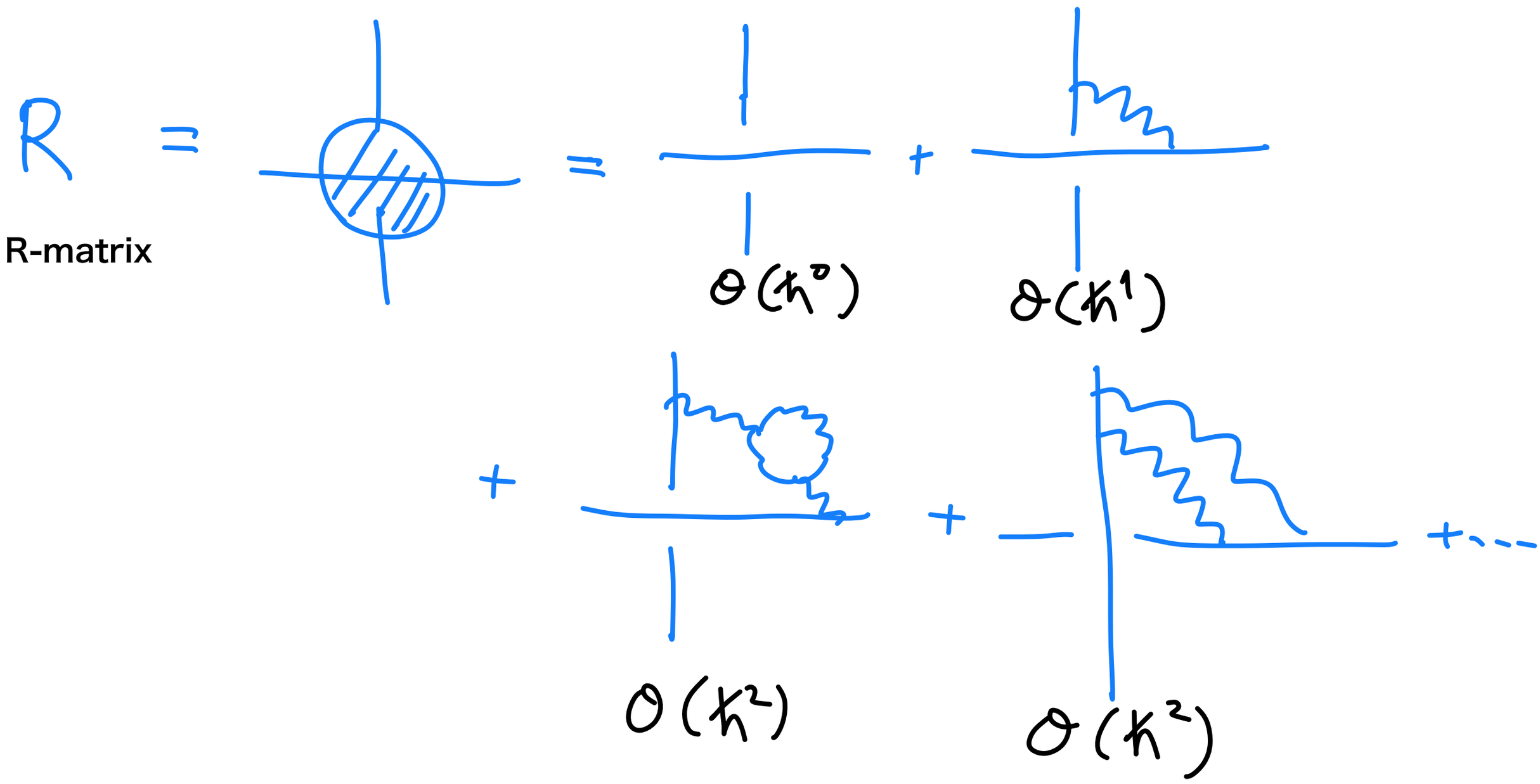
Yang-Baxter equation automatic
from topological invariance





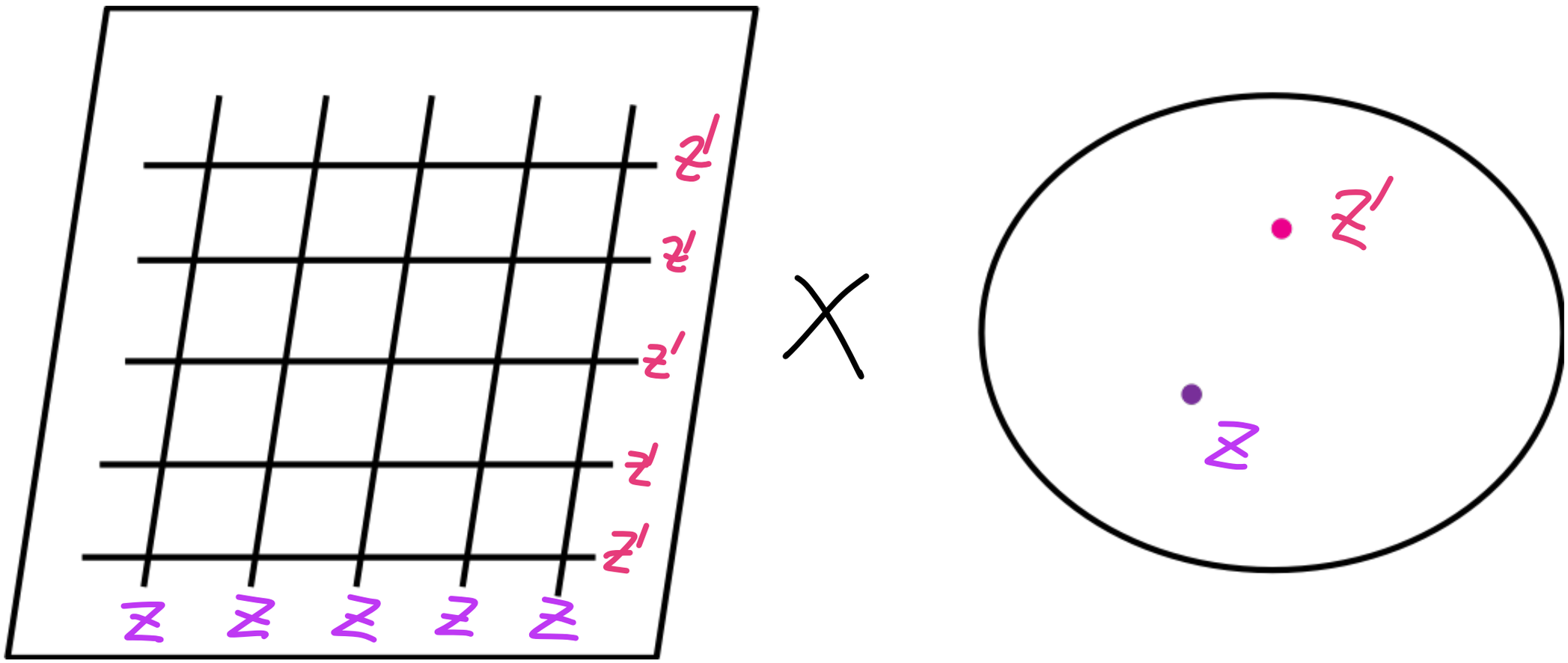
QFT perturbation techniques (re)produce many results in integrable models

[Costello-Witten-MY '17, '18]

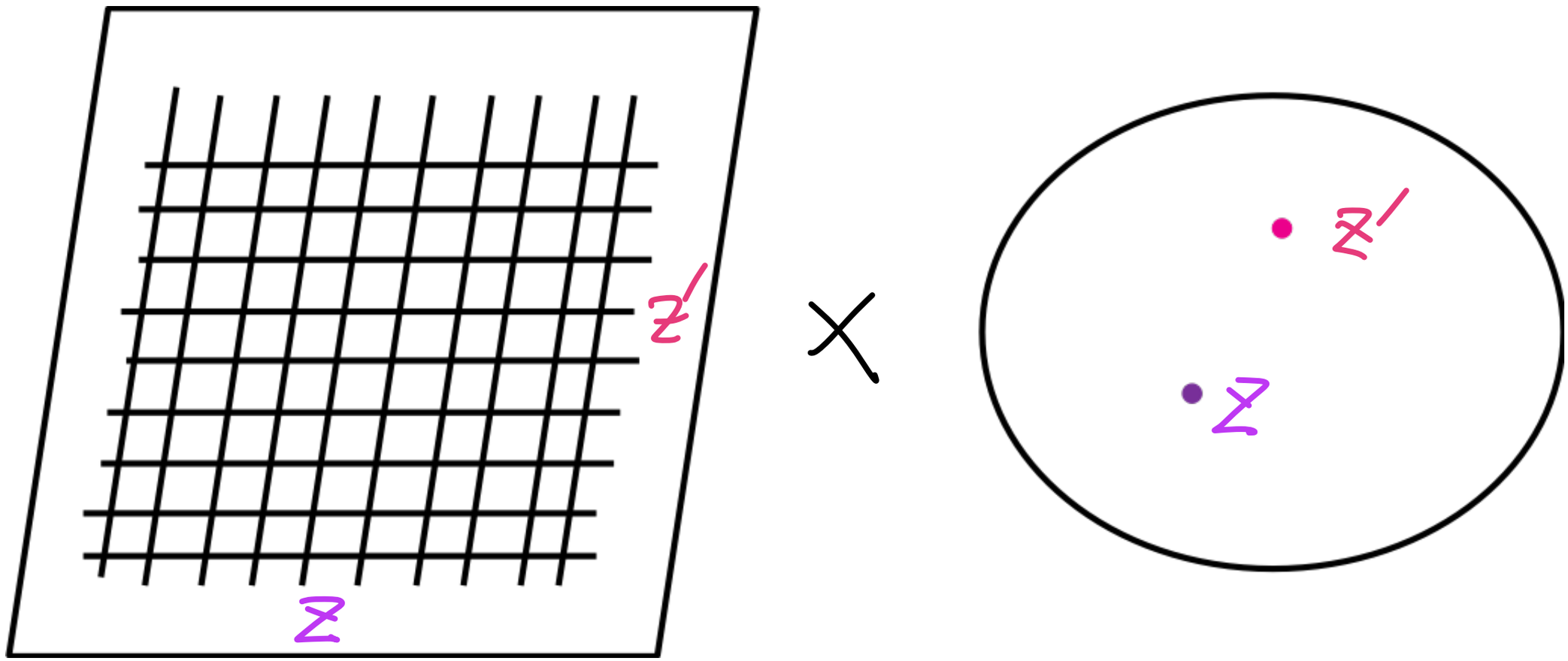


Coming back to thermodynamic limit…

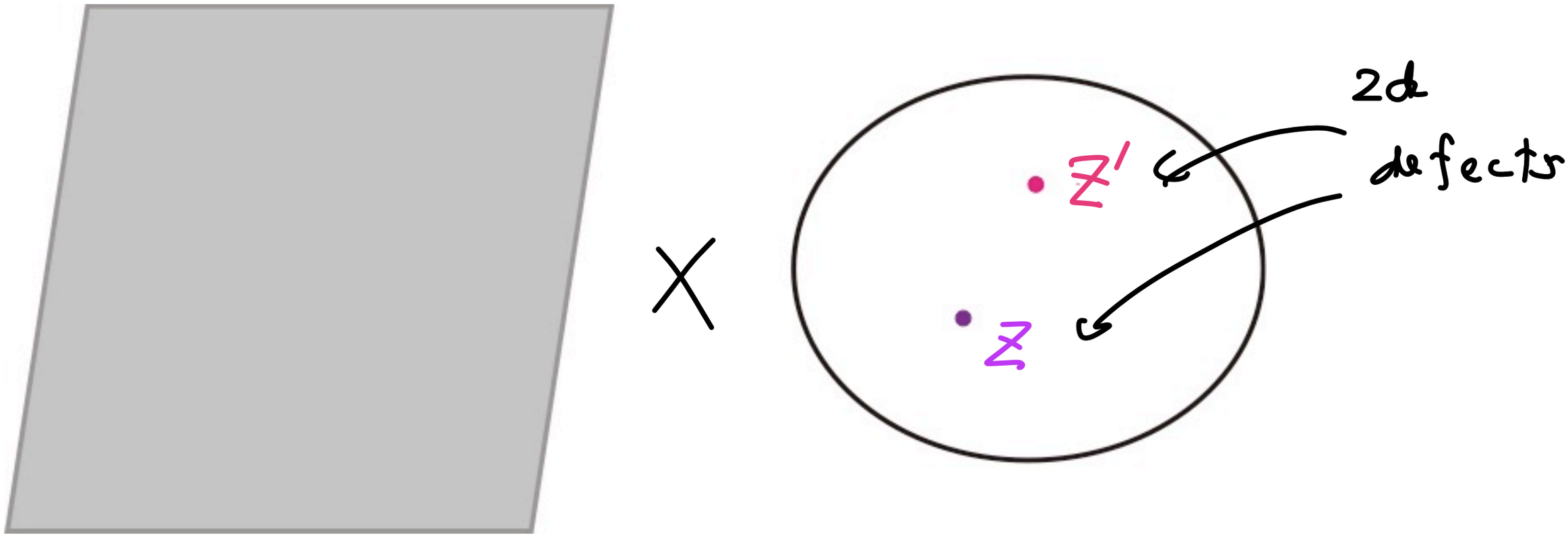
lattice model



refine the mesh…

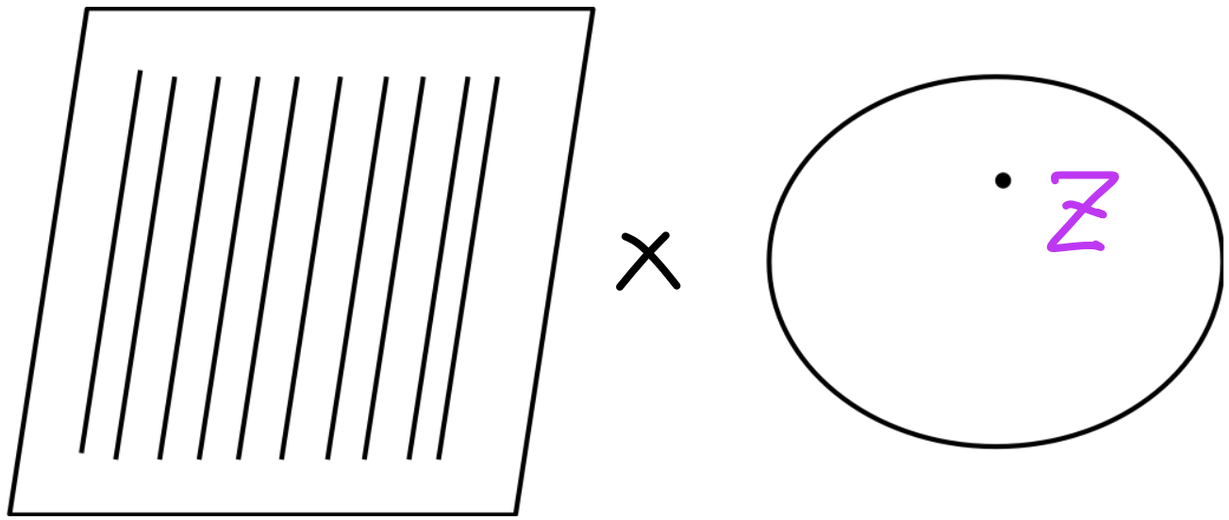


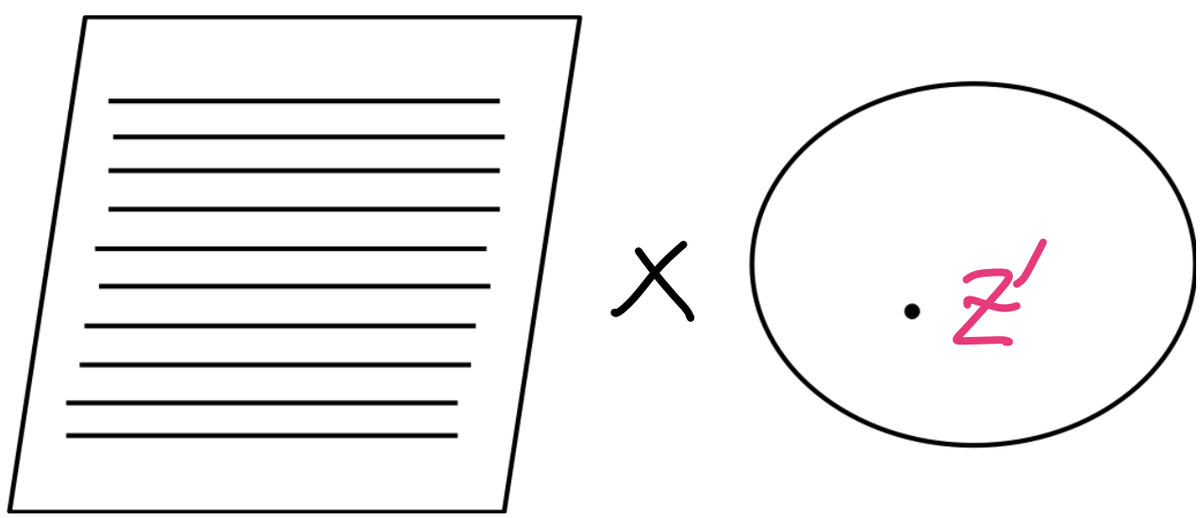
Thermodynamic limit



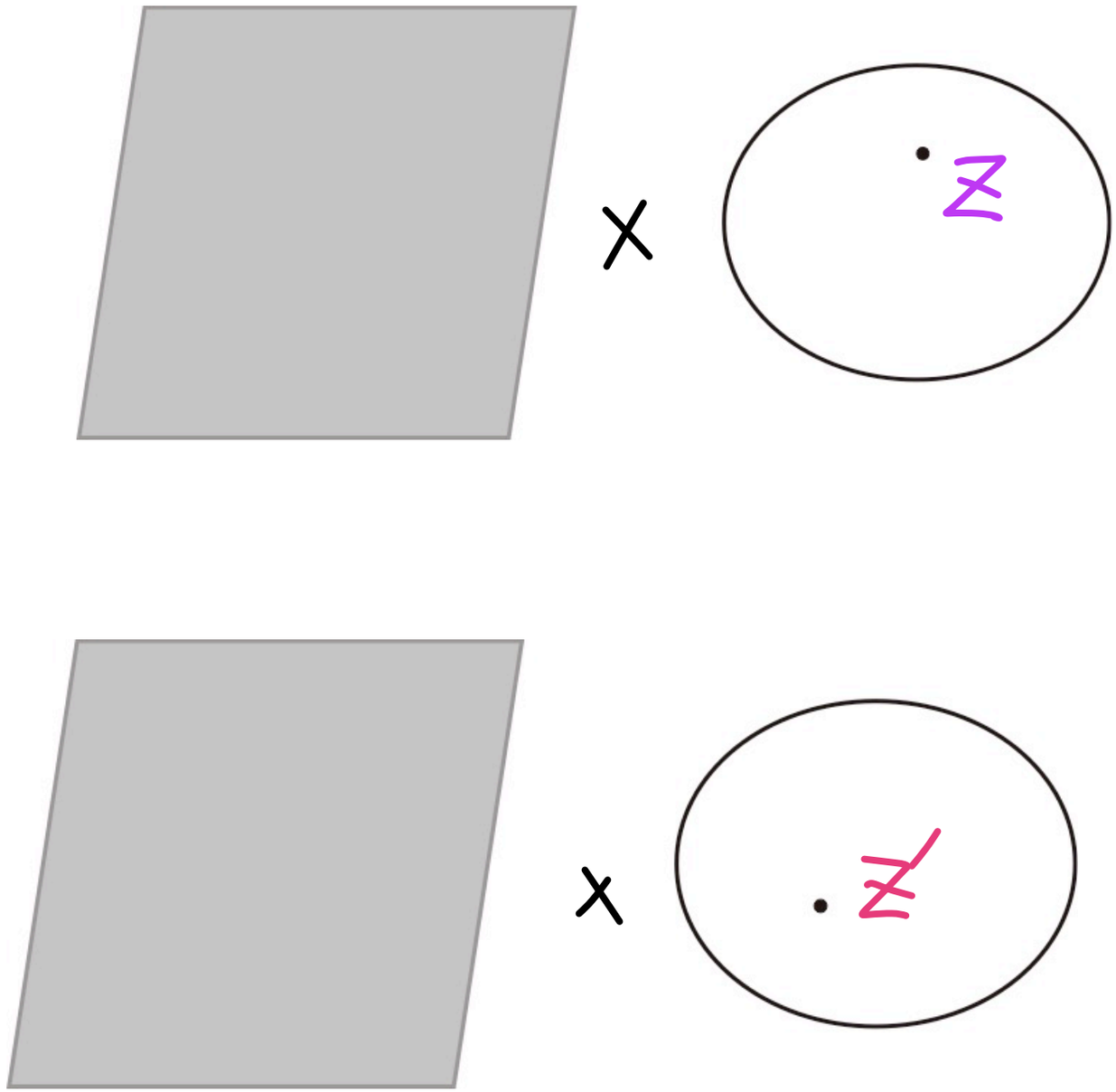
We obtain 2d surface defects
coupled with the bulk 4d theory

Two defects: horizontal and vertical



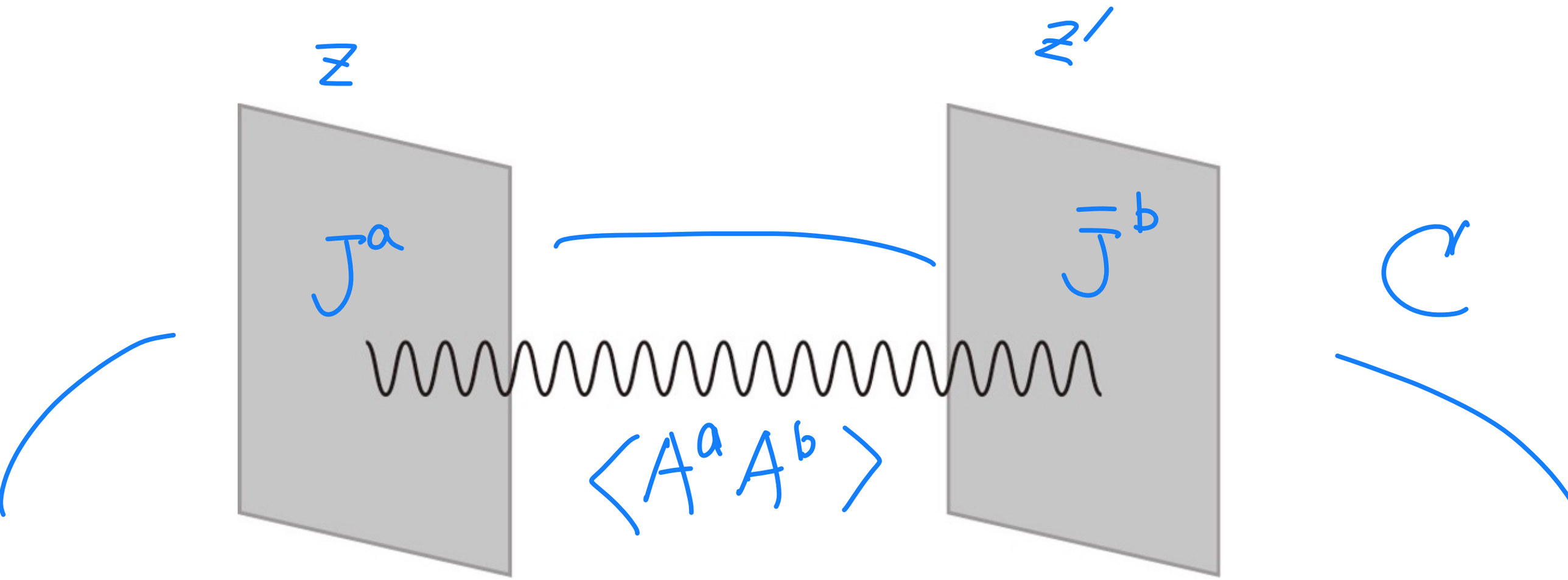


Two defects: horizontal and vertical



The two defects seem to be decoupled, but interact through extra dimensions

[Costello-MY '19]



Simple example: chiral/anti-chiral free fermions

$\psi, \bar{\psi}$

$$\mathcal{L} = \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi}$$

$$+ r_{ab}(z_1 - z_2) \left(\psi t^a \psi(z_1)\right)\left(\bar{\psi} t^b \bar{\psi}(z_2)\right)$$

$\propto (z_1 - z_2)^{-1}$

Green's function

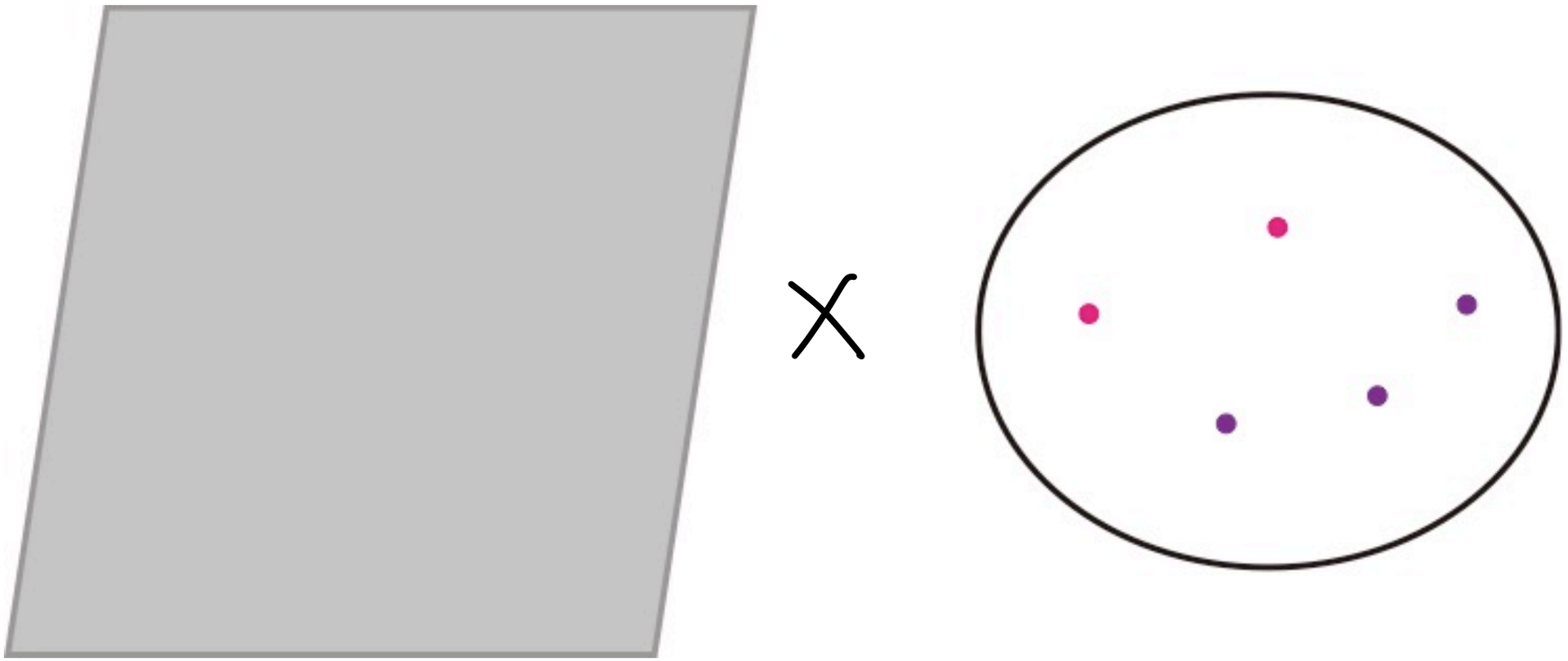
Reproduce Gross-Neveu and Thirring models

ψ $\quad$ ψ

$G = SO(N)$ $\qquad$ $G = SU(N)$

We can systematically construct many new integrable field theories

[Costello-MY ’19]

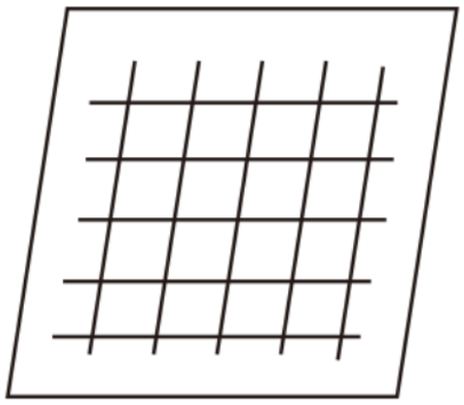


e.g. we can have multiple defects, higher genus spectral curves…

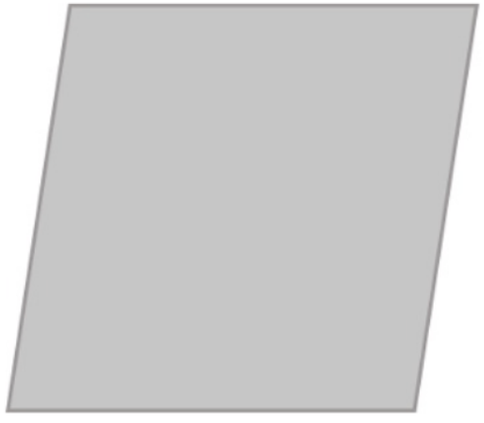
Different discretizations

[Ashwinkuma-Sakamoto-MY (to appear)]

Wilson line R



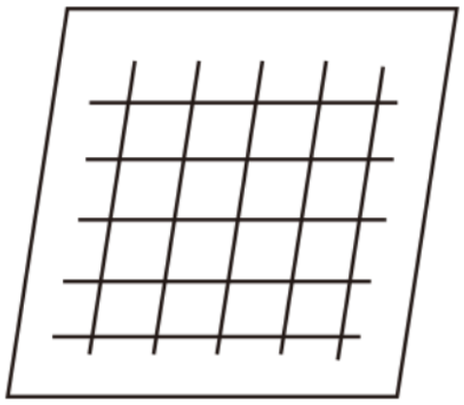
2d QFT



Different discretizations

[Ashwinkuma-Sakamoto-MY (to appear)]

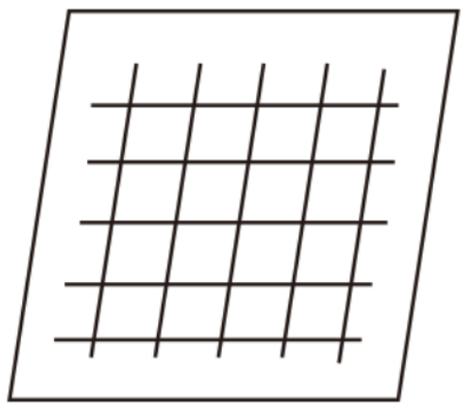
Wilson line R



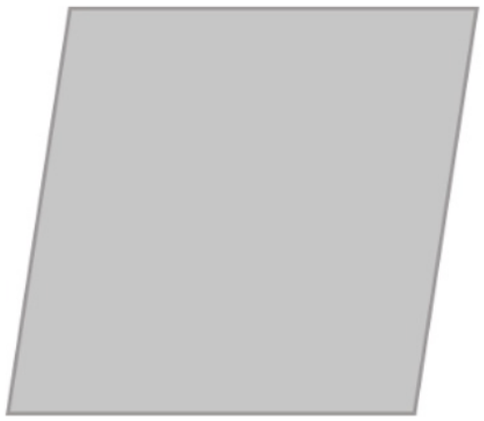
$\Longleftrightarrow$

Borel-Weil-Bott theory

1d QFT $\mathfrak{g}$

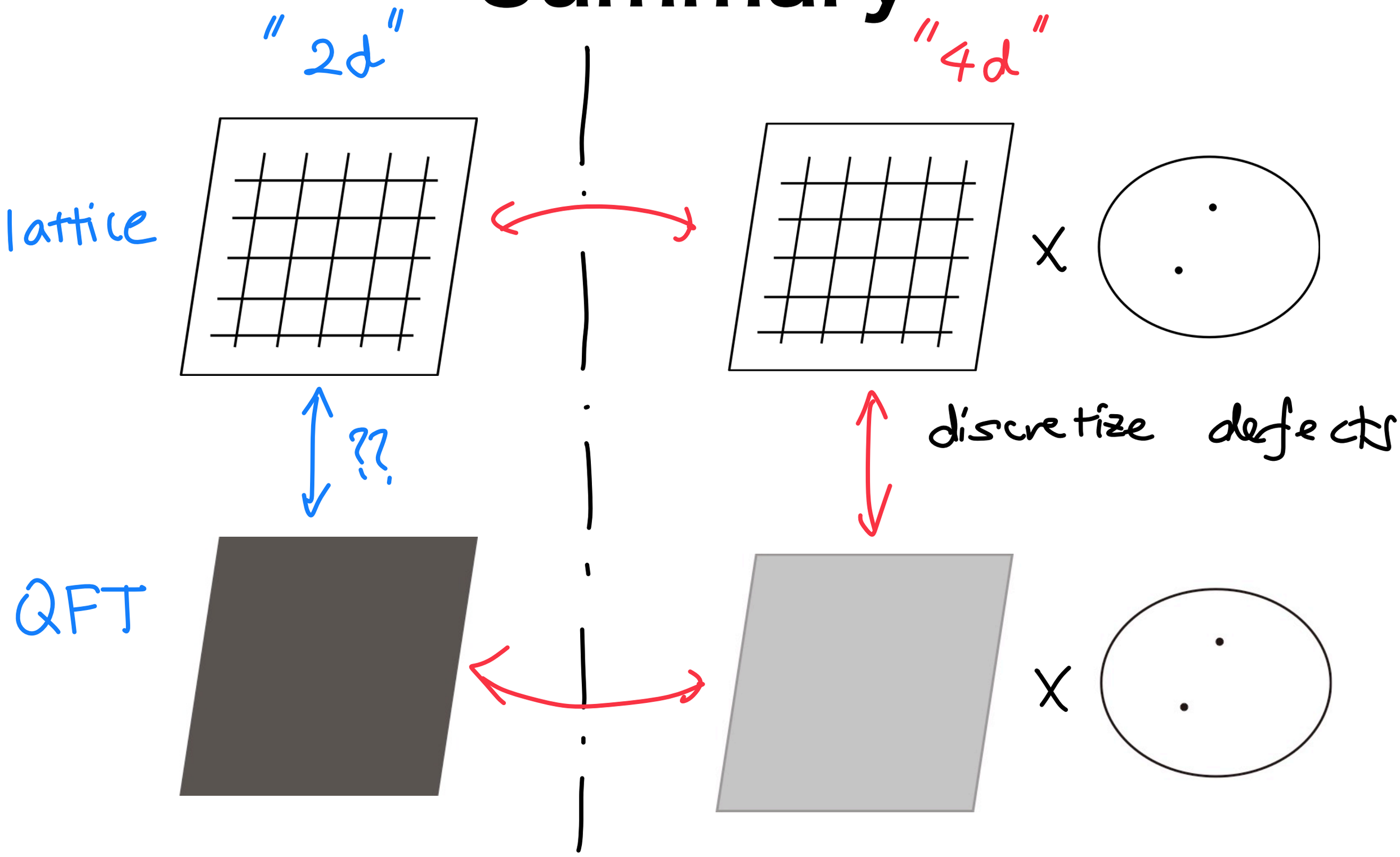


2d QFT



Dualities among defects lead to a web of dualities

Summary



Extra dimensions help! (Einsteinian "Geometrization")

topological/holomorphic phases of matter

Cf. “Field theory T-duality” [Taylor ’96,⋯, MY ’19];
dimensional oxidation vs. dimensional reduction

applications to quantum simulations
“integrable Trotterization” as benchmarks?

[Okuda-Maruyoshi-Suzuki-Yamazaki-Yoshida ’22]

engineering extra dimensions via
“synthesic dimensions”?