4d SU(N) Yang-Mills

vs

2d $\mathbb{CP}^{N-1}$-Model

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KEK

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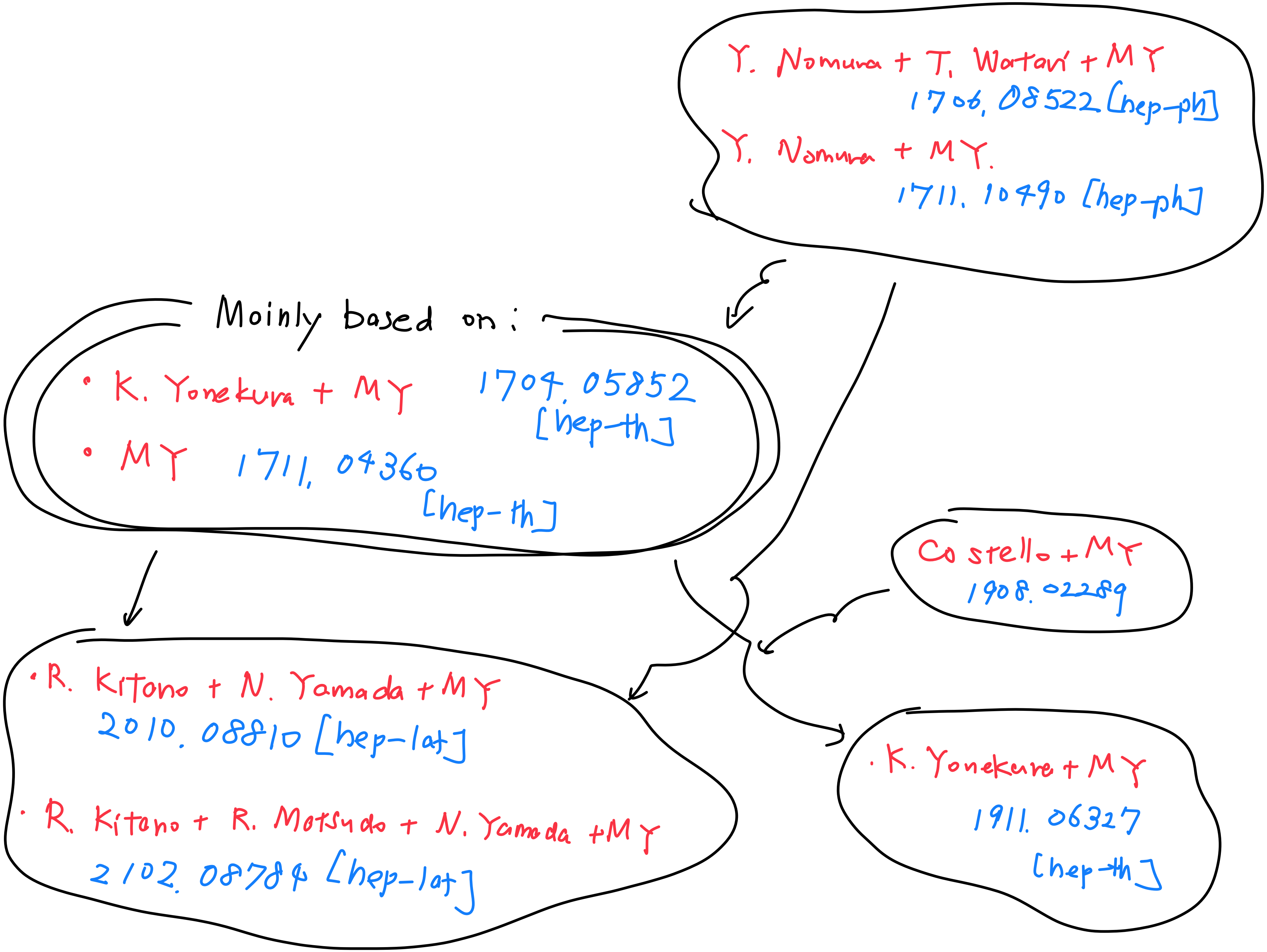
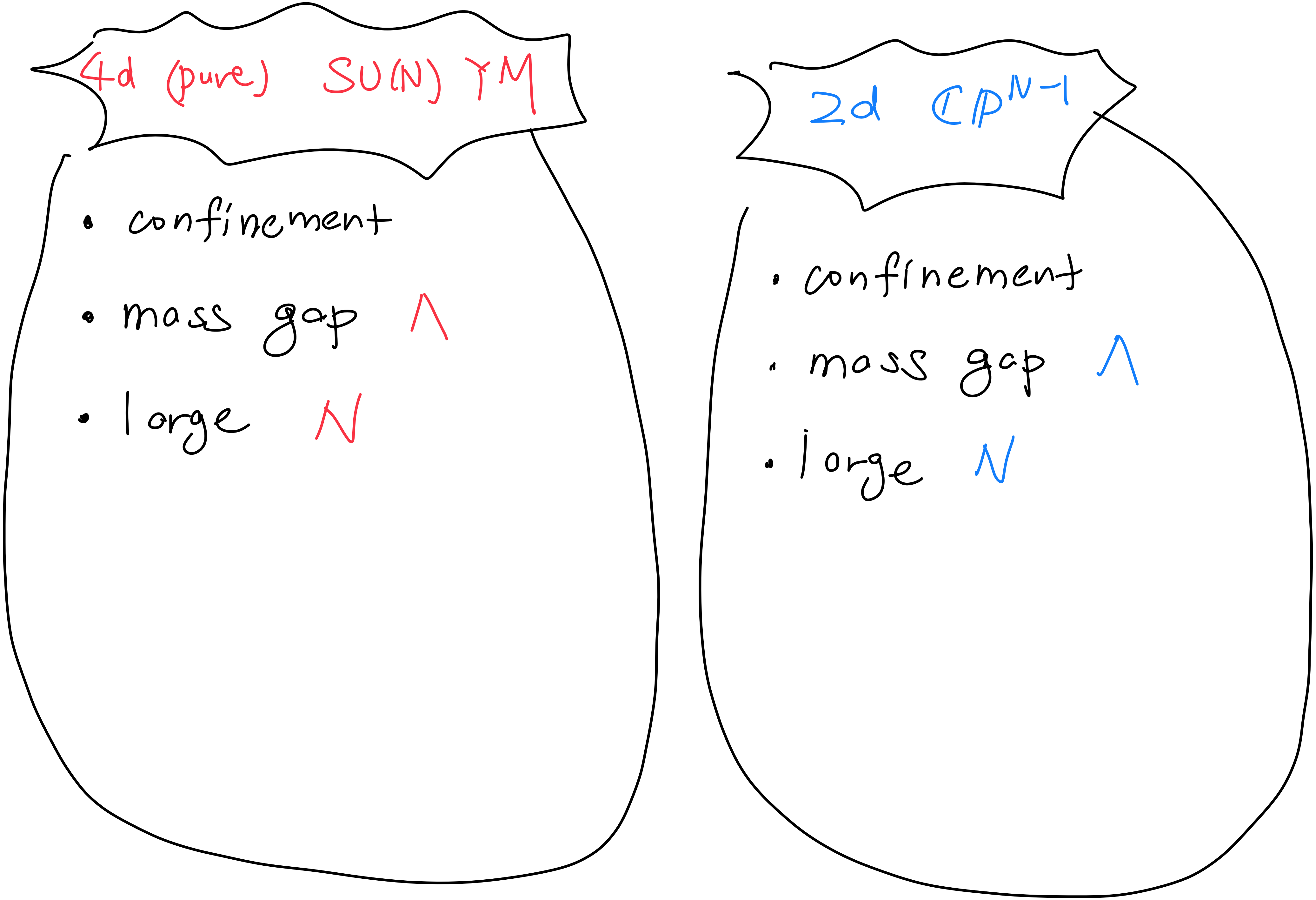

Y. Nomura + T. Watari + MY
1706.08522 [hep-ph]
Y. Nomura + MY.
1711.10490 [hep-ph]
Mainly based on:
• K. Yonekura + MY 1704.05852 [hep-th]
• MY 1711.04360 [hep-th]
Costello + MY
1908.02289
• R. Kitano + N. Yamada + MY
2010.08810 [hep-lat]
• R. Kitano + R. Matsudo + N. Yamada + MY
2102.08784 [hep-lat]
• K. Yonekura + MY
1911.06327 [hep-th]

Similar but different ....



Similar but different ....

4d (pure) SU(N) YM

- confinement
- mass gap Λ
- large N
- $SU(N)$ gauge sym.

  $\cup$

  $\mathbb{Z}_N$ "center sym." one-form sym.

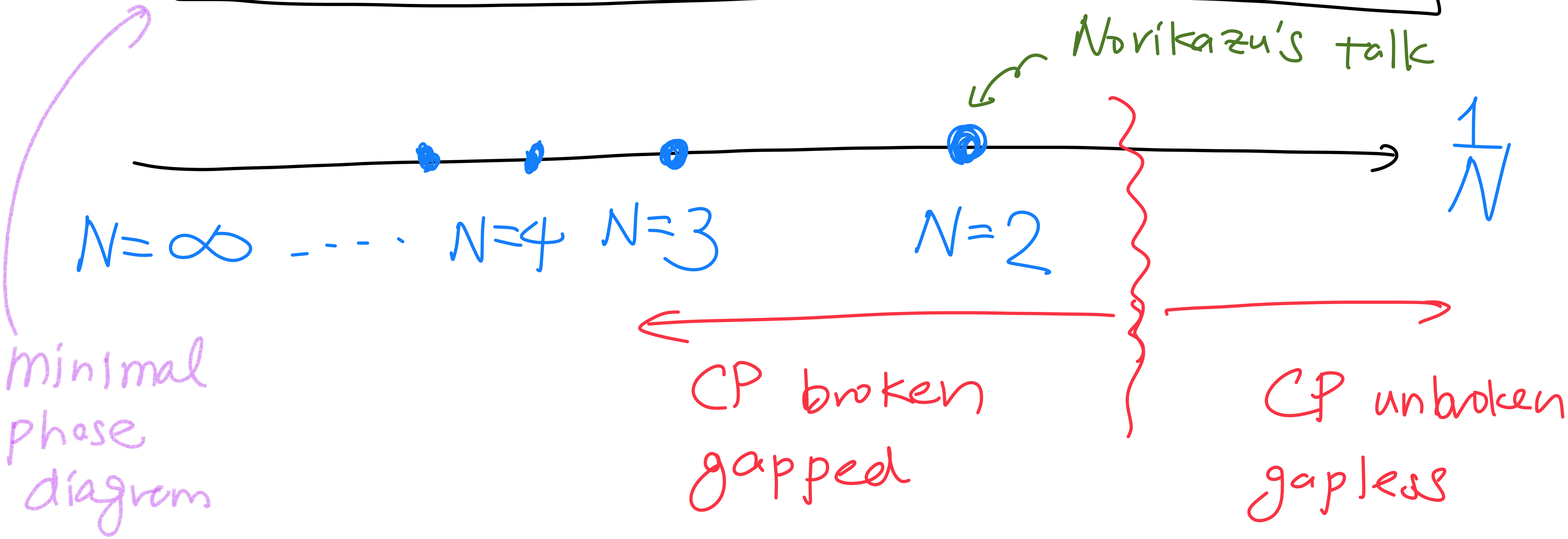
2d $\mathbb{C}P^{N-1}$

- confinement
- mass gap Λ
- large N
- $SU(N)$ flavor sym.

  $\cup$

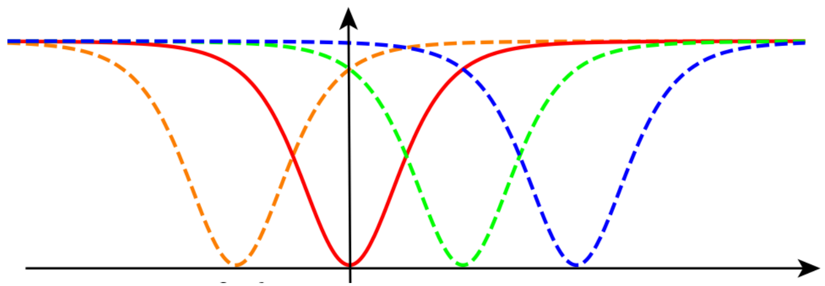
  $\mathbb{Z}_N$ flavor sym.

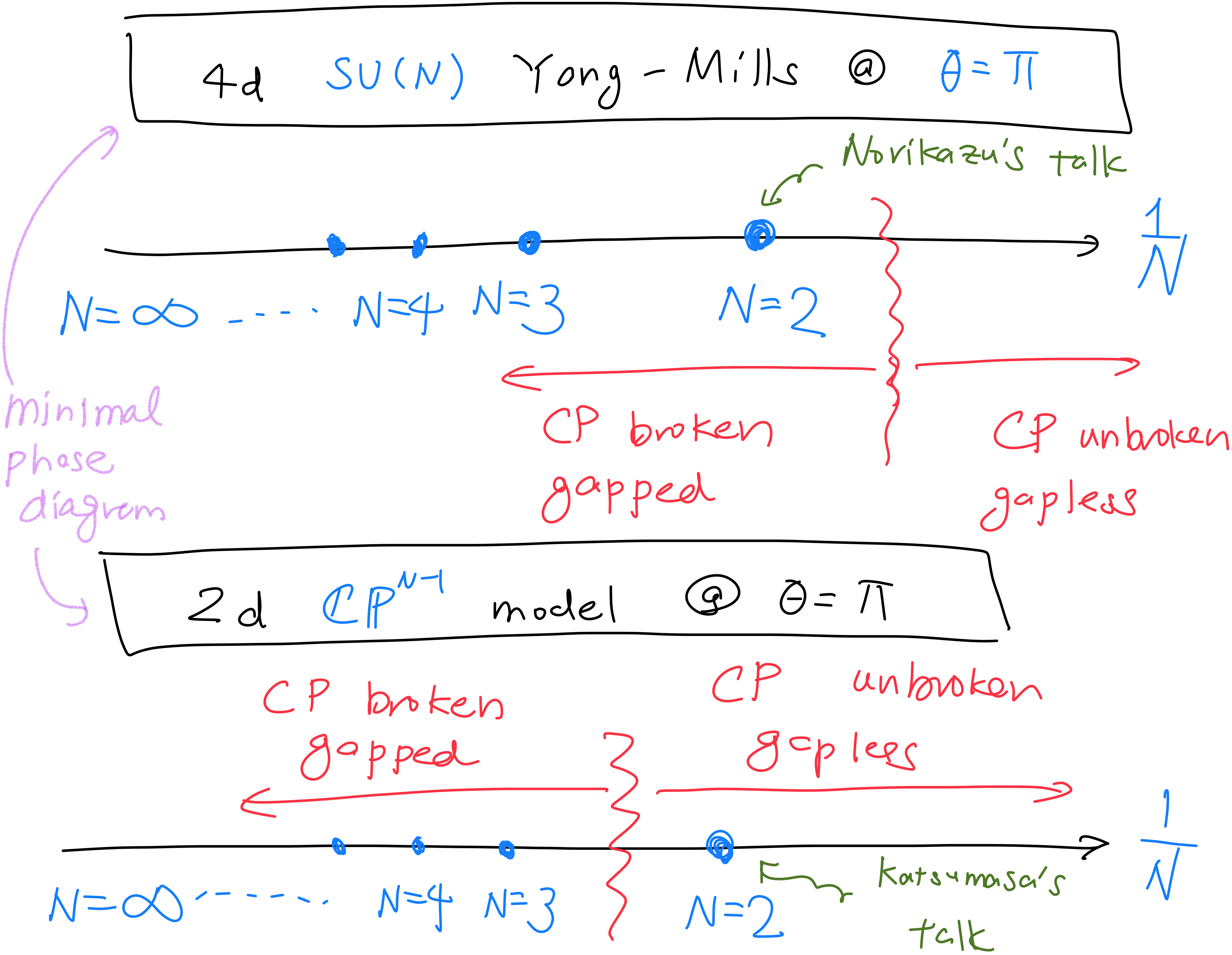
4d SU(N) Yang-Mills @ $\theta = \pi$



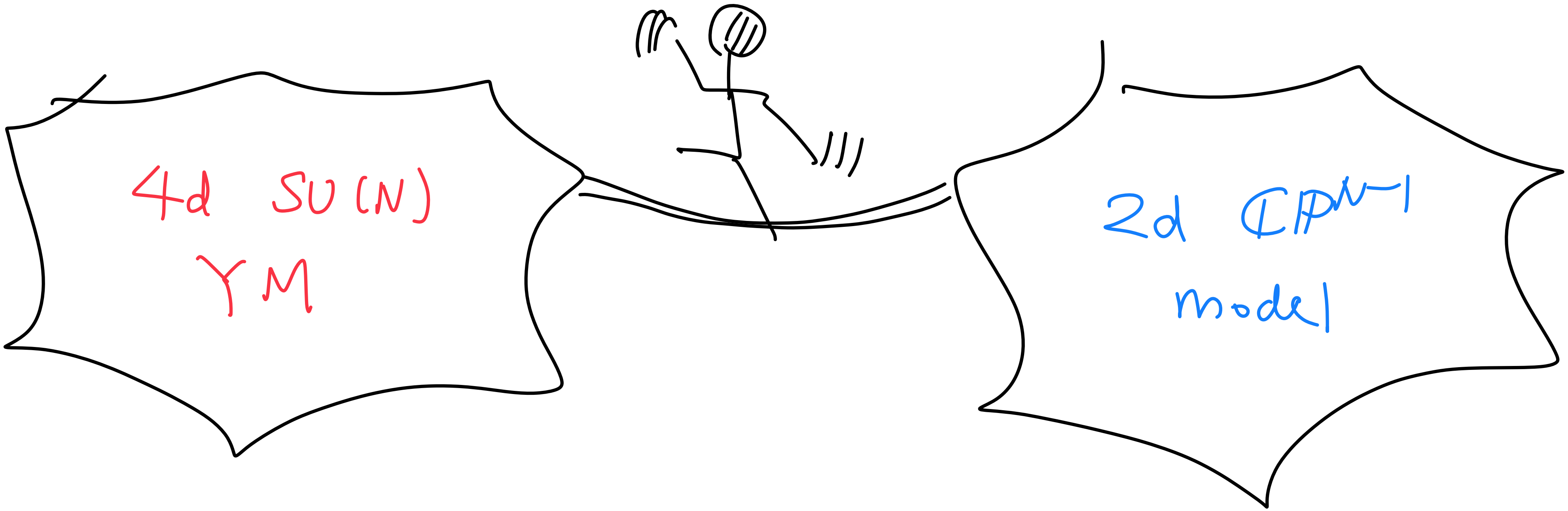
※ ~~CP~~ by multiple branches? [Dashen, Witten, ...]

fractional instanton?




4d SU(N) Yang-Mills @ θ=π
Norikazu's talk
1/N
N=∞ ..... N=4 N=3 N=2
CP broken
gapped
CP unbroken
gapless
minimal
phase
diagram
2d CP^{N-1} model @ θ=π
CP broken
gapped
CP unbroken
gapless
1/N
N=∞ ...... N=4 N=3 N=2
Katsumasa's
talk

Relations subtle?



…. as "anticipated" by

[Yonekura – MY ('17), …]

Basic idea: compactification

4d $SU(N)$ pure YM on $T^2 \times \Sigma$ (T^2 small)

$\downarrow$ $F_{\mu\nu} = 0$ along T^2

$\mathcal{M}_{flat}$: flat connection moduli

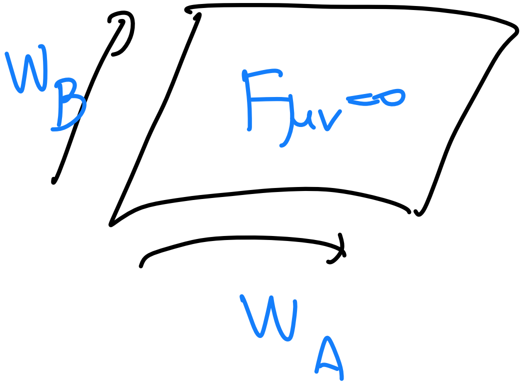
$\cong$ [Looijenga]

$\mathbb{CP}^{N-1}$

$\longrightarrow$ 2d "$\mathbb{CP}^{N-1}$" model on Σ

[cf. Atiyah, Friedman-Morgan-Witten]

For $SU(2)$:

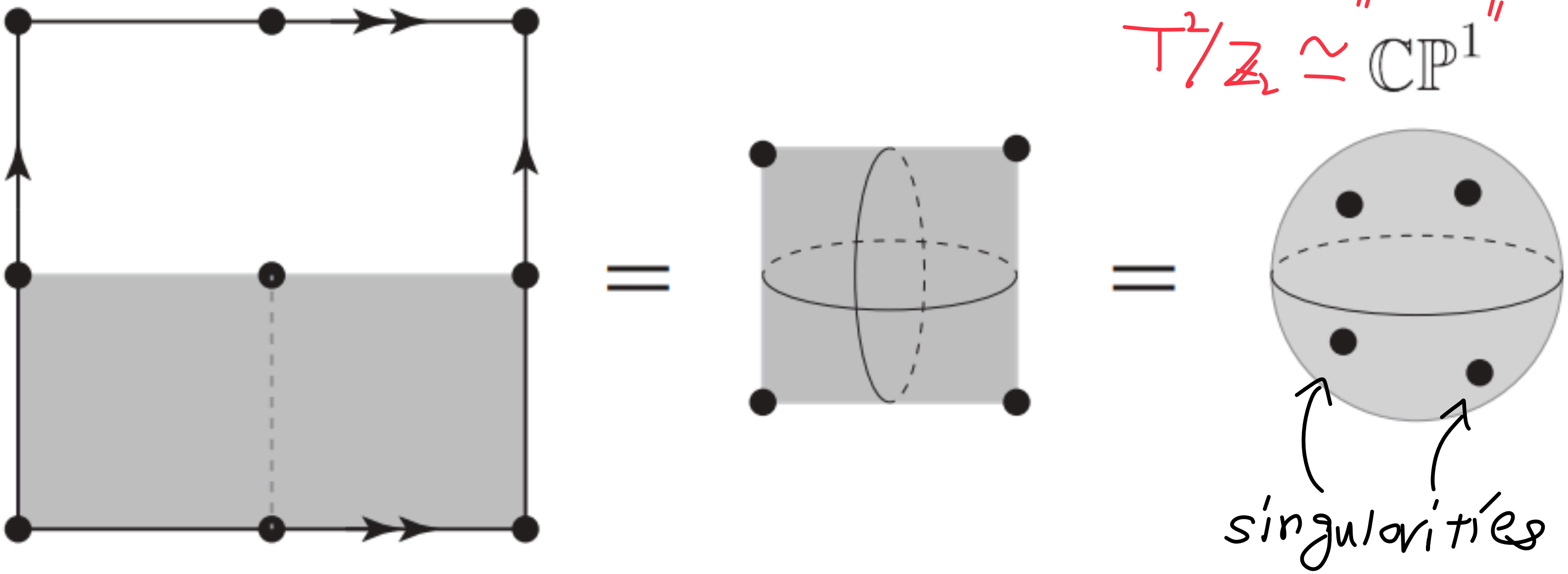


$$W_A = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} \qquad W_B = \begin{pmatrix} e^{i\psi} & 0 \\ 0 & e^{-i\psi} \end{pmatrix}$$

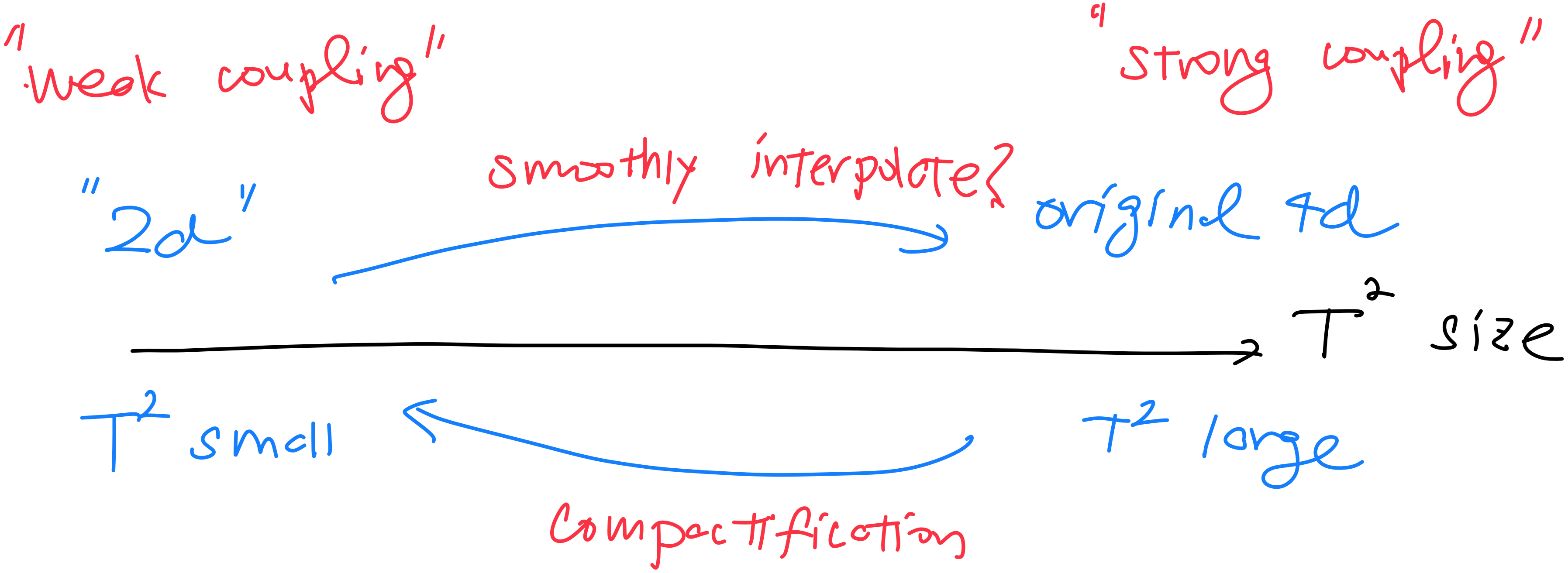
$T^2/\mathbb{Z}_2$: $(\phi, \psi) \simeq (-\phi, -\psi)$ Weyl group

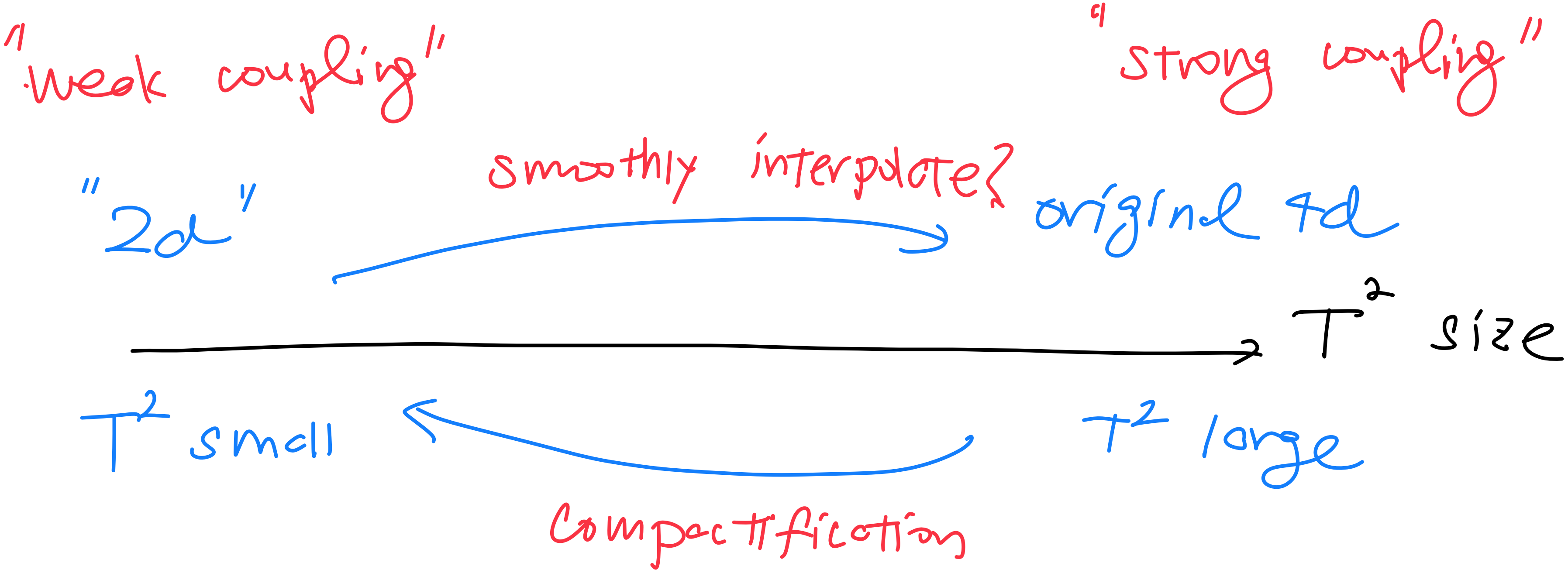
$\mathbb{T}^2/\mathbb{Z}_2$

$T^2/\mathbb{Z}_2 \simeq$ "$\mathbb{CP}^1$"



4d YM		2d $\mathcal{M}_{flat}$ "$\simeq \mathbb{C}P^{N-1}$"
$\mathcal{M}_{flat}$ ψ		$\mathbb{C}P^{N-1}$ ψ
Wilson line	$\longrightarrow$	coordinate / field $[z_1, \ldots z_N]$
↻		↻
$\mathbb{Z}_N$ one-form sym $B_{\mu\nu}$	$\longrightarrow$	$\mathbb{Z}_N$ flavor sym, $A_\mu^{(A/B)}$
instanton Q	$\longrightarrow$	instanton Q


"weak coupling"
"strong coupling"
smoothly interpolate?
"2d"
original 4d
T^2 size
T^2 small
T^2 large
compactification



Issue: — need to be in the same phase
($\mathbb{Z}_N$ sym. preserved)

— eliminate zero mode
(for well-defined perturbation theory (mathematically exact))

In [Yonekuro - Y] more elaborate setup:

$$\boxed{\text{4d YM}} \quad \overbrace{\mathbb{R} \times S^1_C}^{\Sigma} \times \overbrace{S^1_B \times S^1_A}^{T^2}$$

$S^1_C \times S^1_B$: 't Hooft magnetic flux

$S^1_B \times S^1_A$ $\downarrow$ $\mathcal{M}_{flat} \simeq \mathbb{C}P^{N-1}$

In [Yonekura-Y] more elaborate setup:

4d YM

$$\mathbb{R} \times \underbrace{S^1_C \times \overbrace{S^1_B \times S^1_A}^{T^2}}_{\mathcal{M}_{\text{flat}} \simeq \mathbb{CP}^{N-1}}$$

$\Sigma = \mathbb{R} \times S^1_C$; $S^1_C \times S^1_B$: 't Hooft magnetic flux

$\downarrow$ $S^1_C \gg S^1_A, S^1_B$

2d "$\mathbb{CP}^{N-1}$"

$\mathbb{R} \times S^1_C$ [cf. Dunne-Ünsal]

S^1_C: twist by $\mathbb{Z}_N^B$:

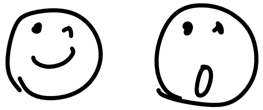
[fractional instantons known
Eto-Isozumi-Nitta-Sakai, Bruckmann...]

In [Yonekuro - Y] more elaborate setup:

strong coupling

4d YM

We can in principle prove confinement



2d "$\mathbb{CP}^{N-1}$"

weak coupling

$$\overbrace{\mathbb{R} \times S^1_C}^{\Sigma} \times \overbrace{S^1_B \times S^1_A}^{T^2}$$

$S^1_C \times S^1_B$: 't Hooft magnetic flux

$S^1_B \times S^1_A$ ↓ $\mathcal{M}_{flat} \simeq \mathbb{CP}^{N-1}$

↓ $S^1_C \gg S^1_A, S^1_B$

$\mathbb{R} \times S^1_C$

S^1_C: twist by $\mathbb{Z}_N^B$:

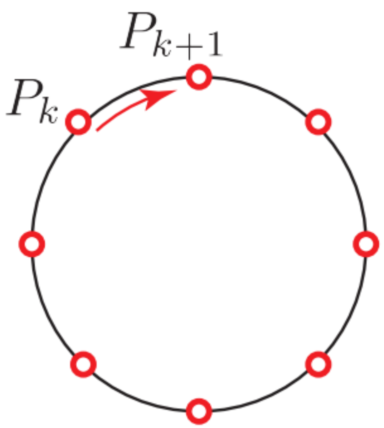
[cf. Dunne - Ünsal]

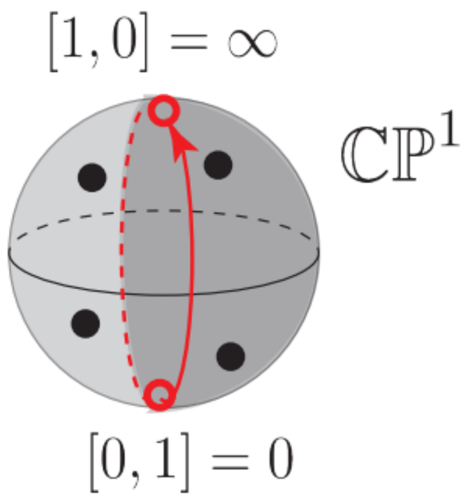
[fractional instantons known Eto - Isozumi - Nitta - Sakai, Brukmann ...]

- In 2d we have N- classical vacua

- $\mathbb{Z}_N^A$ - sym. recovered by quantum tunneling
  (fractional instantons)

$U_A = \omega^k \rightsquigarrow$





- Same in 4d ?

We can compare anomalies @ $\theta = \pi$

(despite $\mathcal{M}_{flat} \neq \mathbb{C}P^{N-1}$)

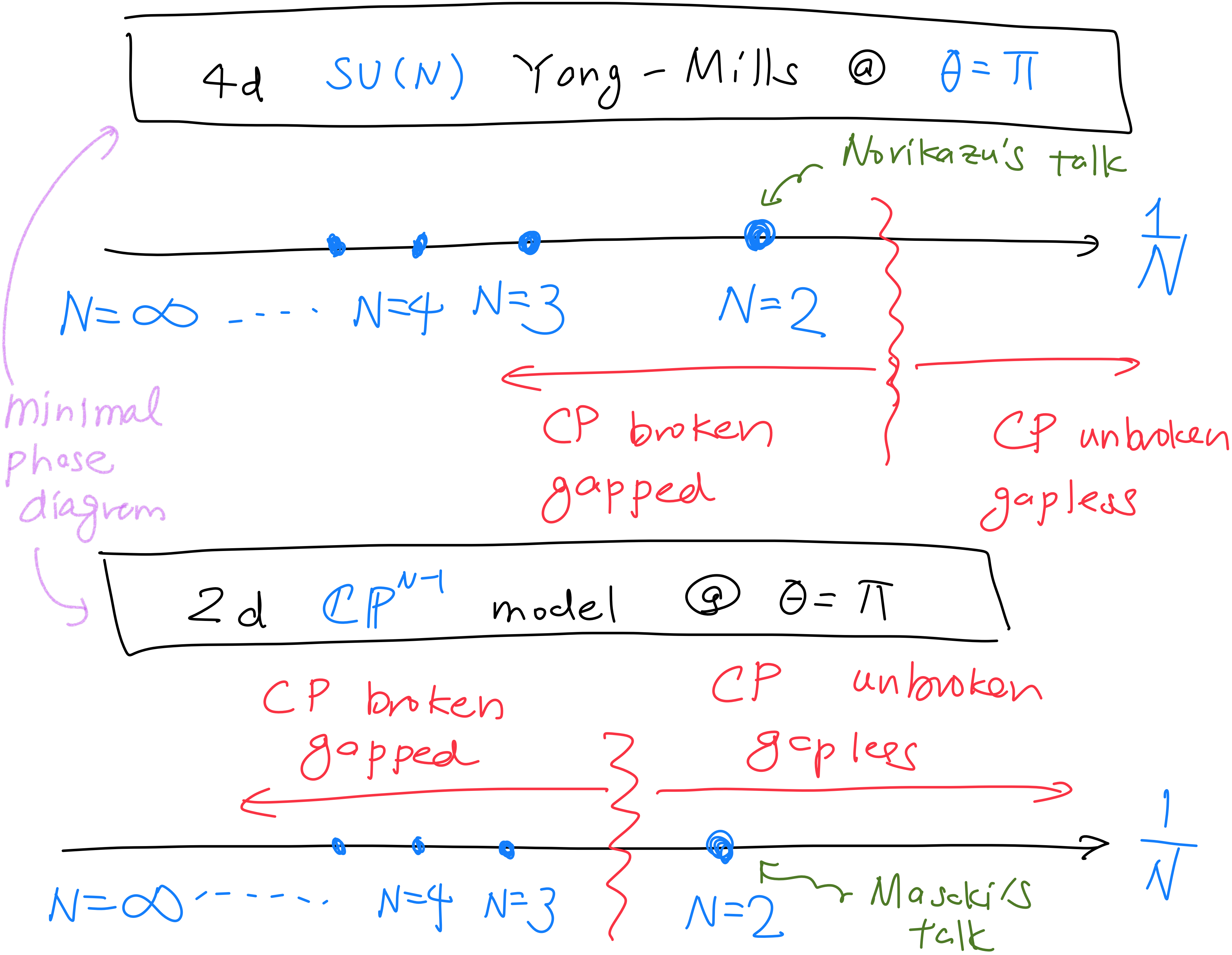
[MY '17, Wan-Wang-Zheng '18]

4d mixed anomaly [Gaiotto-Kapustin-Komargodski-Seiberg '14]

- $\mathbb{Z}_N$ one-form sym. ← confinement
- CP sym. ← multiple branches θ-vacua

2d mixed anomaly [Tanizaki-Misumi-Sakai '17]

- $\mathbb{Z}_N$ zero-form sym.
- CP sym.



4d SU(N) Yang-Mills @ θ = π
Norikazu's talk
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CP broken
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minimal phase diagram
2d CP^{N-1} model @ θ = π
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1/N
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Masaki's talk