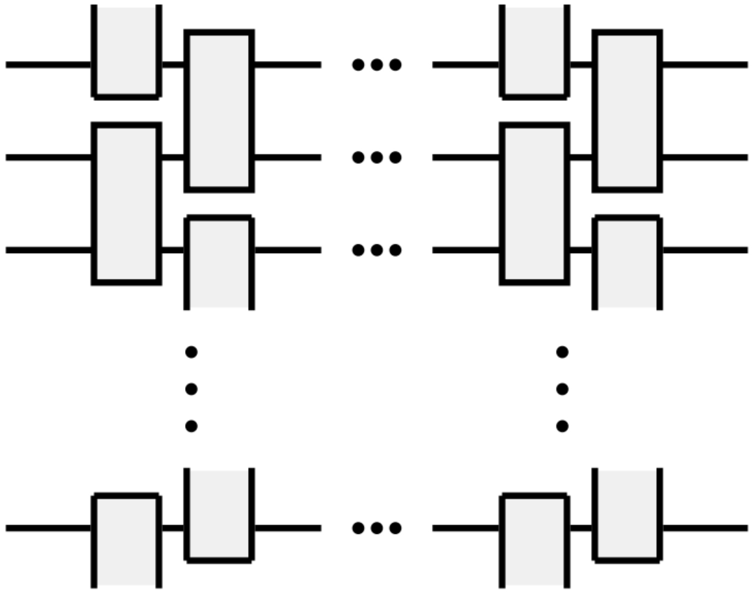
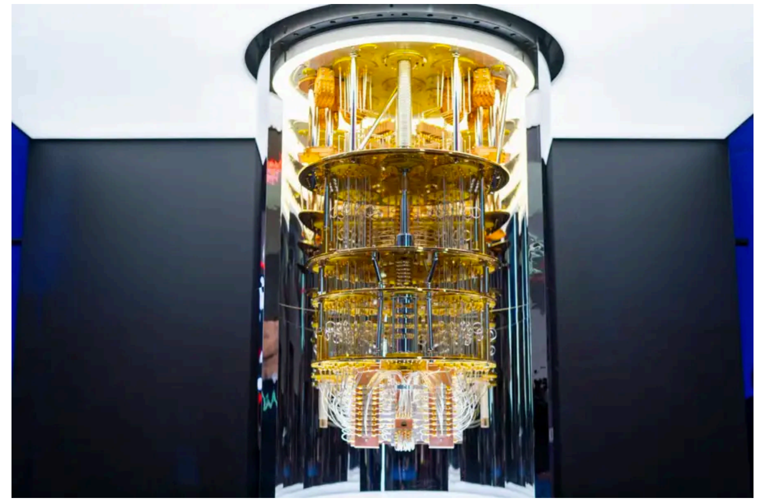
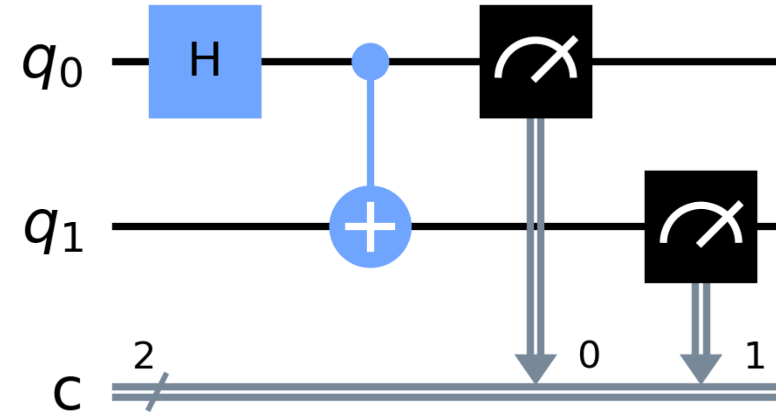






Quantum Simulations of Integrable Models

Masahito Yamazaki

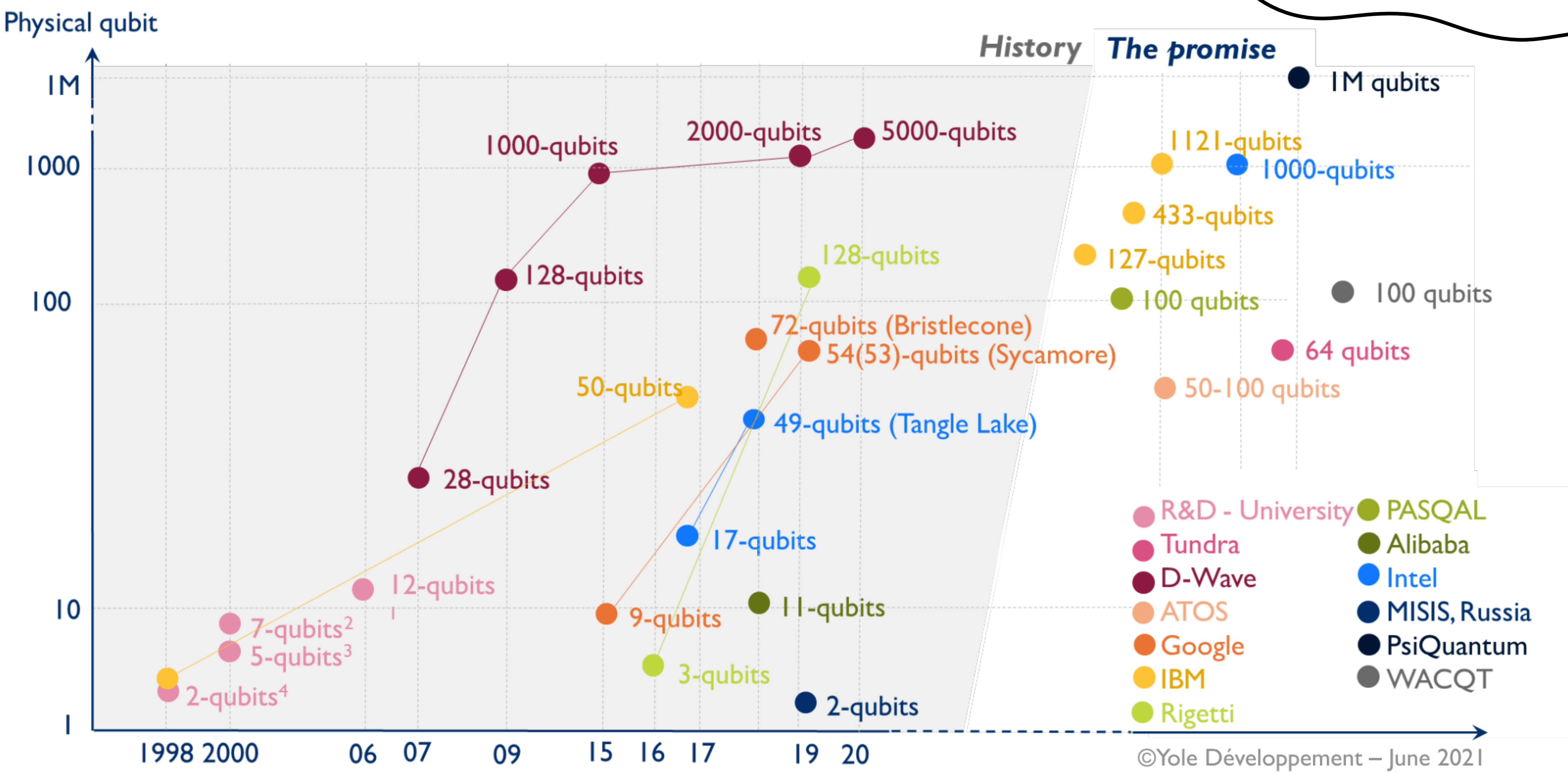
IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Frontiers in Theoretical Physics, Jeju

May 24, 2023

"Quantum Era"?

100,000 ~2033
IBM roadmap



2021, Yale development report

- \# qubits $\gg 30 \sim 40$

(in principle)

quantum advantage

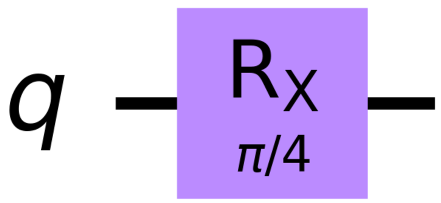
- rapid experimental progress

- anybody has cloud access

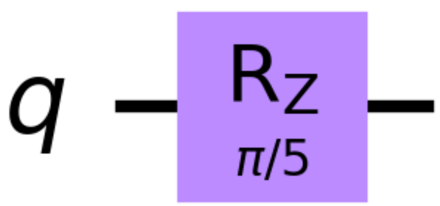
- # qubits $\gg 30 \sim 40$
  (in principle)
  quantum advantage
- rapid experimental progress
- anybody has cloud access

- (Almost) no proofs
- Noisy
  error correction overhead huge
- "Not practically useful"
  ... at least not obviously

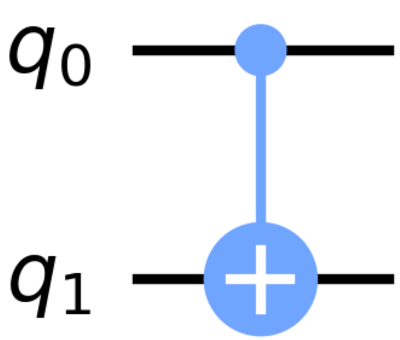
even undergrads can use quantum computers!



$$e^{-i\frac{\theta}{2}X} = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

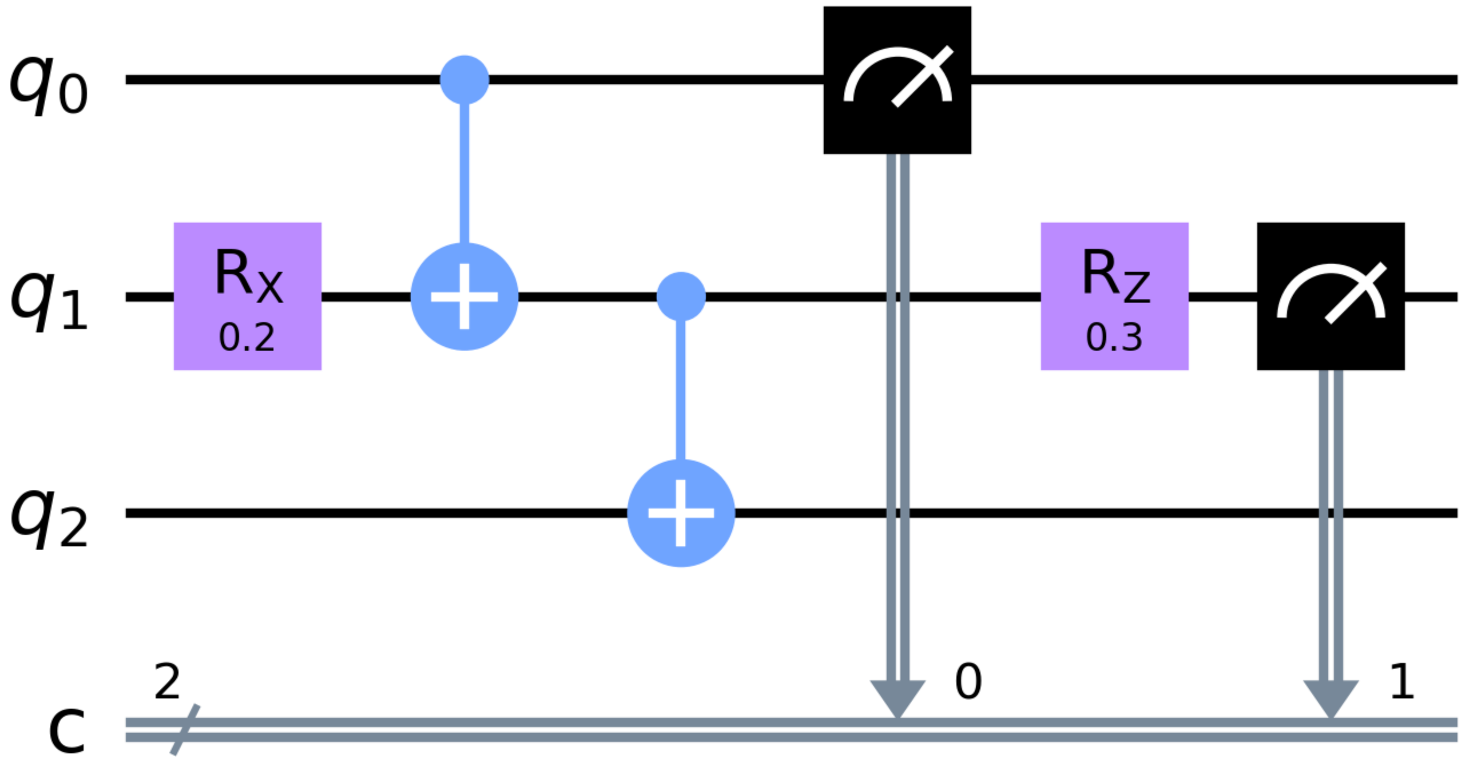


$$e^{-i\frac{\theta}{2}Z} = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$



$$\begin{array}{cc} & \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} \\ \begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} & \begin{pmatrix} 1 & 0 & & \\ 0 & 1 & & 0 \\ & 0 & 0 & 1 \\ & & 1 & 0 \end{pmatrix} \end{array}$$

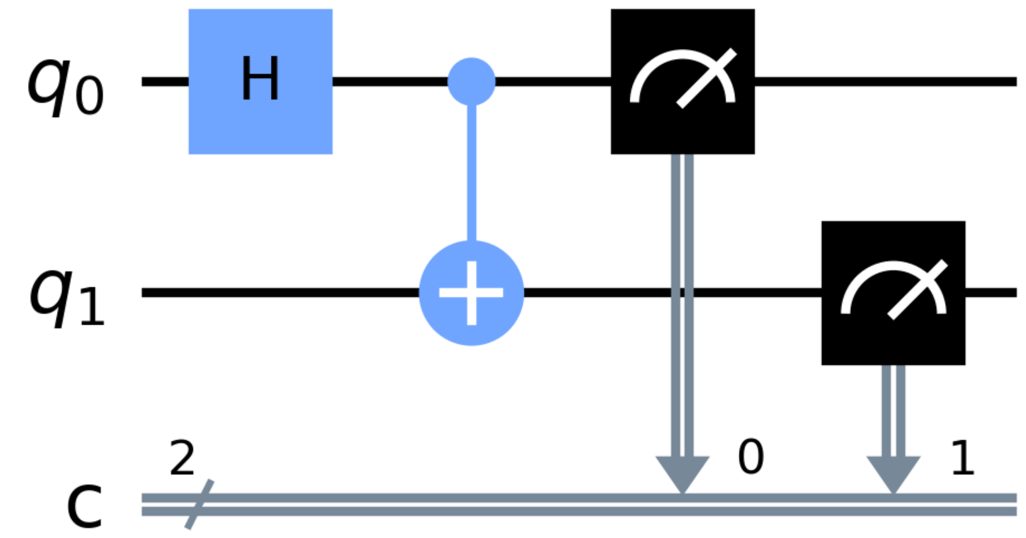
even undergrads can use quantum computers!



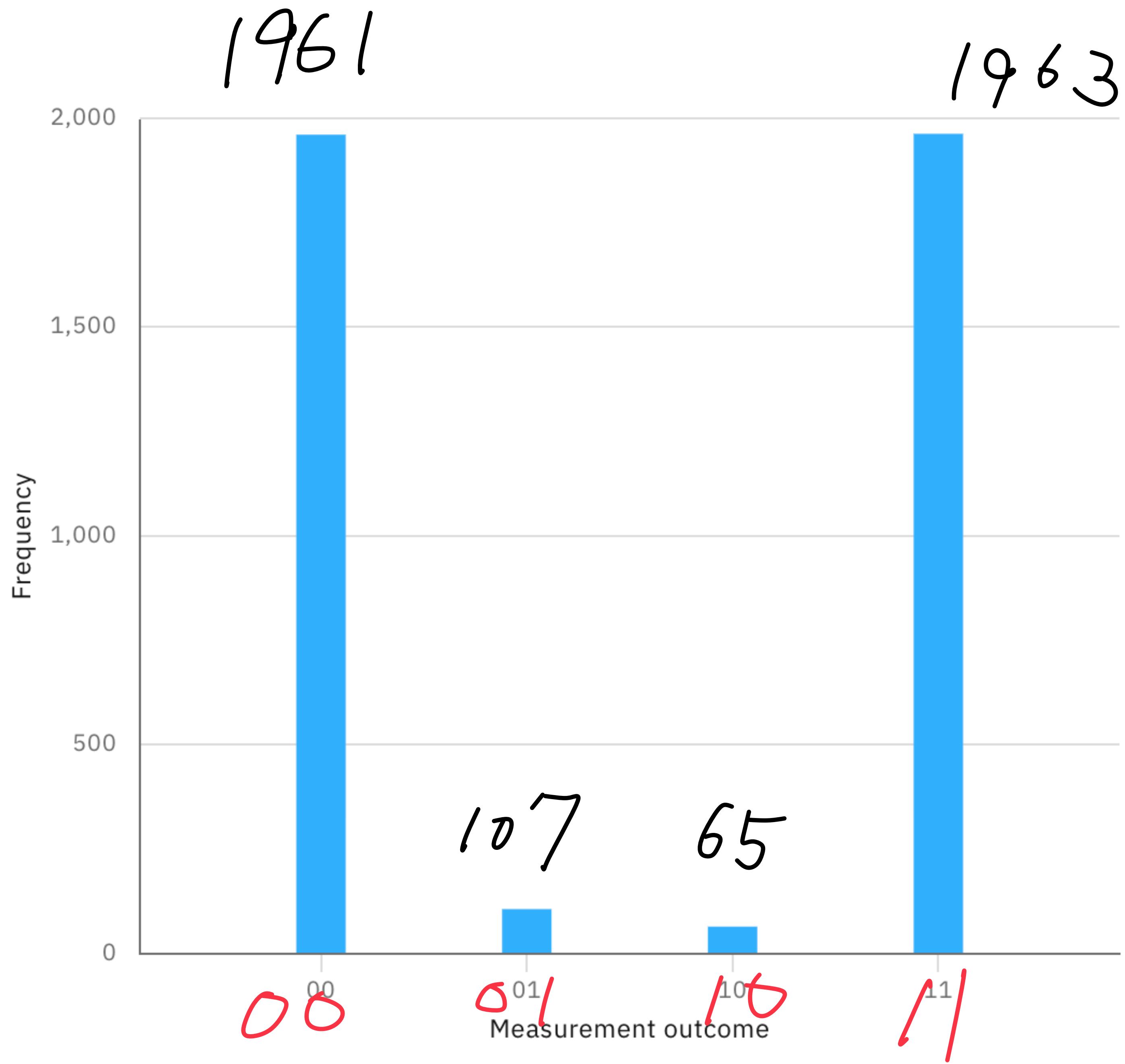
Can realize Any unitary matrix from basic gates

Example: Bell state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big)$$



$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



e.g. analyze noise
open/non-equilibrium system

Integrable Model

Emerging Area

Quantum Computing

e.g. solve Bethe Ansatz equation

Integrable Conserved Charges

on quantum computers?

arXiv > quant-ph > arXiv:2208.00576

Search...

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Quantum Physics

[Submitted on 1 Aug 2022 (v1), last revised 3 Apr 2023 (this version, v2)]

Conserved charges in the quantum simulation of integrable spin chains

Kazunobu Maruyoshi, Takuya Okuda, Juan William Pedersen, Ryo Suzuki, Masahito Yamazaki, Yutaka Yoshida

When simulating the time evolution of quantum many-body systems on a digital quantum computer, one faces the challenges of quantum noise and of the Trotter error due to time discretization. The Trotter error in integrable spin chains can be under control if the discrete time evolution preserves integrability. In this work we implement, on a real quantum computer and on classical simulators, the integrable Trotterization of the spin-1/2 Heisenberg XXX spin chain. We study how quantum noise affects the time evolution of several conserved charges, and observe the decay of the expectation values. We in addition study the early time behaviors of the time evolution, which can potentially be used to benchmark quantum devices and algorithms in the future. We also provide an efficient method to generate the conserved charges at higher orders.

Based on

arXiv:2208.00576 [quant-ph]

e.g. XXX spin chain

$$H = \sum_{j=1}^{N} \sigma_i \sigma_{i+1} = \sum_{j=1}^{N} X_i X_{i+1} + Y_i Y_{i+1} + Z_i Z_{i+1}$$

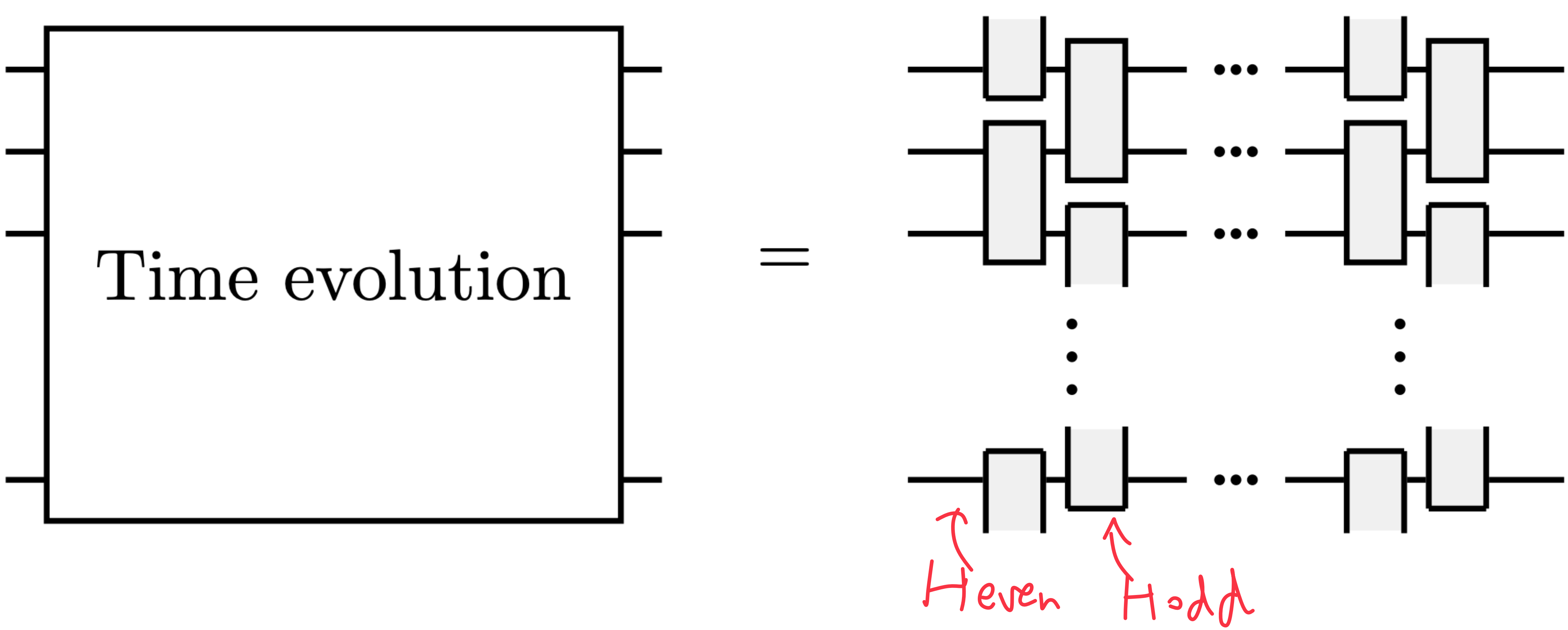
$$= \sum_{j=1}^{N} \left(2 P_{i,i+1} - I_i I_{i+1} \right)$$

Permutation

$$= \underbrace{\sum_{j:\text{even}} \left(2 P_{i,i+1} - I_{i,i+1} \right)}_{H_{\text{even}}} + \underbrace{\sum_{j:\text{odd}} \left(2 P_{i,i+1} - I_{i,i+1} \right)}_{H_{\text{odd}}}$$

"standard" technique : Trotterization

$$\underbrace{e^{iHt}}_{\text{time evolution}} \underset{N\to\infty}{\sim} \left(e^{i\frac{t}{N}H_{even}} \, e^{i\frac{t}{N}H_{odd}} \right)^N \left\{ 1 + \underbrace{\mathcal{O}\left(\left(\tfrac{t}{N}\right)^2\right)}_{\text{breaks integrability}} \right\}$$



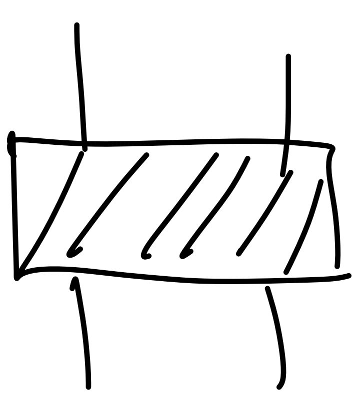
Integrable Trotterization:

(Destri-de Vega, ...
Vanicat-Zadnik-Prosen
cf. Ashwinkumar-Sakamoto-MY)

Building Block: R-matrix

$$\check{R}_{ij}(\lambda) = \frac{1+i\lambda P_{ij}}{1+i\lambda}$$

Permutation



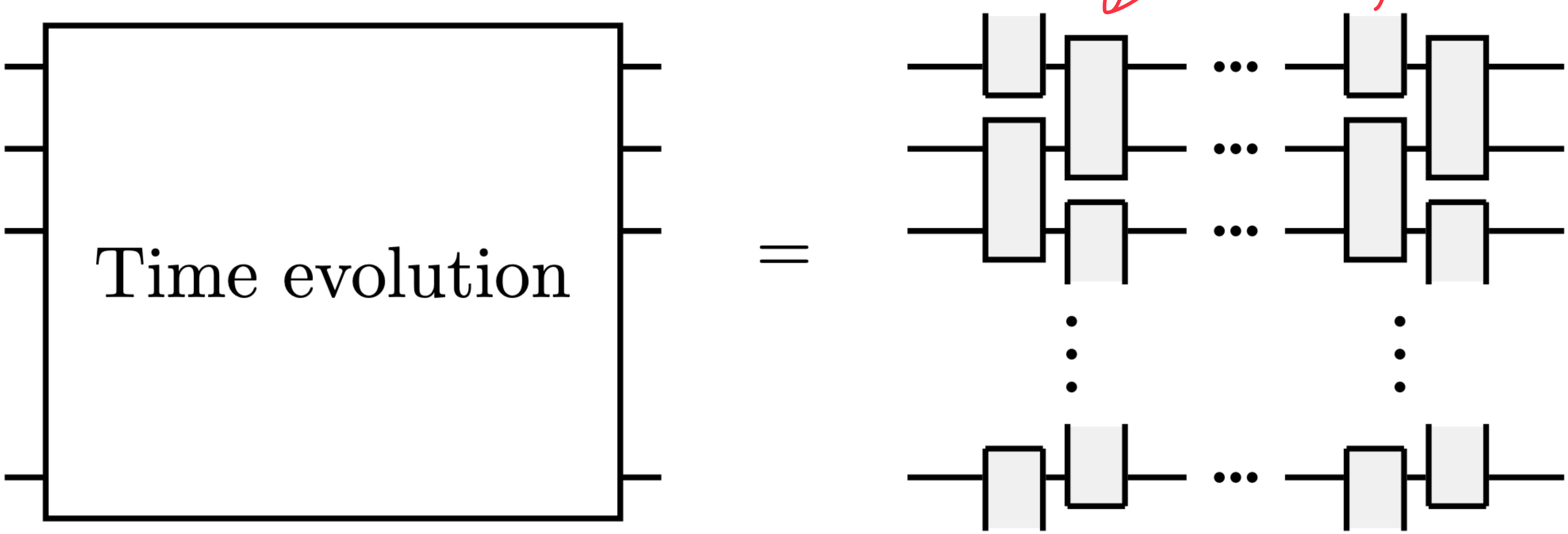
$\check{R}$

$$= \frac{1}{1+i\lambda} \begin{array}{c} \\ |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{array}{c} \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} \\ \begin{pmatrix} 1 & & & \\ & 1 & i\lambda & \\ & i\lambda & 1 & \\ & & & 1 \end{pmatrix} \end{array}$$

Integrable Trotterization:

$$\mathcal{U}(\delta) = \left(\prod_{j=1}^{N/2} \check{R}_{2j-1,2j}(\delta)\right)\left(\prod_{j=1}^{N/2} \check{R}_{2j,2j+1}(\delta)\right)$$

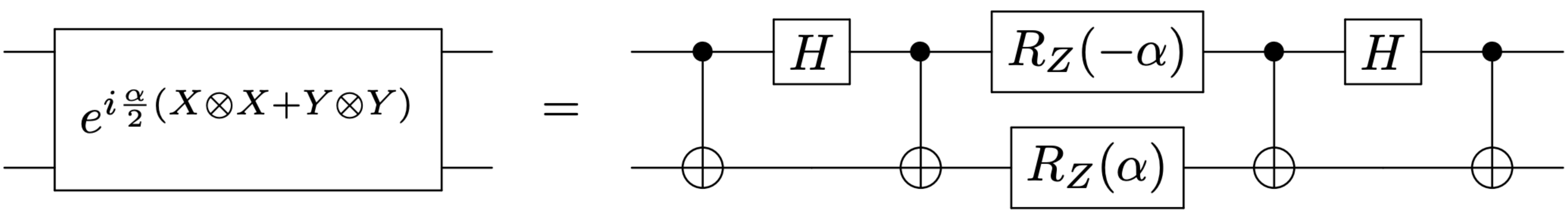
R-matrix

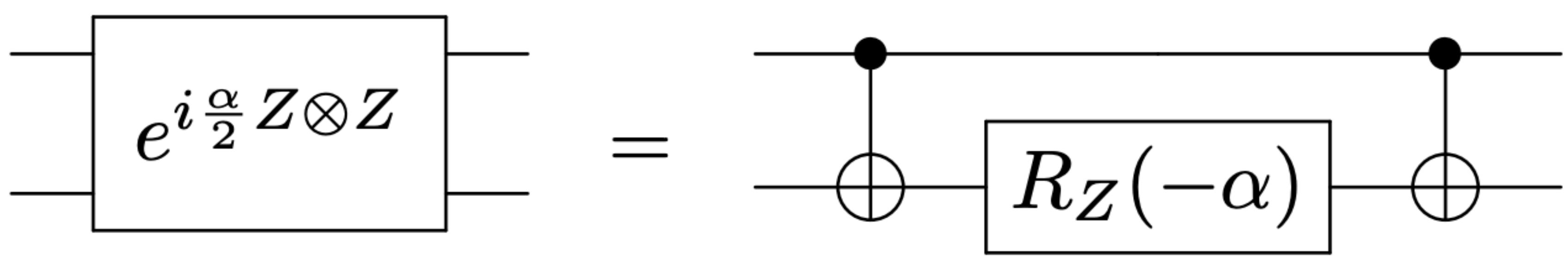


"time evolution from R-matrix"

$$\check{R}_{j,j+1}(\lambda) = e^{-i\frac{\alpha}{2}}\, e^{i\frac{\alpha}{2}(X_j X_{j+1} + Y_j Y_{j+1})}\, e^{i\frac{\alpha}{2} Z_j Z_{j+1}}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



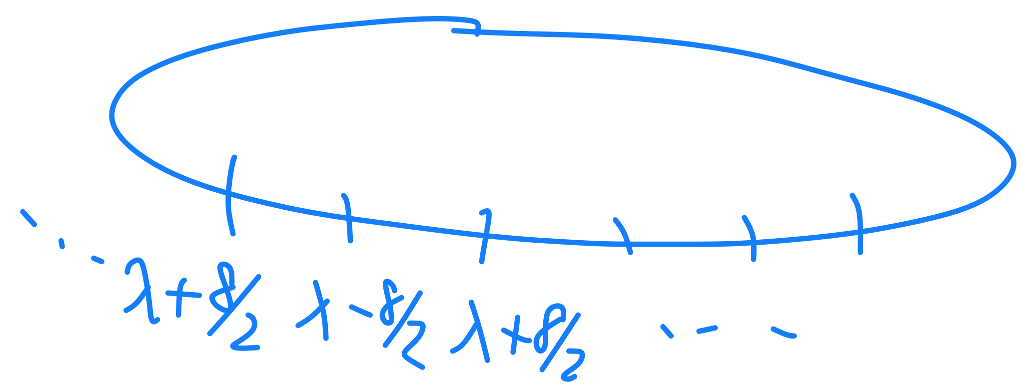


- $\mathcal{U}(\delta) = T\left(-\frac{\delta}{2}\right)^{-1} T(\delta/2)$

$$\underbrace{T(\lambda)}_{\text{transfer matrix}} := \mathrm{tr}_0 \left(\prod_{1 \le j \le N} R_{0j}\Big(\lambda - \underbrace{(-1)^j \frac{\delta}{2}}_{\text{inhomogenity}}\Big) \right)$$

$XXX \to XXX_\delta$

$$\big([T(\lambda), T(\mu)] = 0 \big)$$



- Conserved charges

$$Q_n^{\pm} = \frac{d^n}{d\lambda^n} \log T(\lambda) \Big|_{\lambda = \pm\delta/2}$$

recursion by boost operator B

$$Q^{\pm}_{n+1} = [B, Q^{\pm}_{n}]$$

determined from R-matrix

$$Q_n^+(\delta) = \sum_{j=1}^{N/2} q^{[n,+]}_{2j-2,2j-1,\ldots,2j+2n-2}(\delta),$$

$$Q_n^-(\delta) = \sum_{j=1}^{N/2} q^{[n,-]}_{2j-1,2j,\ldots,2j+2n-1}(\delta),$$

$$q^{[1,\pm]}_{1,2,3}(\delta) = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3 \mp \delta \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3) + \delta^2 \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3 \,,$$

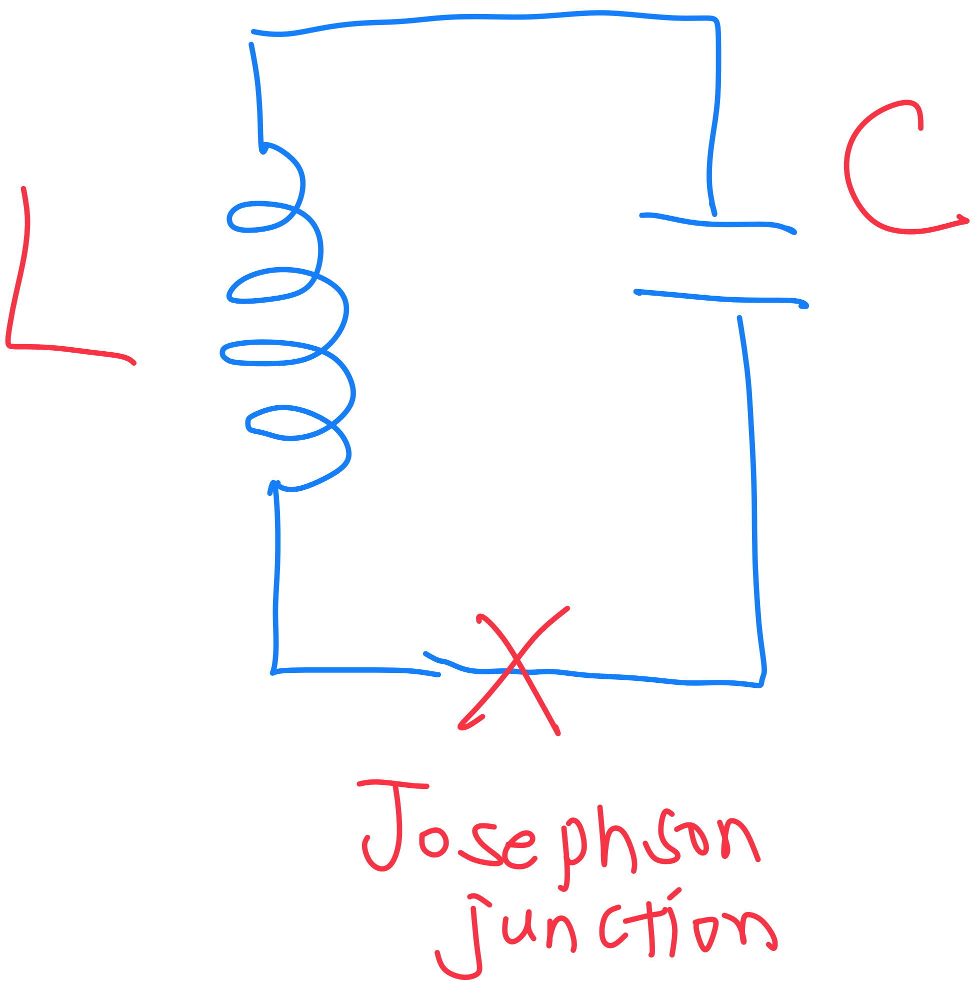
$$\begin{aligned} q^{[2,\pm]}_{1,2,3,4,5}(\delta) = &\mp 2\delta(\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_4 + \boldsymbol{\sigma}_4 \cdot \boldsymbol{\sigma}_5 - \boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_5) - (1-\delta^2)\boldsymbol{\sigma}_3 \cdot (\boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5) - \boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4) - \delta^2 \boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5) \\ &- \delta^2 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4) - \delta^4 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5) \pm \delta \boldsymbol{\sigma}_2 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5) \pm \delta \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4) \\ &\pm \delta^3 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5) \pm \delta^3 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_5) - \delta^2 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3 \times \boldsymbol{\sigma}_4 \times \boldsymbol{\sigma}_5). \end{aligned}$$

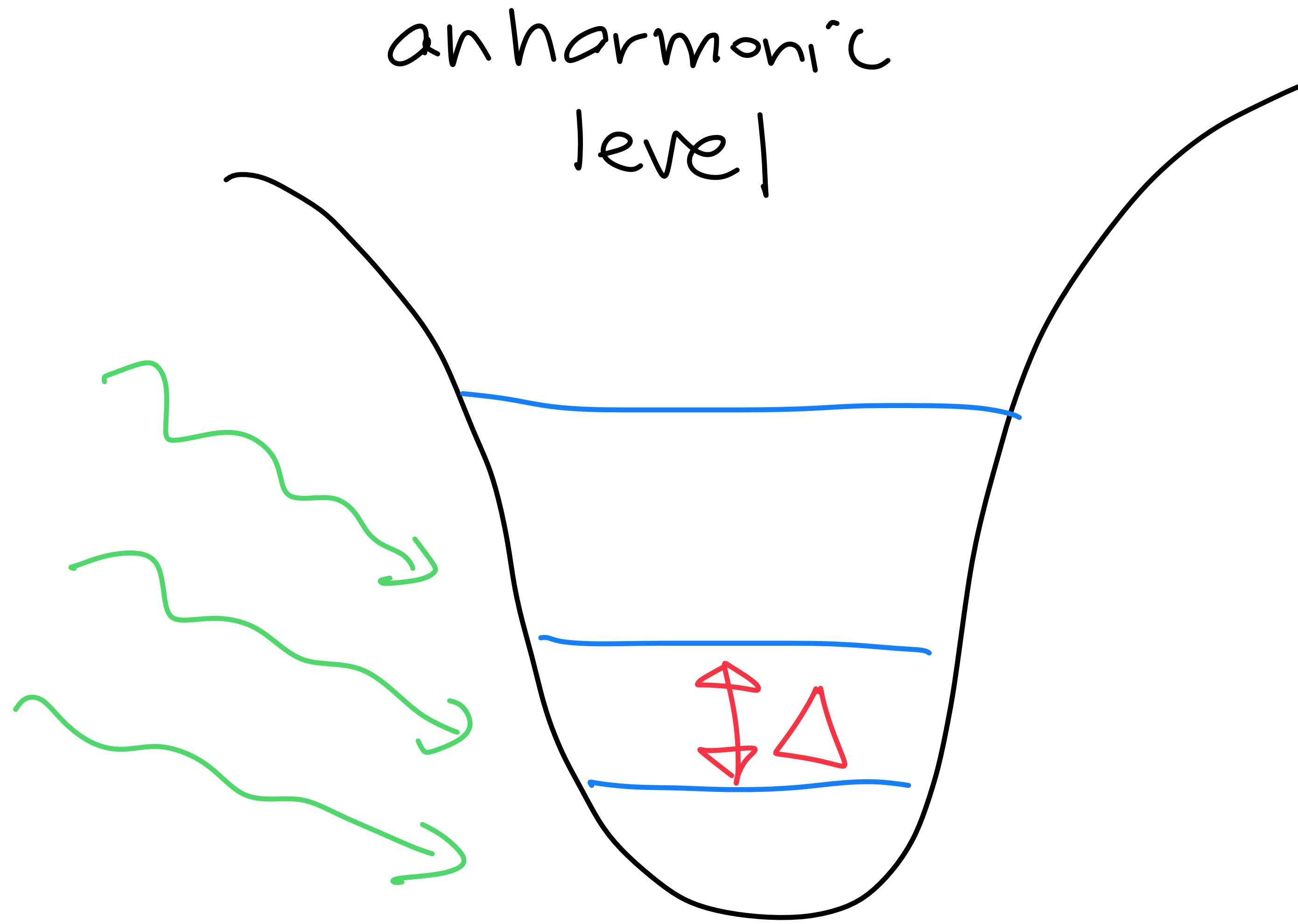
$$
\begin{aligned}
q^{[3,+]}_{1,2,3,4,5,6,7} = & -4\boldsymbol{\sigma}_6\cdot\boldsymbol{\sigma}_7+2\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_7-4\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_6+2\boldsymbol{\sigma}_4\cdot\boldsymbol{\sigma}_6+2\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)+2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\\
& +\delta\Big(10\boldsymbol{\sigma}_5\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)-4\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)+8\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)-4\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\\
& -2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_6)-4\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\Big)\\
& +\delta^2\Big(2\boldsymbol{\sigma}_6\cdot\boldsymbol{\sigma}_7-10\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_7+2\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_6+2\boldsymbol{\sigma}_4\cdot\boldsymbol{\sigma}_7+2\boldsymbol{\sigma}_4\cdot\boldsymbol{\sigma}_6+2\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_6\\
& -6\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)+6\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)+2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)+6\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)\\
& -6\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)+2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\\
& +2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)+2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\Big)\\
& +\delta^3\Big(6\boldsymbol{\sigma}_5\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)+4\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)-8\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)\\
& -2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_6)+4\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)-2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_7)+4\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)\\
& -2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\\
& -2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)-2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)\Big)\\
& +\delta^4\Big(-2\boldsymbol{\sigma}_6\cdot\boldsymbol{\sigma}_7-8\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_7-2\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_6+2\boldsymbol{\sigma}_4\cdot\boldsymbol{\sigma}_7+2\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_6+2\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_7-2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)\\
& +2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)+2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)+2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\\
& +2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)+2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)+2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)\Big)\\
& +\delta^5\Big(4\boldsymbol{\sigma}_5\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)\\
& -2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)-2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)\Big)\\
& +\delta^6\Big(-4\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_7+2\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_7+2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)\Big).
\end{aligned}
$$

Results

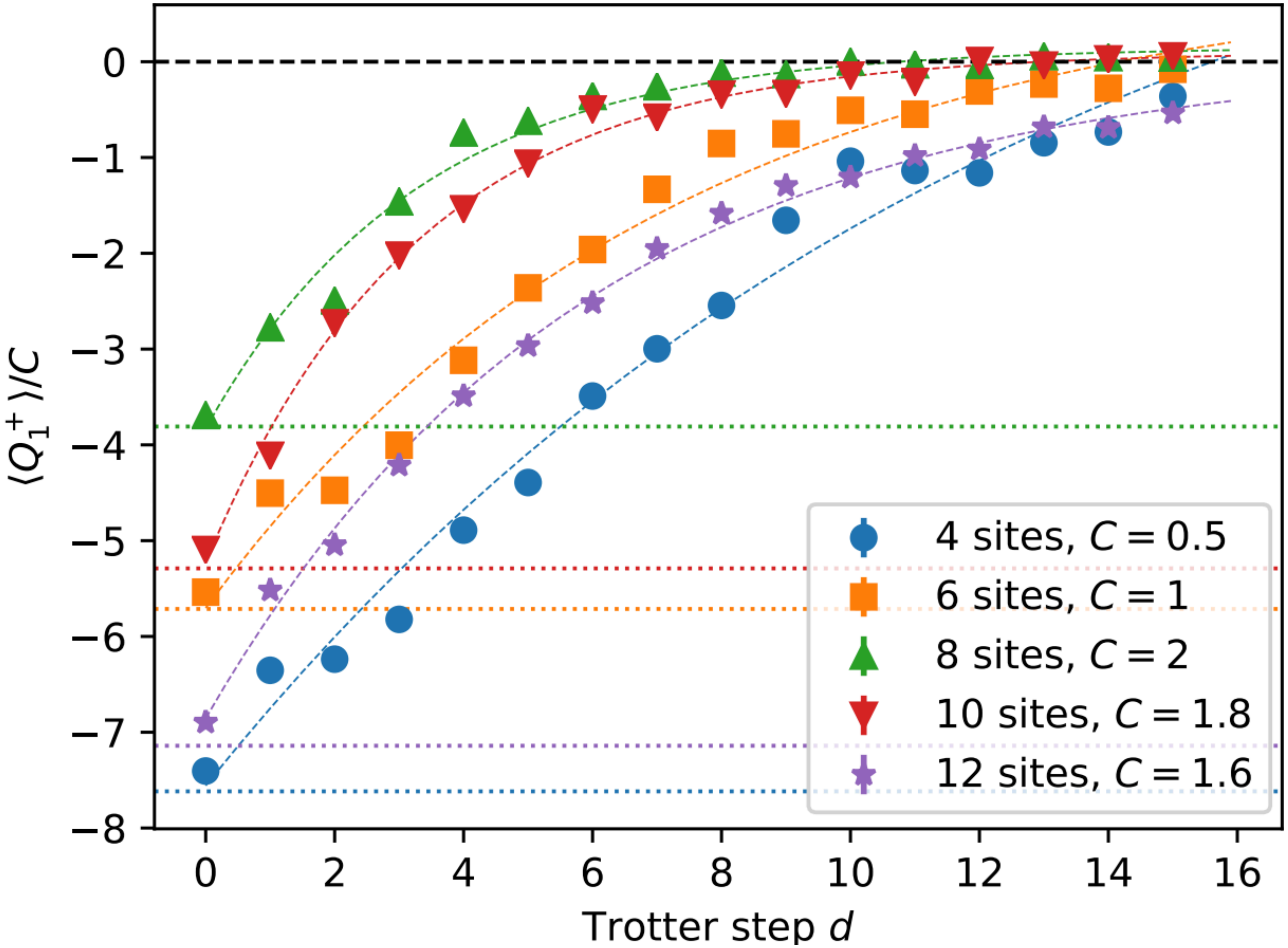
Superconducting transmon qubit

ibm_kawasaki @ UTokyo





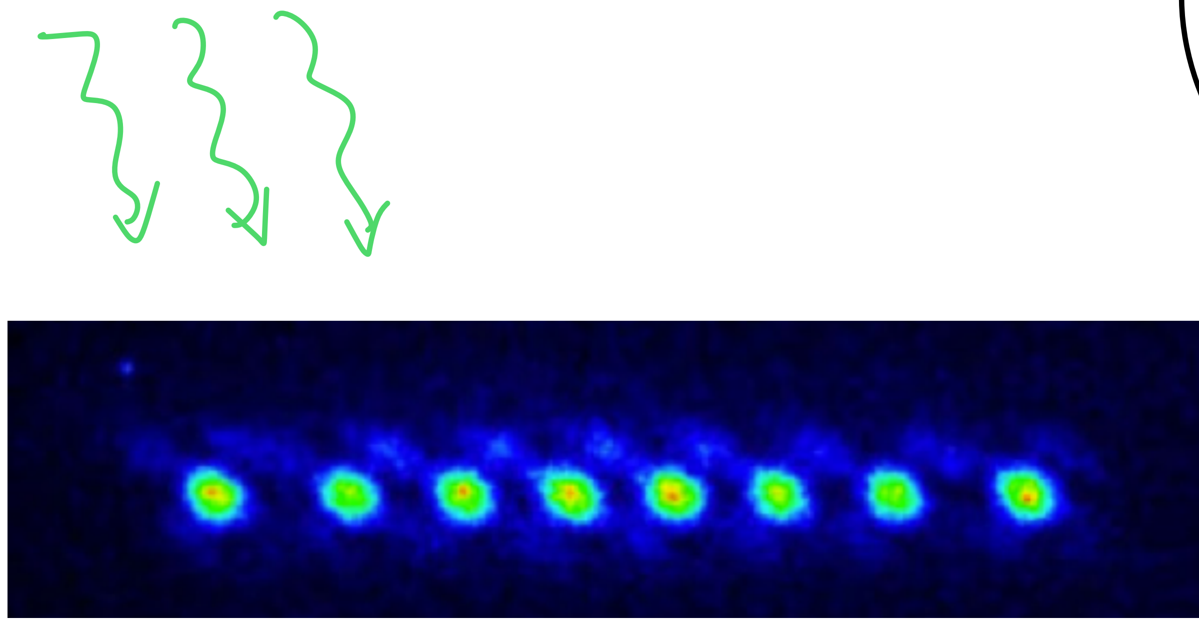
IBM Q

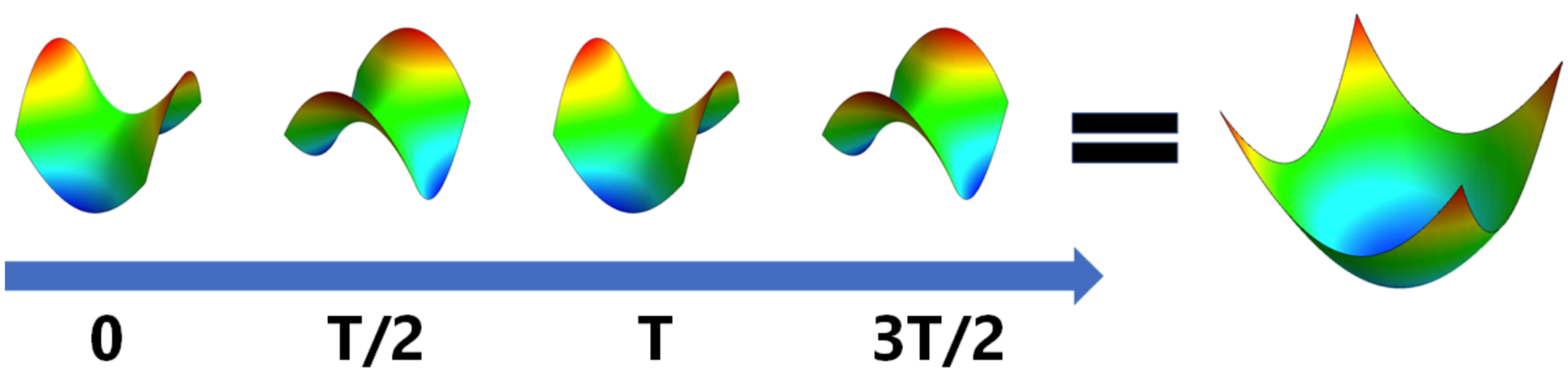


IBM-Q (superconducting qubits)

$\sim 10^5$ shots

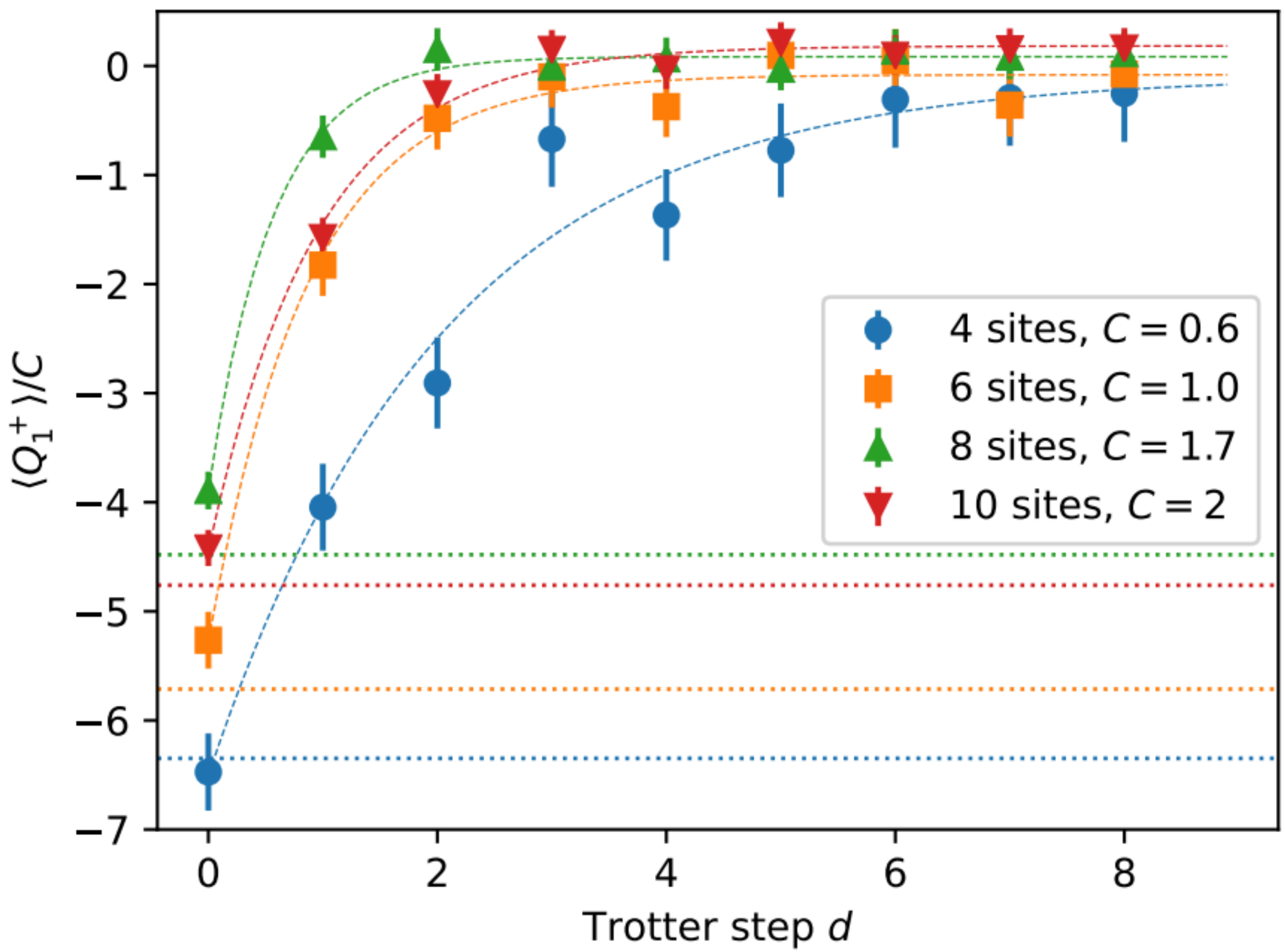
Ion Trap





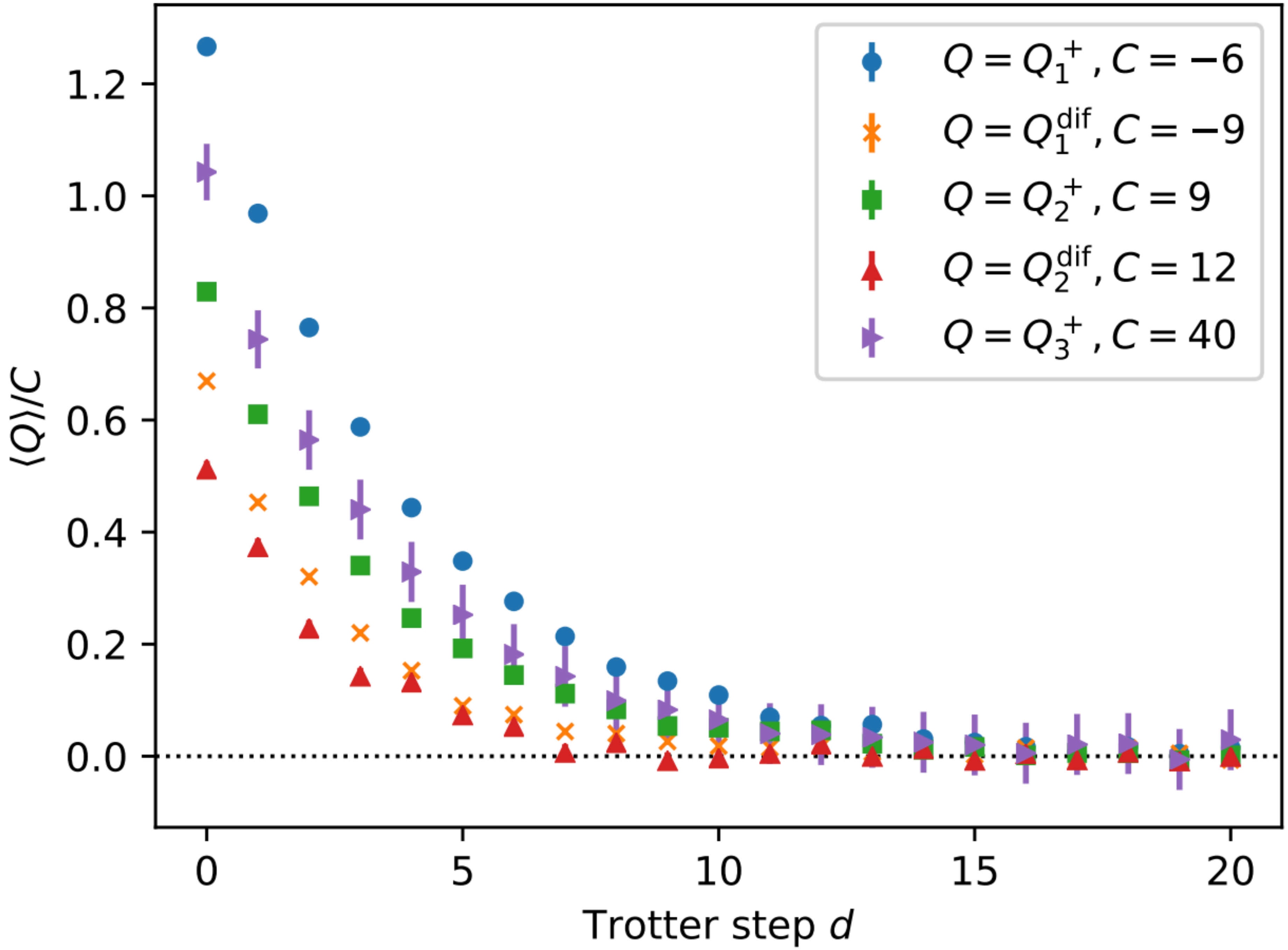
Paul trap

borrowed from https://www.qmedia.jp/basic-of-iontrap/



IonQ (ion trap) ~ 2000 shots

Classical Simulator
w/ depolarization error



Late-Time Behavior

(depolarization

$\rho \to (1-p)\rho + \frac{pI}{2}$

$p = 0.013$

2-qubit)

- $\rho \rightsquigarrow \underbrace{\Phi^N}(\rho) \xrightarrow[N\to\infty]{} \underbrace{\rho_0}$

noise

q-channel (CPTP map)

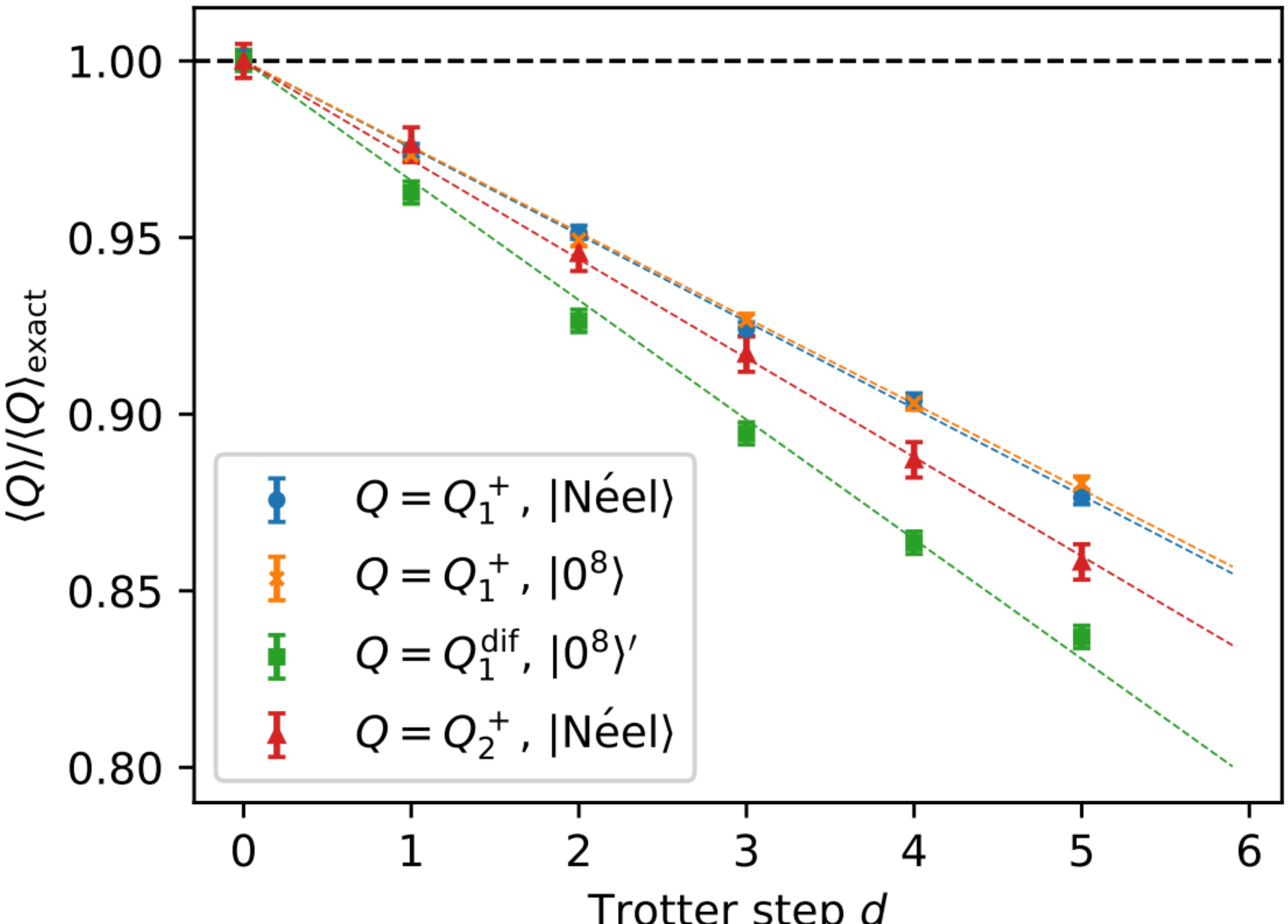
Φ eigenstate for largest eigenvalue

- $\langle Q \rangle_{d \text{ steps}} \sim c_1 e^{-\gamma d} + c_2$

charge	Q_1^+	Q_1^{dif}	Q_2^+	Q_2^{dif}	Q_3^+
γ	0.26	0.38	0.29	0.39	0.30

$(\gamma_{\text{dep}} = 0.20)$

Early-Time Behavior (p=0.0013 2-qubit)



benchmark quantum devices?

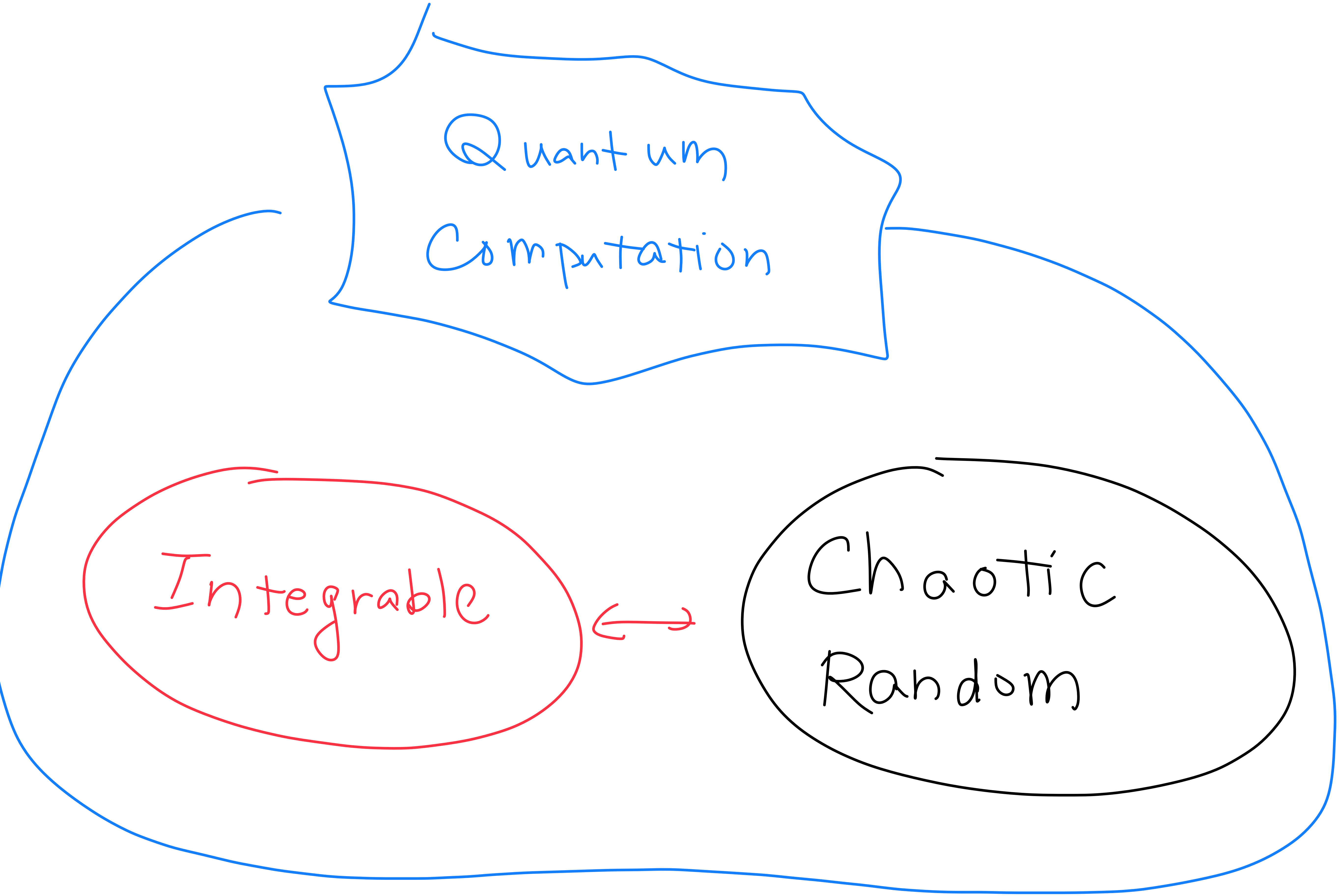
Summary

- Quantum Computers : already available
  promising ?

- Integrable model on actual quantum computers !
  ($Q_n^{\pm}$)

- Noisy. Diagnostic of error ?

- Phys / Math questions to aim for ???


Quantum Computation
Integrable
Chaotic
Random

unofficial annoucement

Les Houches Summer School 2025

"Exact Solvability and Quantum Information"

(S. Ouvry, D. Serban, T. Prosen, M. Yamazaki)

ÉCOLE DE
PHYSIQUE
DES HOUCHES