

Gravitational Positivity For Dark Gauge Bosons



Masahito Yamazaki

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THE UNIVERSITY OF TOKYO

K. Aoki (Kyoto), T. Noumi (Kobe->Tokyo), R. Saito (Yamaguchi),
S. Sato (Kobe/Tokyo), S. Shirai (IPMU), J. Tokuda (IBS) + MY,
arXiv: 2305.10058 [hep-ph]

How dark is “dark”?

Gravity to Rescue?

$$\| \left(\text{other forces} \right) \geq \left(\text{gravity} \right) \|$$

cf. • (gauge) weak gravity conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06

• scalar weak gravity conjecture

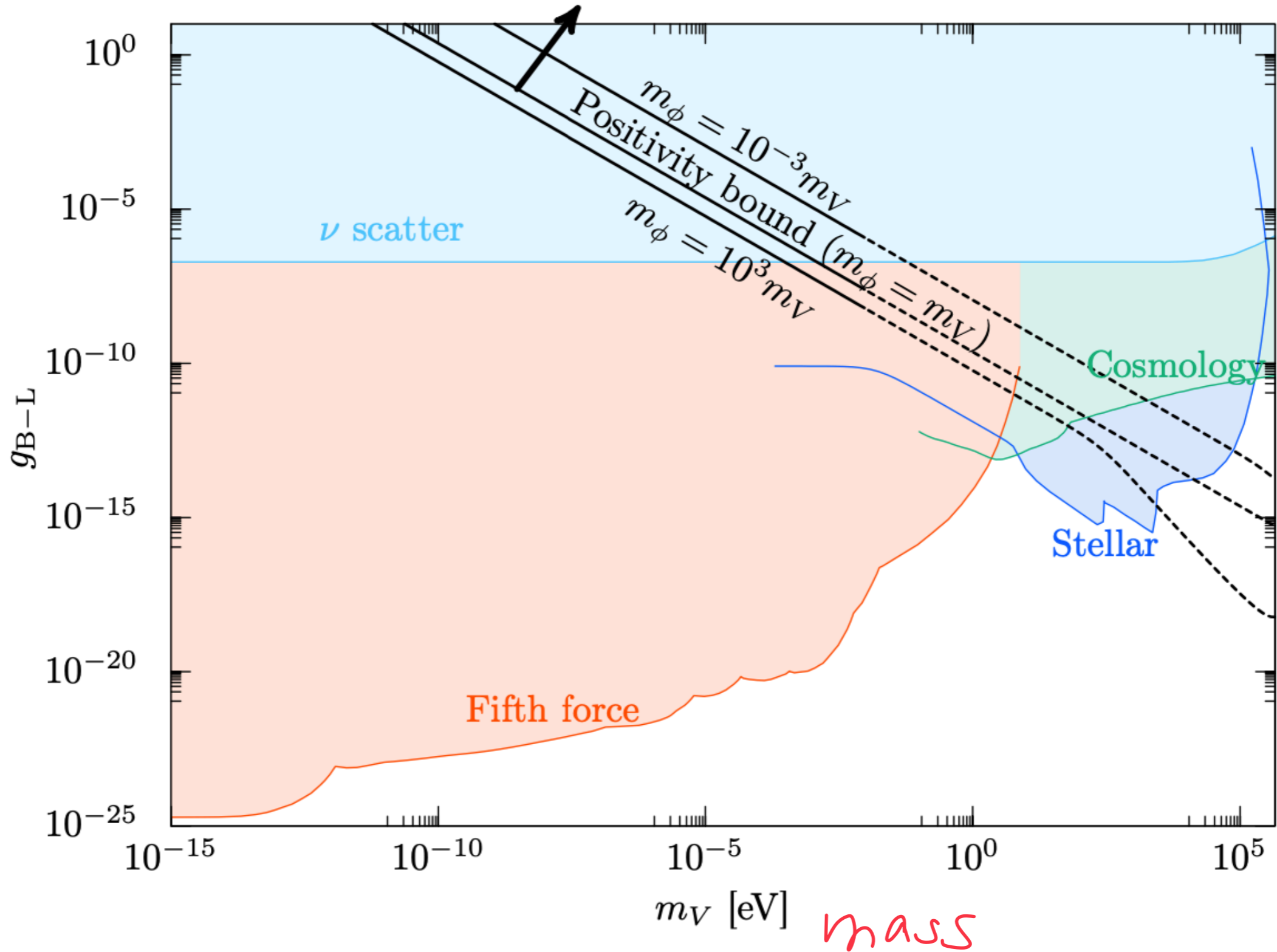
Palti '17, Shirai-MY '19, Kuzenko-Takhistov-Yamada-MY '19

• gravitational positivity

Tokuda, Aoki, Hirano '20,

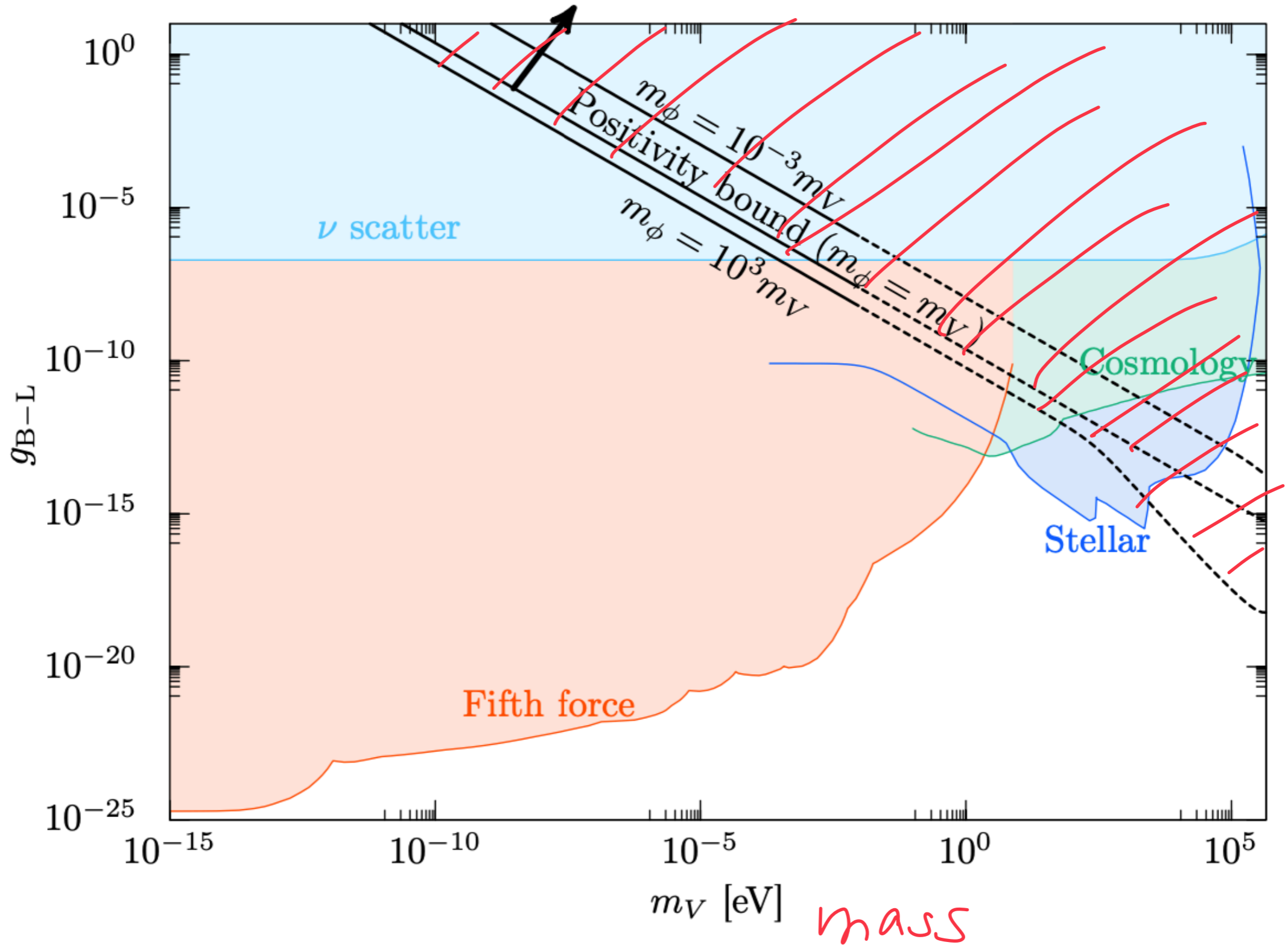
"Result" of gauged $U(1)_{B-L}$

Coupling



"Result" of gauged $U(1)_{B-L}$

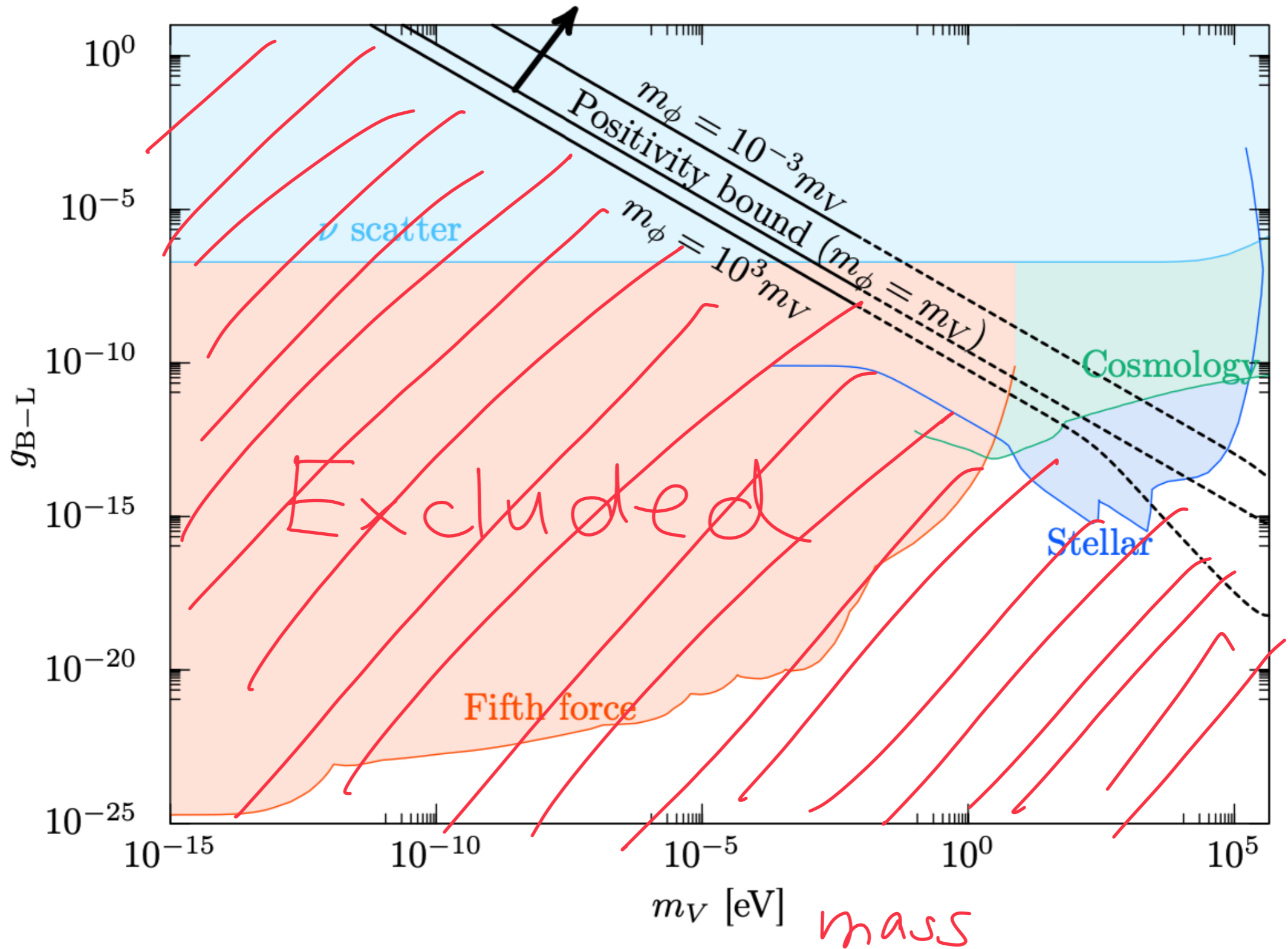
Coupling



m_ν [eV] mass

Result of gauged $U(1)_{B-L}$

Coupling



$$i\mathcal{M}_{\text{non-grav}}(s, t) = \begin{array}{c} \vee \\ A \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ B \\ \vee \end{array} + \dots$$

Assumptions

(i) analyticity

(ii) unitarity ($\text{Im } \mu \geq 0$)

(iii) s^2 -boundedness $\lim_{|s| \rightarrow \infty} \frac{|\mu(s, 0)|}{s^2} = 0$

["old" S-matrix theory since '60s]

$$i\mathcal{M}_{\text{non-grav}}(s, t) = \begin{array}{ccc} A & & A \\ & \diagdown \quad \diagup & \\ & \text{---} \text{---} \text{---} & \\ & \diagup \quad \diagdown & \\ B & & B \end{array} + \dots$$

$$a_{2i} = \left. \frac{\partial^2 \mathcal{M}(s, t=0)}{\partial s^2} \right|_{s=2m_V^2}$$

IR data

$$B_2 := a_2 - \frac{2 \cdot 2!}{\pi} \int_{m_V^2}^{\Lambda^2} ds \frac{\text{Im } \mathcal{M}(s, t=0)}{(s - 2m_V^2)^3}$$

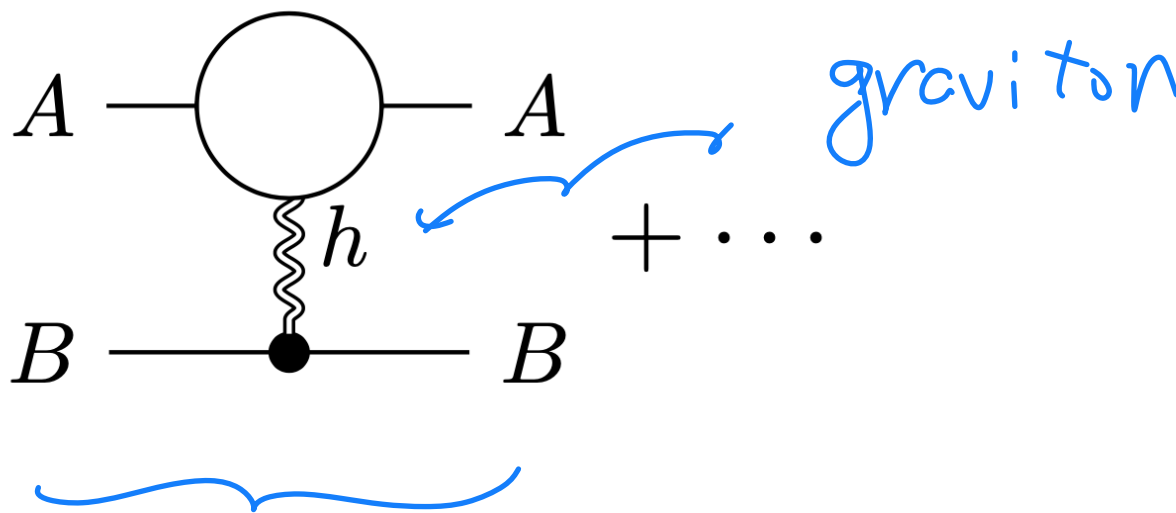
$$= \frac{2 \cdot 2!}{\pi} \int_{\Lambda^2}^{\infty} ds \frac{\text{Im } \mathcal{M}(s, t=0)}{(s - 2m_V^2)^3} \geq 0$$

dispersion rel.

analyticity + s^2 -boundedness

UV data

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06
 Bellazzini '16 de Rham, Melville, Tolley, Zhou '17, ...]

$$i\mathcal{M}_{\text{grav},t\text{-channel}}(s, t) =$$


graviton

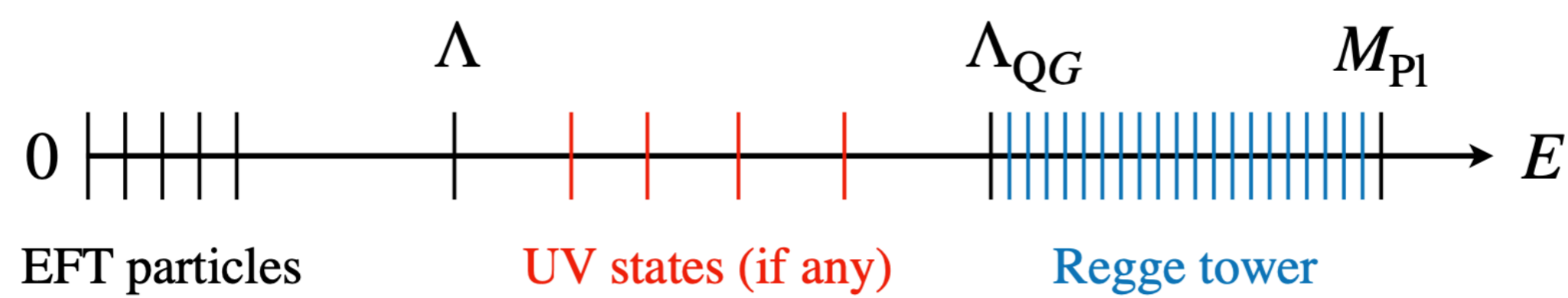
+ ...

↓

$\frac{s^2}{t}$: violates s^2 -boundedness?

$$i\mathcal{M}_{\text{grav},t\text{-channel}}(s, t) =$$

$\frac{s^2}{t}$: violates s^2 -boundedness?



recovers s^2 -boundedness

$$B_{\text{non-grav}}(\Lambda) \simeq \frac{4}{\pi} \int_{\Lambda^2}^{\infty} ds \frac{\text{Im } \mathcal{M}_{\text{non-grav}}(s, t=0)}{(s - 2m_V^2)^3}$$

$$B_{\text{grav}}(\Lambda) \simeq \lim_{t \rightarrow -0} \left[\frac{\partial^2 \mathcal{M}_{\text{grav}, t\text{-channel}}(s, t)}{\partial s^2} + \frac{2}{M_{\text{Pl}}^2 t} - (\text{kinematic singularity}) \right]_{s=2m_V^2}$$

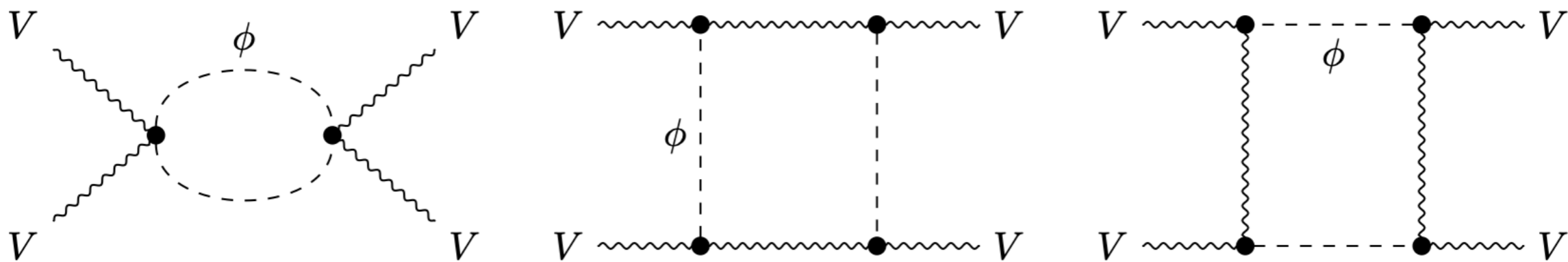
sum

$$B(\Lambda) \gtrsim \mathcal{O}\left(\frac{1}{M_{\text{pl}}^2 M^2}\right)$$

related to Λ_{QG}

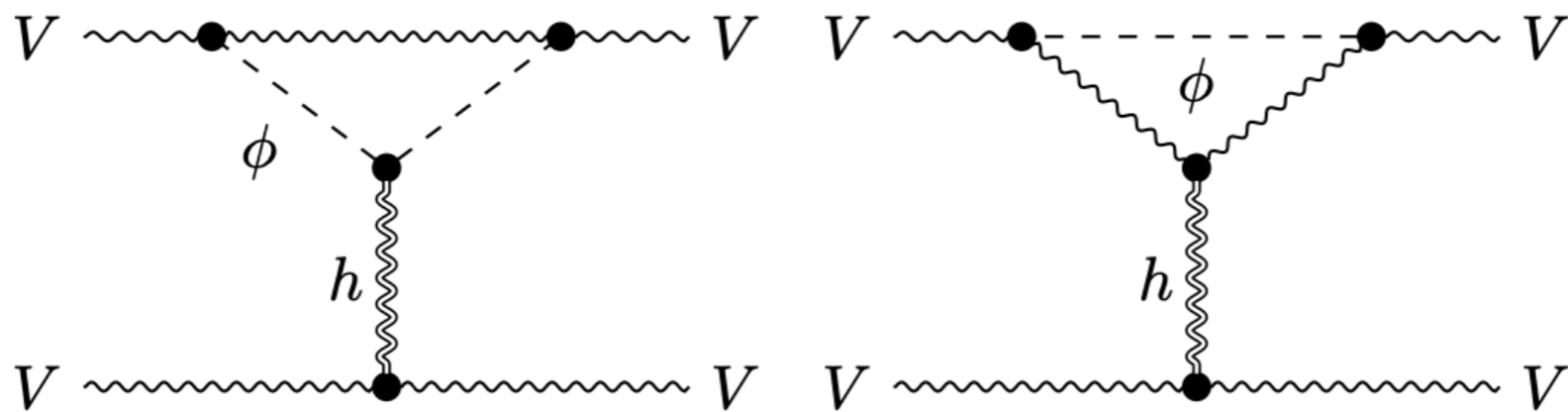
[Tokuda, Aoki, Hirano '20
cf. Alberte, de Rham, Jaitly, Tolley '20]

Example: U(1) gauge boson with Higgs mass



non-gravitational

$$B_{\text{non-grav}} \stackrel{\text{e.g.}}{\sim} + \frac{g^4}{\Lambda^4} + \dots$$

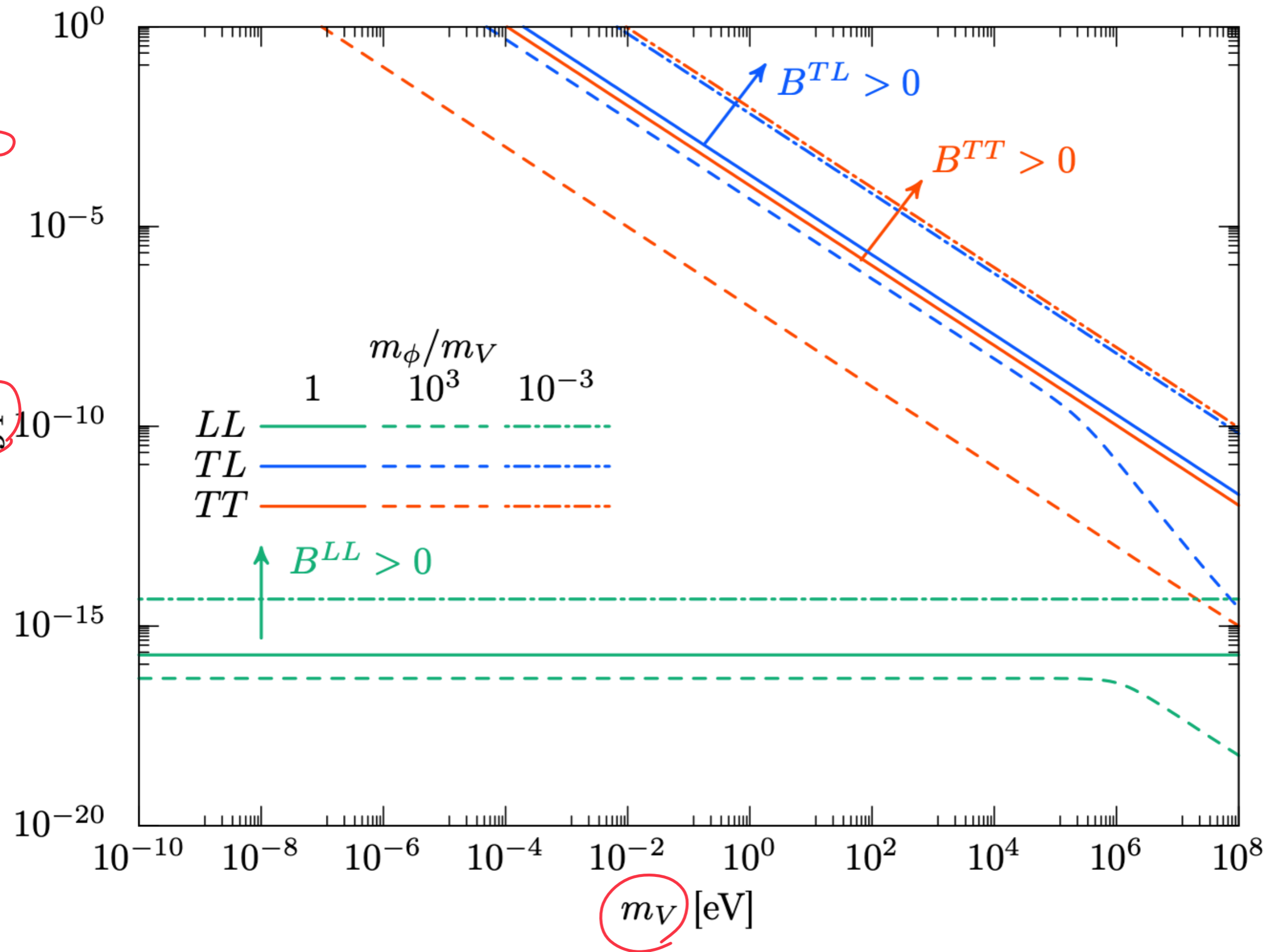


gravitational

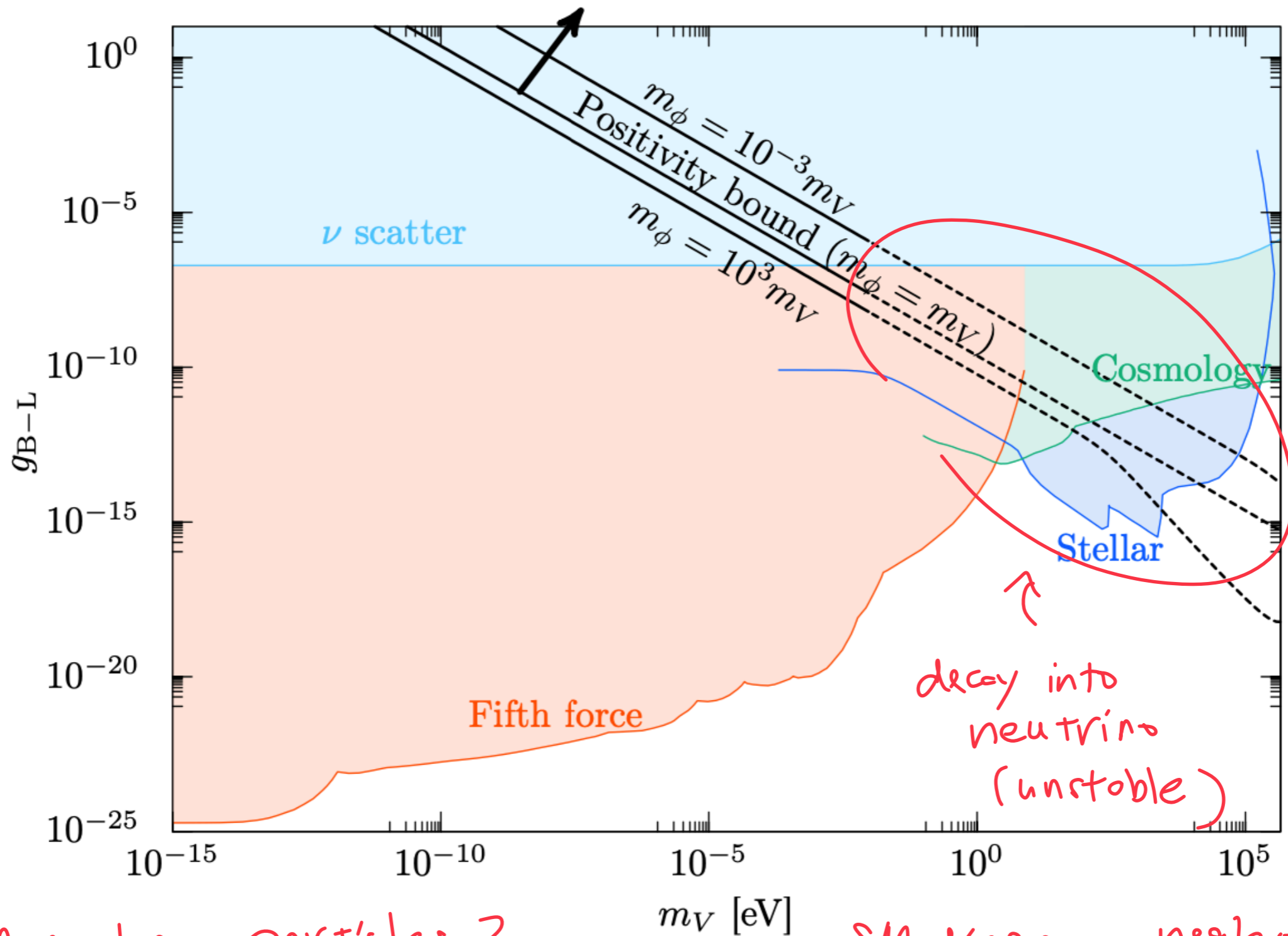
$$B_{\text{grav}} \stackrel{\text{e.g.}}{\sim} - \frac{g^2}{m_\Phi^2 M_{\text{pl}}^2}$$

$U(1)$ charge of Φ

g_Φ



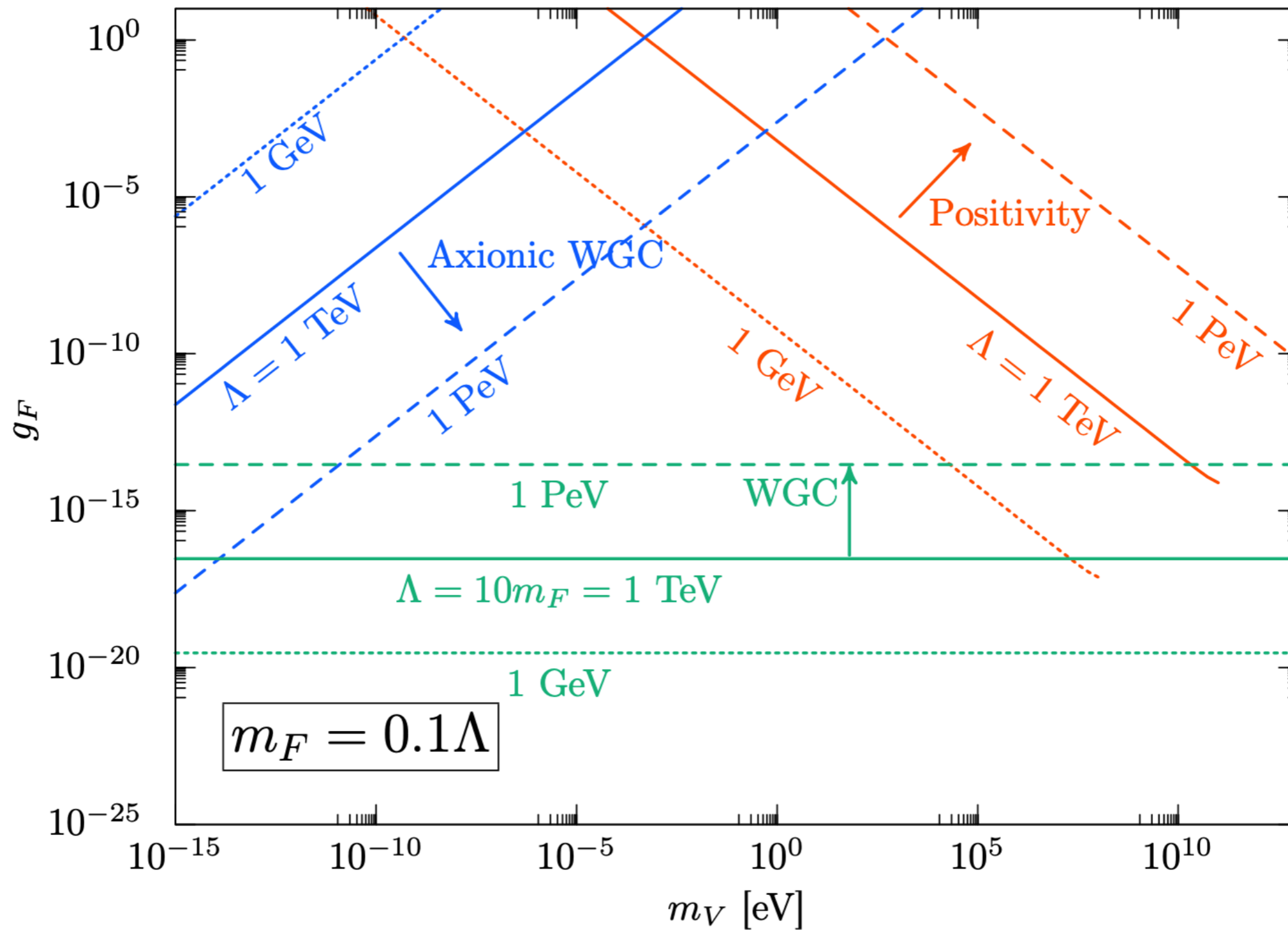
Realistic Cases: Theoretically Subtle



massless particles?

SM diagrams neglected

Example: Stückelberg $U(1)$



Summary

(Gravity
UV completion)



practical
recipes

(lower bound
on dark sector
couplings)

Dark side of Universe

= Quantum - Gravity side of Universe