Emergent Global Symmetries in Holography for Swamplanders

Masahito Yamazaki

KAVLI IPMU

THE UNIVERSITY OF TOKYO

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Based on works with IPMU alumini

Meer Ashwinkumar

Matthew Dodelson

Abhiram Kidambi

Jacob M Leedom

Based on

- Ashwinkumar, Leedom + MY 2305.1022

See also

- Ashwinkumar, Dodelson, Kidambi, Leedom + MY 2104.14710
- Ashwinkumar, Kidambi, Leedom + MY to appear

Ensemble Averages in $\overbrace{\text{Holography}}^{\text{"exotic"}}$

Emergent Global Symmetry

"exotic"

Ensemble Averages in Holography

Emergent Global Symmetry

Q: Ensemble vs. Swampland?

initial inspiration:

[Afkhami-Jeddi, Cohen, Hartman, Tajdini ('20)
Maloney - Witten ('20)]

(standard) Narain theory

: even self-dual lattice

generalized Narain theory [Our works]

: general even (non-self-dual) lattice

Generalized Narain Theories

[Ashwinkumar - Dodelson - Kidambi - Leedom - MY ('21)
Ashwinkumar - Kidambi - Leedom - MY (to appear)]

recall: S^1 Narain theory

$$P = \left\{ \left(p_L = \frac{n}{2R} + wR \,,\; p_R = \frac{n}{2R} - w\underbrace{R}_{\substack{\| \\ \text{moduli} \hookrightarrow \text{T-duality group}}} \right) \;\middle|\; (n, w) \in \mathbb{Z} \right\}$$

$R = 1/2$ moduli space $R = \infty$

$$Q(\{n, w\}) = p_L^2 - p_R^2 = \underbrace{2nw}_{\text{even quadratic form}} \in 2\mathbb{Z}$$

$$H = p_L^2 + p_R^2 = \frac{n^2}{2R^2} + 2w^2R^2$$

Data

- Q : even quadratic form of signature (p,q) $\quad Q = p_L^2 - p_R^2$

  lattice Λ

- H : Hamiltonian $\quad H = p_L^2 + p_R^2$

  moduli dependent

- $\mathcal{M}_Q$: CFT moduli space

$$\mathcal{M}_Q = O_Q(p,q;\mathbb{Z}) \backslash O(p,q) / O(p) \times O(q)$$

(T-duality group)

Theta function

$$\vartheta_Q(\tau, \bar{\tau}; m) = \sum_{\ell \in \Lambda} e^{\pi i \tau_1 Q(\ell) - \pi \tau_2 \overbrace{H(\ell)}^{m\text{-dependence}}}$$

$$= \sum_{\ell \in \Lambda} \underbrace{q}_{e^{2\pi i \tau}}{}^{p_L^2(\ell)/2} \, \underbrace{\bar{q}}_{e^{2\pi i \bar{\tau}}}{}^{p_R^2(\ell)/2}$$

$(\tau = \tau_1 + i\tau_2)$

(※ m : CFT moduli ; τ : spacetime torus moduli)

Theta function

$$\vartheta_{Q,\alpha}(\tau,\bar\tau;m) = \sum_{\ell\in\Lambda} e^{\pi i \tau_1 Q(\ell+\alpha) - \pi\tau_2 H(\ell+\alpha)}$$

$$= \sum_{\ell\in\Lambda} \underbrace{q}_{e^{2\pi i\tau}}^{p_L^2(\ell+\alpha)/2}\, \underbrace{\bar q}_{e^{2\pi i\bar\tau}}^{p_R^2(\ell+\alpha)/2}$$

a set of basis functions

$$\alpha \in \mathcal{D} := \Lambda^*/\Lambda \quad \text{(discriminant)}$$

modular transformation

$$T:\ \vartheta_{Q,\alpha}(\tau+1;m) = e^{i\pi Q(\alpha,\alpha)}\,\vartheta_{Q,\alpha}(\tau,m)$$

$$S:\ \vartheta_{Q,\alpha}\left(-\frac{1}{\tau},m\right) = \frac{e^{-i\pi\frac{r-s}{4}}}{\sqrt{|\det Q|}}\,\tau^{\frac{r}{2}}\,\bar{\tau}^{\frac{s}{2}}\sum_{\beta\in D} e^{-2\pi i Q(\alpha,\beta)}\,\vartheta_{Q,\beta}(\tau;m)$$

T^2 partition function

$$Z_{Q,\alpha}(\tau,\bar{\tau};m) = \frac{\vartheta_{Q,\alpha}(\tau,\bar{\tau};m)}{\eta(\tau)^{p}\,\bar{\eta}(\bar{\tau})^{q}}$$

$$T : Z_{Q,\alpha}(\tau+1; m) = e^{i\pi Q(\alpha,\alpha)} e^{-\frac{2\pi i (p-8)}{24}} Z_{Q,\alpha}(\tau, m)$$

$$S : Z_{Q,\alpha}\left(-\frac{1}{\tau}, m\right)$$

$$= \frac{1}{\sqrt{|\det Q|}} \sum_{\beta\in\mathcal{D}} e^{-2\pi i Q(\alpha,\beta)} Z_{Q,\beta}(\tau; m)$$

"Weil representation" of metaplectic group

Ensemble Average

[Ashwinkumar, Dodelson, Kidambi, Leedom, MY ('21)]

Ensemble Average

Haar measure / Zamolodchikov metric

$$\underbrace{\frac{1}{\mathrm{Vol}(\mathcal{M}_{Q,\alpha})}\int_{\mathcal{M}_{Q,\alpha}} \overbrace{[dm]}\,\vartheta_{Q,\alpha}(\tau,\bar{\tau};m)}_{\text{ensemble average over CFT moduli space}} = ??$$

T-duality preserving α

$\boxed{\text{Siegel-Weil formula}}$ [Siegel ('51)
Weil ('64)]

$$\underbrace{\frac{1}{\mathrm{Vol}(\mathcal{M}_{Q,\alpha})}\int_{\mathcal{M}_{Q,\alpha}}[dm]\,\vartheta_{Q,\alpha}(\tau,\bar{\tau};m)}_{\substack{\text{ensemble average} \\ \text{over CFT moduli space}}} = \underbrace{E_{Q,\alpha}(\tau,\bar{\tau})}_{\substack{\text{non-hol.} \\ \text{Eisenstein series}}}$$

Poincaré sum

$$\begin{pmatrix} * & * \\ c & d \end{pmatrix} : \; SL(2,\mathbb{Z}) / \Gamma_\infty \qquad \Gamma_\infty \cong \mathbb{Z}$$

$$E_{Q,\alpha}(\tau) := \delta_{\alpha\in\Lambda} + \overbrace{\sum_{\substack{(c,d)=1 \\ c>0}}} \frac{\gamma_{Q,\alpha}(c,d)}{(c\tau+d)^{\frac{p}{2}} (c\bar{\tau}+d)^{q/2}}$$

$$\gamma_{Q,\alpha}(c,d) = \frac{e^{\frac{\pi i (p-q)}{4}}}{\sqrt{|\det Q|}} \, c^{-\frac{p+q}{2}} \sum_{\ell \in \Lambda / c\Lambda} \exp\left[-\pi i \frac{d}{c} Q(\ell+\alpha) \right]$$

In fact, modular form expected

$$\vartheta_{Q,\alpha}(\tau+1;m) = \underbrace{e^{i\pi Q(\alpha,\alpha)}}\,\vartheta_{Q,\alpha}(\tau,m)$$

$$\vartheta_{Q,\alpha}\left(-\frac{1}{\tau}, m\right) = \underbrace{\frac{e^{-i\pi\frac{p-q}{4}}}{\sqrt{|\det Q|}}\,\tau^{\frac{p}{2}}\,\bar{\tau}^{\frac{q}{2}}\sum_{\beta\in\mathcal{D}} e^{-2\pi i Q(\alpha,\beta)}}_{\text{moduli independent!}}\,\vartheta_{Q,\beta}(\tau;m)$$

$$\left.\begin{aligned}
\langle\vartheta_{Q,\alpha}\rangle(\tau+1) &= e^{i\pi Q(\alpha,\alpha)}\,\langle\vartheta_{Q,\alpha}\rangle(\tau) \\
\langle\vartheta_{Q,\alpha}\rangle\left(-\frac{1}{\tau}\right) &= \frac{e^{-i\pi\frac{p-q}{4}}}{\sqrt{|\det Q|}}\,\tau^{\frac{p}{2}}\,\bar{\tau}^{\frac{q}{2}}\sum_{\beta\in\mathcal{D}} e^{-2\pi i Q(\alpha,\beta)}\,\langle\vartheta_{Q,\beta}\rangle(\tau)
\end{aligned}\right\} \text{modular form for } \Gamma\subset SL(2,\mathbb{Z})$$

$$\left\langle \vartheta_{Q,\alpha} \right\rangle(\tau+1) = \overbrace{e^{i\pi Q(\alpha,\alpha)} \left\langle \vartheta_{Q,\alpha} \right\rangle(\tau)}$$

$$\left\langle \vartheta_{Q,\alpha} \right\rangle\left(-\frac{1}{\tau}\right) = \underbrace{\frac{e^{-i\pi\frac{p-q}{4}}}{\sqrt{|\det Q|}} \tau^{\frac{p}{2}} \bar{\tau}^{\frac{q}{2}} \sum_{\beta\in\mathcal{D}} e^{-2\pi i Q(\alpha,\beta)} \left\langle \vartheta_{Q,\beta} \right\rangle(\tau)}$$

reproduce behavior at cusps

$$E_{Q,\alpha}(\tau) := \delta_{\alpha\in\Lambda} + \sum_{\substack{(c,d)=1 \\ c>0}} \frac{\overbrace{\gamma_{Q,\alpha}(c,d)}}{(c\tau+d)^{\frac{p}{2}} (c\bar{\tau}+d)^{q/2}} \qquad \tau \sim -\frac{d}{c}$$

Holographic Dual

[Ashwinkumar, Dedelson, Kidambi, Leedom, MY ('21)]

Holographic dual after averaging:

$$S_{cs} = \sum_{I,J=1}^{p+q} \frac{1}{4\pi} Q_{IJ} \int A_I \wedge d\underbrace{A_J}_{U(1)}$$

* defined from Q

* Sum over geom: $SL(2,\mathbb{Z})$ BHs

"exotic gravity" (solid torus)

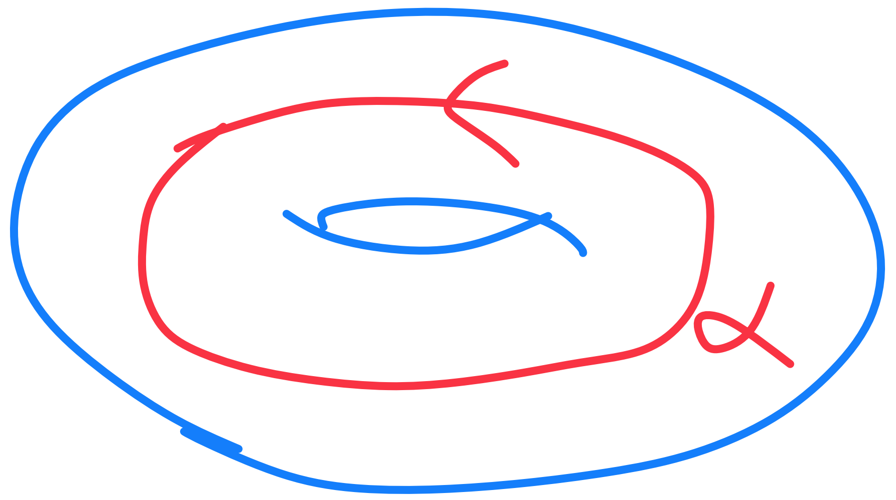
thermal AdS_3, BTZ BH, ...

* $|E_{Q,\alpha}(\tau)\rangle$: wavefunction of Abelian CS on $T^2 = \partial(D^2 \times S^1)$

$* \quad |E_{Q,\underset{\smile}{\alpha}}(\tau)\rangle$: Wavefunction of

Abelian CS on $T^2 = \partial(D^2 \times S^1)$

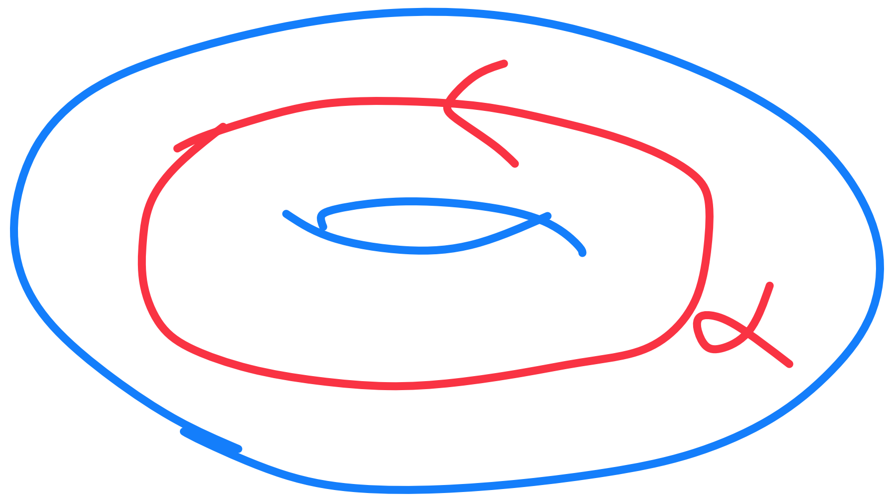
Wilson line insertion



anyon $\quad \alpha \in \underbrace{\Lambda}_{\text{charge}}{}^* / \underbrace{\Lambda}_{\text{gauge equiv.}}$

* $|E_{Q,\underbrace{\alpha}}(\tau)\rangle$: Wavefunction of Abelian CS on $T^2 = \partial(D^2 \times S^1)$

Wilson line insertion



anyon $\alpha \in \underbrace{\Lambda^*}_{\text{charge}} / \underbrace{\Lambda}_{\text{gauge equiv.}}$

(Supplemented by $\frac{1}{\eta^p \bar{\eta}^{\bar{p}}}$: Brown-Henneaux Virasoro modes)

$$
* \begin{cases} T : Z_{Q,\alpha}(\tau+1) = e^{i\pi Q(\alpha,\alpha)} e^{-\frac{2\pi i (p-q)}{24}} Z_{Q,\alpha}(\tau, m) \\ S : Z_{Q,\alpha}(-\frac{1}{\tau}) = \frac{1}{\sqrt{|\det Q|}} \sum_{\beta\in\mathcal{D}} e^{-2\pi i Q(\alpha,\beta)} Z_{Q,\beta}(\tau; m) \end{cases}
$$

matched by modular T/S matrix
for Abelian anyons

$$
T_{\alpha\beta} = \underbrace{e^{i\pi Q(\alpha,\alpha)}}_{\text{topological spin}} e^{-\frac{2\pi i (p-q)}{24}} \delta_{\alpha\beta} \; ; \; S_{\alpha\beta} = e^{-2\pi i Q(\alpha,\beta)}
$$

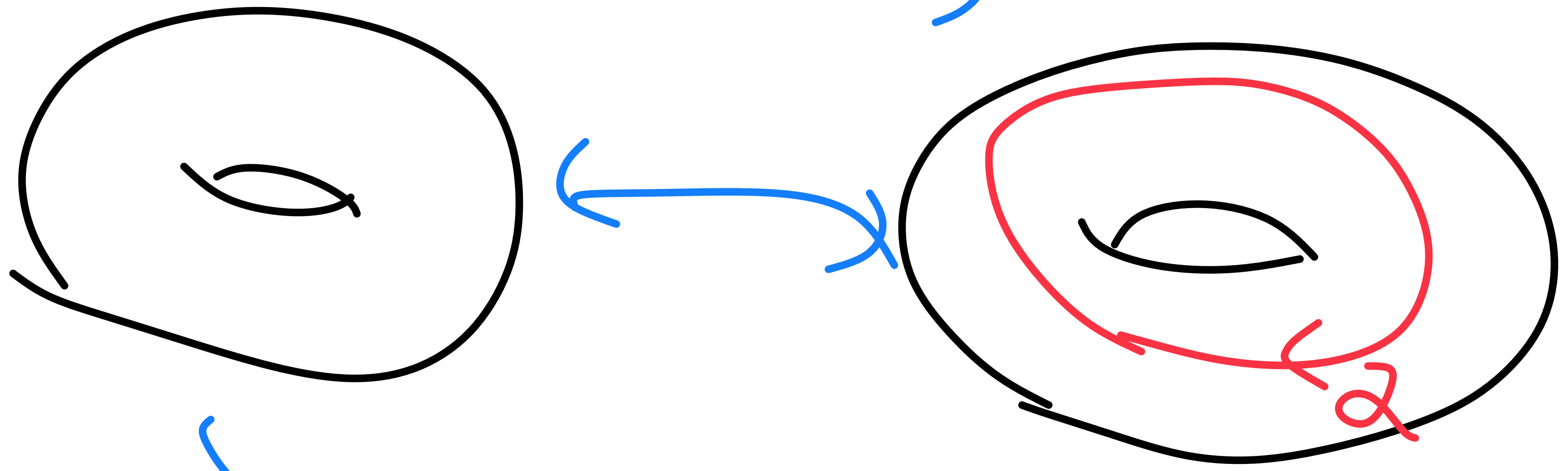
$M_{(c,d)}$

sum over

$SL(2,\mathbb{Z})$ BH

lens space partition function

$$\langle \psi_0 | \begin{pmatrix} * & * \\ c & d \end{pmatrix} | \psi_\alpha \rangle$$



$M_{(c,d)}$

$$E_{Q,\alpha}(\tau) := \delta_{\alpha\in\Lambda} + \sum_{\substack{(c,d)=1 \\ c>0}} \frac{\gamma_{Q,\alpha}(c,d)}{(c\tau+d)^{\frac{p}{2}}(c\bar{\tau}+d)^{\frac{q}{2}}}$$

$$\gamma_{Q,\alpha}(c,d) = \frac{e^{\frac{\pi i(p-q)}{4}}}{\sqrt{|\det Q|}}\, c^{-\frac{p+q}{2}} \sum_{\ell\in\Lambda/c\Lambda} \exp\left[-\pi i \frac{d}{c} Q(\ell+\alpha)\right]$$

※ Holography works before / after average

	CFT $\longleftrightarrow$	bulk
before average	gen. Narain theories	Abelian Maxwell – CS
after average	$\langle$ gen. Narain theories $\rangle$	Abelian CS

※ Holography works before / after average

	CFT $\longleftrightarrow$	bulk
before average	$\vartheta_{Q,\alpha}(\tau,\bar{\tau};m)$	$S_{CS}+S_M$, $\quad S_M = \left(\frac{1}{g^2}\right)_{IJ}\int F_I\wedge * F_J$
$\downarrow$ after average	$E_{Q,\alpha}(\tau,\bar{\tau})$	$S_{CS}=\sum\int Q_{IJ}\,A_I\wedge dA_J$

Our setup: proposed for

$AdS_3 \times K_7$ compactifications in

actual string theory!

[Gukov - Martinec - Moore - Strominger '04]

Ideal setup for precision

Swampland / Ensemble discussion?

Emergent Global Symmetries

[Ashwinkumar, Leedom, MY ('23)]

QG : "No exact global symmetries"

Swampland conjecture!

However, Abelian CS theories have

$\underbrace{\text{zero-form / one-form}}_{\text{global}}$ symmetries

(∵ not an immediate contradiction

since bulk is not Einstein gravity)

Symmetries of anyons

- $\alpha \in \mathcal{D} = \Lambda^*/\Lambda$; anyons w/ spin $\theta(\alpha) = e^{\pi i Q(\alpha)}$

- symmetry : $\sigma \in \mathrm{Aut}(\mathcal{D})$

  $\alpha \longmapsto \sigma \cdot \alpha$ s.t. $\theta(\alpha) = \theta(\sigma \cdot \alpha)$

$$\alpha \xrightarrow{\ \sigma\ } \sigma(\alpha)$$

$\sigma \in \mathrm{Aut}(\mathcal{D})$ then

After average

- $E_{Q,\, \sigma \cdot \alpha}(\tau, \bar{\tau}) = E_{Q,\, \alpha}(\tau, \bar{\tau})$

$\sigma \in \mathrm{Aut}(\mathcal{D})$ then

After average

- $E_{Q,\sigma\cdot\alpha}(\tau,\bar{\tau}) = E_{Q,\alpha}(\tau,\bar{\tau})$

Before average

$$\vartheta_{Q,\sigma\cdot\alpha}(\tau,\bar{\tau};m) \neq \vartheta_{Q,\alpha}(\tau,\bar{\tau};m)$$??

NOT a symmetry of a given CFT

$\sigma \in \mathrm{Aut}(\mathcal{D})$ then

After average

- $E_{Q,\sigma\cdot\alpha}(\tau,\bar{\tau}) = E_{Q,\alpha}(\tau,\bar{\tau})$

Before average

$$\vartheta_{Q,\sigma\cdot\alpha}(\tau,\bar{\tau};m) \neq \vartheta_{Q,\alpha}(\tau,\bar{\tau};m)$$

??

$$\vartheta_{Q,\sigma\cdot\alpha}(\tau,\bar{\tau};\underbrace{\sigma\cdot m}) = \vartheta_{Q,\alpha}(\tau,\bar{\tau};m)$$

if T-duality origin relation between different theories

In general,

"ensemble sym."

average over $\mathcal{M} \circlearrowleft \sigma \in G$ s.t. $[d(\sigma(m))] = [dm]$

of $\mathcal{O}(m,\alpha)$ s.t. $\mathcal{O}(\sigma\cdot m, \sigma\cdot\alpha) = \mathcal{O}(m,\alpha)$

$$\Rightarrow \langle \mathcal{O}\rangle(\alpha) = \int [dm]\, \mathcal{O}(m,\alpha) = \langle \mathcal{O}\rangle(\sigma\cdot\alpha)$$

$$\langle \mathcal{O} \rangle(\sigma \cdot \alpha) = \int [dm]\, \mathcal{O}(m, \sigma \cdot \alpha)$$

$\mathcal{O}$ transformation

$$= \int [dm]\, \mathcal{O}(\sigma^{-1} m, \alpha)$$

change $m \to \sigma m$

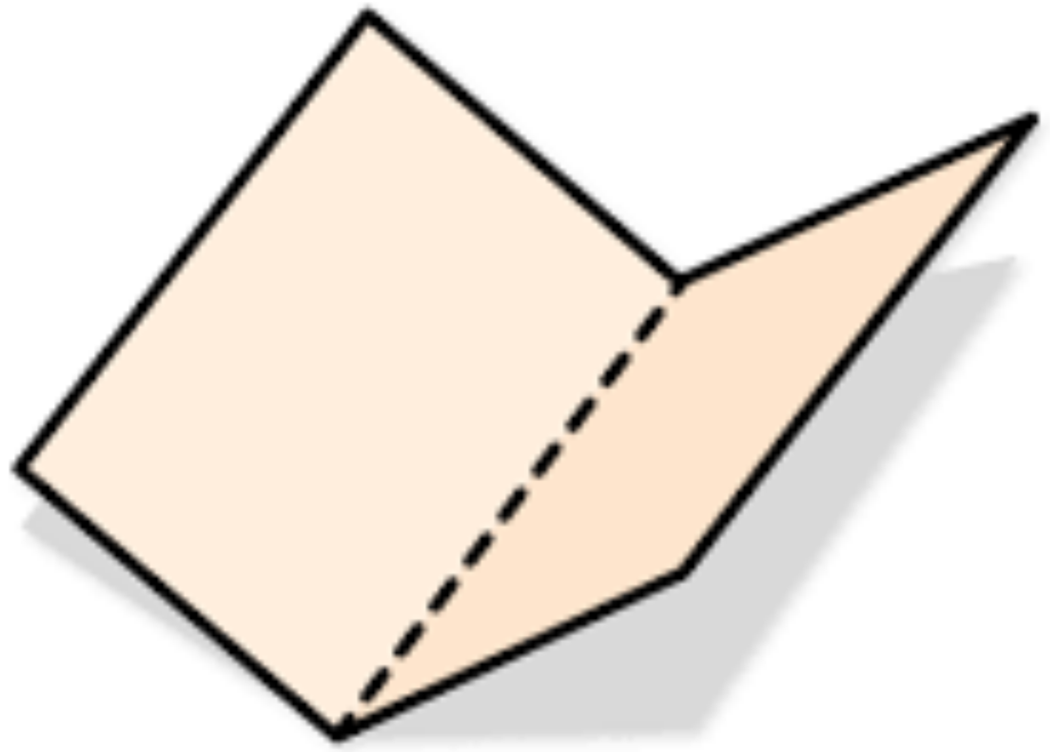
$$= \int [d(\sigma \cdot m)]\, \mathcal{O}(m, \alpha)$$

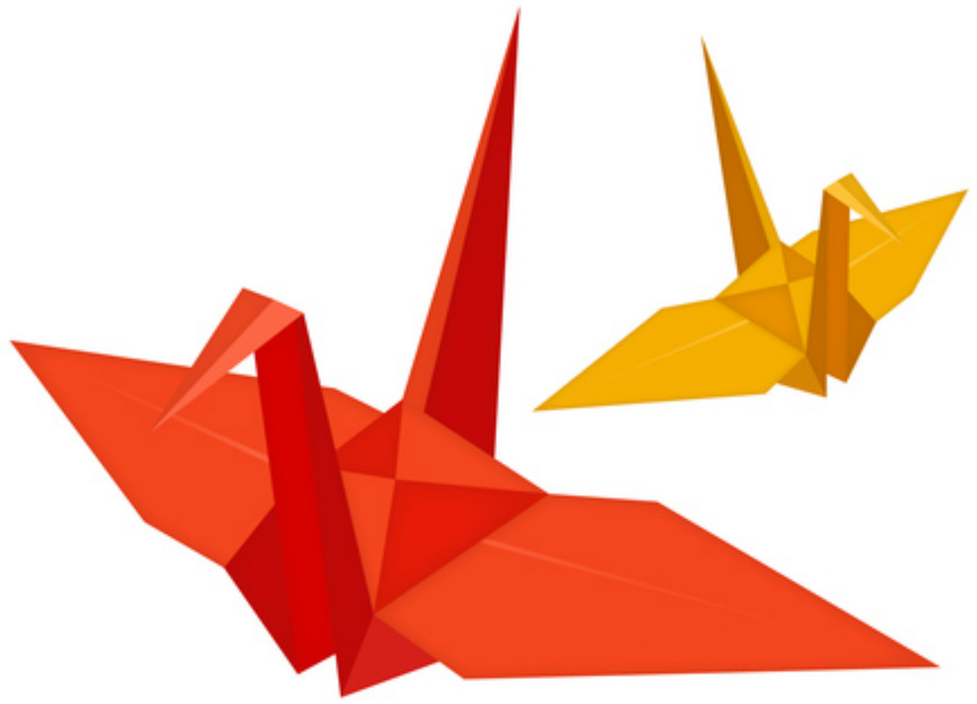
inv. of $[dm]$

$$= \int [dm]\, \mathcal{O}(m, \alpha)$$

$$= \langle \mathcal{O} \rangle(\alpha)$$

"duality origami"

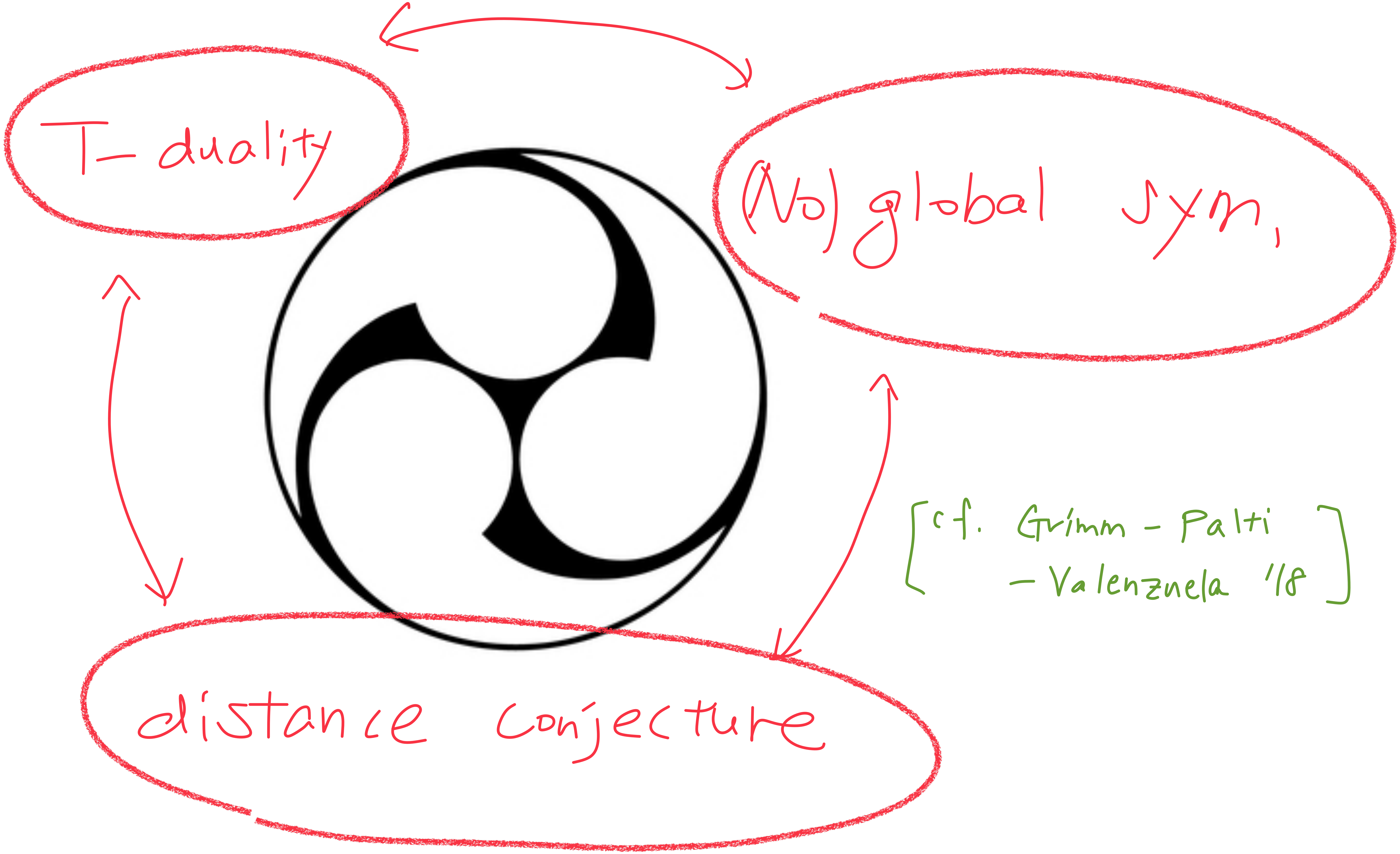




ensemble symmetry $\xrightarrow{\text{ensemble average}}$ global symmetry

(e.g. T-duality)

Swampland ?


T-duality
(No) global sym.
distance conjecture
[cf. Grimm - Palti - Valenzuela '18]

$\vartheta_{Q,\alpha}(\tau)$

asymptotic at cusps as $\tau \to -d/c$

∞-distance

[cf. distance conj.]

lens inv.

$\gamma_{Q,\alpha}(c,d)$

ensemble average

extract

gather all

$$\langle \vartheta_{Q,\alpha} \rangle(\tau) = \delta_{\alpha\in\Lambda} + \sum_{\substack{(c,d)=1 \\ c>0}} \frac{\gamma_{Q,\alpha}(c,d)}{(c\tau+d)^{\frac{p}{2}}\,(c\bar{\tau}+d)^{q/2}}$$

Deviations from ensemble average?

Spectral Decomposition [Benjamin, Collier, Fritzpatrick, Maloney, Perlmutter ('21)]

$$f(\tau,\bar\tau) := \tau_2^{\frac{p+q}{2}}\left(\vartheta_{Q,\alpha}(\tau,\bar\tau;m) - E_{Q,\alpha}(\tau,\bar\tau)\right)$$

$$= \sum_{i=1}^{\infty}\langle f, u_i\rangle u_i(\tau) + \sum_{j=1}^{N}\langle f, v_j\rangle v_j(\tau) + \sum_{\mathfrak{R}} \frac{1}{4\pi}\int_{-\infty}^{\infty}\left\langle f, E_{\mathfrak{a}}\left(\tau,\tfrac{1}{2}+it\right)\right\rangle E_{\mathfrak{a}}\left(\tau,\tfrac{1}{2}+it\right)dt$$

fluctuation from av. break emergent global sym.

Gauging Global Symmetries

[Ashwinkumar, Kidambi, Leedom, MY (to appear)]

cf. [Dong, Hartman, Jiang ('21)]

* Gauge global sym. in the bulk?

- $G \in \mathrm{Aut}(\mathcal{D})$ anyon sym.

- $\mathcal{D} \to$ " $\underbrace{\left(\mathcal{D}/\sim\right)}_{\text{mod by } G} \oplus \underbrace{\mathcal{D}_G}_{G\text{-anyons}}$ "

* Gauge global sym. in the bulk?

  - $G \in \mathrm{Aut}(\mathcal{D})$ anyon sym.

  - $\mathcal{D} \to$ " $\underbrace{(\mathcal{D}/\sim)}_{\text{mod by } G} \oplus \underbrace{\mathcal{D}_G}_{G\text{- anyons}}$ "

* orbifolding generalized Narain theories

  - $G \in \mathrm{Aut}(\mathcal{Q})$ lattice symmetry

  - $\underbrace{(\text{untwisted sector})}_{\text{projected by } G} + \underbrace{(\text{twisted sectors})}_{\text{rep. of } G}$

We can orbifold ensemble average stories ☺

Ashwinkumar - Kidambi - Leedom - M.Y.

(to appear)

New generalized $\left\{ \begin{array}{l} \text{Eisenstein series} \\ \text{Siegel-Weil formula} \end{array} \right.$

(cf. Zemels ('21) for theta functions)

Summary

"Precision Ensemble Average"

Generalized Narain theory

v.s. Abelian CS in bulk

"Emergent Global Symmetries"

by "folding" ensemble sym (e.g. T-duality)

Lessons from swampland ?

Can we fit everything?