Crystal Melting and Double Quiver Algebras from Jeffrey-Kirwan Residues

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Jan. 7, Strings 2025 @ NYU Abu Dhabi


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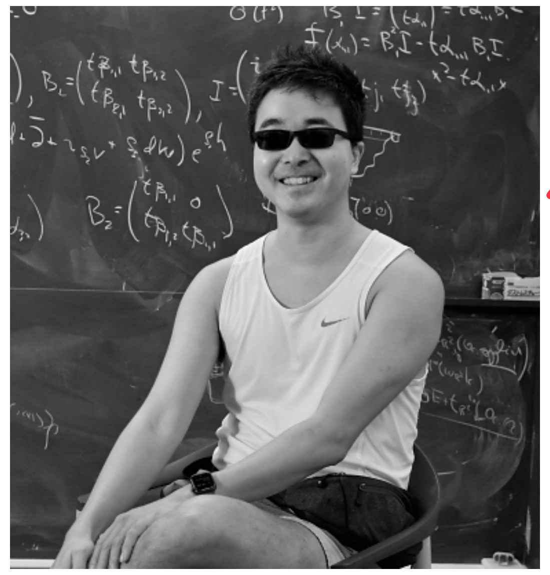
الصبر مفتاح الفرج

“Patience is the key to relief.”



Counting of BPS States !!

- dualities in gauge / string theory
- microstates of black holes
- mathematics of enumerative invariants

⋮

Based on:

Jiakang Bao + MY (To Appear Tonight!!)

See also

Jiakang Bao, Rak-Kyeong Seong + MY (2401.02792)

and earlier papers with

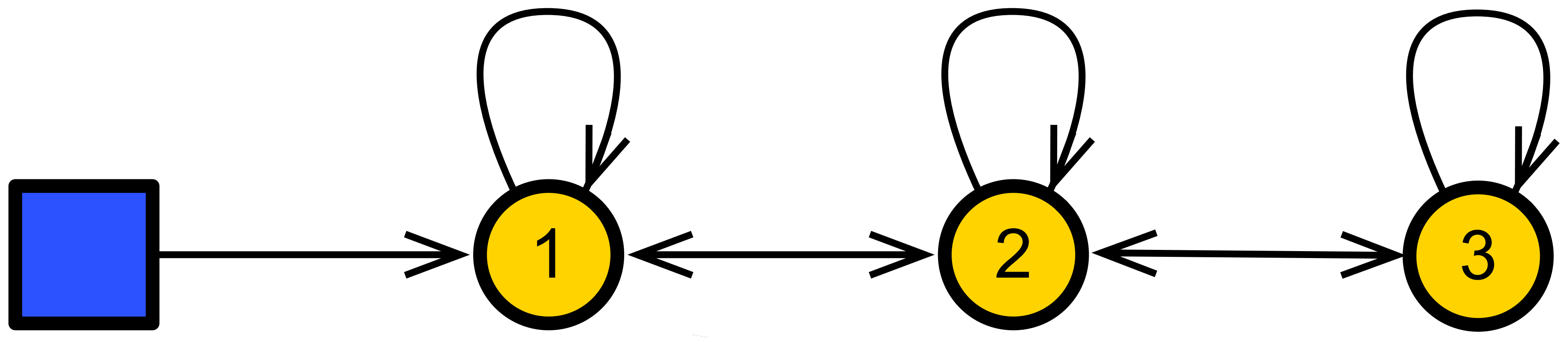
Dimitry Galakhov, Wei Li, Hirosi Ooguri, ... (2008–)

Today: BPS / Witten indices

for $\mathcal{N} \geq 2$ quiver gauge theories

4d $\mathcal{N}=1$

$\mathcal{N}=4$

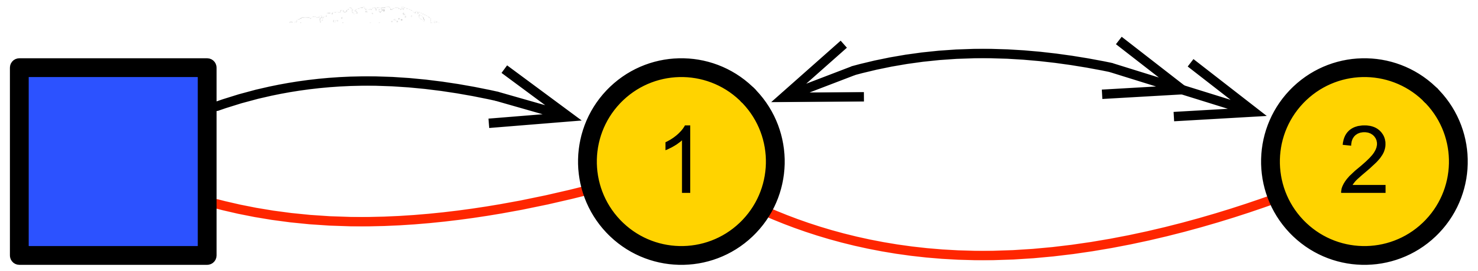


$+$ F - term

($\longrightarrow$ chiral)

2d $\mathcal{N}=(0,2)$

$\mathcal{N}=2$



$+$ J / E - term

($\longrightarrow$ chiral — Fermi)

Main Results in [Bao-MY]

① Counting

$$Z_{BPS} = Z_{crystal}$$

$$= \sum_{e} \omega(e)$$

(e : crystal state

combinatorial description

$\omega(e)$: weight)

② Algebra

$$Z_{BPS} = \chi_{\tilde{Y}}$$

$$\tilde{Y} \supseteq \mathcal{H} = \{ |e\rangle \}$$

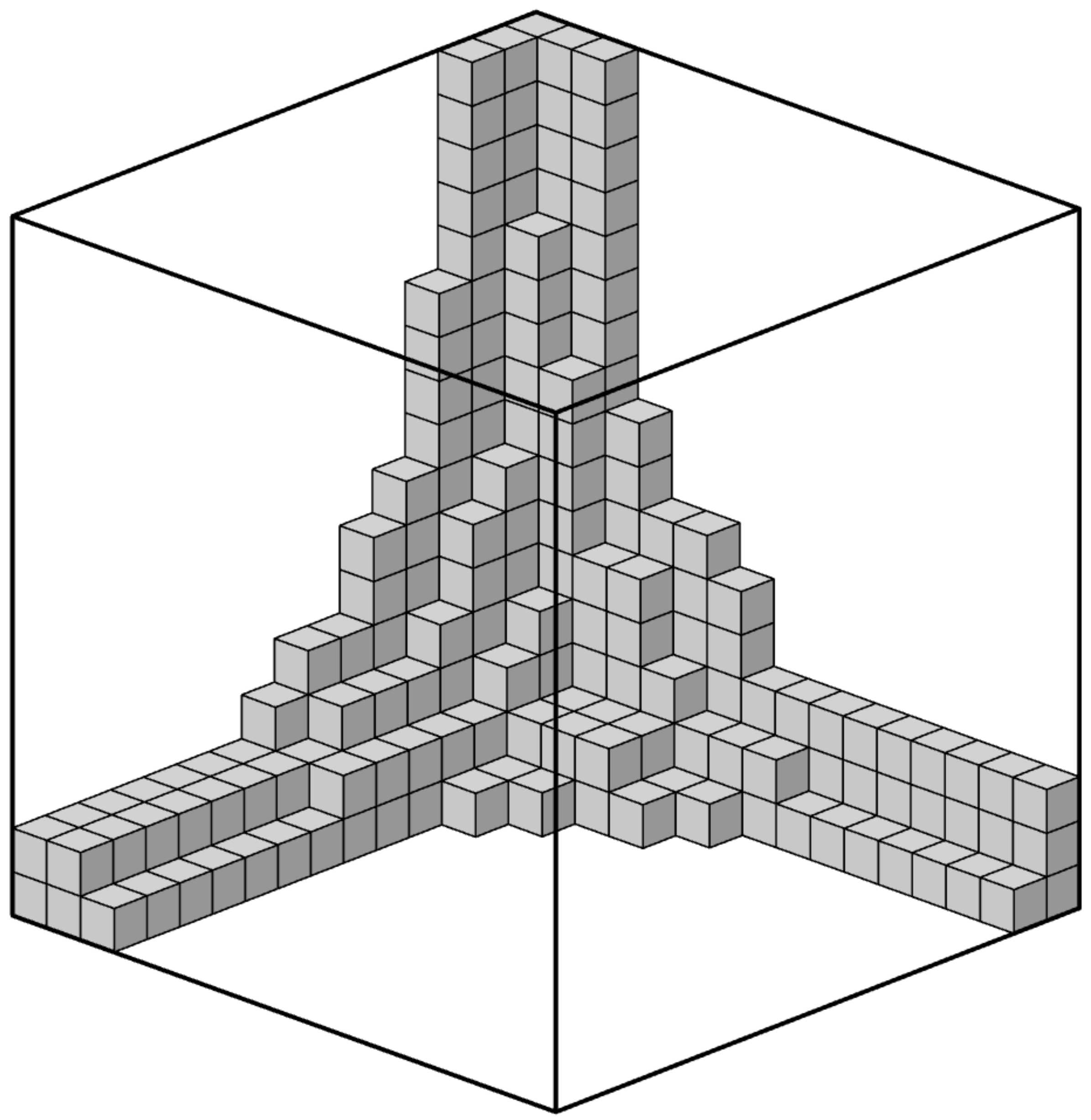
representation R

($\tilde{Y}$: new algebra

double quiver algebra)

Counting

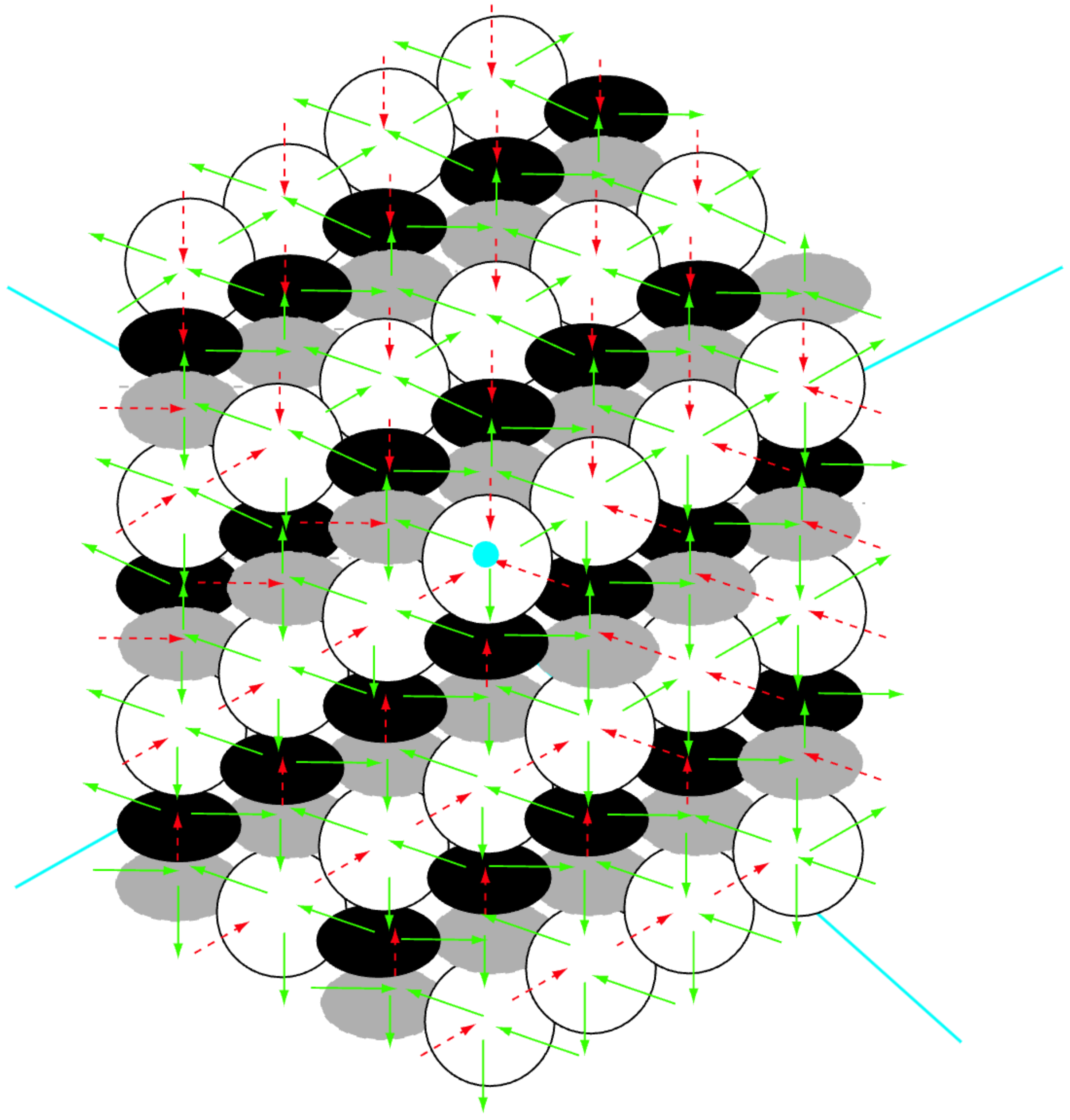
< Hint 1: Crystal Melting for toric CY3 / CY4 >



$\mathbb{C}^3$

[Okounkov-Reshetikhin-Vafa '03]

C. Vafa @ Strings 2004

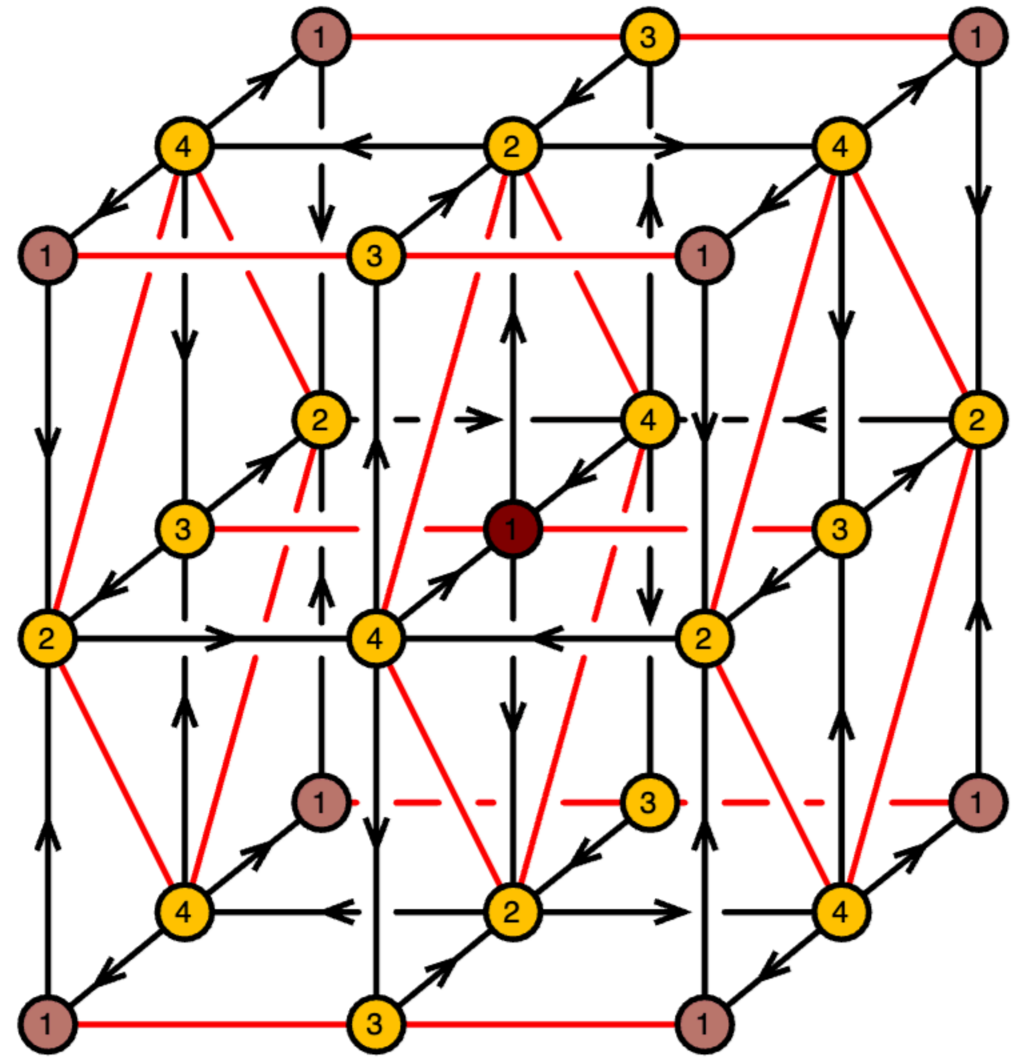


Toric CY3

[Ooguri - MY '08]

H. Ooguri @ Strings 2009

3d crystal



toric CY4

[Bao-Soong-MY '24]

4d crystal

⟨Hint 2 : JK residue formula for index⟩

[2d on T^2 : Benini - Eager - Hori - Tachikawa '13
1d on S^1 : Hori - Kim - Yi, Cordova - Shao '14]

$$Z(\epsilon) = \frac{1}{|W|} \sum_{u^* \in \mathcal{M}^*} \underset{u=u^*}{\text{JK-Res}}(\eta)\, Z_{\text{1-loop}}(u, \epsilon)$$

$\langle$ Hint 2 : JK residue formula for index $\rangle$

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1d on S^1 : Hori - Kim - Yi, Cordova - Shao '14]

$$Z(\epsilon) = \frac{1}{|W|} \sum_{u^* \in \mathcal{M}^*} \underbrace{\underset{u=u^*}{\text{JK-Res}}(\eta)}_{\text{iterative residue}} Z_{1\text{-loop}}(u, \epsilon)$$

[Szenes - Vergne '03]

⟨Hint 2 : JK residue formula for index⟩

[2d on T^2 : Benini-Eager-Hori-Tachikawa '13
1d on S^1 : Hori-Kim-Yi, Cordova-Shao '14]

$$Z(\epsilon) = \frac{1}{|W|} \sum_{u^* \in \mathcal{M}^*} \underset{u=u^*}{\text{JK-Res}}(\underbrace{\eta}) \, Z_{1\text{-loop}}(u,\epsilon)$$

"separates particle/anti-particle"

(moduli dependence)

⟨Hint 2 : JK residue formula for index⟩

[2d on T^2 : Benini–Eager–Hori–Tachikawa '13
1d on S^1 : Hori–Kim–Yi, Cordova–Shao '14]

$$Z(\epsilon) = \frac{1}{|W|} \sum_{u^* \in \mathcal{M}^*} \underset{u=u^*}{\text{JK-Res}}(\eta)\, Z_{1\text{-loop}}(u, \epsilon)$$

flavor chemical potential

singular point in the plane of $u = \{u_i\}_{i=1}^N \in \mathfrak{h} \subset \mathfrak{g}$

1-loop determinant

⟨Hint 2 : JK residue formula for index⟩

[2d on T^2 : Benini-Eager-Hori-Tachikawa '13
1d on S^1 : Hori-Kim-Yi, Cordova-Shao '14]

$$Z(\epsilon) = \frac{1}{|W|} \sum_{u^* \in \mathcal{M}^*} \underbrace{\underset{u=u^*}{\text{JK-Res}}(\eta)}_{\text{iterative residue}} Z_{1\text{-loop}}(u, \epsilon)$$

iterative residue [Szenes-Vergne '03]

Sum over crystal state

growth of crystal molecules from atoms

$Z_{\text{1-loop}}$ contains contributions from different multiplets

e.g. bifundamental $\mathcal{N}=2$ chiral w/ charge ϵ_χ

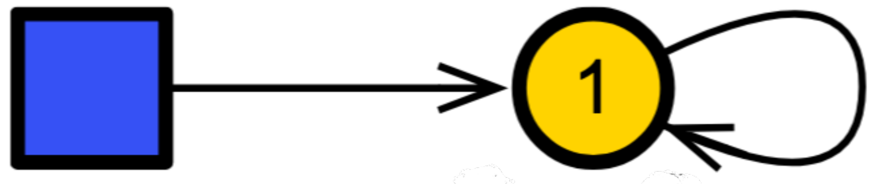
$$
\begin{array}{ccc}
U(N_s) & \xrightarrow{\chi} & U(N_t) \\
\{u_i^{(s)}\} & & \{u_j^{(t)}\}
\end{array}
$$

$$Z_\chi(\epsilon, u) = \prod_{i=1}^{N_s} \prod_{j=1}^{N_t} \frac{1}{\zeta\left(u_j^{(t)} - u_i^{(s)} - \epsilon_\chi\right)}$$

Pole @ $u_j^{(t)} - u_i^{(s)} - \epsilon_\chi = 0$

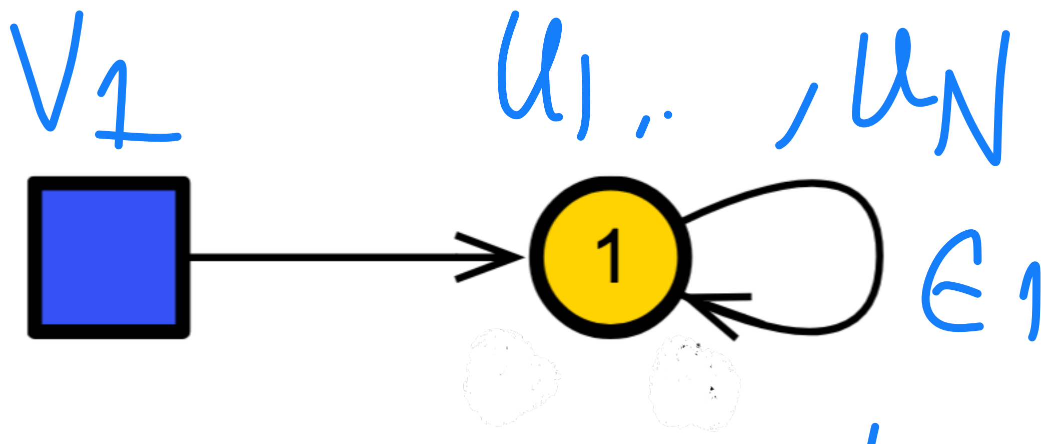
$$\left(\zeta(z) = \begin{cases} \dfrac{\mathrm{i}\theta_1(\tau, z)}{\eta(\tau)}, & \leftarrow \text{ell.} \quad (\text{2d on } T^2) \\ 2\mathrm{i}\sin(\pi z), & \leftarrow \text{trig.} \quad (\text{1d on } S^1) \\ z, & \leftarrow \text{rational} \quad (\text{0d}) \end{cases} \right)$$

Example



($\mathcal{N}=4$ quiver, $W=0$)

Example



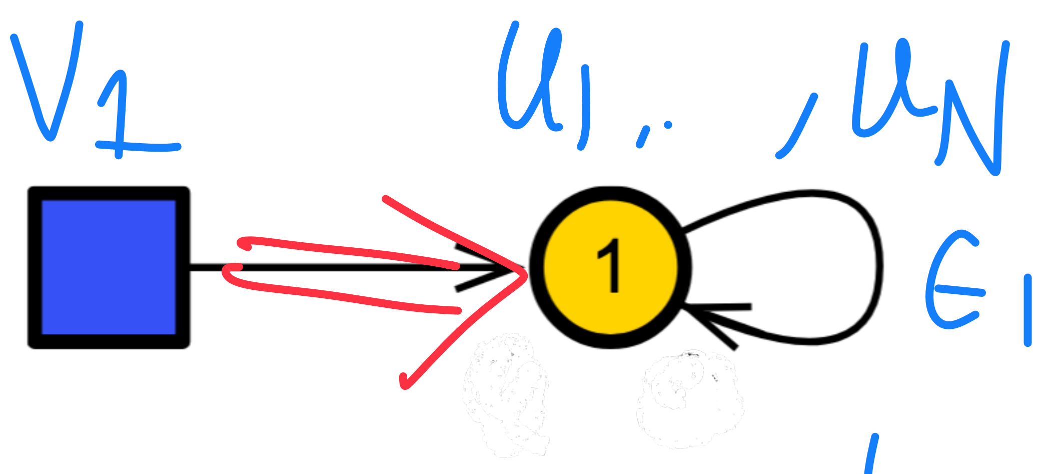
($\mathcal{N}=4$ quiver, $W=0$)

$$\prod_{i=1}^{N} \frac{-\zeta(u_i+\epsilon-v_1)}{\zeta(u_i-v_1)}$$

$$\frac{-\zeta(\epsilon-\epsilon_1)}{\zeta(-\epsilon_1)} \prod_{i \neq j}^{N} \frac{-\zeta(u_i-u_j+\epsilon-\epsilon_1)}{\zeta(u_i-u_j-\epsilon_1)}$$

charge of chiral.

Example



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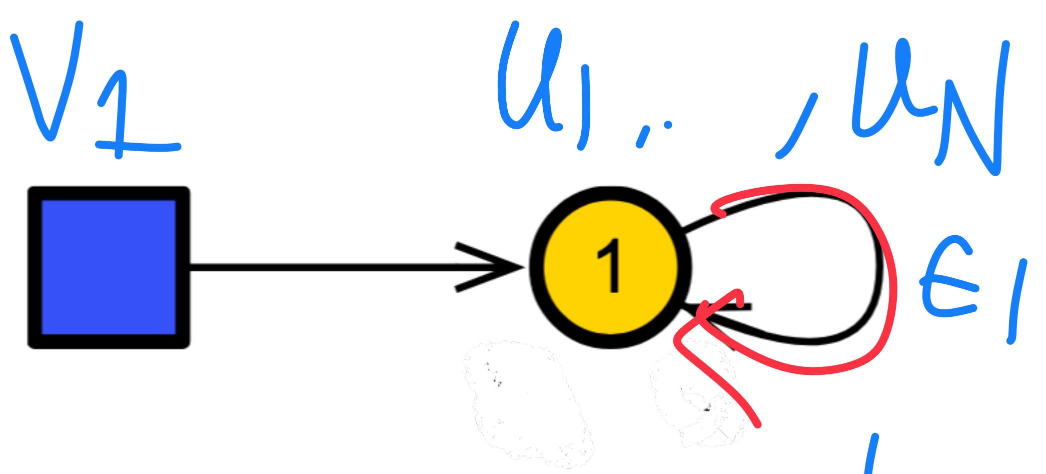
$N=1$ $\quad \{u_1 - v_1 = 0\}$ $\quad u_1^* = v_1$

Example



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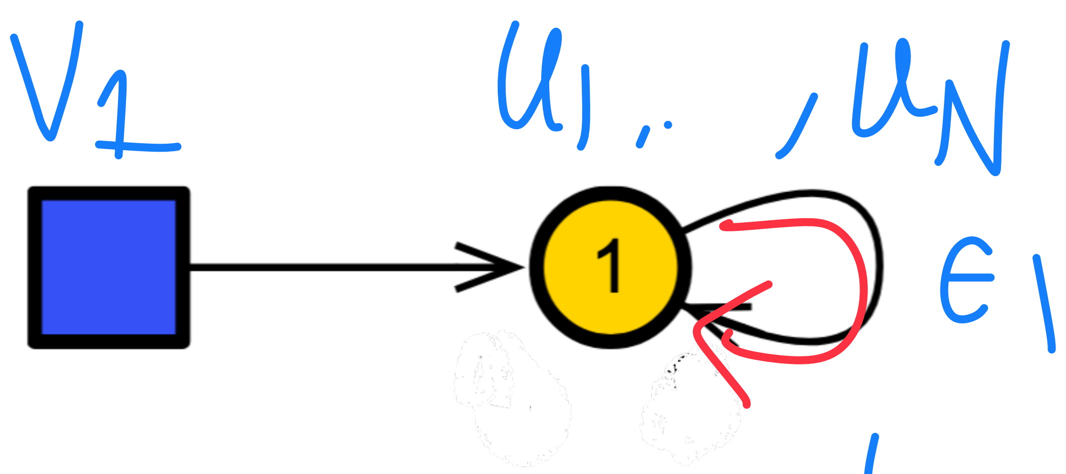
$N=2$ $\quad \{u_1 - v_1 = 0,\ u_2 - u_1 = \epsilon_1\}$ $\quad u_2^* = v_1 + \epsilon_1$

Example



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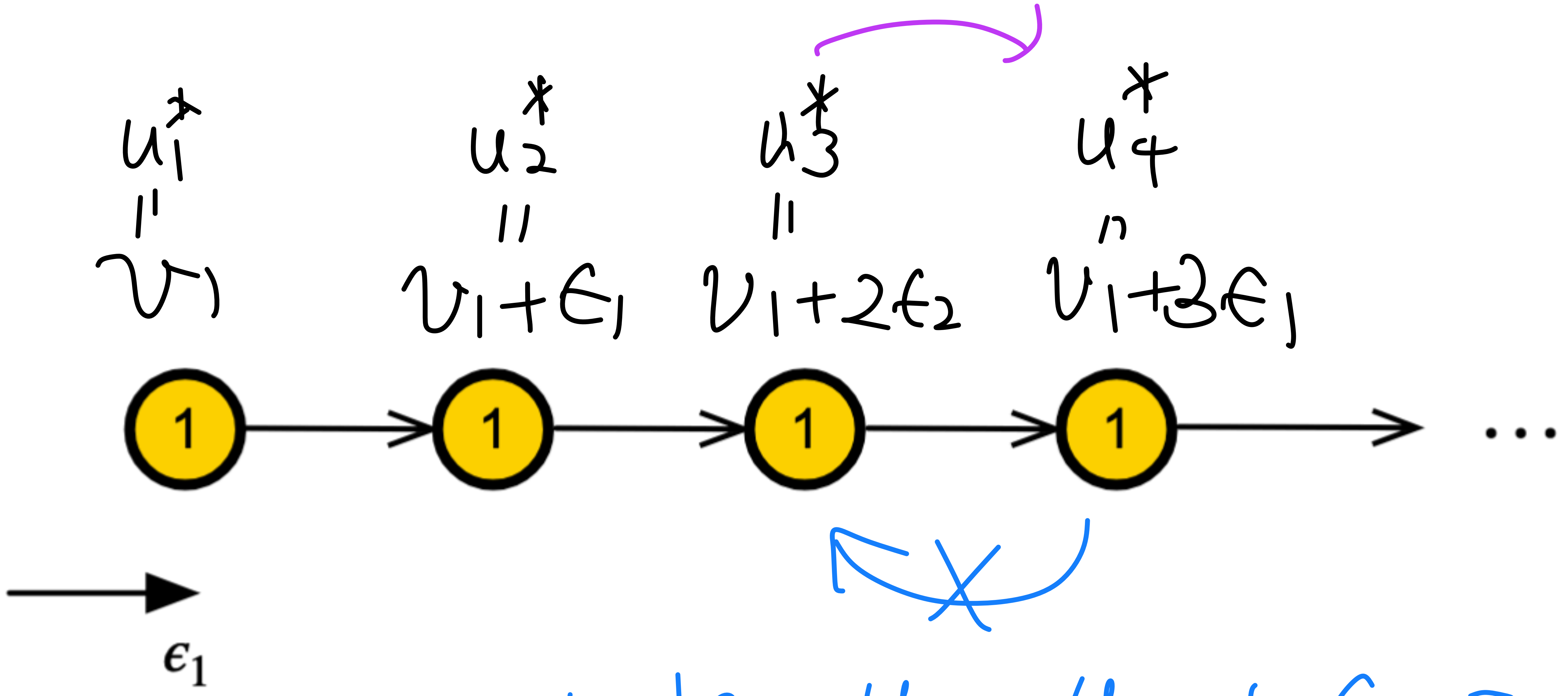
$N=2$ $\quad \{u_1 - v_1 = 0,\ u_2 - u_1 = \epsilon_1\}$ $\quad u_2^* = v_1 + \epsilon_1$

$N=3$ $\quad \{u_1 - v_1 = 0,\ u_2 - u_1 = \epsilon_1,\ u_3 - u_2 = \epsilon_1\}$ $\quad u_3^* = v_1 + 2\epsilon_1$

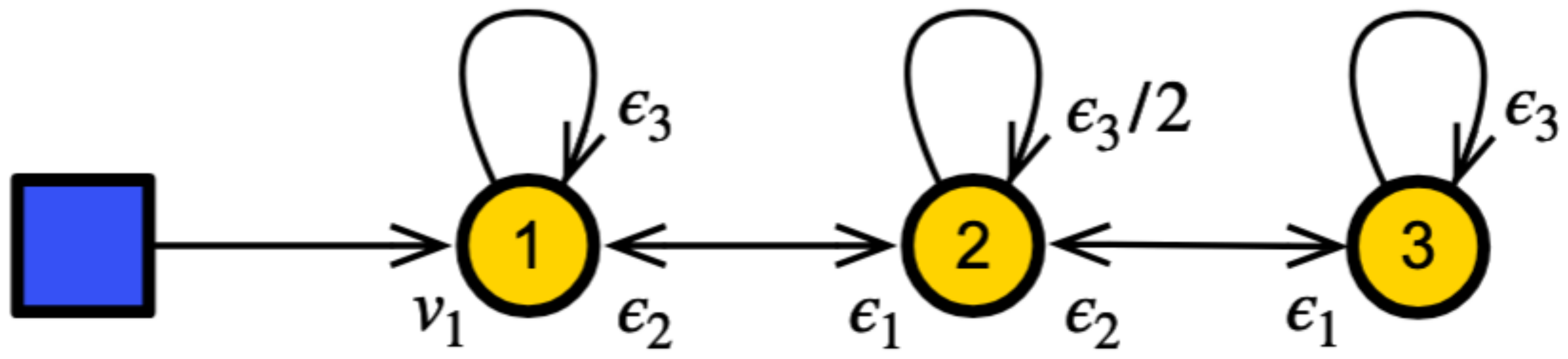
pole $u_4 - u_3 - \epsilon_1 = 0$ "particle"



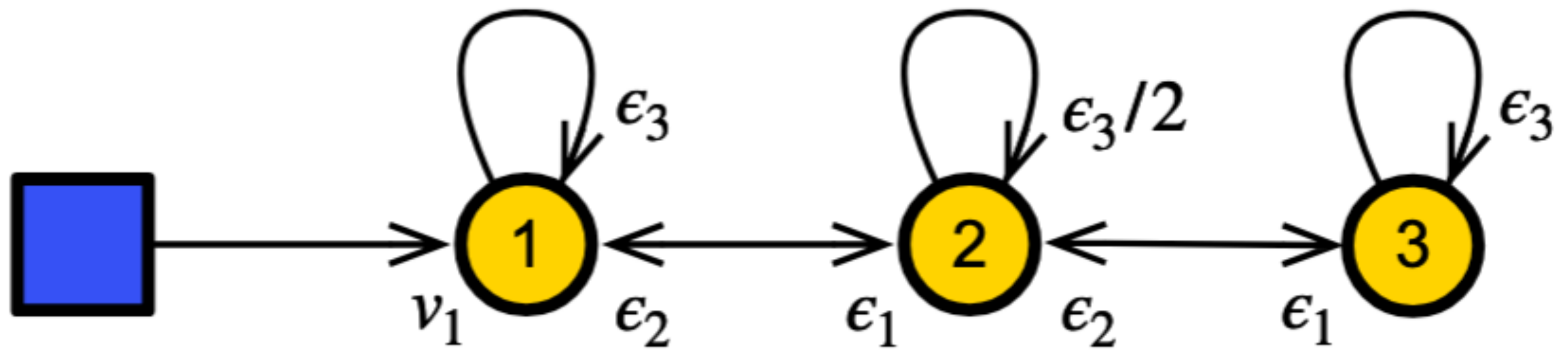
pole $u_4 - u_3 + \epsilon_1 = 0$ "anti-particle"

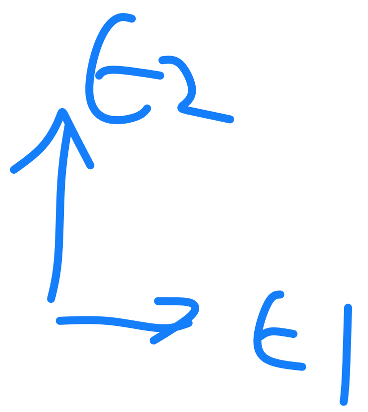
but not allowed under ?

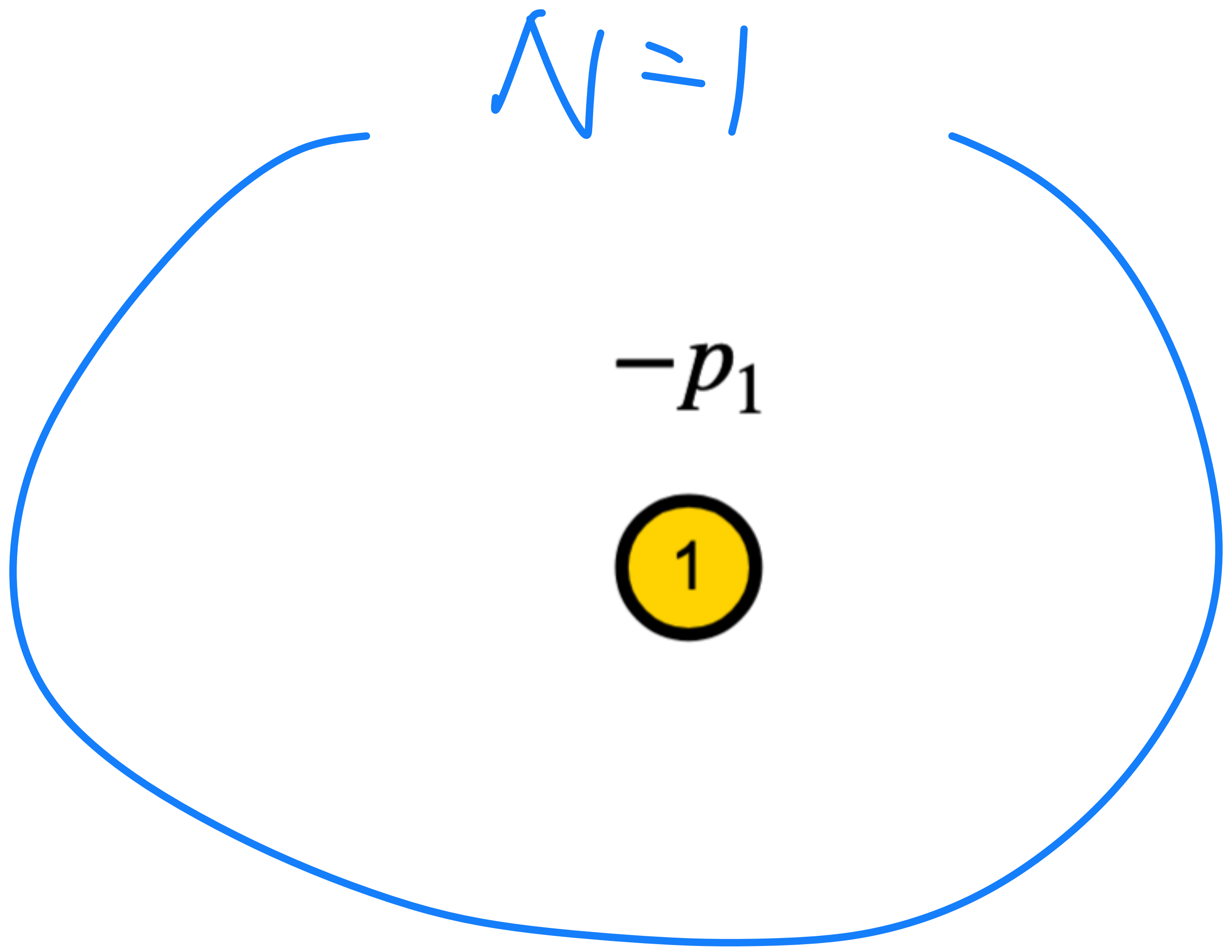
Example : $C_2^{(1)}$ Theory

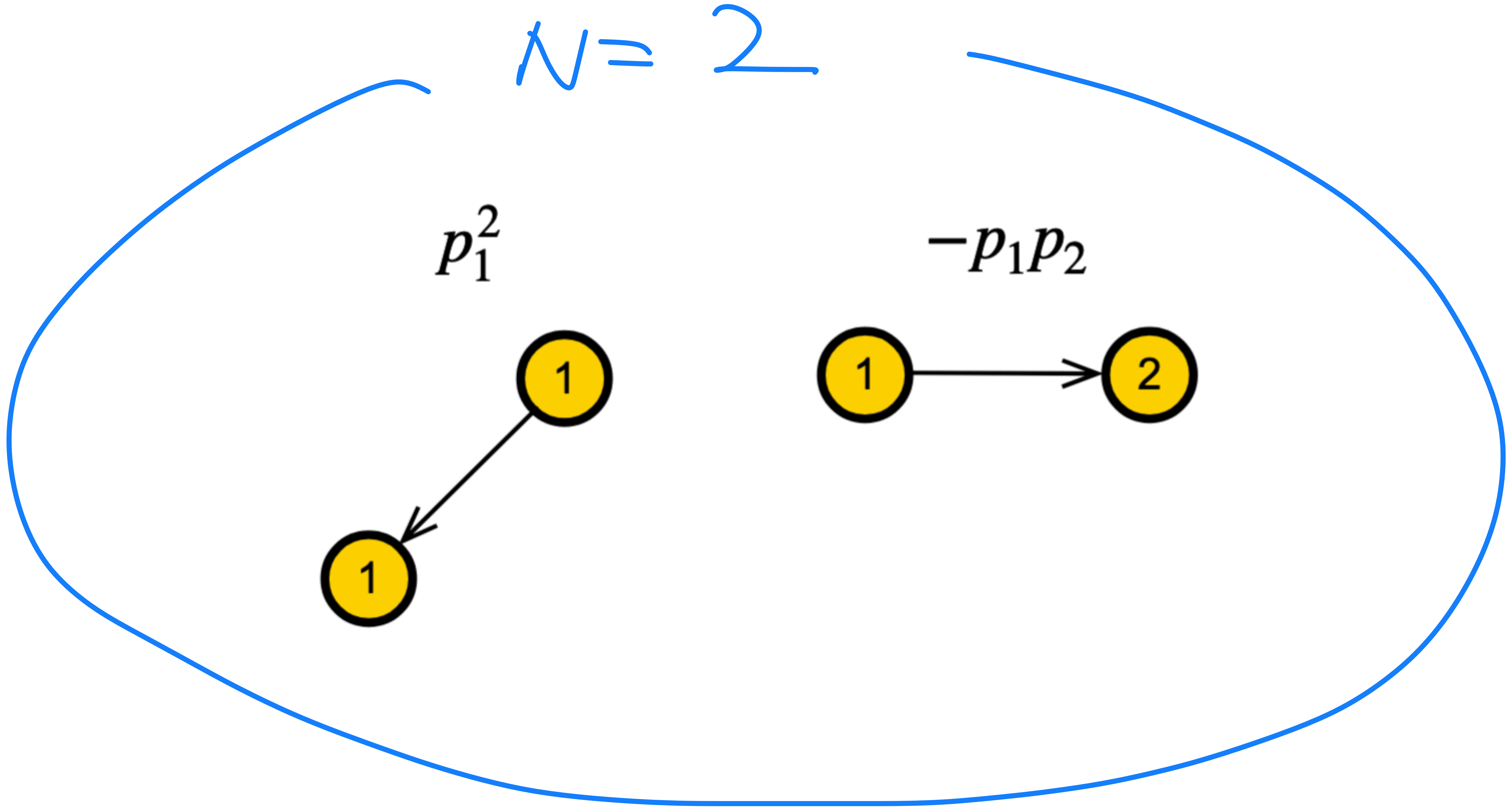


Example : $C_2^{(1)}$ Theory

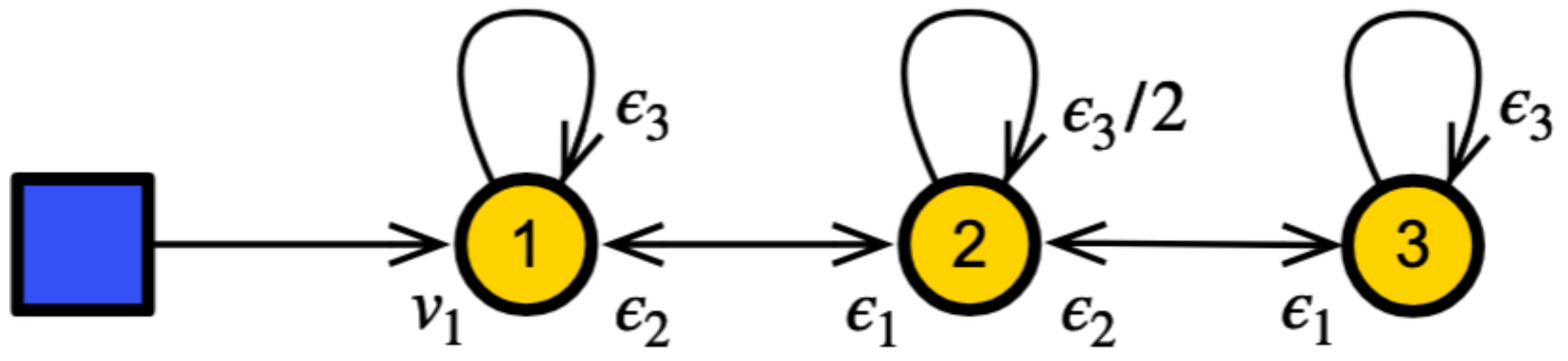






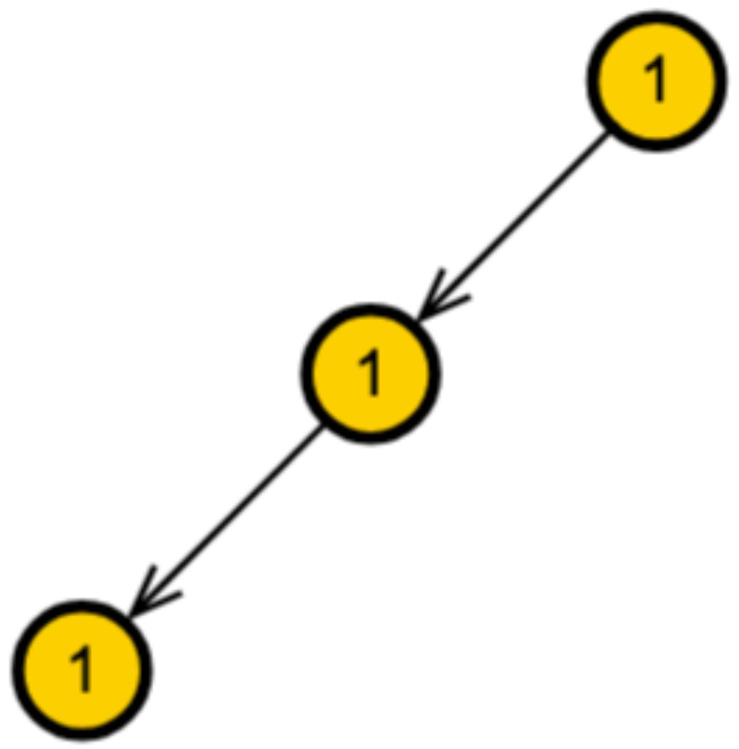


Example: $C_2^{(1)}$ Theory

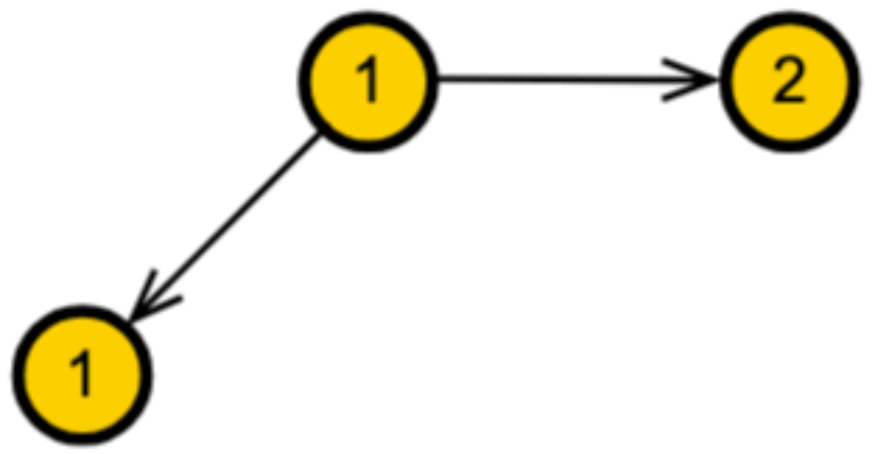


$N = 3$

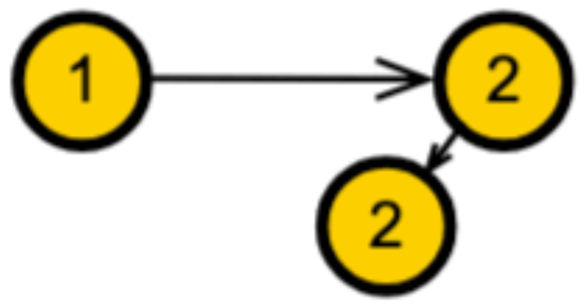
$-p_1^3$



$p_1^2 p_2$



$-p_1 p_2^2$



$-p_1 p_2 p_3$

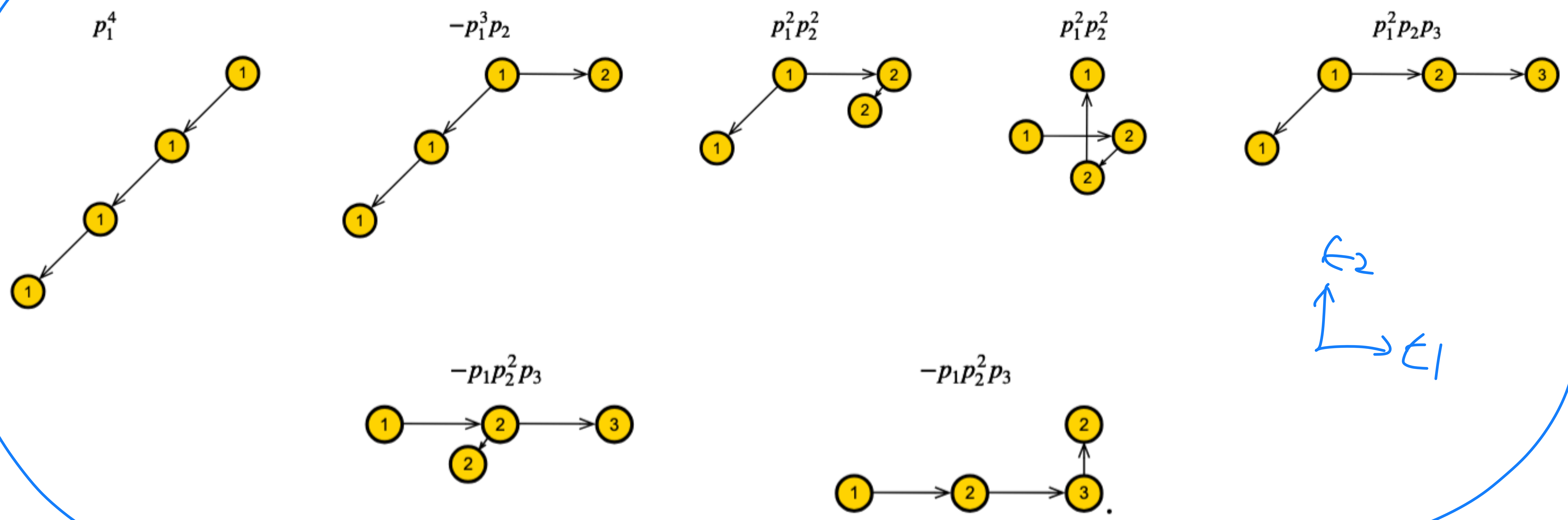
ϵ_2

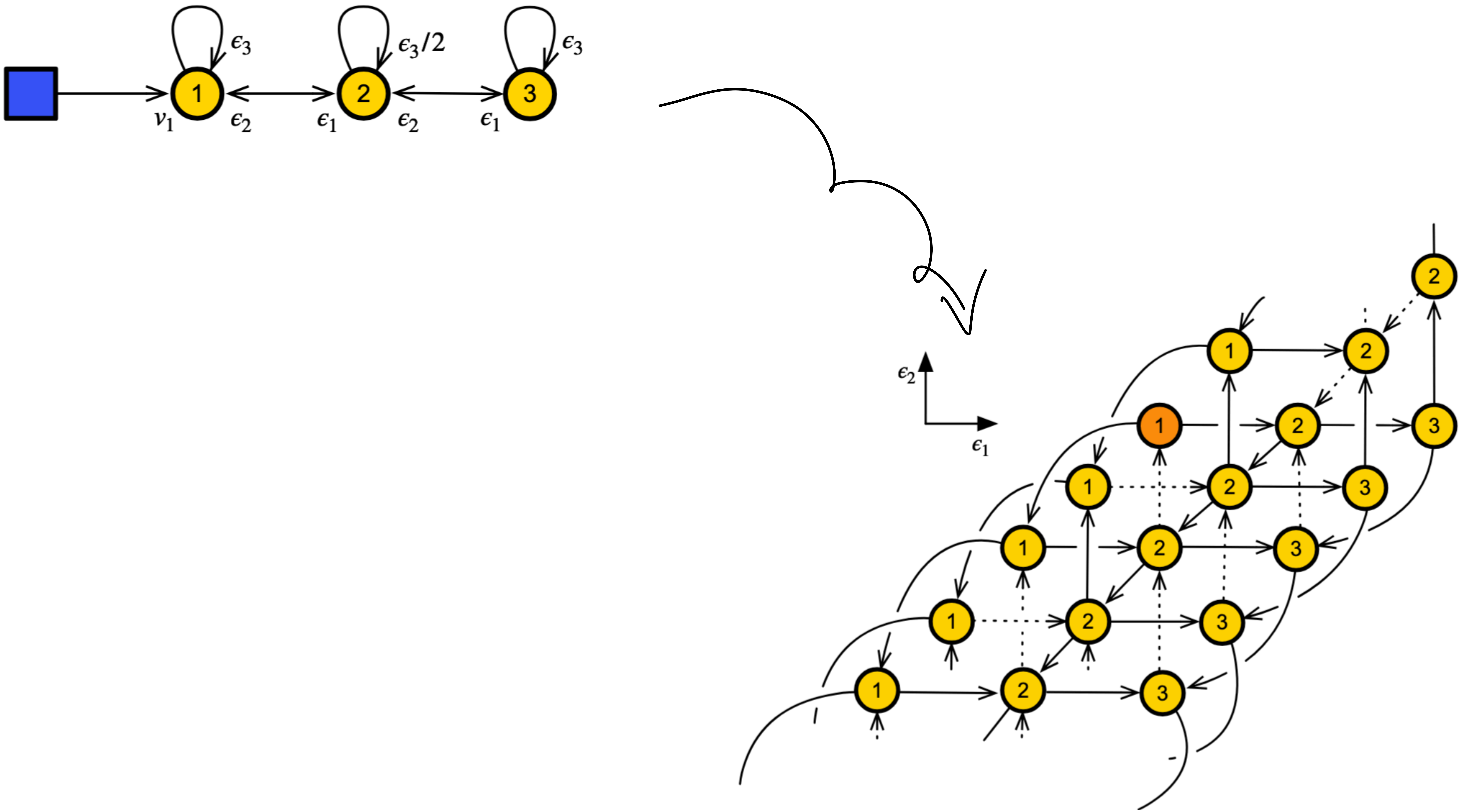
$\uparrow$

$\to \epsilon_1$

Example: $C_2^{(1)}$ Theory

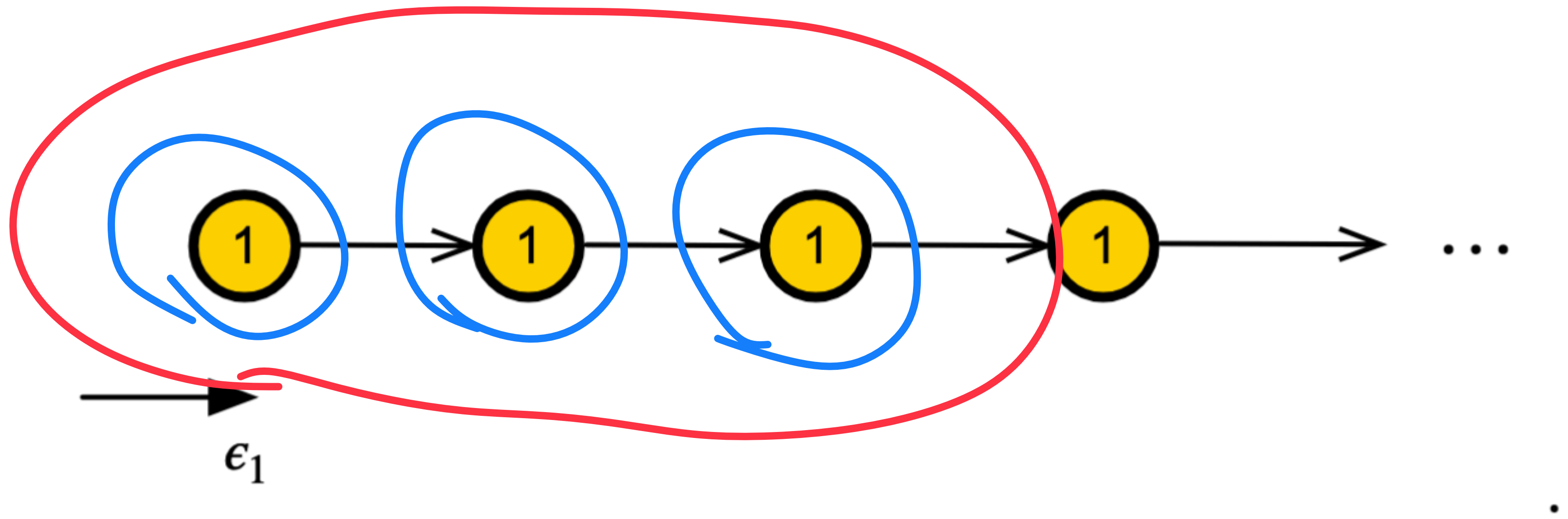
$N = 4$

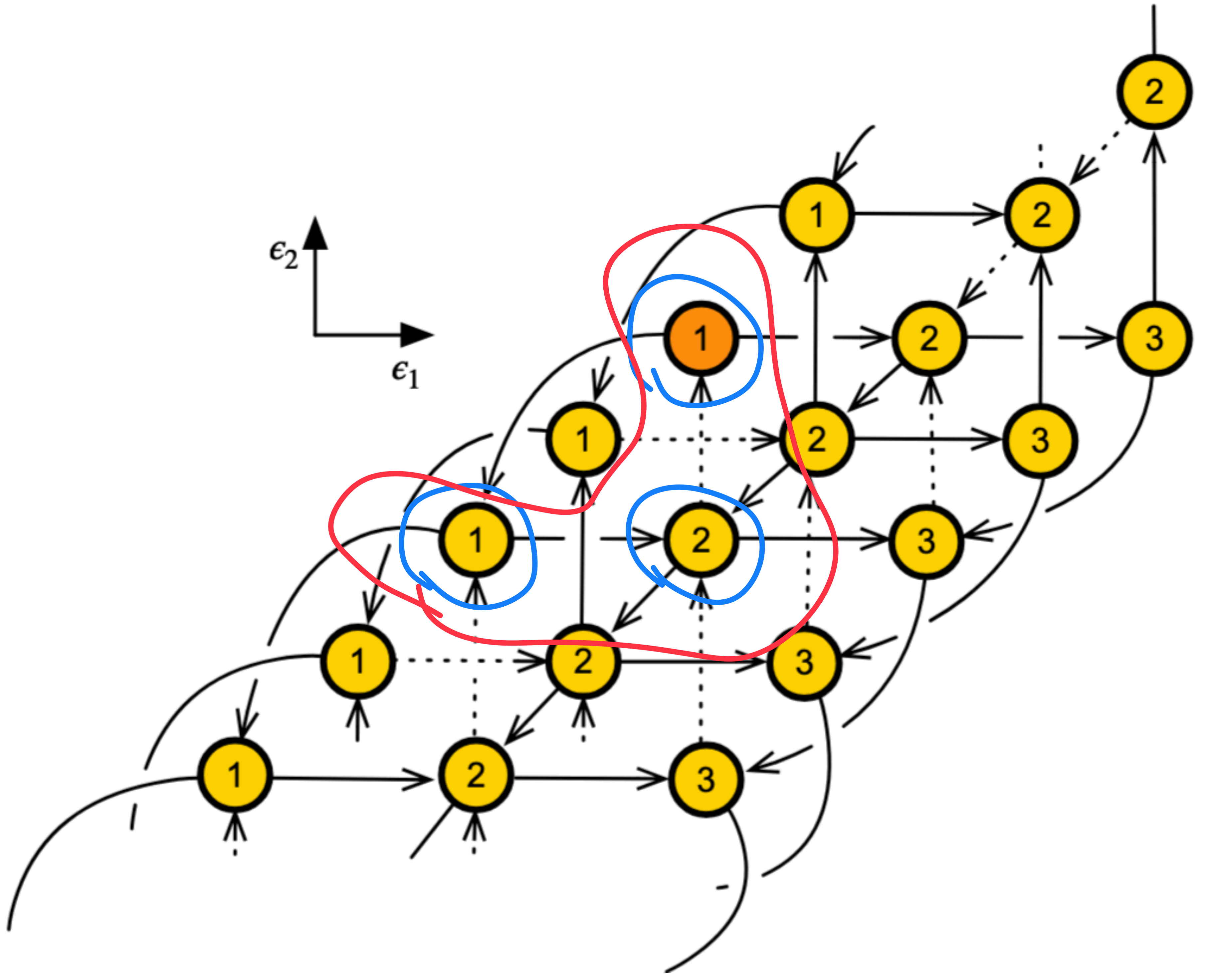




ϵ_3
$\epsilon_3/2$
ϵ_3
1
2
3
v_1
ϵ_2
ϵ_1
ϵ_2
ϵ_1
ϵ_2
ϵ_1

BPS state = ("molecules growing from origin")





- This should NOT work for arbitrary quiver + relations

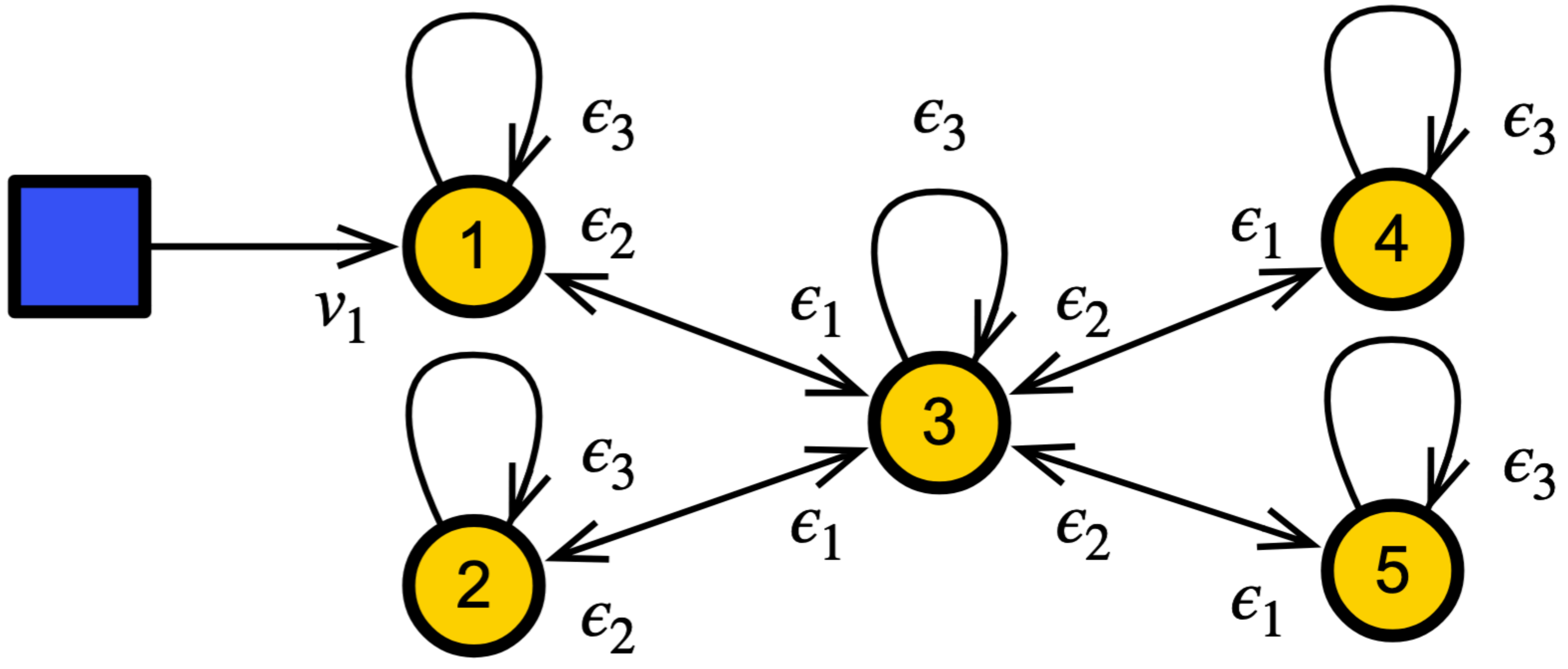
  (e.g. no flavor sym, ~~SUSY~~, ...)

- We here assume non-degeneracy of the residue

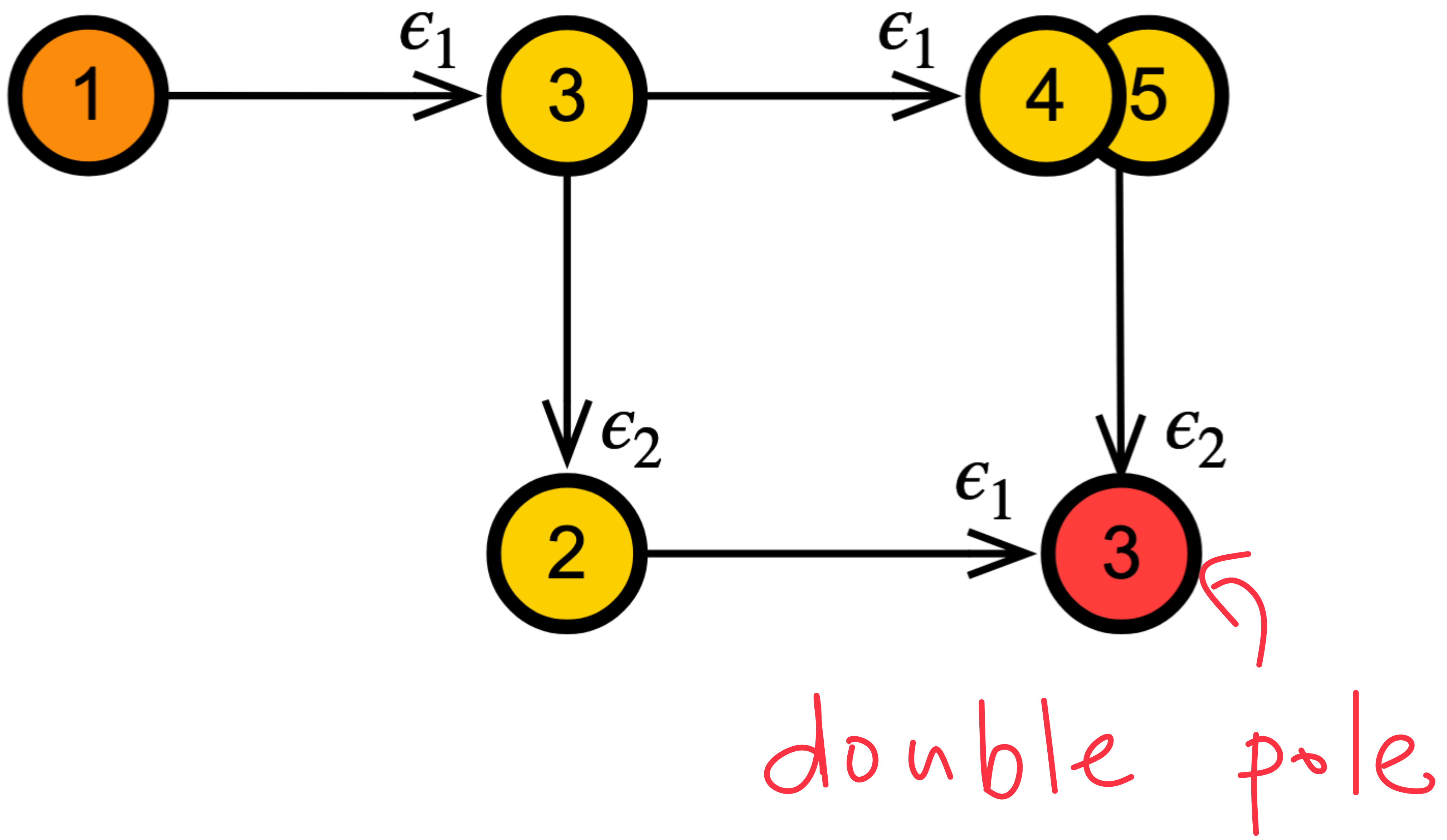
  ("no higher-order pole")

This is violated when $\exists$ "overlaps in crystal"

"No - Overlap Condition"



$D_4^{(1)}$



Double Quiver Algebra

< Hint : Single Quiver Algebra > [Li-MY '20, Galakhov-Li-MY '21]

For $\mathcal{N}=4$ quivers associated w/ toric CY_3

$\exists$ known crystal rep. of single quiver algebra Y

[generalizes (affine Yangian / quantum toroidal alg) for $\mathfrak{gl}_N$]

known algebra:

(single) quiver algebra Y

$\rightsquigarrow$

new algebra:

double quiver algebra $\tilde{\mathsf{Y}}$

more direct connection to JK residue

"particle"

$\rightsquigarrow$

"particle + anti-particle"

Double Quiver Algebra $\tilde{Y}$

- generators : $\tilde{e}^a(z)$, $\tilde{f}^a(z)$, $\tilde{\psi}^a(z)$, $\tilde{\omega}^a(z)$ (a : quiver vertex)

- "bond factor"

$$\tilde{\varphi}^{a\Leftarrow b}(z) := \begin{cases} \dfrac{\zeta(z)\zeta(-z)}{\zeta(z+\epsilon)\zeta(-z+\epsilon)} \left(\displaystyle\prod_{I\in\{a\to a\}} \frac{-\zeta\left(\epsilon-\epsilon_I\right)}{\zeta(-\epsilon_I)} \frac{\zeta(z+\epsilon-\epsilon_I)\zeta(z-\epsilon+\epsilon_I)}{\zeta(z-\epsilon_I)\zeta(z+\epsilon_I)}\right), & b=a\,, \\ \left(\displaystyle\prod_{I\in\{a\to b\}} \frac{-\zeta(z-\epsilon+\epsilon_I)}{\zeta(z+\epsilon_I)}\right)\left(\displaystyle\prod_{I\in\{b\to a\}} \frac{-\zeta(z+\epsilon-\epsilon_I)}{\zeta(z-\epsilon_I)}\right), & b\neq a\,. \end{cases}$$

$$\zeta(z) = \begin{cases} \dfrac{\mathrm{i}\theta_1(\tau,z)}{\eta(\tau)}\,, \\ 2\mathrm{i}\sin(\pi z)\,, \\ z\,, \end{cases}$$

OPE-like relations

$$\widetilde{\omega}^{(a)}(z)\widetilde{\omega}^{(b)}(w) \simeq \widetilde{\omega}^{(b)}(w)\widetilde{\omega}^{(a)}(z)\,,$$
$$\widetilde{\psi}_{+}^{(a)}(z)\widetilde{\psi}_{+}^{(b)}(w) \simeq \widetilde{\psi}_{+}^{(b)}(w)\widetilde{\psi}_{+}^{(a)}(z)\,,$$
$$\widetilde{\psi}_{-}^{(a)}(z)\widetilde{\psi}_{-}^{(b)}(w) \simeq \widetilde{\phi}^{a\Leftarrow b}(z-w+2c)\widetilde{\phi}^{a\Leftarrow b}(z-w-2c)^{-1}\widetilde{\psi}_{-}^{(b)}(w)\widetilde{\psi}_{-}^{(a)}(z)\,,$$
$$\widetilde{\psi}_{+}^{(a)}(z)\widetilde{\psi}_{-}^{(b)}(w) \simeq \widetilde{\phi}^{a\Leftarrow b}(z-w+c)\widetilde{\phi}^{a\Leftarrow b}(z-w-c)^{-1}\widetilde{\psi}_{-}^{(b)}(w)\widetilde{\psi}_{+}^{(a)}(z)\,,$$
$$\widetilde{\psi}_{\pm}^{(a)}(z)\widetilde{\omega}^{(b)}(w) \simeq \widetilde{\phi}^{a\Leftarrow b}(z-w+c\mp c/2)^{-1}\widetilde{\phi}^{a\Leftarrow b}(z-w-c\pm c/2)\widetilde{\omega}^{(b)}(w)\widetilde{\psi}_{\pm}^{(a)}(z)\,,$$
$$\widetilde{\psi}_{\pm}^{(a)}(z)\widetilde{e}^{(b)}(w) \simeq \widetilde{\phi}^{a\Leftarrow b}(z-w+c\mp c/2)\widetilde{e}^{(b)}(w)\widetilde{\psi}_{\pm}^{(a)}(z)\,,$$
$$\widetilde{\psi}_{\pm}^{(a)}(z)\widetilde{f}^{(b)}(w) \simeq \widetilde{\phi}^{a\Leftarrow b}(z-w-c\pm c/2)^{-1}\widetilde{f}^{(b)}(w)\widetilde{\psi}_{\pm}^{(a)}(z)\,,$$

bond factors

OPE-like relations

$$\delta(z-w)\widetilde{\phi}^{d\Leftarrow a}(u-z-c)\widetilde{e}^{(d)}(u)\widetilde{\omega}^{(a)}(z)+\delta(u-w)\widetilde{\phi}^{a\Leftarrow d}(z-w-c)\widetilde{e}^{(a)}(z)\widetilde{\omega}^{(d)}(w)$$
$$\simeq \delta(z-w)\widetilde{\omega}^{(a)}(z)\widetilde{e}^{(d)}(u)+\delta(u-w)\widetilde{\omega}^{(d)}(u)\widetilde{e}^{(a)}(z)\ ,$$
$$\delta(z-w)\widetilde{\phi}^{d\Leftarrow a}(u-z-c)^{-1}\widetilde{f}^{(d)}(u)\widetilde{\omega}^{(a)}(z)+\delta(u-w)\widetilde{\phi}^{a\Leftarrow d}(z-w-c)^{-1}\widetilde{f}^{(a)}(z)\widetilde{\omega}^{(d)}(w)$$
$$\simeq \delta(z-w)\widetilde{\omega}^{(a)}(z)\widetilde{f}^{(d)}(u)+\delta(u-w)\widetilde{\omega}^{(d)}(u)\widetilde{f}^{(a)}(z)\ ,$$
$$\widetilde{e}^{(a)}(z)\widetilde{e}^{(b)}(w)\simeq(-1)^{|a||b|}\widetilde{e}^{(b)}(w)\widetilde{e}^{(a)}(z)\ ,$$
$$\widetilde{f}^{(a)}(z)\widetilde{f}^{(b)}(w)\simeq(-1)^{|a||b|}\widetilde{f}^{(b)}(w)\widetilde{f}^{(a)}(z)\ ,$$
$$\widetilde{\phi}^{a\Leftarrow b}(z-w-c)\widetilde{e}^{(a)}(z)\widetilde{f}^{(b)}(w)-(-1)^{|a||b|}\widetilde{f}^{(b)}(w)\widetilde{e}^{(a)}(z)$$
$$\simeq\delta_{ab}\left(\delta(z-w-c)\widetilde{\psi}_{+}^{(a)}(w+c/2)-\delta(z-w+c)\widetilde{\psi}_{-}^{(a)}(z+c/2)-\delta(z-w)\widetilde{\omega}^{(a)}(z)\right)\ .$$

Representation of $\check{Y}$ on crystal

$$
\left\{
\begin{aligned}
\widetilde{\psi}^{(a)}_{\pm}(z)|\mathscr{C}\rangle &= \begin{cases} \widetilde{\Psi}^{(a)}_{\mathscr{C}}(z)|\mathscr{C}\rangle\,, & \text{rational}\,, \\ \left[\widetilde{\Psi}^{(a)}_{\mathscr{C}}(Z)\right]_{\pm}|\mathscr{C}\rangle\,, & \text{trigonometric/elliptic}\,, \end{cases} \\
\widetilde{\omega}^{(a)}(z)|\mathscr{C}\rangle &= \sum_{\mathrm{Inad}\left(\Psi^{(a)}_{\mathscr{C}}(z),\eta\right)} \delta(z-\epsilon_{\mathfrak{a}}) \lim_{x=\epsilon_{\mathfrak{a}}} \zeta(x-\epsilon_{\mathfrak{a}})\widetilde{\Psi}^{(a)}_{\mathscr{C}}(x)|\mathscr{C}\rangle\,, \\
\widetilde{e}^{(a)}(z)|\mathscr{C}\rangle &= \sum_{\mathfrak{a}\in\mathrm{Add}(\mathscr{C})} \pm\delta(z-\epsilon_{\mathfrak{a}})) \left(\pm \lim_{x=\epsilon_{\mathfrak{a}}} \zeta(x-\epsilon_{\mathfrak{a}})\widetilde{\Psi}^{(a)}_{\mathscr{C}}(x)\right)^{1/2} |\mathscr{C}+\mathfrak{a}\rangle\,, \\
\widetilde{f}^{(a)}(z)|\mathscr{C}\rangle &= \sum_{\mathfrak{a}\in\mathrm{Rem}(\mathscr{C})} \pm\delta(z-\epsilon_{\mathfrak{a}}) \left(\pm \lim_{x=\epsilon_{\mathfrak{a}}} \zeta(x-\epsilon_{\mathfrak{a}})\widetilde{\Psi}^{(a)}_{\mathscr{C}}(x)\right)^{1/2} |\mathscr{C}-\mathfrak{a}\rangle\,,
\end{aligned}
\right.
$$

diagonal

create

annihilate

poles inadmissible by η

$$\left\{\begin{aligned} \widetilde{\psi}^{(a)}_{\pm}(z)|\mathscr{C}\rangle &= \begin{cases} \widetilde{\Psi}^{(a)}_{\mathscr{C}}(z)|\mathscr{C}\rangle\,, & \text{rational}\,,\\ \left[\widetilde{\Psi}^{(a)}_{\mathscr{C}}(Z)\right]_{\pm}|\mathscr{C}\rangle\,, & \text{trigonometric/elliptic}\,,\end{cases}\\ \widetilde{\omega}^{(a)}(z)|\mathscr{C}\rangle &= \sum_{\mathrm{Inad}\left(\Psi^{(a)}_{\mathscr{C}}(z),\eta\right)} \delta(z-\epsilon_{\mathfrak{a}}) \lim_{x=\epsilon_{\mathfrak{a}}} \zeta(x-\epsilon_{\mathfrak{a}})\widetilde{\Psi}^{(a)}_{\mathscr{C}}(x)|\mathscr{C}\rangle\,,\\ \widetilde{e}^{(a)}(z)|\mathscr{C}\rangle &= \sum_{\mathfrak{a}\in\mathrm{Add}(\mathscr{C})} \pm\delta(z-\epsilon_{\mathfrak{a}}))\left(\pm\lim_{x=\epsilon_{\mathfrak{a}}}\zeta(x-\epsilon_{\mathfrak{a}})\widetilde{\Psi}^{(a)}_{\mathscr{C}}(x)\right)^{1/2}|\mathscr{C}+\mathfrak{a}\rangle\,,\\ \widetilde{f}^{(a)}(z)|\mathscr{C}\rangle &= \sum_{\mathfrak{a}\in\mathrm{Rem}(\mathscr{C})} \pm\delta(z-\epsilon_{\mathfrak{a}})\left(\pm\lim_{x=\epsilon_{\mathfrak{a}}}\zeta(x-\epsilon_{\mathfrak{a}})\widetilde{\Psi}^{(a)}_{\mathscr{C}}(x)\right)^{1/2}|\mathscr{C}-\mathfrak{a}\rangle\,,\end{aligned}\right.$$

poles allowed by η

Representation of $\widetilde{Y}$ on crystal

defined from JK residue Z 1-loop

$$
\left\{
\begin{aligned}
\widetilde{\psi}^{(a)}_{\pm}(z)|\mathscr{C}\rangle &= \begin{cases} \widetilde{\Psi}^{(a)}_{\mathscr{C}}(z)|\mathscr{C}\rangle\,, & \text{rational}\,,\\ \left[\widetilde{\Psi}^{(a)}_{\mathscr{C}}(Z)\right]_{\pm}|\mathscr{C}\rangle\,, & \text{trigonometric/elliptic}\,, \end{cases}\\
\widetilde{\omega}^{(a)}(z)|\mathscr{C}\rangle &= \sum_{\mathrm{Inad}\left(\Psi^{(a)}_{\mathscr{C}}(z),\eta\right)} \delta(z-\epsilon_{\mathfrak{a}}) \lim_{x=\epsilon_{\mathfrak{a}}} \zeta(x-\epsilon_{\mathfrak{a}})\widetilde{\Psi}^{(a)}_{\mathscr{C}}(x)|\mathscr{C}\rangle\,,\\
\widetilde{e}^{(a)}(z)|\mathscr{C}\rangle &= \sum_{\mathfrak{a}\in\mathrm{Add}(\mathscr{C})} \pm\delta(z-\epsilon_{\mathfrak{a}})) \left(\pm \lim_{x=\epsilon_{\mathfrak{a}}} \zeta(x-\epsilon_{\mathfrak{a}})\widetilde{\Psi}^{(a)}_{\mathscr{C}}(x)\right)^{1/2} |\mathscr{C}+\mathfrak{a}\rangle\,,\\
\widetilde{f}^{(a)}(z)|\mathscr{C}\rangle &= \sum_{\mathfrak{a}\in\mathrm{Rem}(\mathscr{C})} \pm\delta(z-\epsilon_{\mathfrak{a}}) \left(\pm \lim_{x=\epsilon_{\mathfrak{a}}} \zeta(x-\epsilon_{\mathfrak{a}})\widetilde{\Psi}^{(a)}_{\mathscr{C}}(x)\right)^{1/2} |\mathscr{C}-\mathfrak{a}\rangle\,,
\end{aligned}
\right.
$$

Summary

JK-residue

$\mathcal{N}=2$ quiver
+ J/E-terms

crystal melting $|\kappa\rangle$

representation

double quiver algebra $\tilde{\mathsf{Y}}$

refined counting

$\tilde{\mathsf{Y}} \subset \tilde{\mathsf{Y}}^{\#}$ enhanced double QA

$\mathcal{N}=4$ quiver
+ F-terms

(single) quiver algebra Y

$\mathsf{Y} \subset \tilde{\mathsf{Y}}^{\#}$

Lessons on QG?

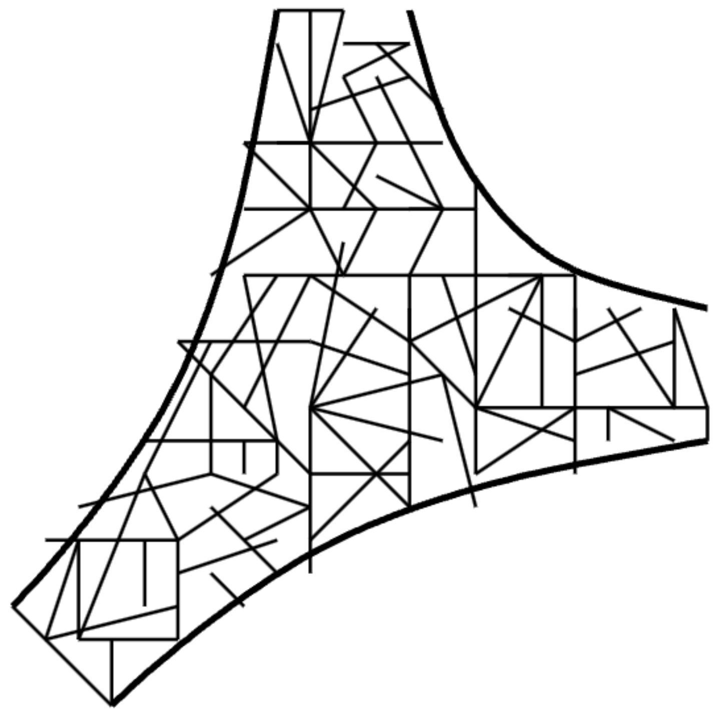
[Iqbal – Nekrasov – Okounkov – Vafa '03]

[Ooguri – MY '09]

Crystal

||

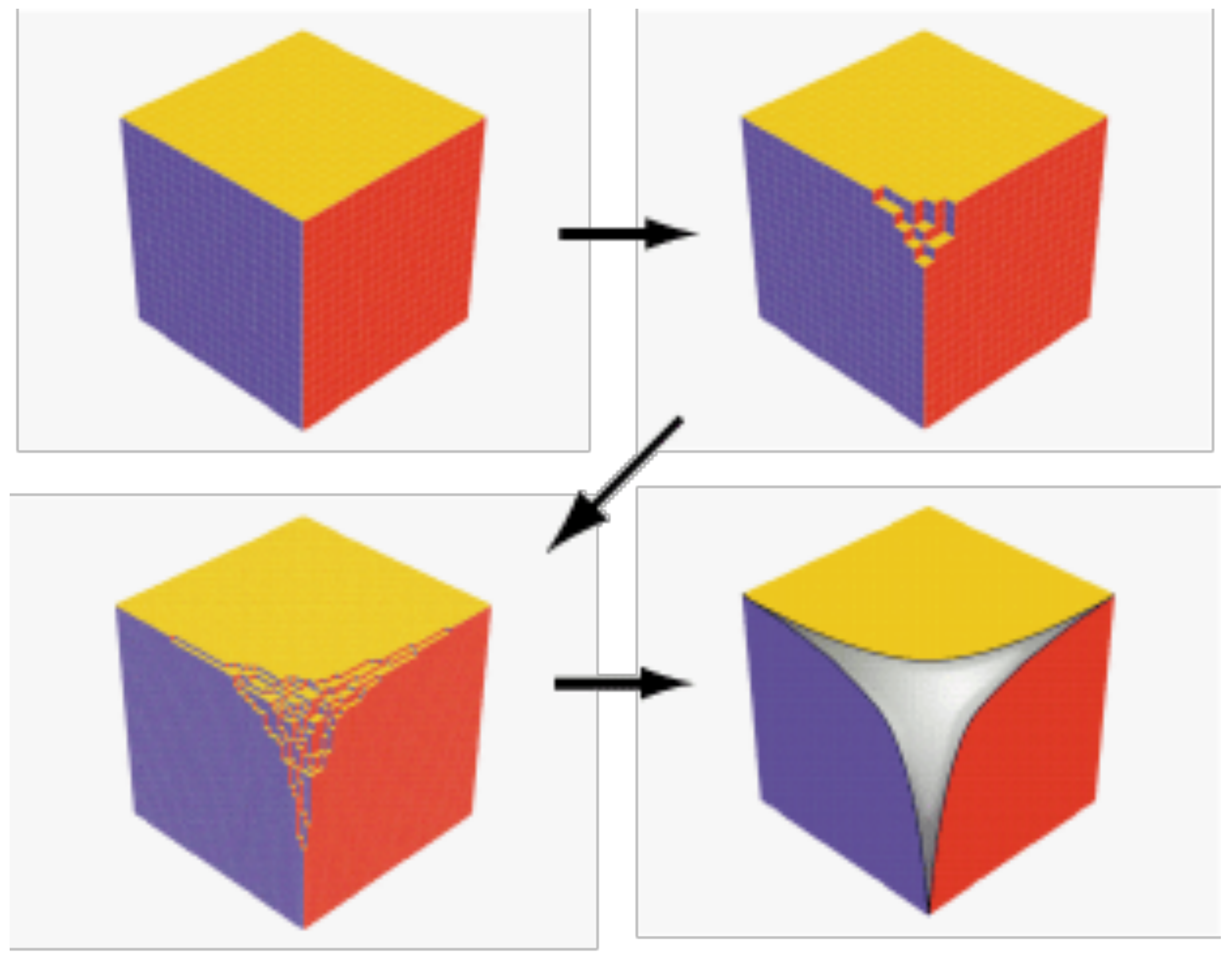
Quantum Foam



(Double) Quiver Algebra

||

Algebra for creating/annihilating foams



"Confinement of Spacetime foam?"

- η : cyclic $\qquad e \to e+a \to e+a+b$

- η : non-cyclic

$$\begin{tikzcd} e \arrow[r, "\times" marking] \arrow[rr, bend right] & e+a \arrow[r, "\times" marking] & e+a+b \end{tikzcd}$$