Basic Numerical Analysis

October 9, 2012

Problem Set 2

1. Write a C program that calculates the definite integral

$$I = \int_0^1 \frac{1}{1+x}\,\mathrm{d}x. \tag{1}$$

Use Trapezoidal rule and Simpson's rule, and compare the results with the true solution $I = \ln 2 = 0.693147....$ Start with the number of bins, say, $N = 16$ and then increase N. Evaluate the total error with respect to $\ln 2$ as a function of N.

2. Write a C program that integrates the following set of ODEs:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= v, \\ \frac{\mathrm{d}v}{\mathrm{d}t} &= \frac{v}{t} - 4kt^2 x, \end{aligned} \tag{2}$$

subject to the boundary conditions $x = \sqrt{k}\,a^2, v = 2\sqrt{k}\,a$ at t=a, where $0 < a \ll 1$.
Hints) First derive the analytic solution. Describe how x varies as a function of time t. Then integrate the equations (numerically) by using Euler's method. Compare the two solutions, i.e., the analytic one and your numerical integration. Explain why Euler's method fails at some point.

3. Lorenz equations
Write a 4-th order Runge-Kutta integrator and solve the following nonlinear coupled equations:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= -10x + 10y, \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= -xz + 28x - y, \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= xy - \frac{8}{3}z. \end{aligned} \tag{3}$$

Try with the initial values $x_0 = 10.0, y_0 = 20.0, z_0 = 30.0$. Show the orbit in 3D and enjoy the attractor, chaos etc.
Advanced) Try with a slightly different value of x_0. Compare the obtained orbit. Explain what is happening to the system, which is apparently deterministic.