

Basic Numerical Analysis

October 23, 2012

Problem Set 3

1. Round-off errors) Write a very simple program that does the following mapping many times:

$$\begin{aligned} \text{Choose a value } & 0 < x < 1. \\ y &= 2x \text{ if } x < 0.5 \\ &= 2 - 2x \text{ if } x \geq 0.5. \\ \text{Then set } & x = y. \end{aligned} \tag{1}$$

Let us try with $x_{\text{initial}} = 0.3$. Do the same calculation by your own hand for, say, 10 cycles. Run your program for 100 cycles and compare the results. Explain what went wrong.

Advanced: (Think about how to) Write a program for the same above mapping such that the code returns *correct* result.

Advanced Try another mapping $y = 4x(1-x)$. Think about to what extent the numerical solution is reliable in this case.

2. Write a C program that solves the 1D diffusion equation

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \tag{2}$$

subject to the boundary conditions

$$\frac{\partial T}{\partial x} = 0 \text{ at } x = \pm L, \tag{3}$$

where the system is evolved on the domain $-L < x < L$. Set L to be sufficiently large. As a simplest case, set the initial condition $T = \exp(-x^2/4)$ at $t = 1$. Compare the numerical solution with analytic one. Try increasing and decreasing the time step width Δt and see how the solution changes.

Advanced: Use the Crank-Nicholson scheme to solve the same equation.