Designing the Next Generation of Extra-Solar Planet Observatories

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Motivation

- Technology advances make dedicated astrometric and direct detection planet-finding space observatories a real possibility in the near future.
- Multiple mission concept studies have been carried out by a variety of groups.
- Multiple different mission scenarios and observing strategies have been proposed.

Basic Questions

- Given an instrument design, what is the expected science yield?
- How can we best use returned data?

Outline

1. Planet Finding Methods
   - Direct Detection
   - Indirect Detection
   - Simulation of Observations
2. Tools
   - Single Visit Completeness
   - Distributions of Orbital Parameters
   - Timing
   - Putting It All Together
3. Integrating Data Streams
   - Dynamic Filtering
   - Applications to Orbit Fitting

Direct Detection Platforms

- Coronagraphs - multiple methods exist for removing light from the star entering a telescope's aperture.
- Occulters - a 'starshade' is flown along with the telescope to block out star-light.

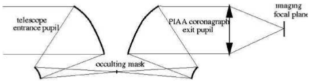


Figure: Schematic of a PIAA system. [Guyon, 2003]

Figure: Pupil mask for high contrast imaging. [Vanderbei et al., 2003]

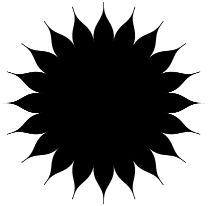


Figure: Proposed star-shade design. [Spergel et al., 2009]

Instrument Outputs and Constraints

- Limiting Δmag - maximum achievable difference in brightness between star and planet.
- Inner working angle (IWA) - minimum angular separation between a star and planet.

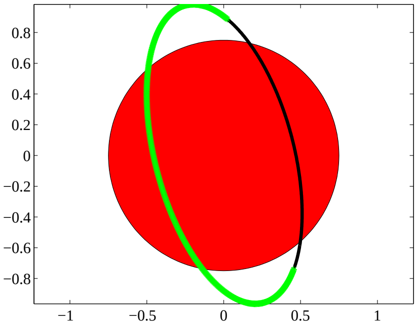


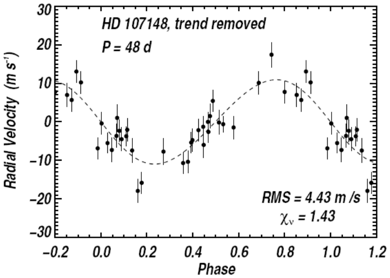
Figure: Schematic of a planetary observation.

- The red circle represents the instrument's projected IWA.
- The planet is sufficiently illuminated only on the green portion of the orbit.
- Detection occurs on the green part of the orbit outside the red circle.

Some Indirect Detection Platforms

Astrometry

- Uses optical interferometry to find angular distance between two objects
- Use set of reference stars to find position of target star with respect to centroid
- Produces target star's position in plane of the sky

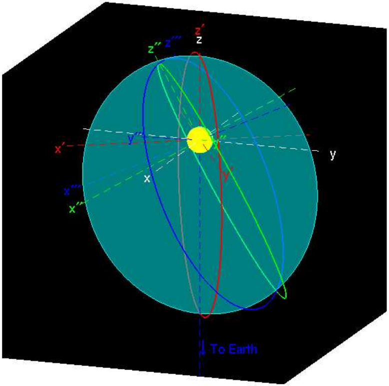


Radial Velocity

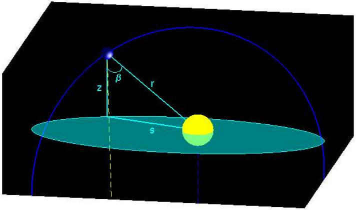
- Uses spectroscopy to find wavelengths of target star's emitted light
- After accounting for other effects, remaining changes in wavelength are attributed to doppler effect
- Produces target star's velocity along the line of sight

Both methods require us to infer the presence of planets from motion of the target star - 'Stellar Wobble'

Direct Detection



3-1-3 (ψ, θ, ϕ) rotation
$\psi, \phi \sim U([0, 2\pi)),\ \theta \sim U(\cos\theta)$

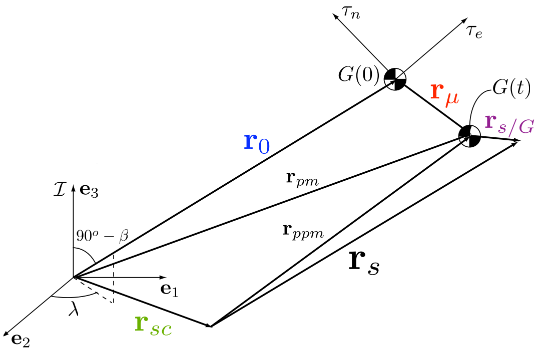


$$r \triangleq \left|\mathbf{r}_{p/s}\right| = \frac{a(1-e^2)}{e\cos(\nu)+1}$$

$$\mathbf{s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{r_{p/s}}$$

$$\Delta\mathrm{mag} = -2.5\log\frac{F_p}{F_\star}$$

Astrometry and Radial Velocity



$$\mathbf{r}_s = \mathbf{r}_0 + \mathbf{r}_\mu - \mathbf{r}_{sc} + \mathbf{r}_{s/G}$$

Fundamental astrometric observation is $\hat{\mathbf{r}}_s$
(can be decomposed into two angles)

Fundamental radial velocity observation is $\dot{\mathbf{r}}_s$

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Obscurational and Photometric Completeness
[Brown, 2005]

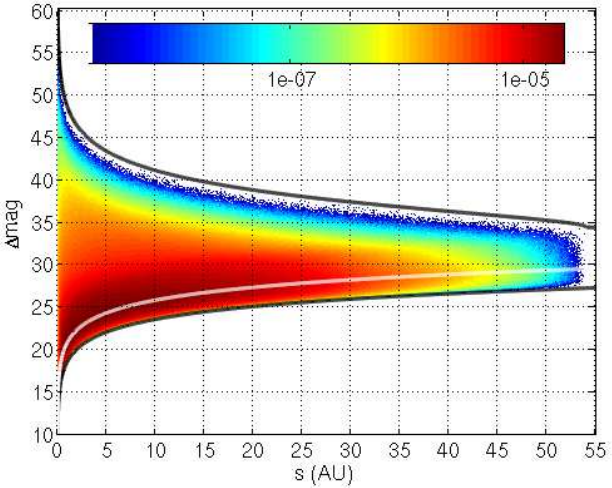


Figure: Probability density function of observable planets.
$a \in [0.4, 30]$, $e \in [0, 0.8]$, $p \in [0.1, 0.5]$, $R \in [4 * 10^{-5}, 4 * 10^{-4}]$

Single Visit Completeness
[Brown, 2005]

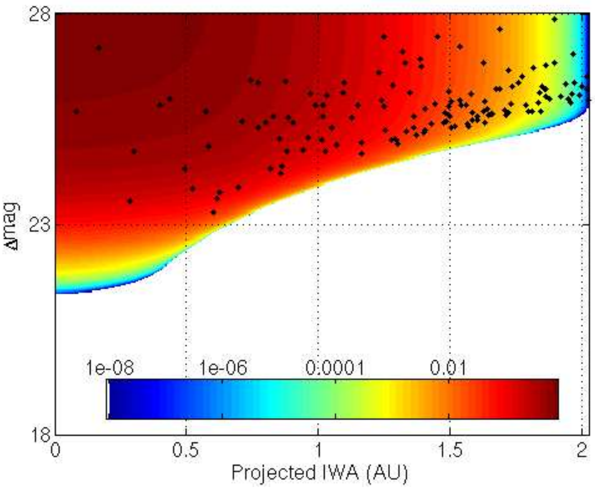


Figure: Candidate stars plotted over the cumulative distribution function for 'Earth-like' planets.

The Distribution of ν

- Let $\nu = g(M, e)$ and $M = h(\nu, e)$. Then, the cumulative distribution function of $\bar{\nu}$ is:

$$F_{\bar{\nu}}(\nu) = P(g(\bar{M}, \bar{e}) \leq \nu) = \int_{-\infty}^{\infty} P(g(\bar{M}, e) \leq \nu | \bar{e} = e) f_{\bar{e}}(e) de$$

  where $f_{\bar{e}}$ is the probability density function of $\bar{e}$.

- $\bar{M}$ and $\bar{e}$ are independent so:

$$F_{\bar{\nu}}(\nu) = \int_{-\infty}^{0} P(\bar{M} \geq h(\nu, e)) f_{\bar{e}}(e) de + \int_{0}^{\infty} P(\bar{M} \leq h(\nu, e)) f_{\bar{e}}(e) de$$

Probability Density Function of ν

$$f_{\bar{\nu}}(\nu) = \int_{0}^{\infty} \frac{\partial h}{\partial \nu} f_{\bar{M}}(M) f_{\bar{e}}(e) de = \frac{1}{2\pi} \int_{0}^{1} \frac{(1 - e^2)^{3/2}}{(1 + e\cos(\nu))^2} f_{\bar{e}}(e) de$$

The Distributions of Other Parameters

We can apply the same procedure to other quantities of interest:

Probability Density Function of E

$$f_{\bar{E}}(E) = \frac{1}{2\pi}\int_0^1 (1 - e\cos(E))\, f_{\bar{e}}(e)de$$

Probability Density Function of r

$$f_{\bar{r}}(r) = \frac{1}{2\pi}\int_0^\infty \int_0^1 Re\left\{ \frac{r\sinh^{-1}\left(\sqrt{-\frac{(ae)^2}{(a-r)^2}}\right)}{a^2} \right\} f_{\bar{e}}(e) f_{\bar{a}}(a)deda$$

Probability Density Function of β

$$f_{\bar{\beta}}(\beta) = -\frac{1}{2\pi^2}\int_0^{2\pi}\int_0^{\pi} \frac{\sin\beta}{\sqrt{\sin^2\theta\sin^2\nu - (\cos\theta\cos\nu - \cos\beta)^2}} d\theta f_{\bar{\nu}} d\nu$$

Improving Sampling for Monte Carlo

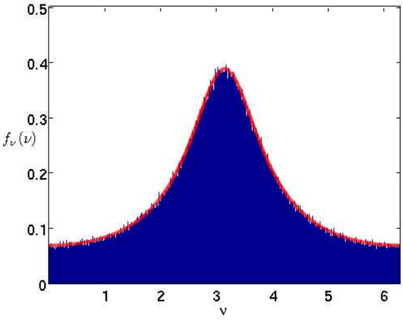


Figure: Analytical and simulated $f_{\bar{\nu}}$.

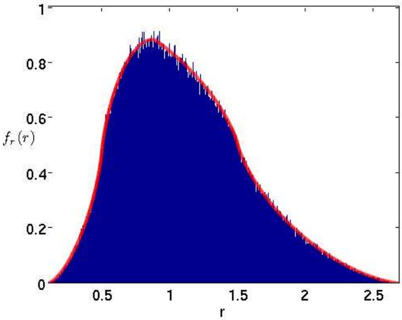


Figure: Analytical and simulated $f_{\bar{r}}$.

Detection Probability Over Multiple Observations

If observations are IID, probability of at least one detection in n visits is:

$$P_n(k > 0) = 1 - \binom{n}{0} p^0 (1-p)^n = 1 - (1-p)^n$$

where p is the single visit completeness.

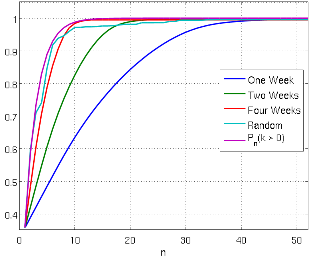


Figure: Fraction of planets found as a function of number of visits, with various intervals between observations.

- For optimal observing schedule, probability of detection approaches unity after 1/p visits.
- For non-optimal schedule, this equation provides an upper bound for detection probability.

Integration Time

[Kasdin and Braems, 2006]

Sufficiently sampled optical systems allow for bayesian detection techniques.
Let the photons received at pixel j be the random variable

$$z_j = C_p \bar{P}_j + C_b + \nu$$

Construct SNR metric as

$$\frac{\hat{C}_p}{\sigma_b}$$

Making a few assumptions

$$t = \frac{1}{b} \frac{\left(K - \gamma \sqrt{1 + \tilde{Q}\Xi/\Psi}\right)^2}{\tilde{Q} T_A \Psi}$$

for

$$\Phi(K) = 1 - \mathrm{FAP} \qquad \Phi(\gamma) = \mathrm{MDP}$$

$\tilde{Q} = C_p/C_b \sum_j \bar{P}_j$. All other parameters are functions of the optical system and the target irradiance.

Integration Time vs. Sampling

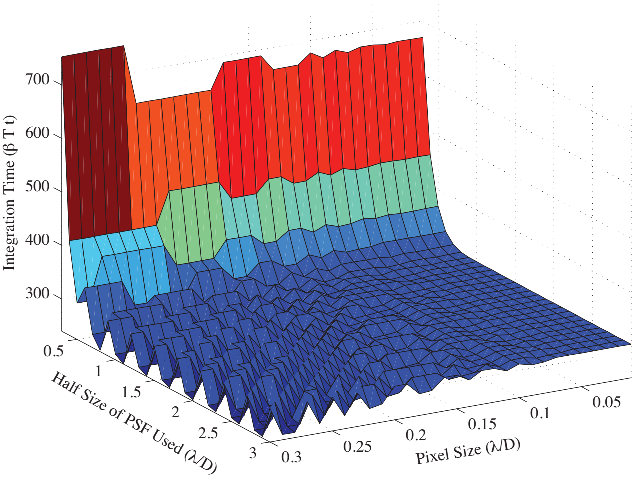


Figure: Normalized integration time as a function of half size PSF used and pixel area for an open circular aperture.

Approximating the Semi-Major Axis

Define parameter Ξ equal to the ratio s/a:

$$\Xi \triangleq \frac{1-e^2}{e\cos\nu+1}\sqrt{(\cos\nu\sin\nu+\cos\theta\cos\psi\sin\nu)^2+(\sin(\nu)\sin(\psi))^2}$$

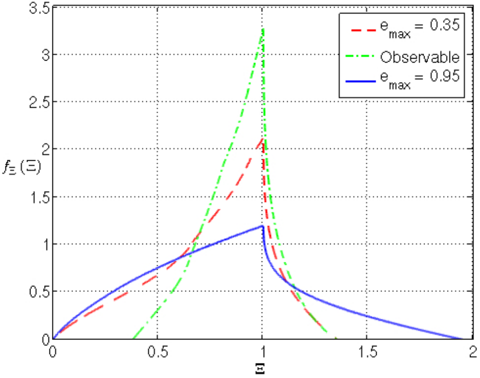


Figure: Probability density functions of Ξ.

- The PDF of Ξ always has a maximum at 1 ($s = a$).
- The observed apparent separation is the best estimate for the semi-major axis.
- The obscurational and photometric limitations actually make $\Xi = 1$ an even better estimate.
- Using these two estimates produces a mean 16% error in orbital period in simulation.

Mission Analysis

- Create descriptions of instruments, planetary orbits/properties and observations.
- Generate full mission simulations (timelines of observations and their outcomes).
- From these mission ensembles, extract distributions of science yield/performance metrics:
  - All Detections - Total number of successful planetary observations throughout a whole mission simulation (includes repeat detections).
  - Unique Detections - Number of individual planets found during a mission simulation.
  - Unique Targets - Number of individual stars observed during a mission simulation.
  - Spectral Characterizations - Number of observations where the planet was observable for sufficient time to integrate to a predefined S/N level.
  - Propellant Used - For occulters, the amount of propellant used by the starshade for slewing and stationkeeping.

Visits as a Graph
[Savransky and Kasdin, 2008]

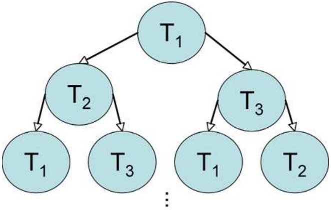


Figure: Visit graph for 3 target pool.

- Each set of possible transitions on the visit graph can be represented as a weighted adjacency matrix.
- The weights of the matrix entries represent the 'cost' of choosing the next star.

The cost of transitioning from target i to target j is calculated as:

$$A_{ij} = \left[\begin{array}{c} a_1 \frac{\cos^{-1}(u_i \cdot u_j)}{2\pi} B_{inst} + a_2 \mathrm{comp}_j - a_3 e^{t-t_f} B_{unvisited} + \\ a_4 B_{visited}(1 - B_{revisit}) - a_5 B_{revisit} \left(\frac{N_j}{N_{req}} \right) (N_j < N_{req}) - a_6 \frac{\tau_j}{\mathrm{vis}_j} \end{array} \right] (1 - B_{ko})$$

Automated Visit Order

- The amount of time spent on any one target depends on whether a planet is detected.
- The adjacency matrix must be continuously updated.

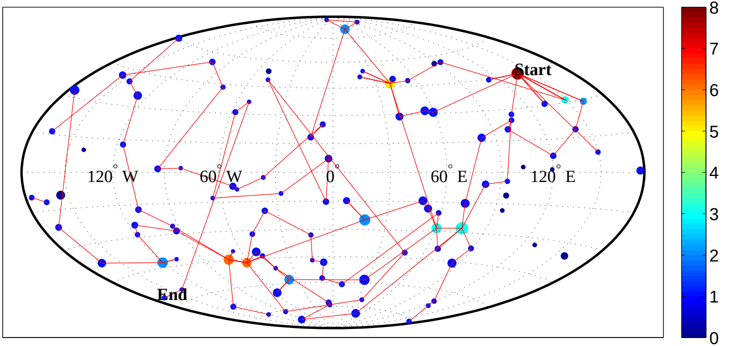
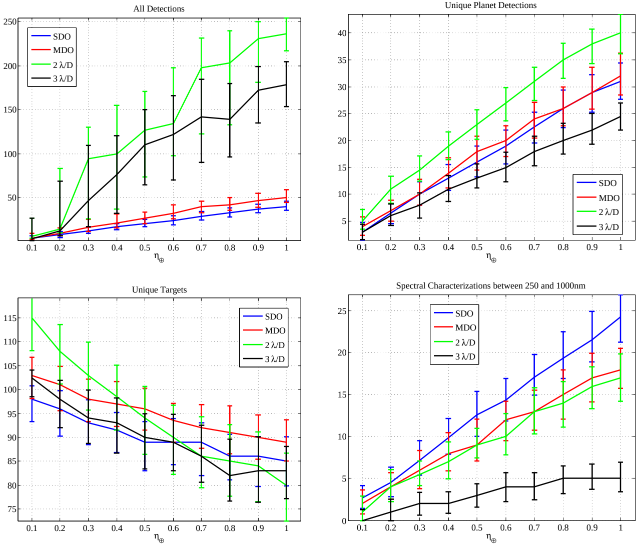


Figure: Automatically generated visit order.

Comparison of Mission Concepts with 4m Telescope



Outline

Dynamic Filtering

- Observations ($\mathbf{z}$) at time k are a function of a state vector ($\mathbf{x}$) describing the positions of all orbiting planets, and time, with added noise $\mathbf{n}$ of covariance R:

$$\mathbf{z}_k = \mathbf{f}(\mathbf{x}_k, k) + \mathbf{n}$$

- The solution to this problem is a minimization with respect to $\mathbf{x}$ for N observations of the cost function:

$$J = \sum_{k=1}^{N} [\mathbf{z}_k - \mathbf{f}(\mathbf{x}_k, k)]^T R^{-1} [\mathbf{z}_k - \mathbf{f}(\mathbf{x}_k, k)]$$

  subject to the constraints of the physical system (i.e. Newtonian dynamics) and any inherent constraints in the formulation of the state (i.e., quaternion definition, eccentricity bounds, etc.).

- We can re-formulate this as a recursive filter, using each observation to update the estimate of the underlying state, and our knowledge of the physical system to propagate the state in time.

Dynamic Filtering (cont.)

Assume a Markov process with state $\mathbf{x}$ and observation $\mathbf{z}$. Then

$$p(\mathbf{x}_0, \mathbf{x}_1 \cdots \mathbf{x}_n, \mathbf{z}_1, \mathbf{z}_2 \cdots \mathbf{z}_n) = p(\mathbf{x}_0) \prod_{j=1}^{n} p(\mathbf{z}_j | \mathbf{x}_j) p(\mathbf{x}_j | \mathbf{x}_{j-1})$$

Predict the next state given the observed history

$$p(\mathbf{x}_j | \mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_j | \mathbf{x}_{j-1}) p(\mathbf{x}_{j-1} | \mathbf{z}_{1:j-1}) d\mathbf{x}_{j-1}$$

Update the state estimate given the current observation

$$p(\mathbf{x}_j | \mathbf{z}_{1:j}) = \frac{p(\mathbf{z}_j | \mathbf{x}_j) p(\mathbf{x}_j | \mathbf{z}_{1:j-1})}{p(\mathbf{z}_j | \mathbf{z}_{1:j-1})}$$

Extended Kalman Filter

$$
\begin{aligned}
\dot{\hat{\mathbf{x}}}(t) &= \mathbf{f}(\hat{\mathbf{x}}(t), t) & \dot{\mathbf{P}}(t) &= \mathbf{F}(t)\mathbf{P}(t) + \mathbf{P}(t)\mathbf{F}^T(t) + \mathbf{Q} \\
\hat{\mathbf{x}}_0 &= E[\mathbf{x}(0)] & \mathbf{P}_0 &= E[(\mathbf{x}(0) - \hat{\mathbf{x}}_0)(\mathbf{x}(0) - \hat{\mathbf{x}}_0)^T] \\
\mathbf{F}(t) &= \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(t)} & \mathbf{Q}(t) &= E[\mathbf{w}(t)\mathbf{w}^T(\tau)]
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathbf{x}}_{k_i}^+ &= \hat{\mathbf{x}}_k^- + \mathbf{K}_{k_i}\left(\mathbf{y}_k - \mathbf{h}(\hat{\mathbf{x}}_{k_{i-1}}^+) - \mathbf{H}_{k_i}(\hat{\mathbf{x}}_k^- - \hat{\mathbf{x}}_{k_{i-1}}^+)\right) & \hat{\mathbf{x}}_{k_0}^+ &= \hat{\mathbf{x}}_k^- \\
\mathbf{K}_{k_i} &= \mathbf{P}_k^- \mathbf{H}_{k_i}^T \left(\mathbf{H}_{k_i}\mathbf{P}_k^-\mathbf{H}_{k_i}^T + \mathbf{R}_k\right)^{-1} & \mathbf{H}_{k_i} &= \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k_i}^+} \\
\mathbf{P}_{k_i}^+ &= \left(\mathbf{I} - \mathbf{K}_{k_i}\mathbf{H}_{k_i}\right)\mathbf{P}_k^- & \mathbf{R}(t) &= E[\mathbf{v}(t)\mathbf{v}^T(\tau)]
\end{aligned}
$$

Position and Velocity state makes it easy to describe open orbits

- Introduce inequality constraints of the form $D\bar{X} \leq d$ to constrain orbital specific energy
- At each time step, solve quadratic programming problem of the form $\min_{\tilde{x}} \left(\tilde{x}^T W \tilde{x} - 2\bar{X}^T W \tilde{x}\right)$ s.t. $D\tilde{x} \leq d$ [Simon and Simon, 2006]

State

Let the state vector for a system of n planets be:

$$X = \begin{bmatrix} \mathbf{r}_1 & \dot{\mathbf{r}}_1 & \ldots & \mathbf{r}_n & \dot{\mathbf{r}}_n & \mathbf{r}_{s/G} & \dot{\mathbf{r}}_{s/G} \end{bmatrix}^T$$

with the state estimate propagation given by

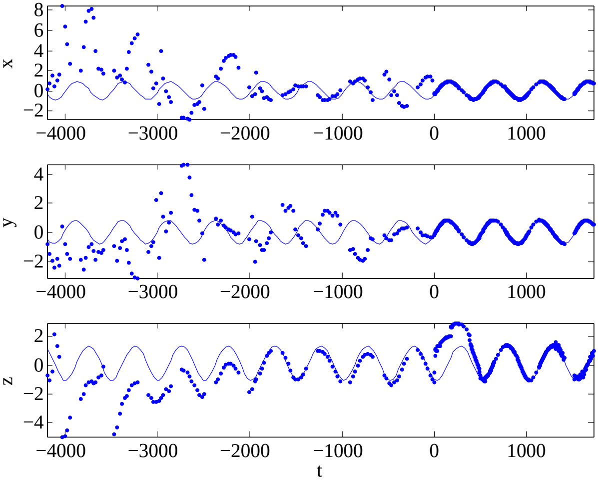
$$\ddot{\mathbf{r}}_j = -\sum_{k \neq j} \frac{\mu_k \mathbf{r}_{k/j}}{|\mathbf{r}_{k/j}|^3} \qquad j = 1, \ldots, n, s/G \qquad \mathbf{r}_{k/j} = \mathbf{r}_k - \mathbf{r}_j$$

Augment state with constant parameters to account for unknown proper motion and stellar distance

$$\bar{X} = \begin{bmatrix} \mathbf{r}_1 & \dot{\mathbf{r}}_1 & \ldots & \mathbf{r}_n & \dot{\mathbf{r}}_n & \mathbf{r}_{s/G} & \dot{\mathbf{r}}_{s/G} & \mathbf{r}_\mu & \varpi \end{bmatrix}^T$$

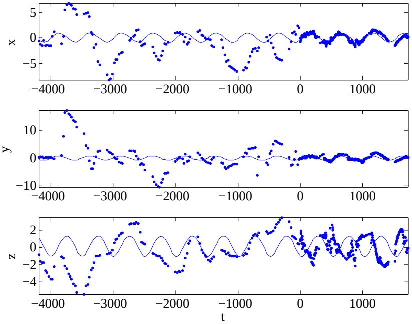
$$\varpi = \frac{a}{\|\mathbf{r}_0\|}$$

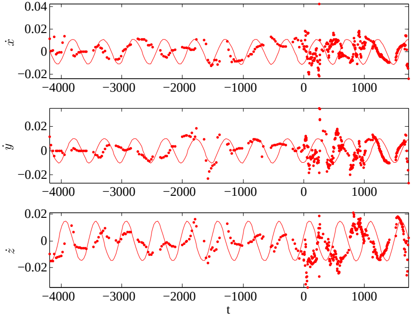
Filtering Astrometric and Radial Velocity Data - M_J

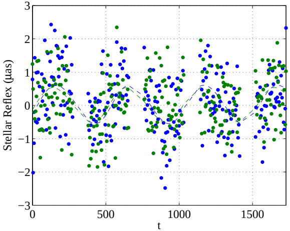


Filtering Astrometric and Radial Velocity Data - $M_\oplus$

[Savransky and Kasdin, 2009]







Adding in Direct Detections

The underlying dynamics remain the same regardless the observation - only the observation changes:

$$\mathbf{z}_{ast} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{r}_{s/G}$$

$$\mathbf{z}_{rv} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{\mathbf{r}}_{s/G}$$

$$\mathbf{z}_{dd} = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{r}_j \\ -2.5 \log \left(p \left(\frac{R}{\|\mathbf{r}_j\|} \right)^2 \Phi \left(\cos^{-1} \frac{\mathbf{r}_j(3)}{\|\mathbf{r}_j\|} \right) \right) \end{bmatrix}$$

Open Questions

- What are the true distributions of exoplanet parameters?
- More realistically, what is the best way to represent what we currently know about exoplanets?
- Is there a better way to calculate the expected number of detections over multiple observations?
- Is astrometric + radial velocity data observable in the dynamic formulation?

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