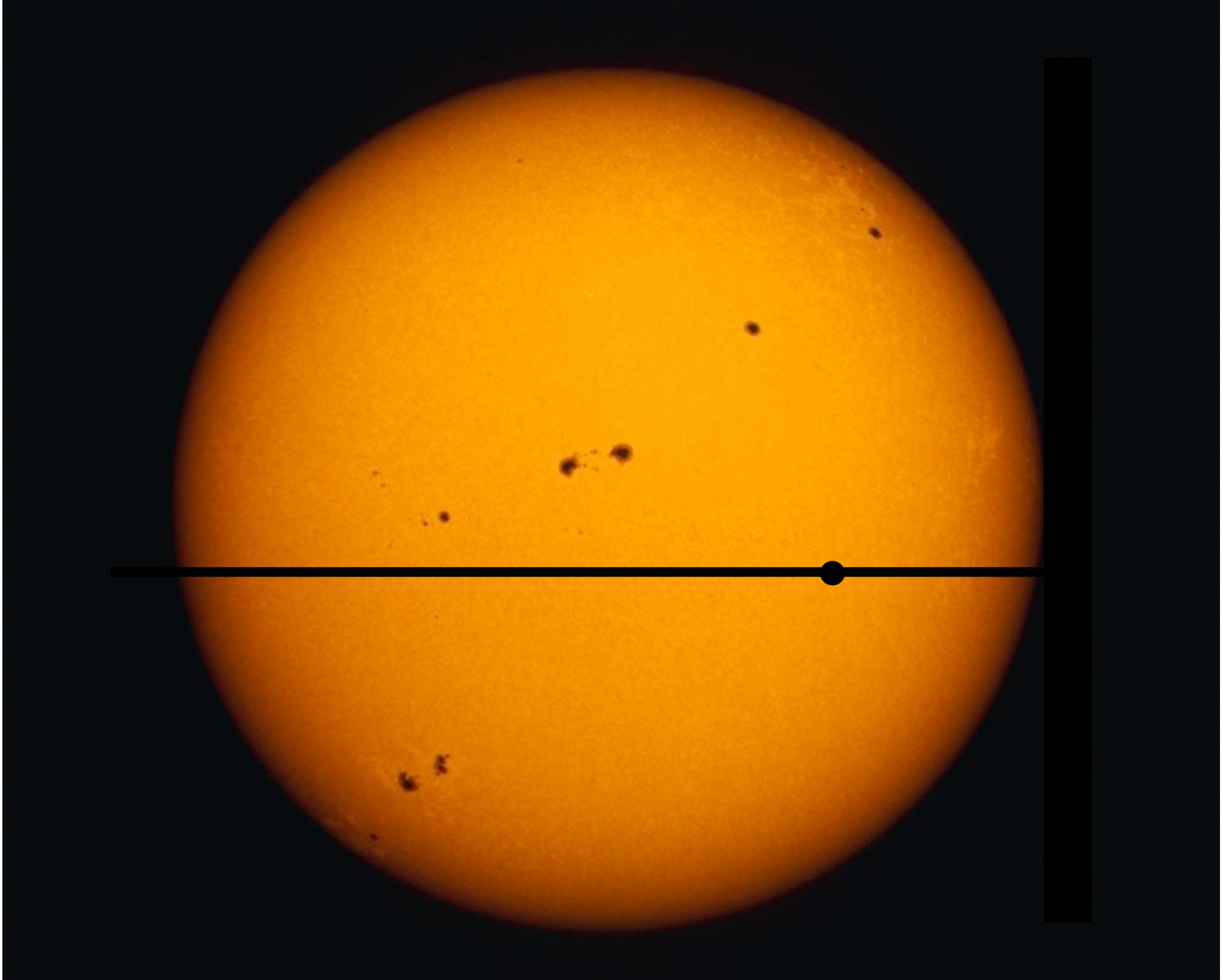
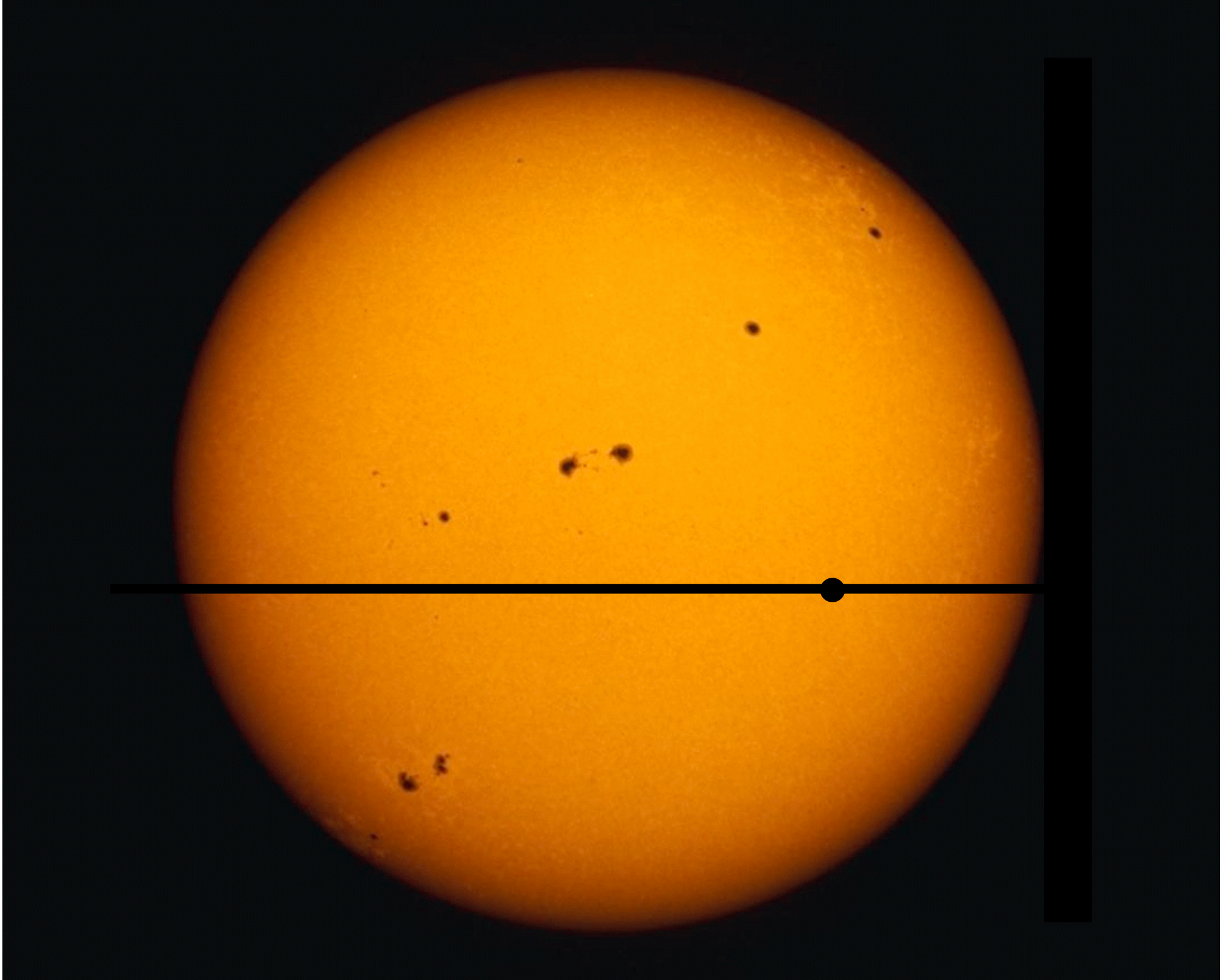

David Cortner

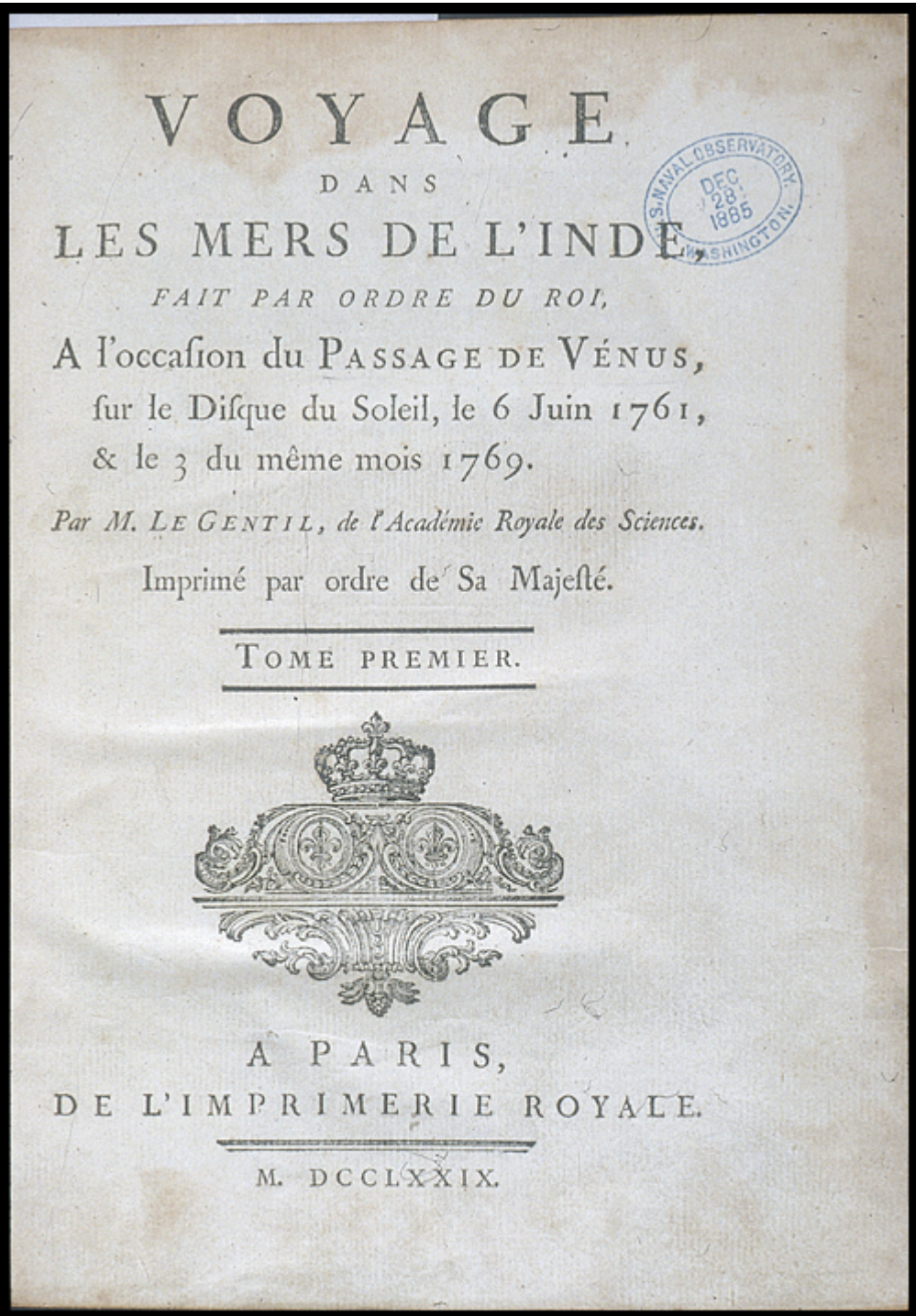
The Transit of Venus

June 8, 2004

David Cortner

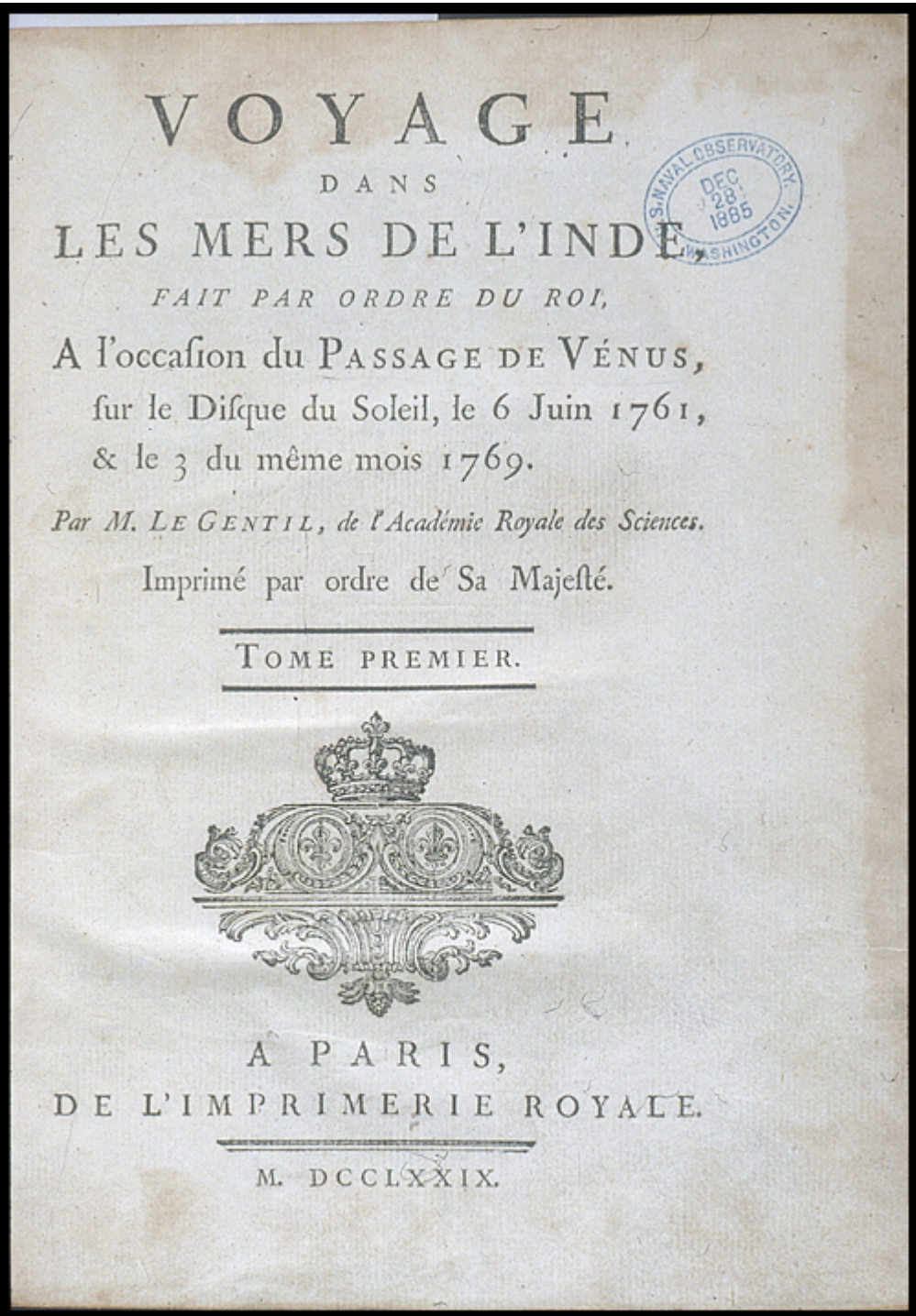






VOYAGE
DANS
LES MERS DE L'INDE,
FAIT PAR ORDRE DU ROI,
A l'occasion du PASSAGE DE VÉNUS,
sur le Disque du Soleil, le 6 Juin 1761,
& le 3 du même mois 1769.
Par M. LE GENTIL, de l'Académie Royale des Sciences.
Imprimé par ordre de Sa Majesté.
TOME PREMIER.
A PARIS,
DE L'IMPRIMERIE ROYALE.
M. DCCLXXIX.

Guillaume Joseph Hyacinthe Jean Baptiste Le Gentil de la Galaisiere (1725-1792)



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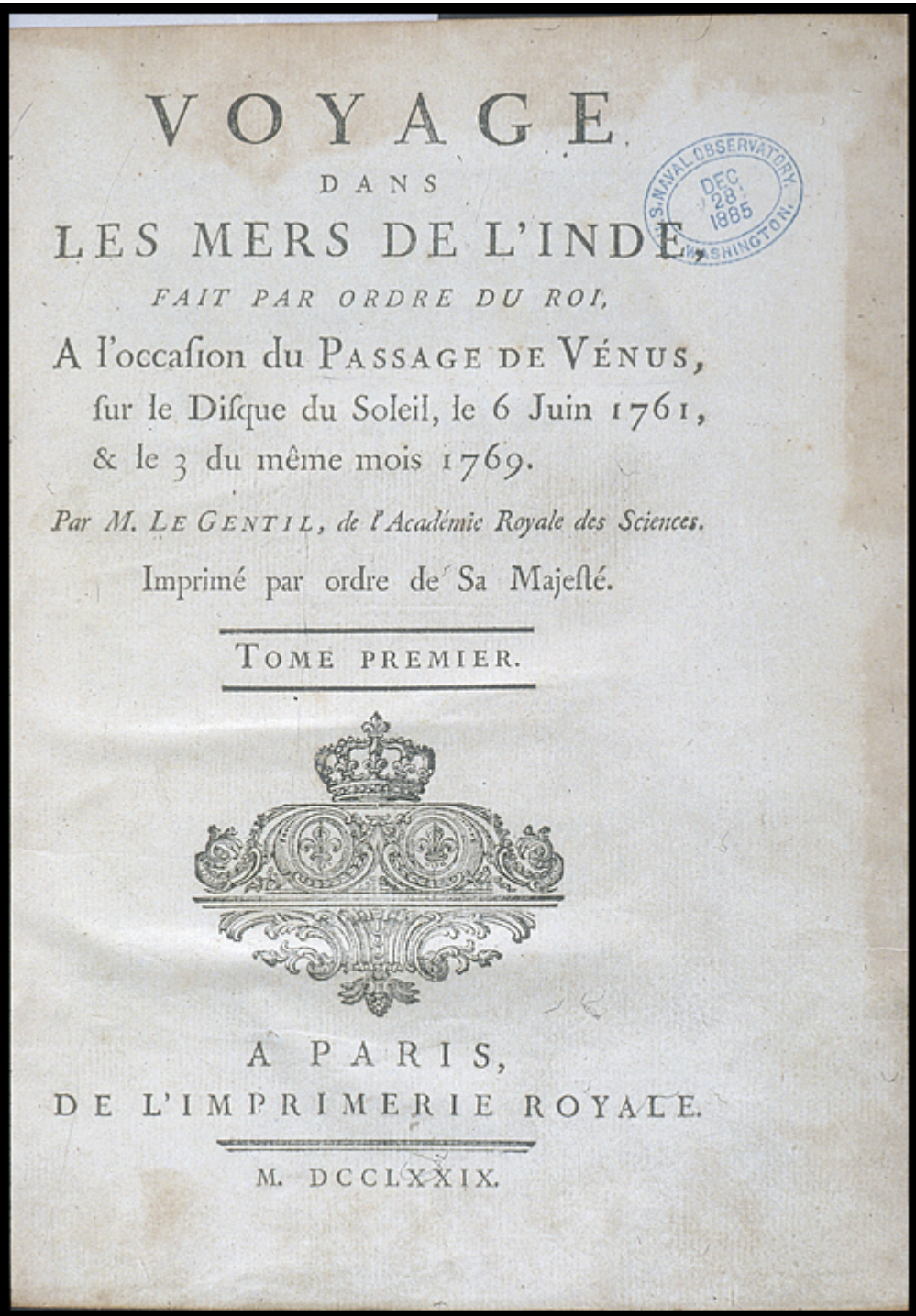
- Destination captured



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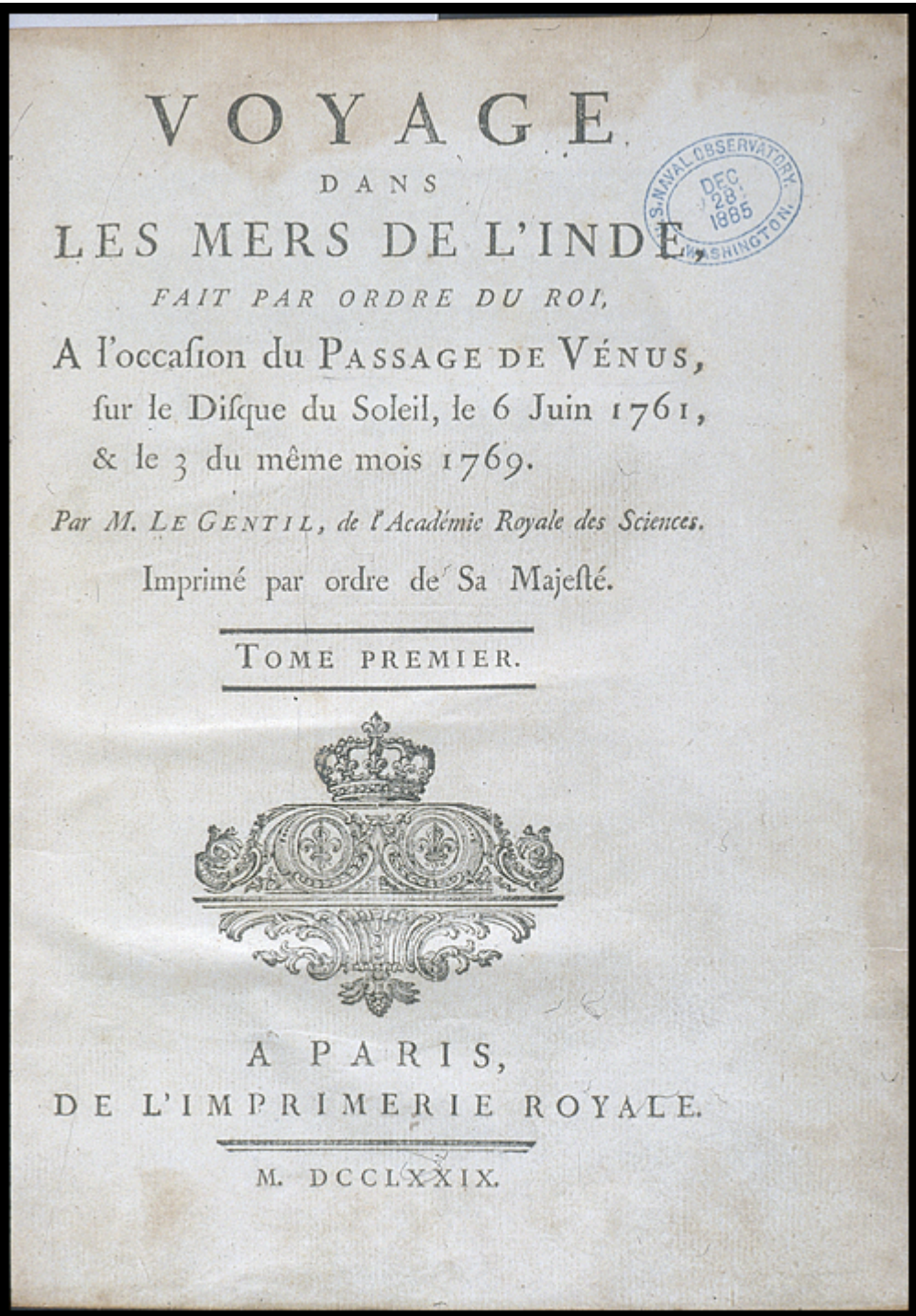
- Destination captured
- Waited 8 years



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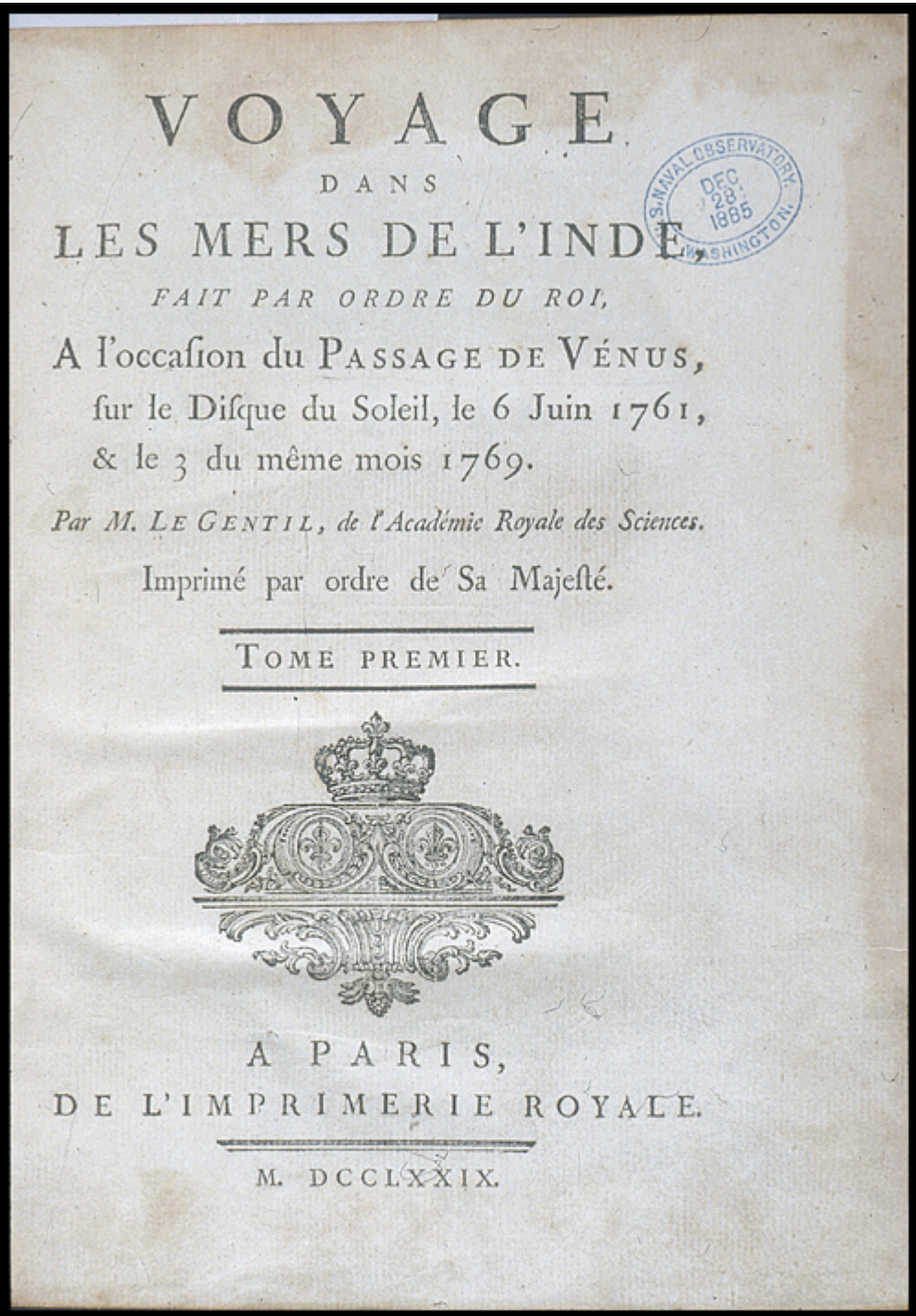
Guillaume Joseph Hyacinthe Jean Baptiste Le Gentil de la Galaisiere (1725-1792)

- Destination captured
- Waited 8 years
- Clouded out



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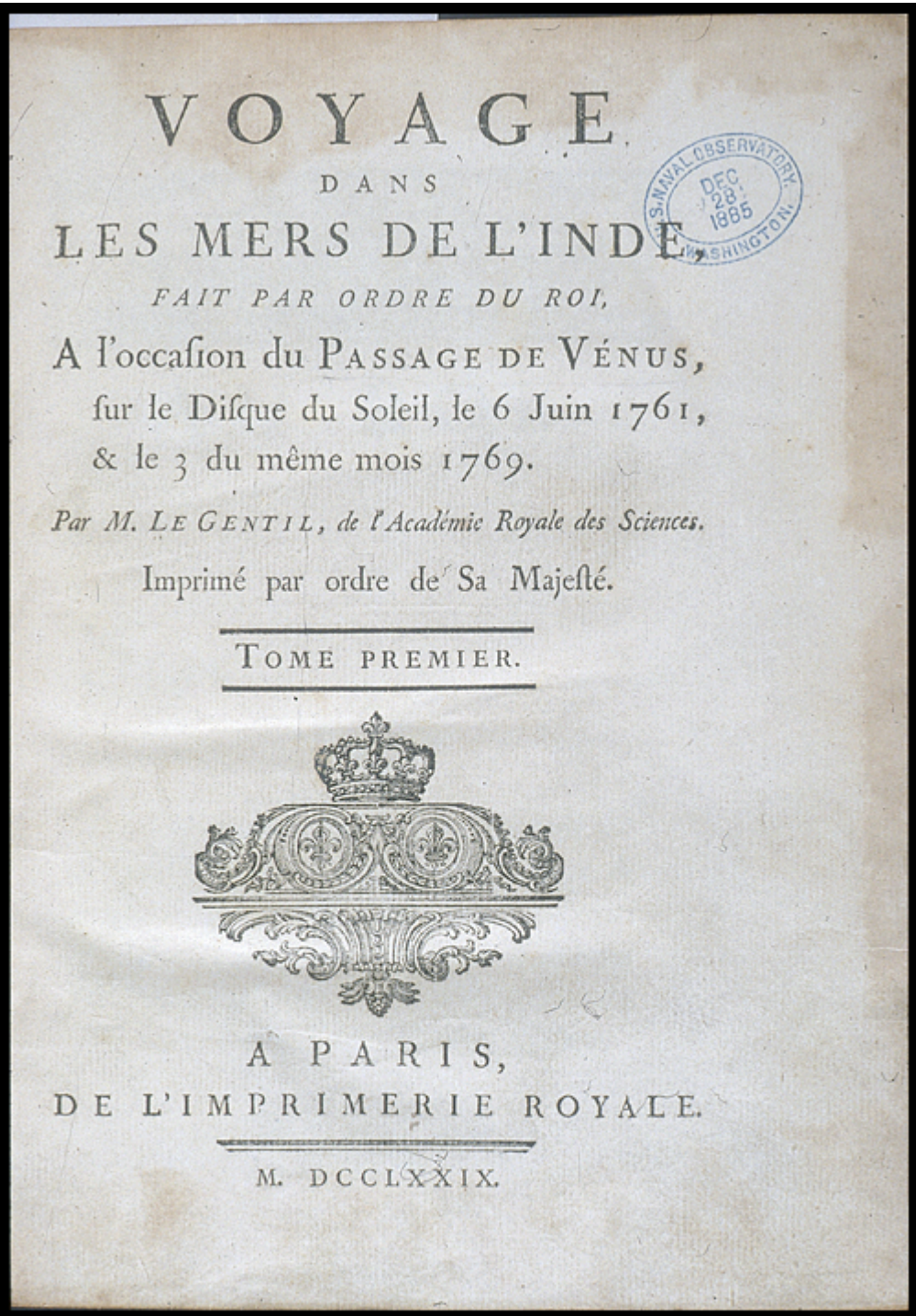
- Destination captured
- Waited 8 years
- Clouded out
- Contracted dysentery



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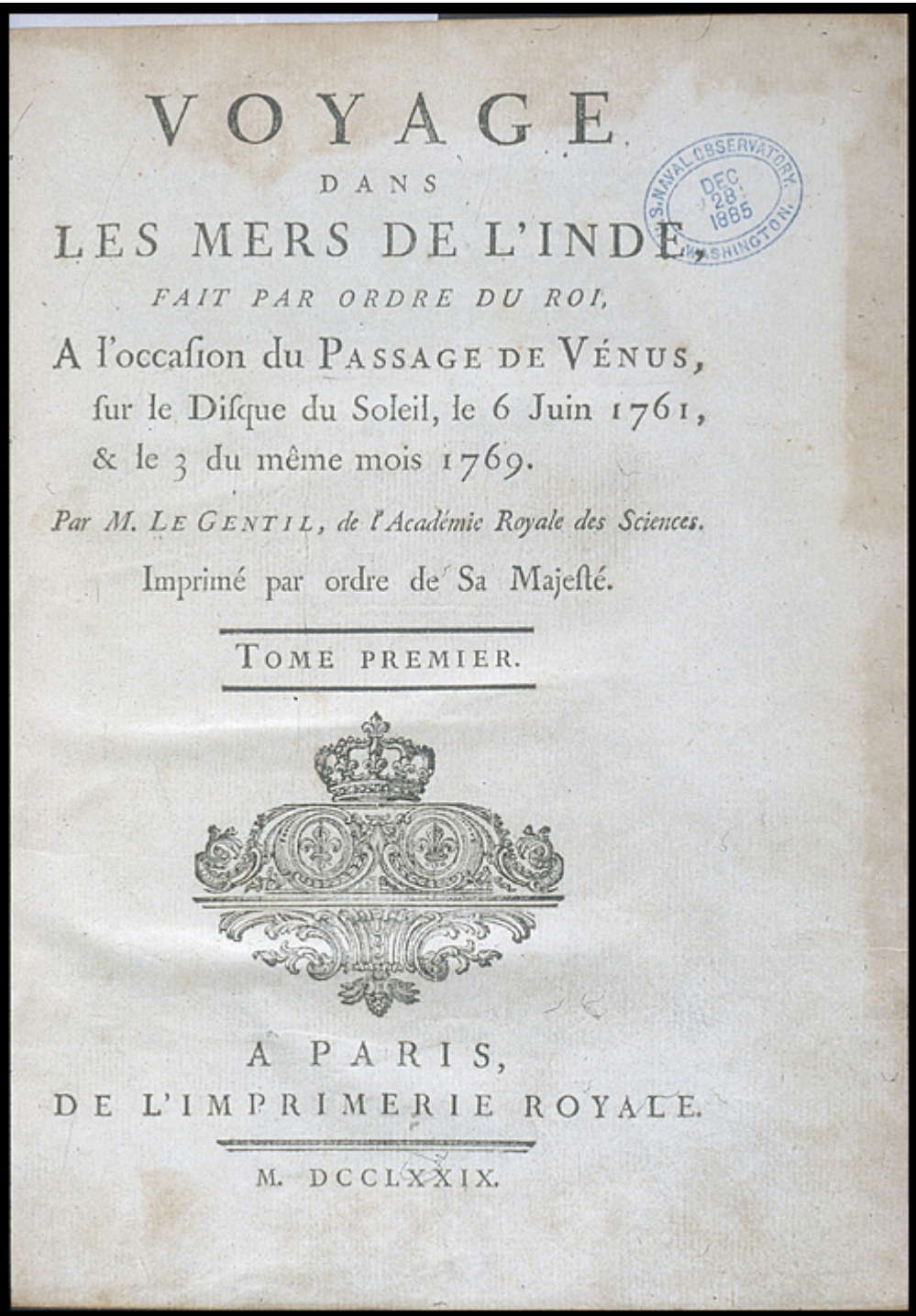
- Destination captured
- Waited 8 years
- Clouded out
- Contracted dysentery
- Shipwrecked



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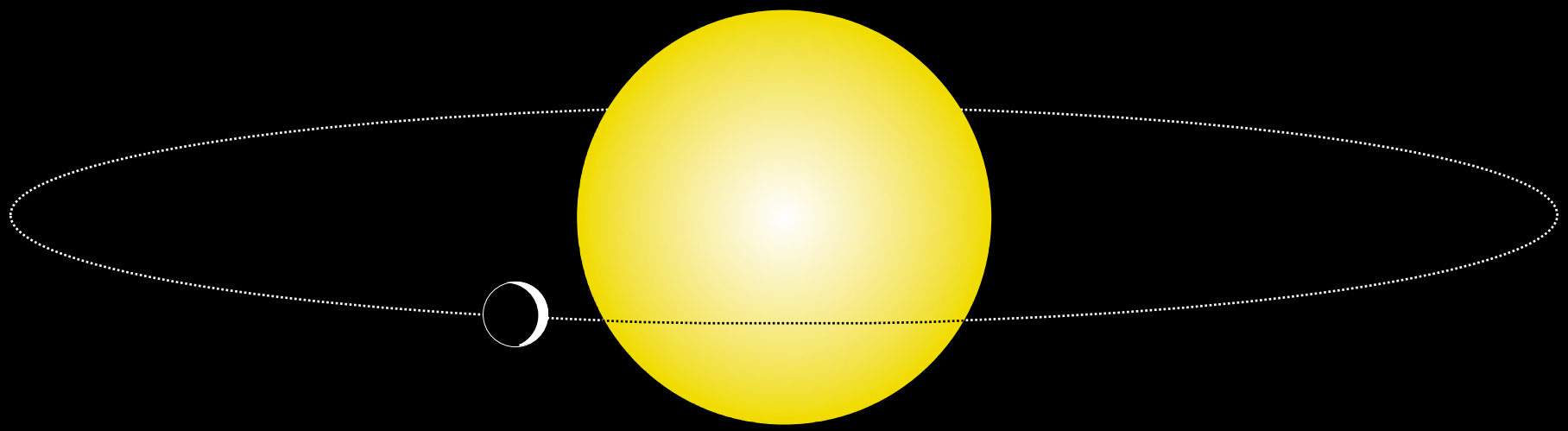
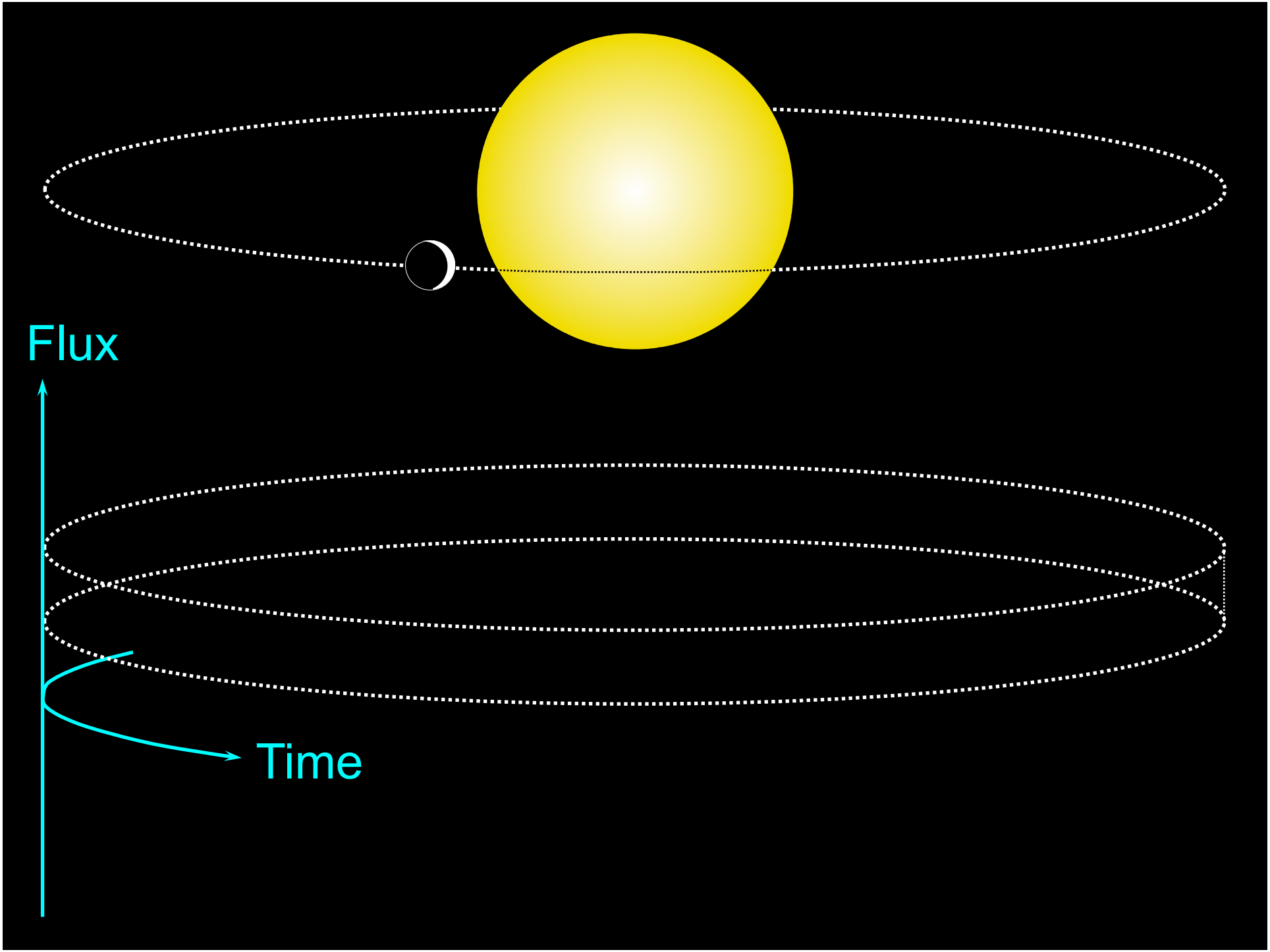
- Destination captured
- Waited 8 years
- Clouded out
- Contracted dysentery
- Shipwrecked
- Declared dead; estate divided up



The Transits of Exoplanets

Josh Winn
Massachusetts Institute of Technology

In collaboration with: Josh Carter (MIT); Matt Holman (CfA); John Johnson (Caltech); Dan Fabrycky (CfA); Geoff Marcy (UCB); Ed Turner (Princeton); Yasushi Suto (Tokyo); Norio Narita (NAOJ)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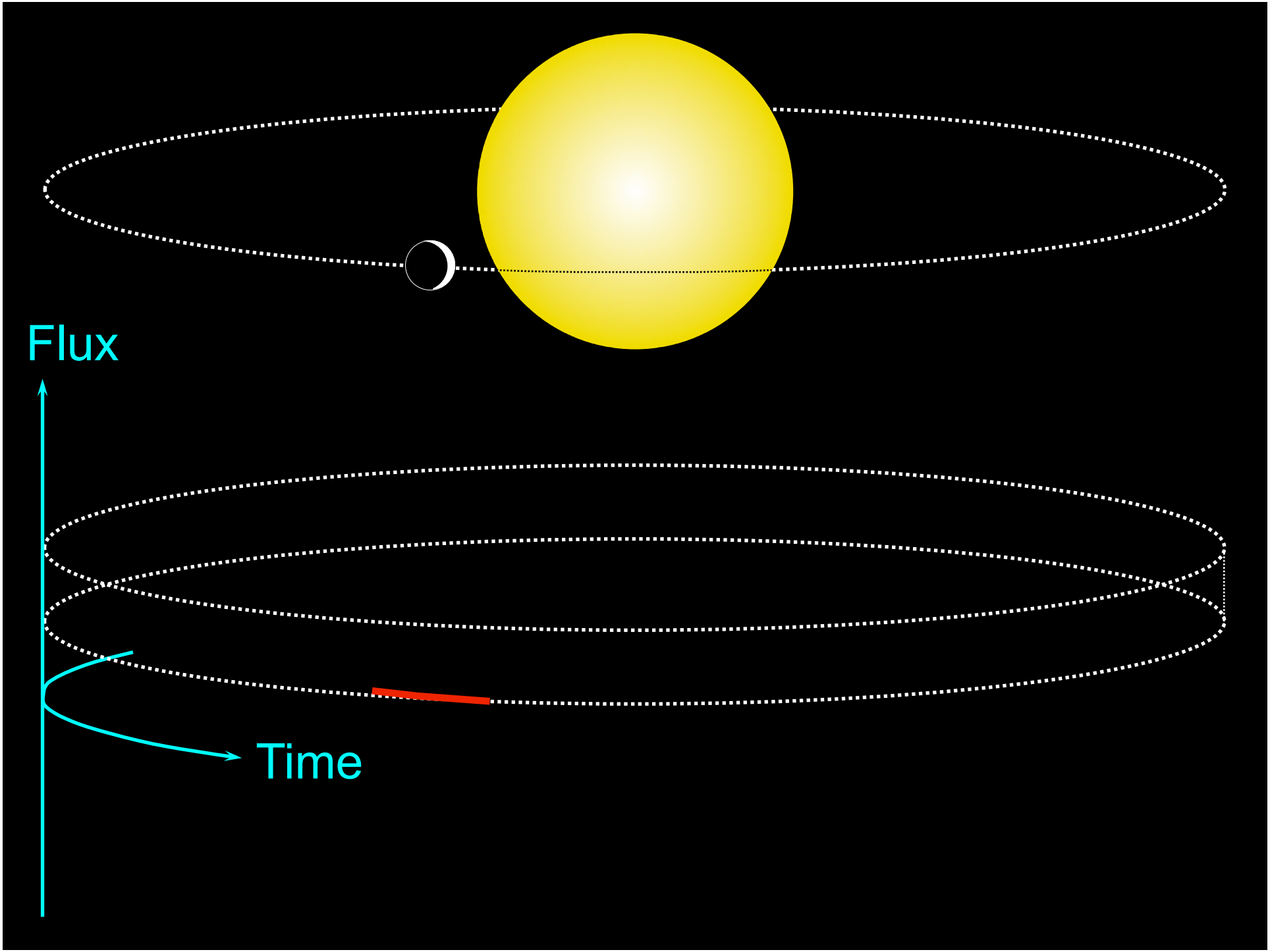
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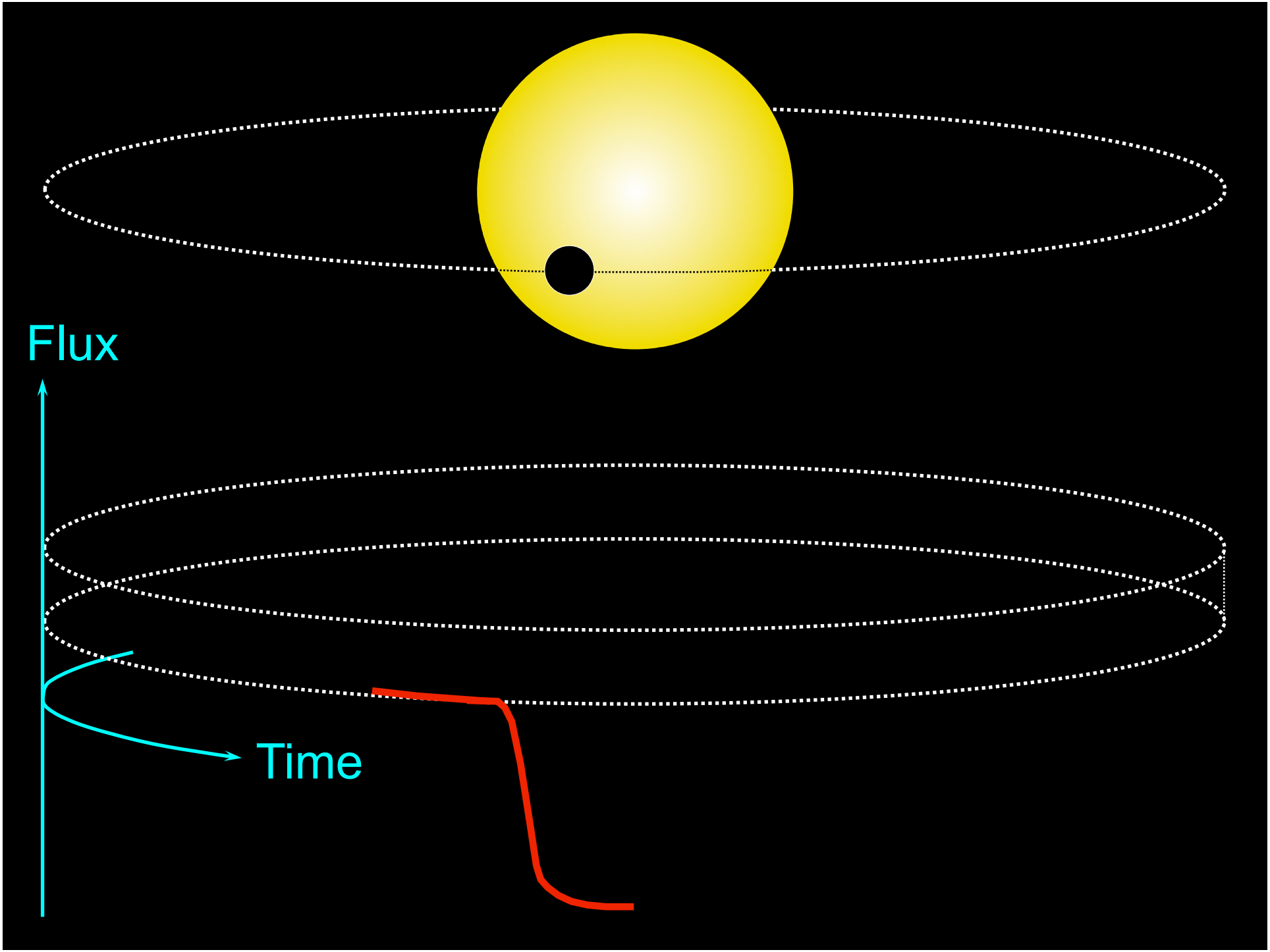
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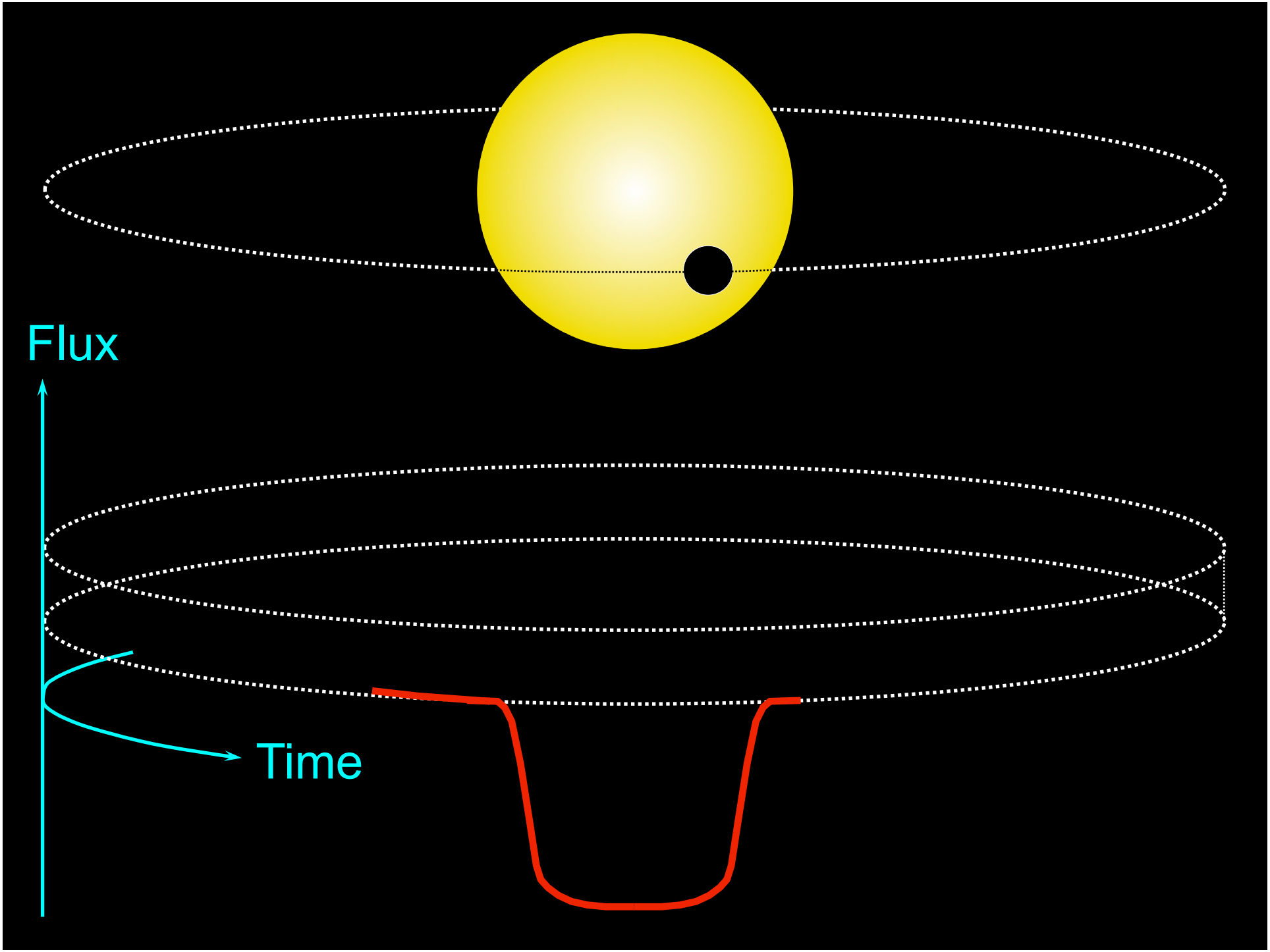
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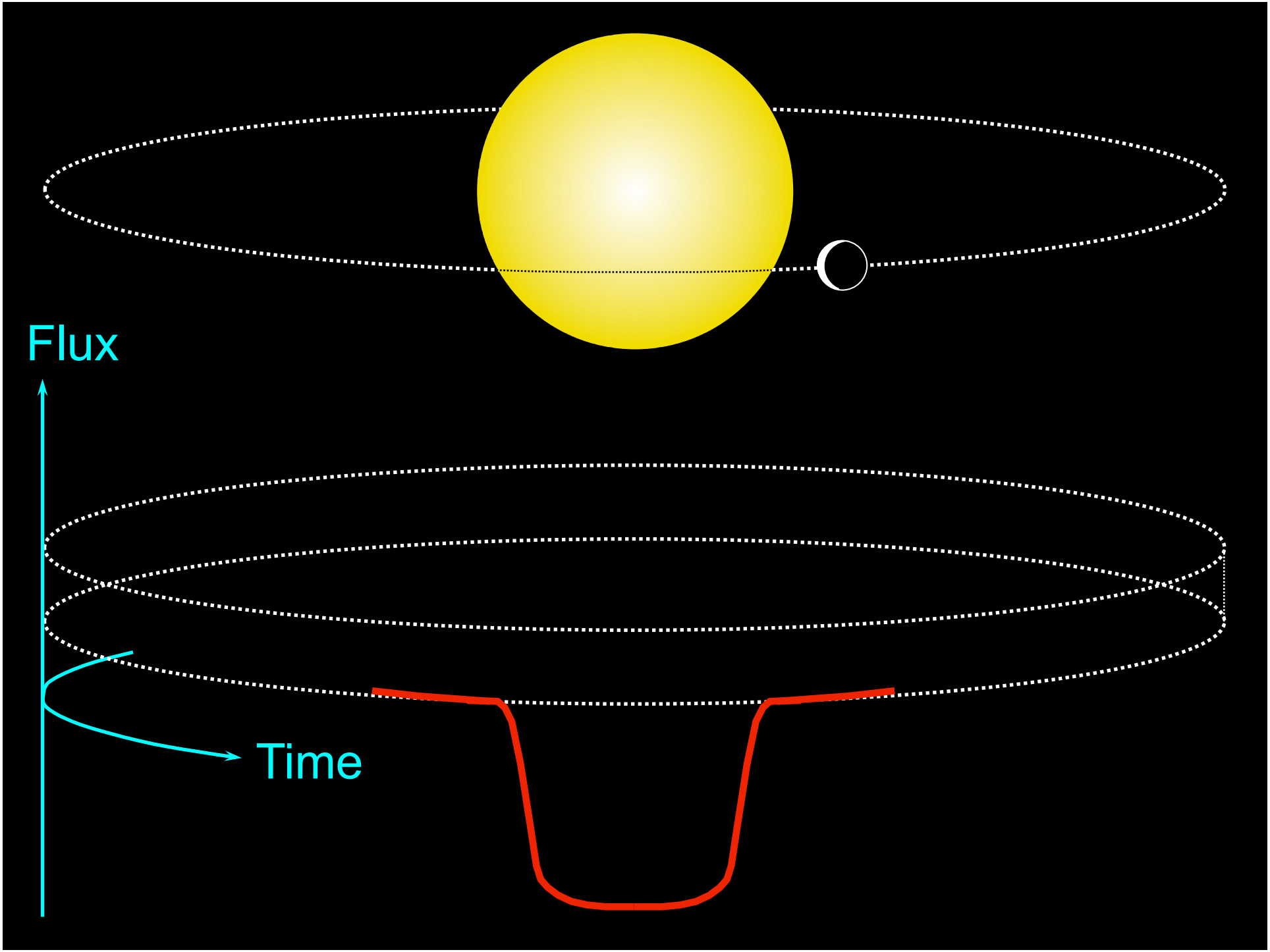
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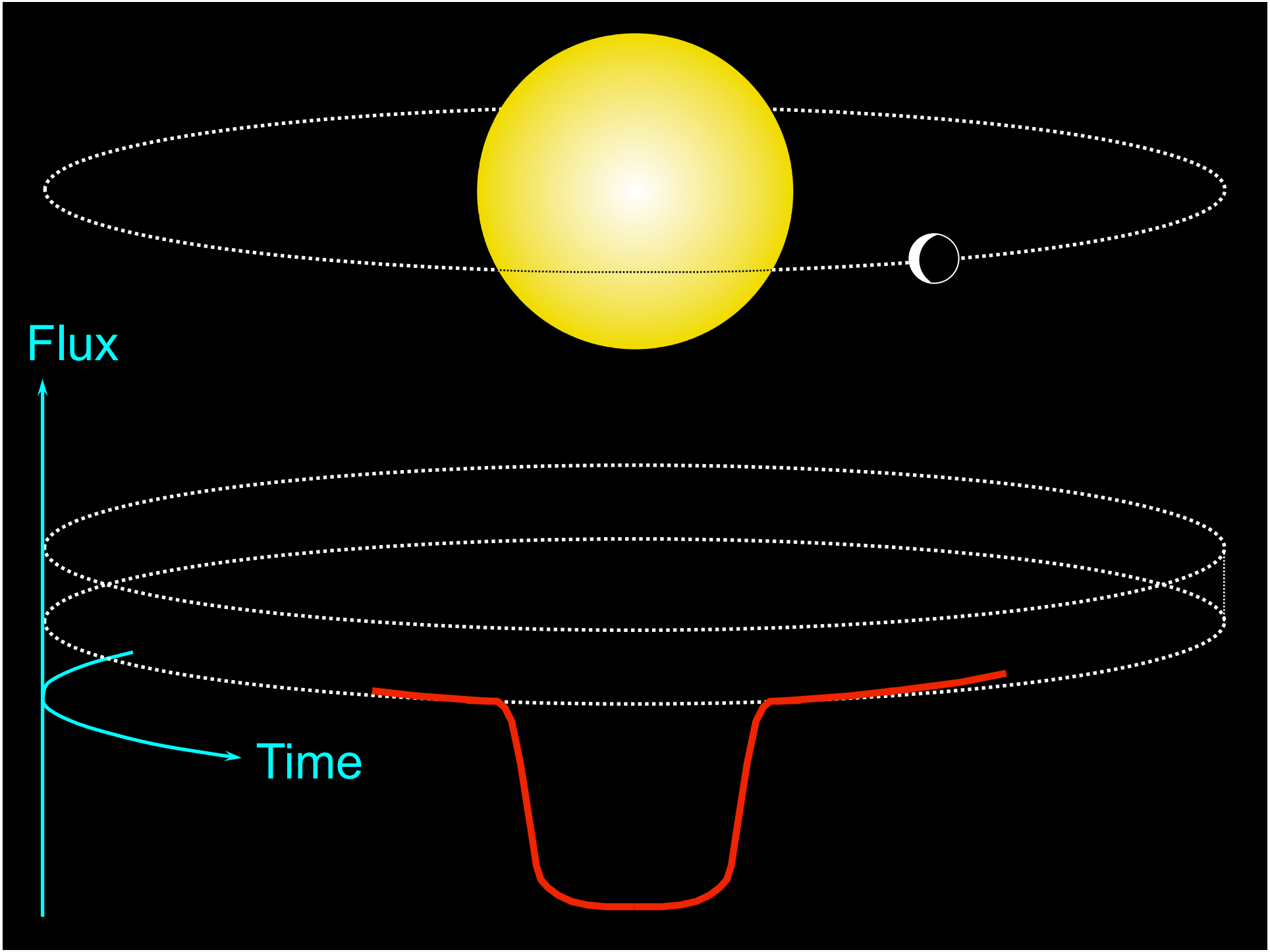
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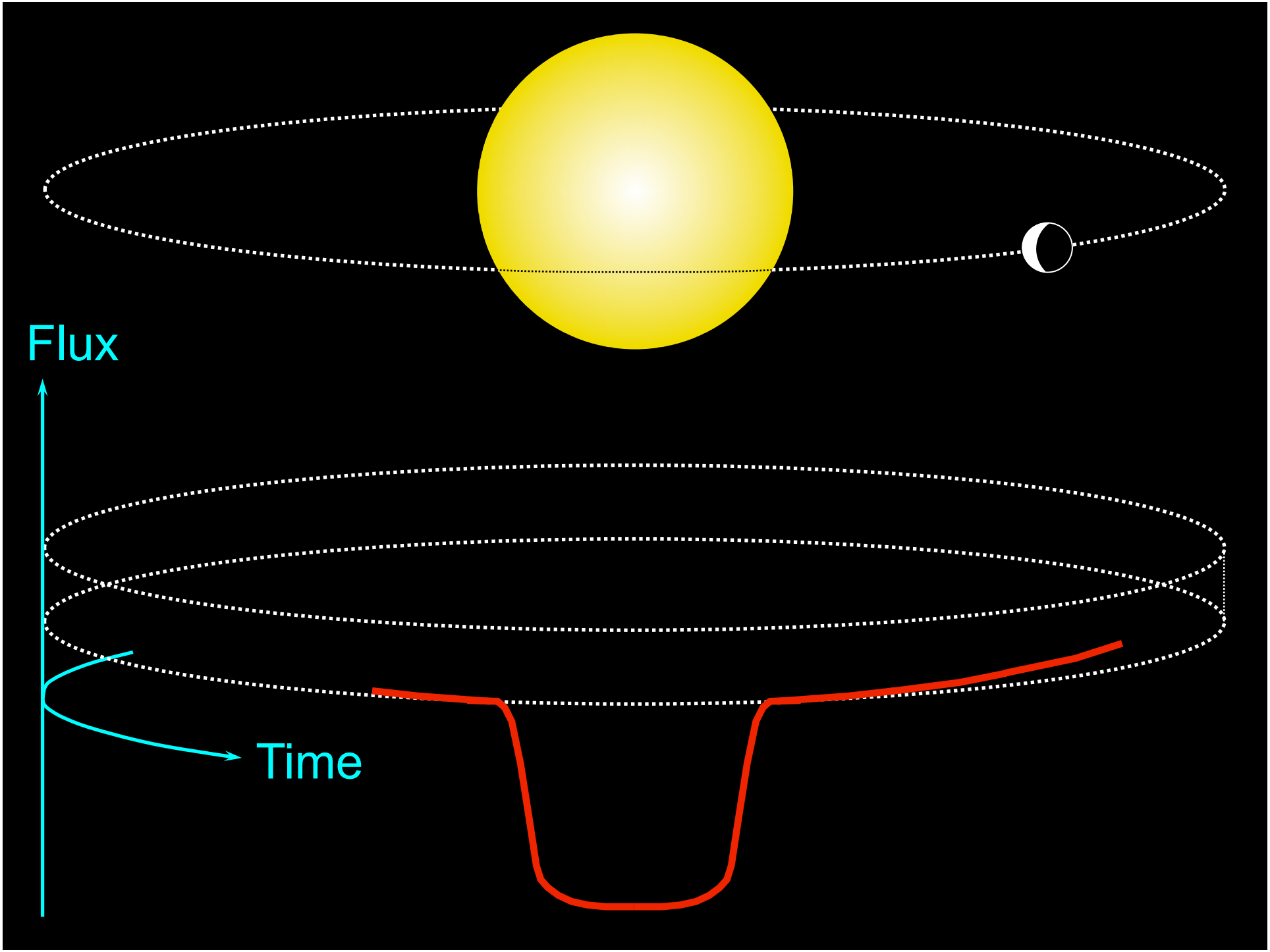
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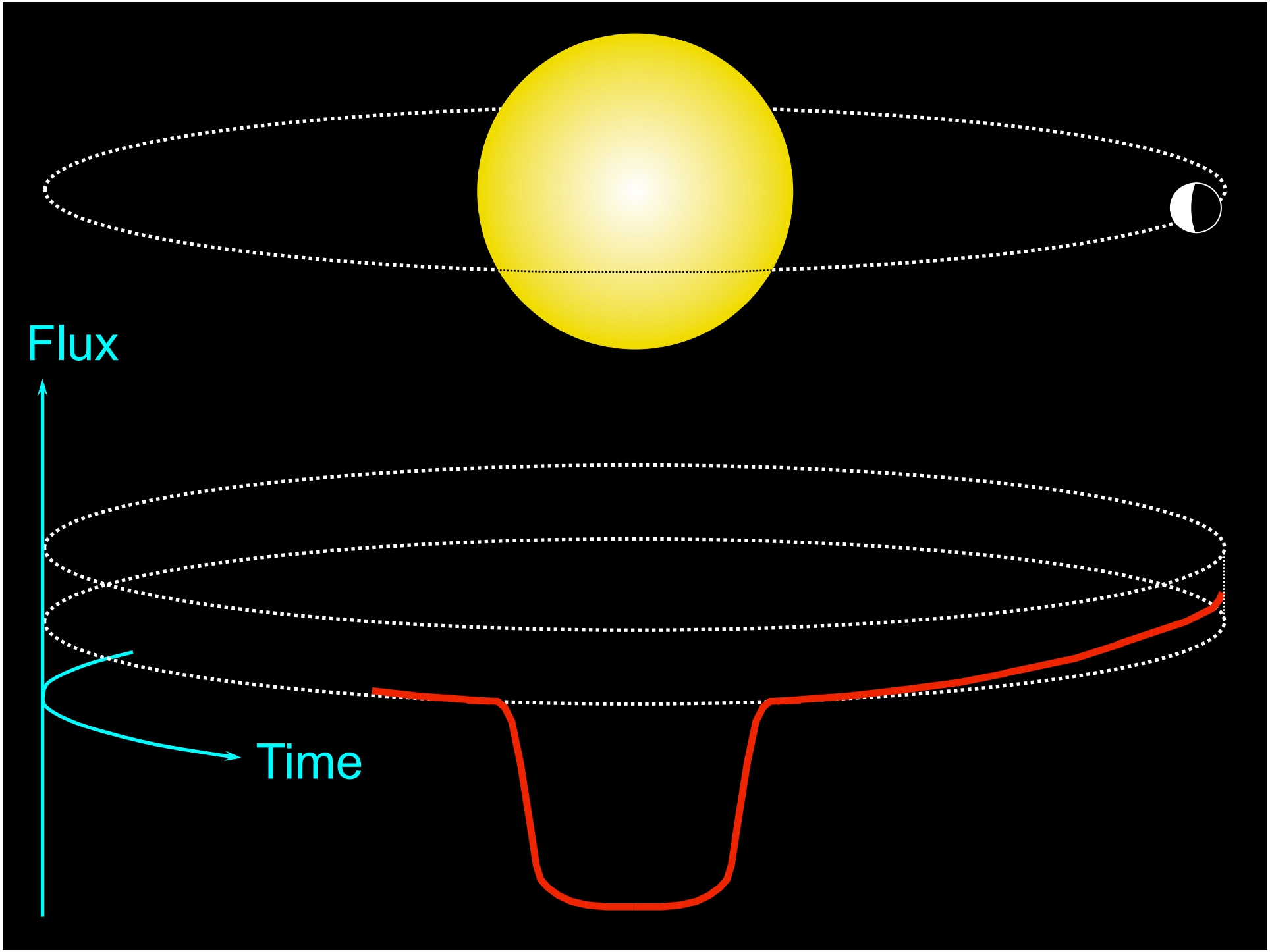
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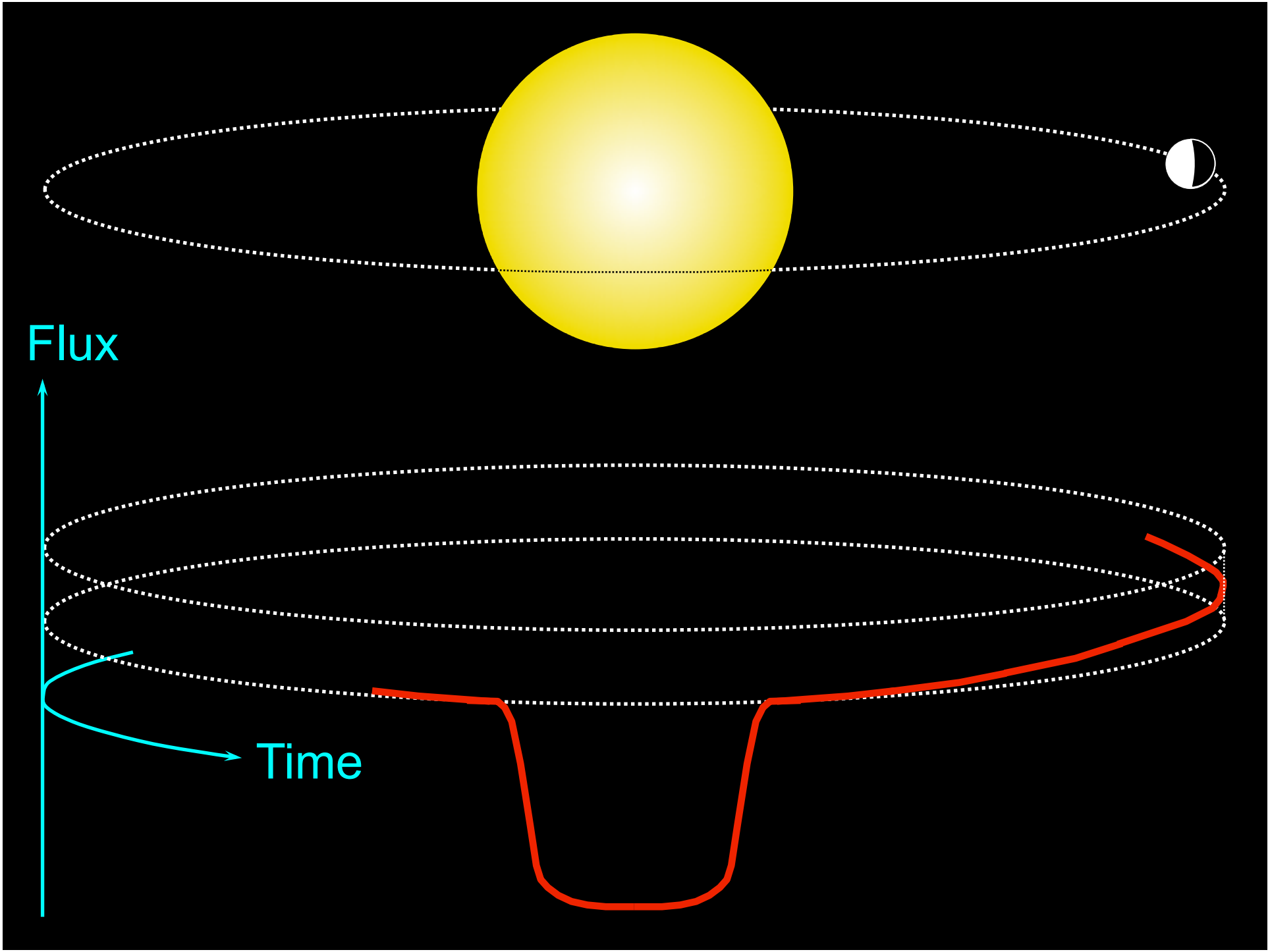
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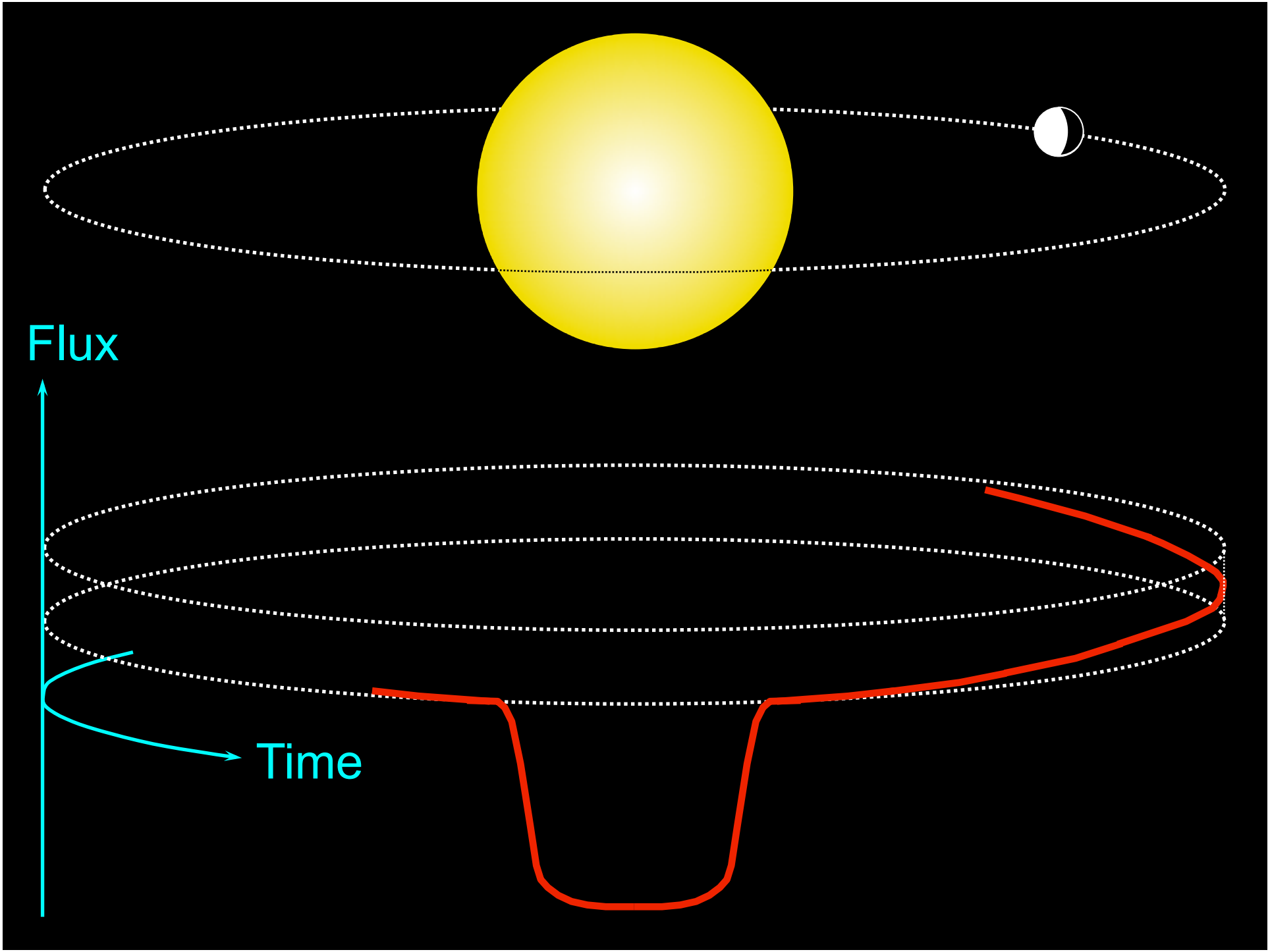
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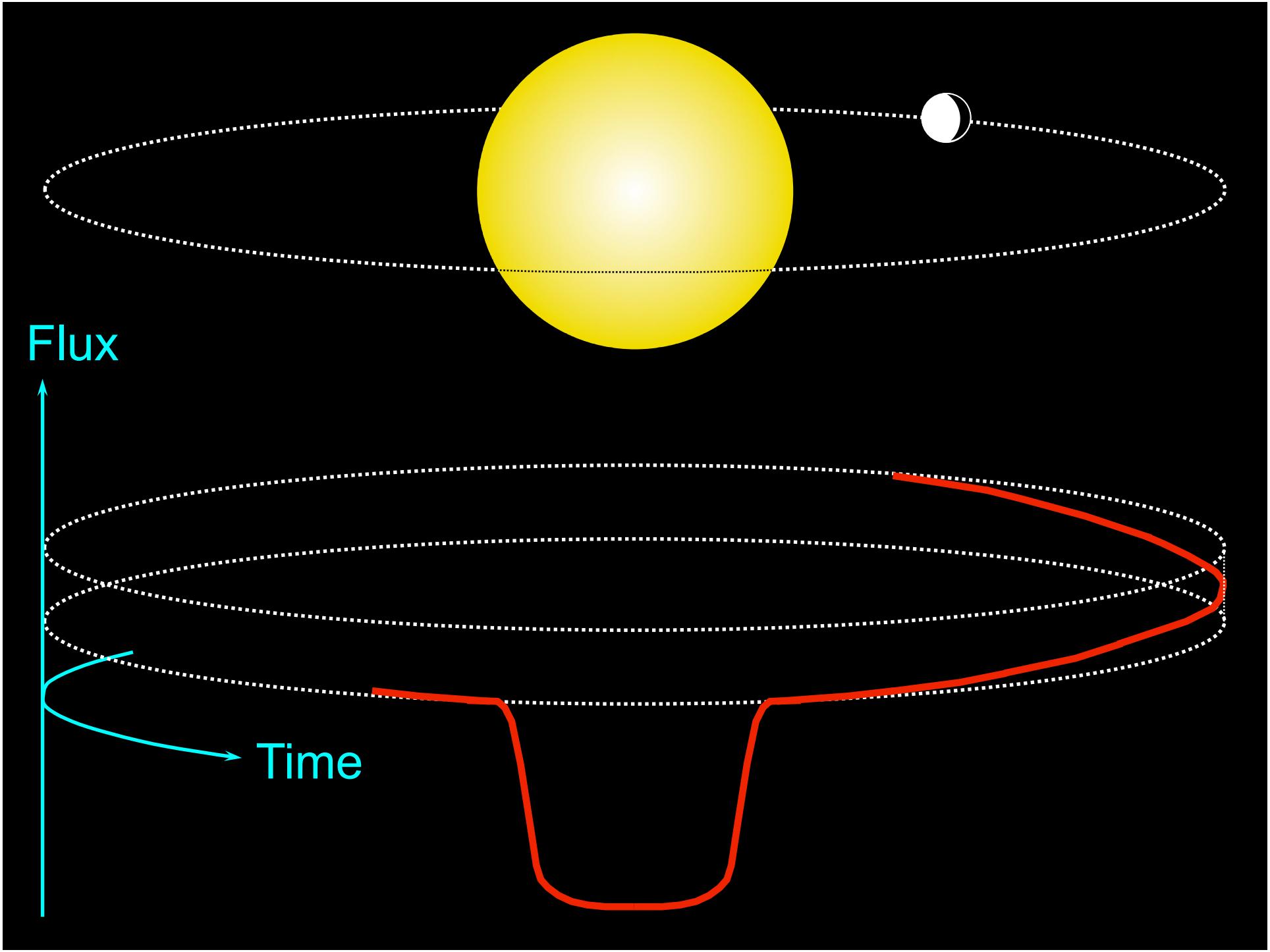
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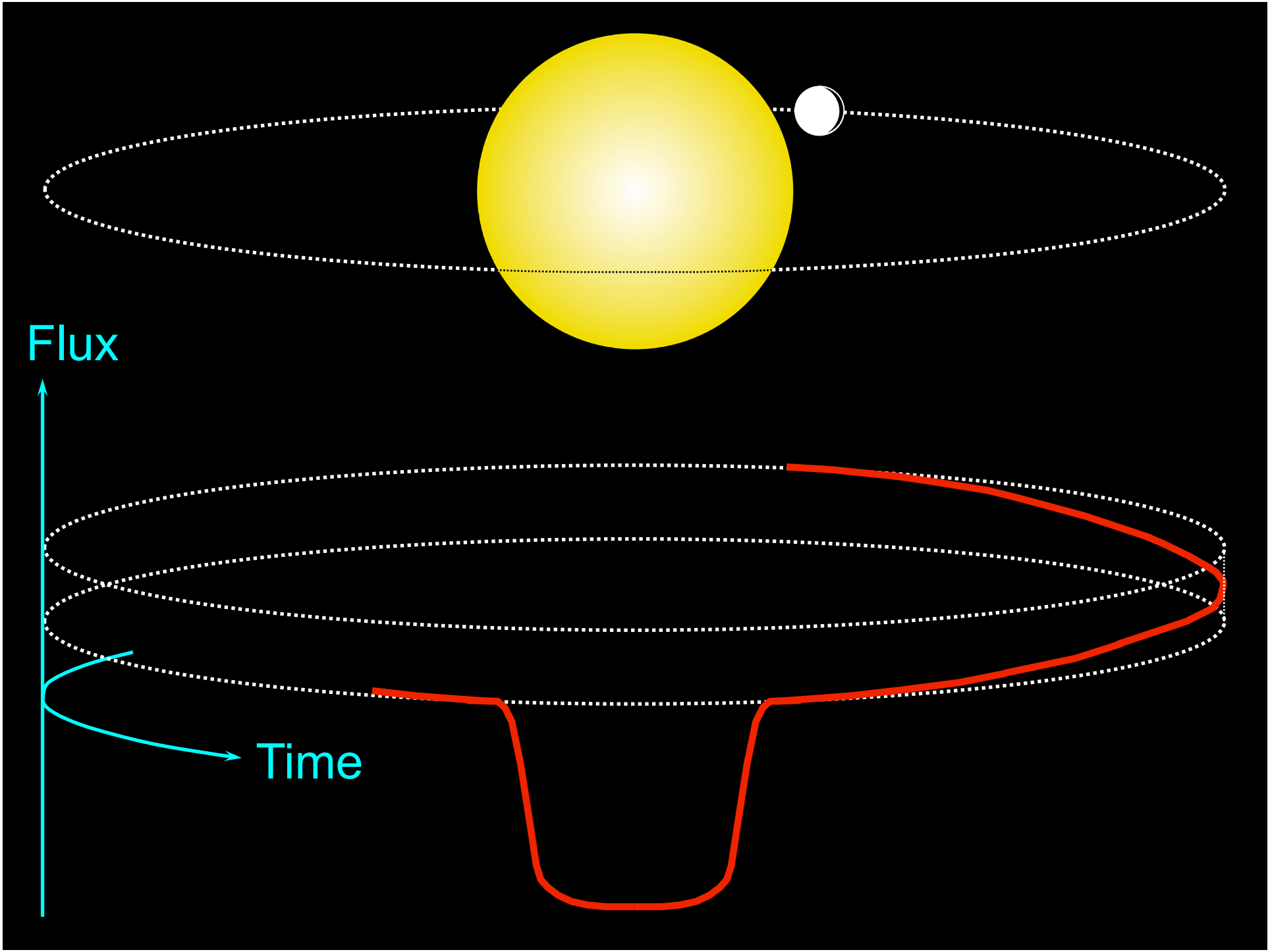
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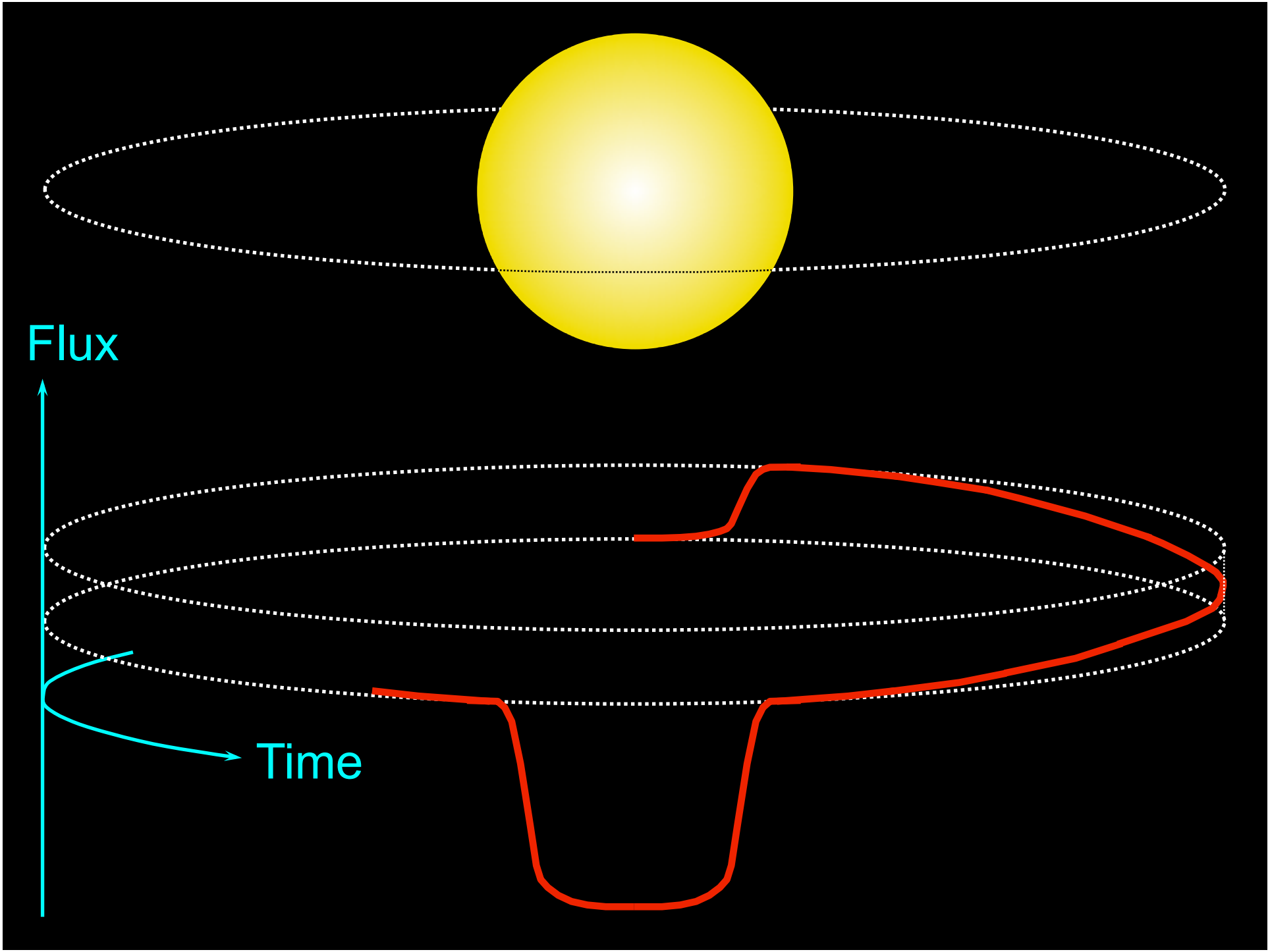
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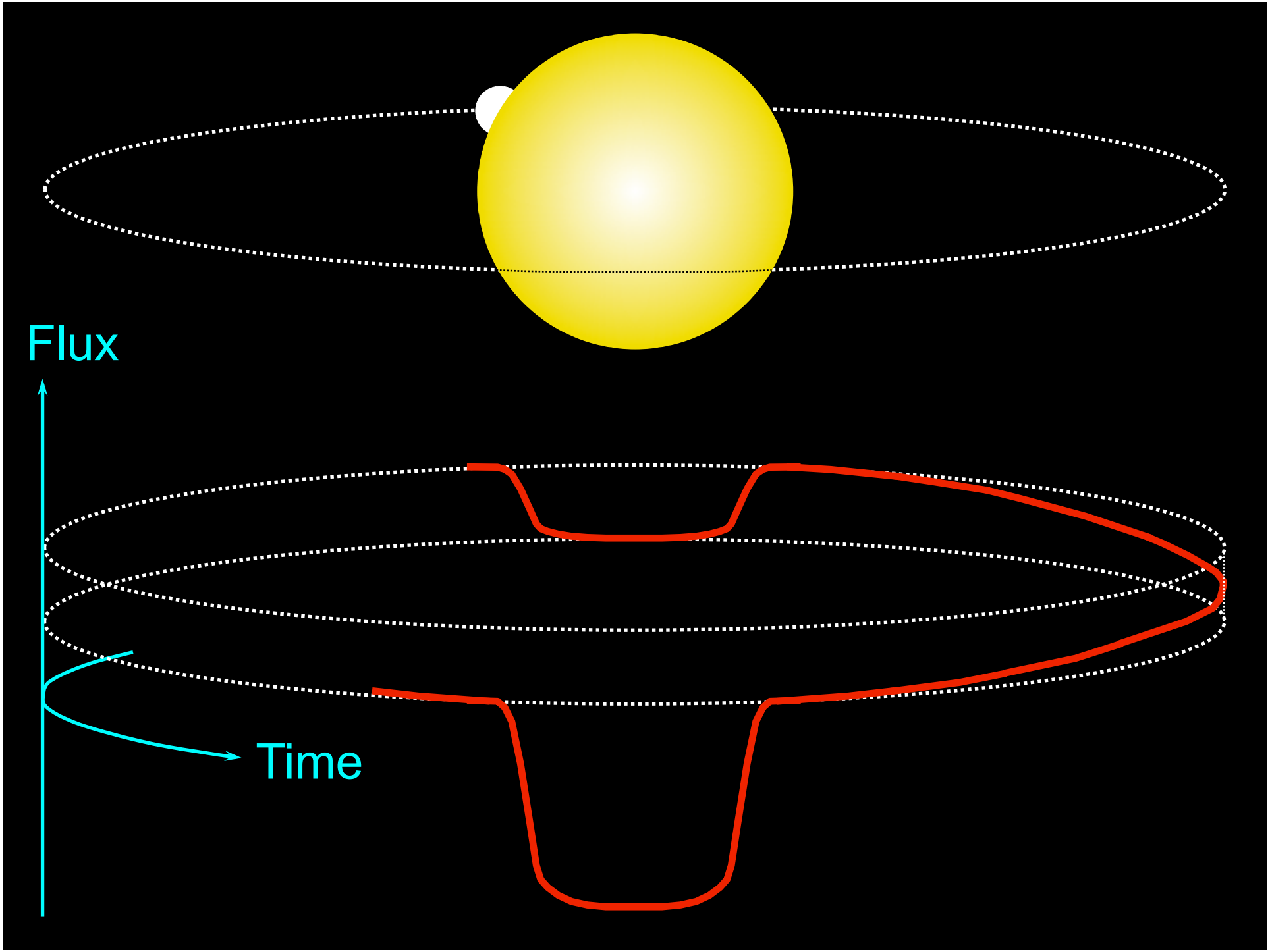
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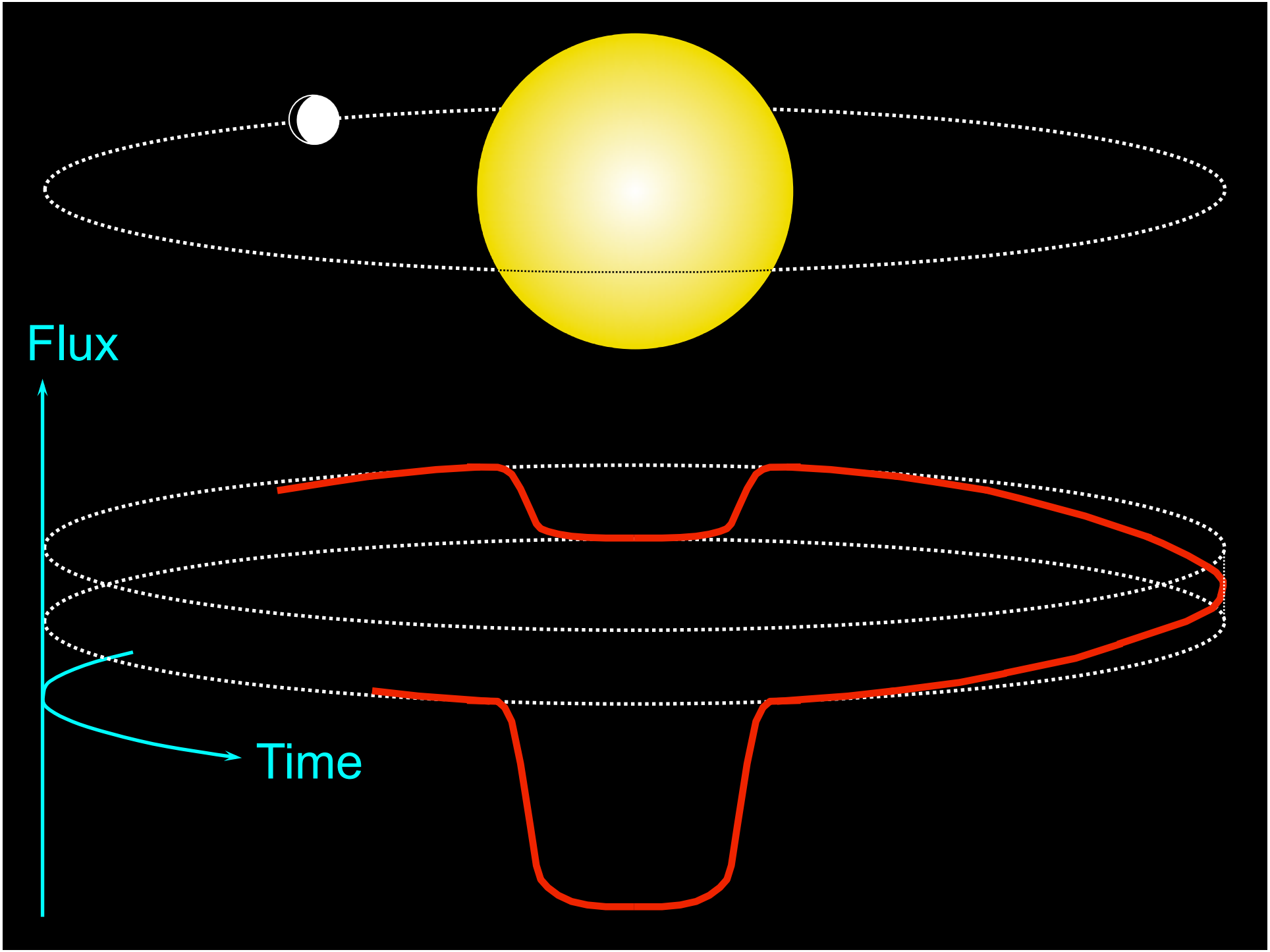
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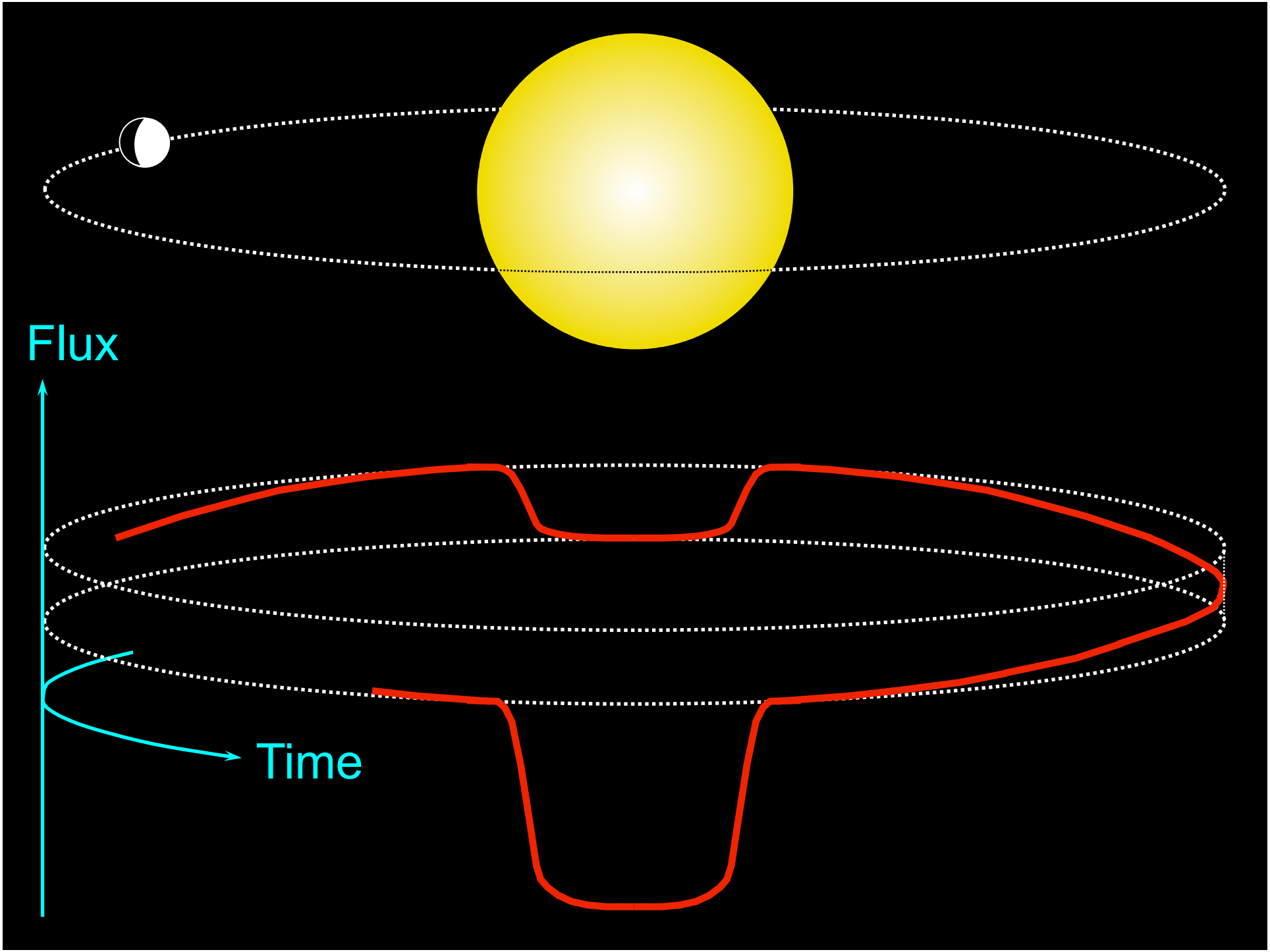
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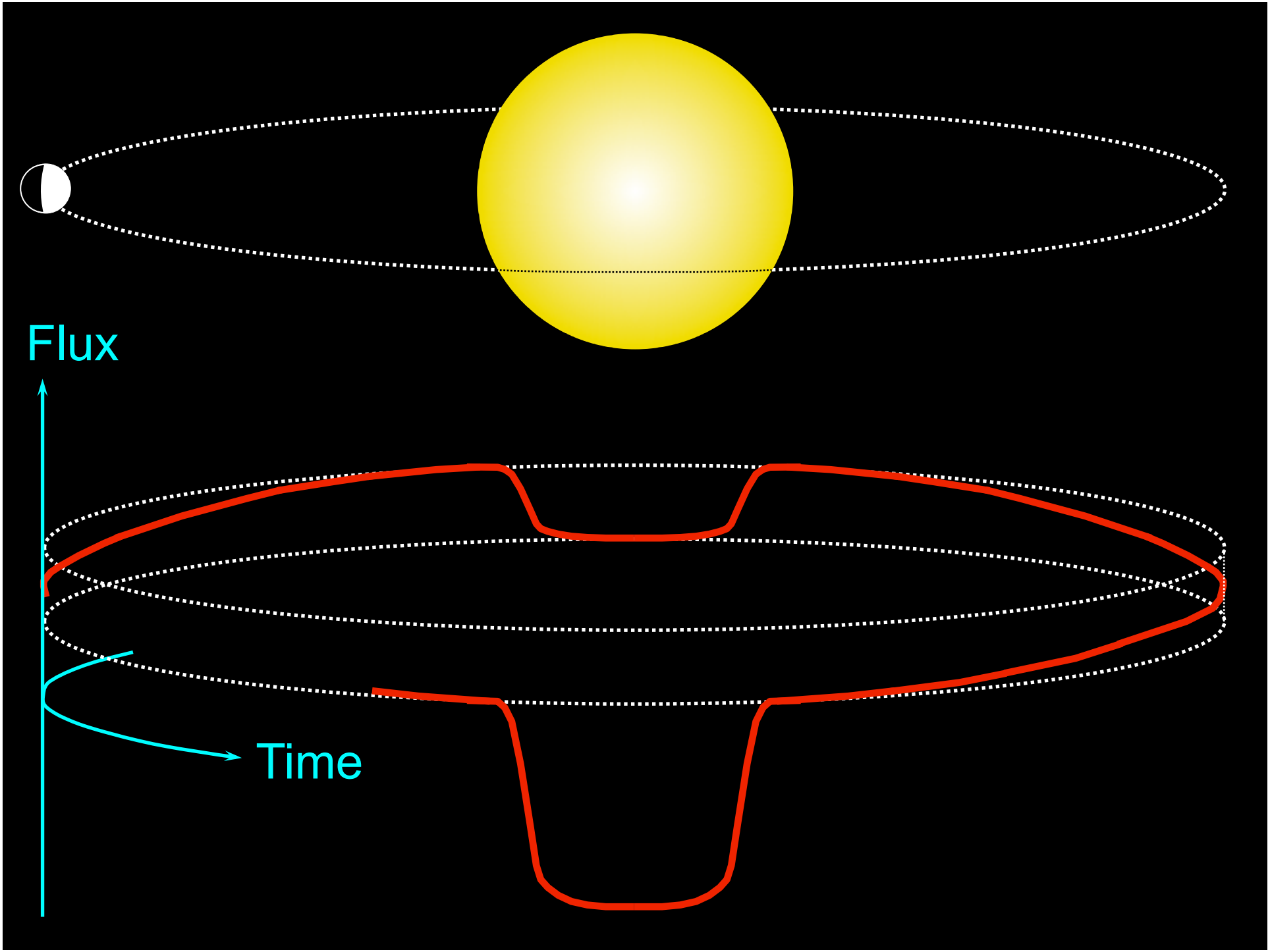
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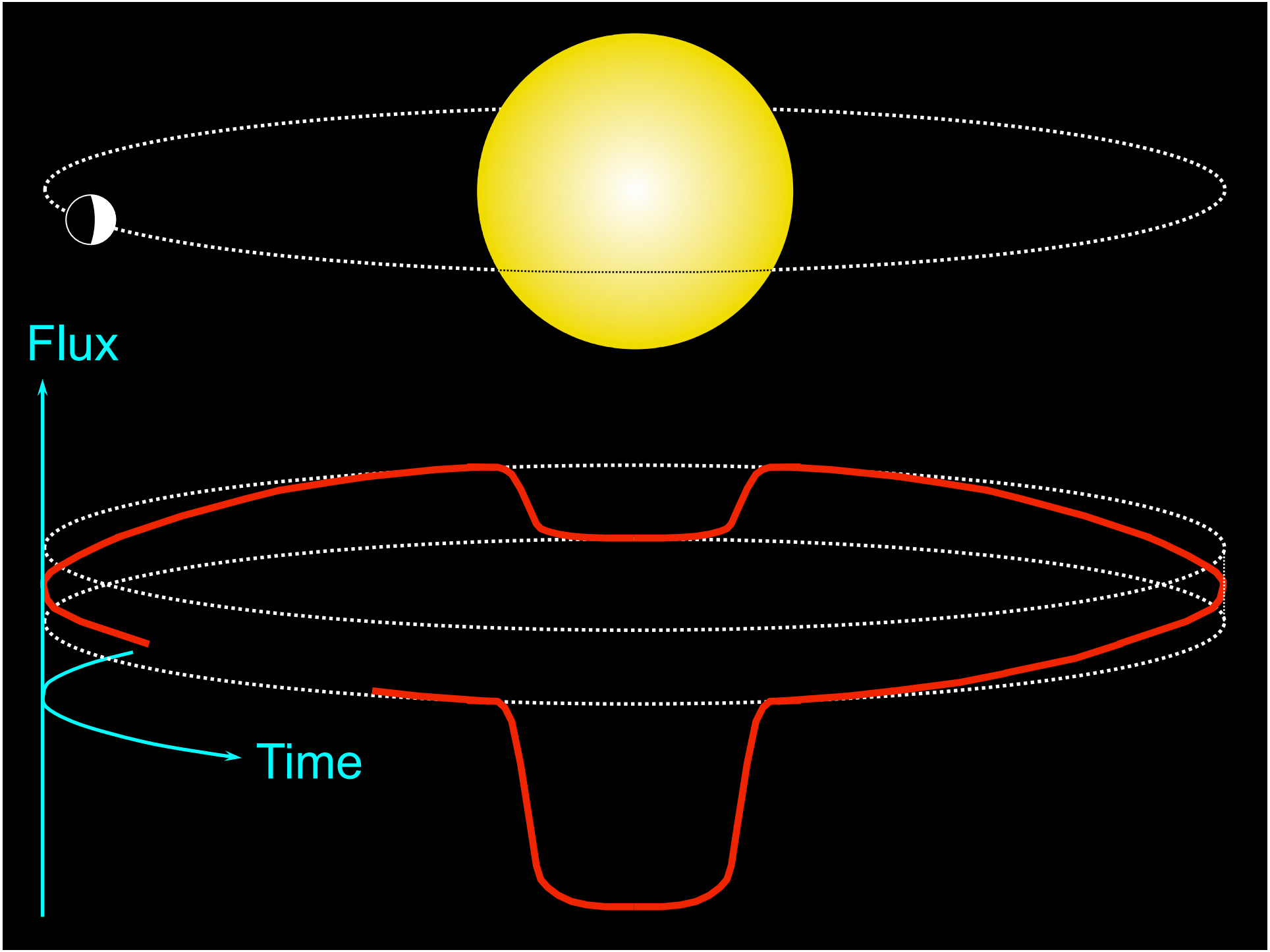
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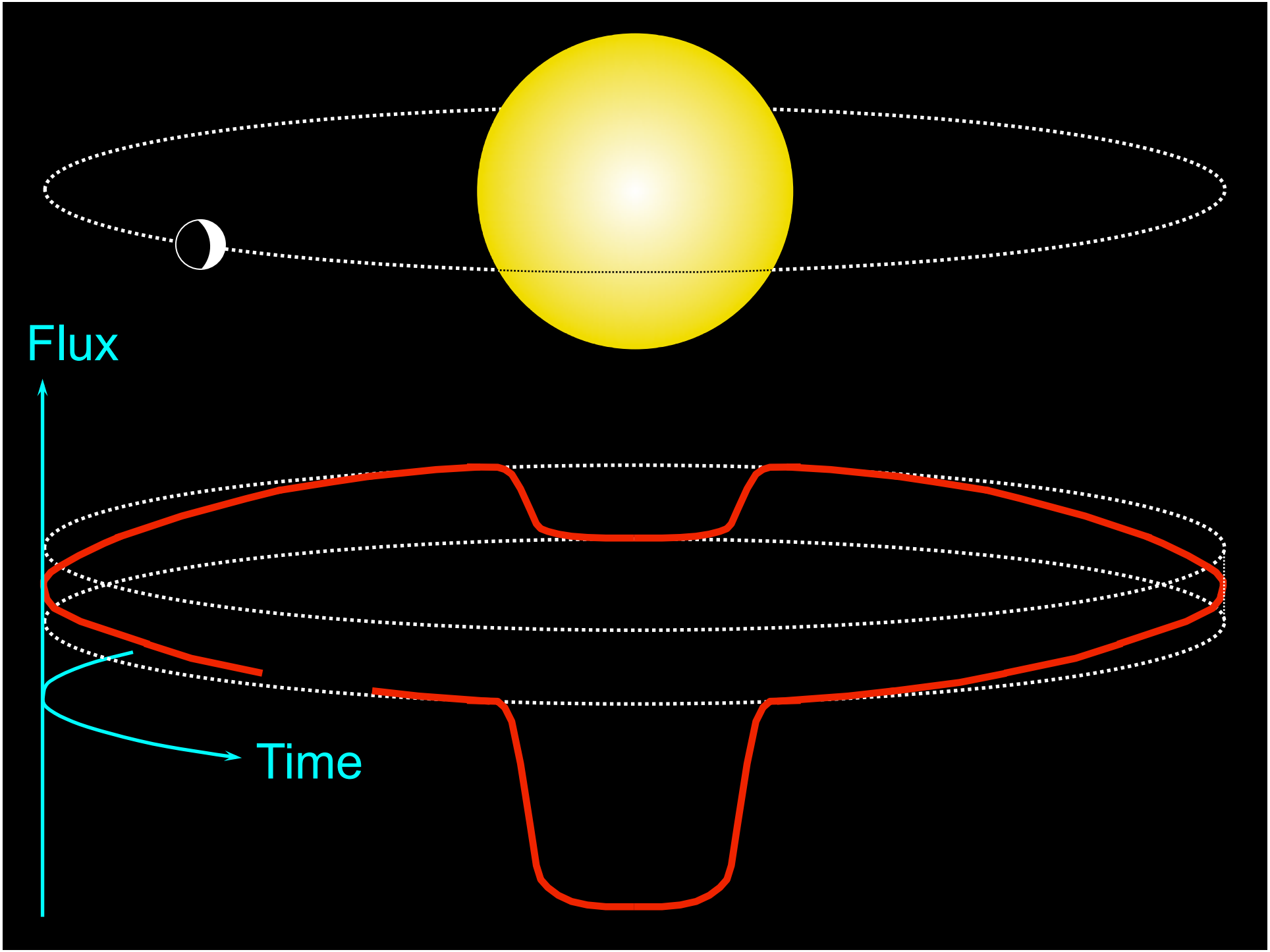
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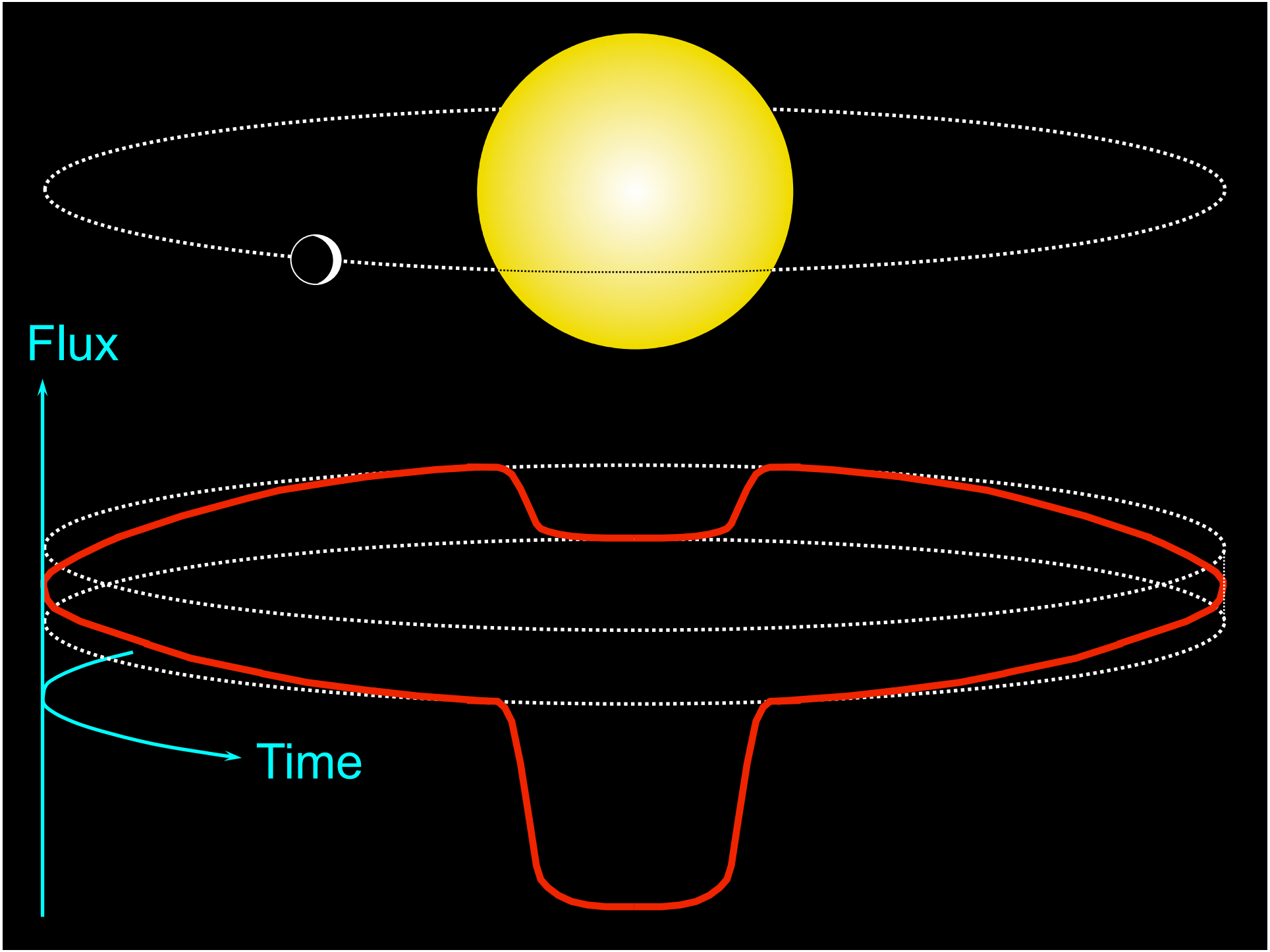
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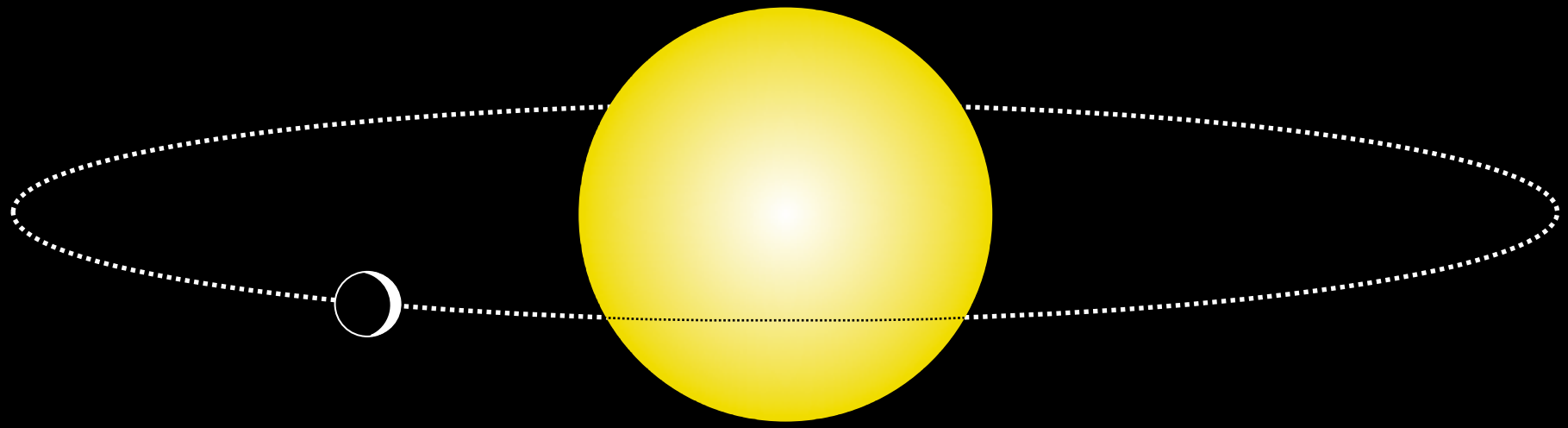


Orbital period

Transit times

Planet mass

Planet radius

Stellar obliquity

Orbital eccentricity

Star spots

Planetary emission spectrum

Planetary absorption spectrum

Planetary phase function

Surface map

Planetary reflectance spectrum

Orbital precession

Moons and rings

Planetary oblateness and obliquity

Planetary rotation rate

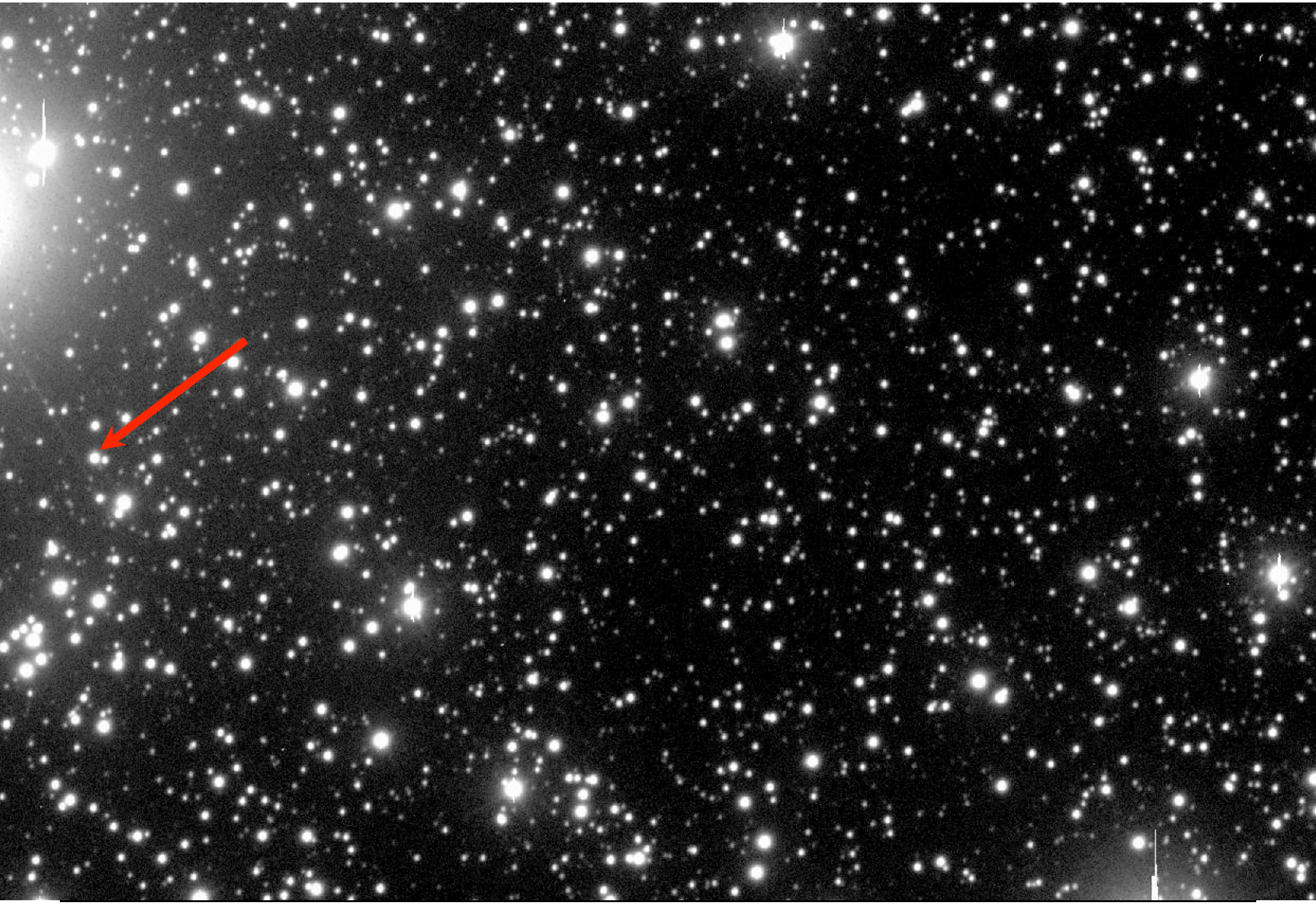
Planetary aurorae

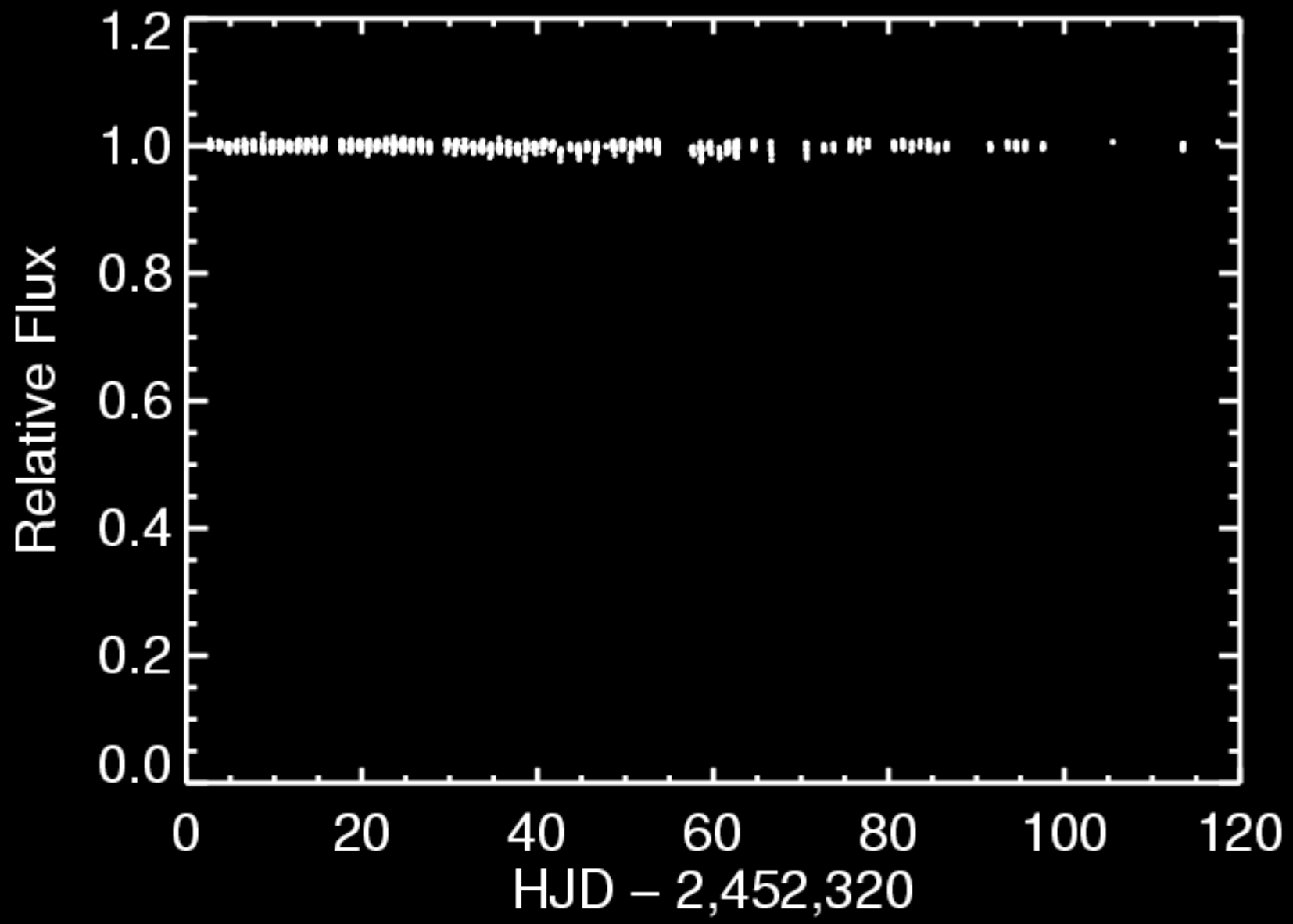
Planetary magnetic field

How do planets form?

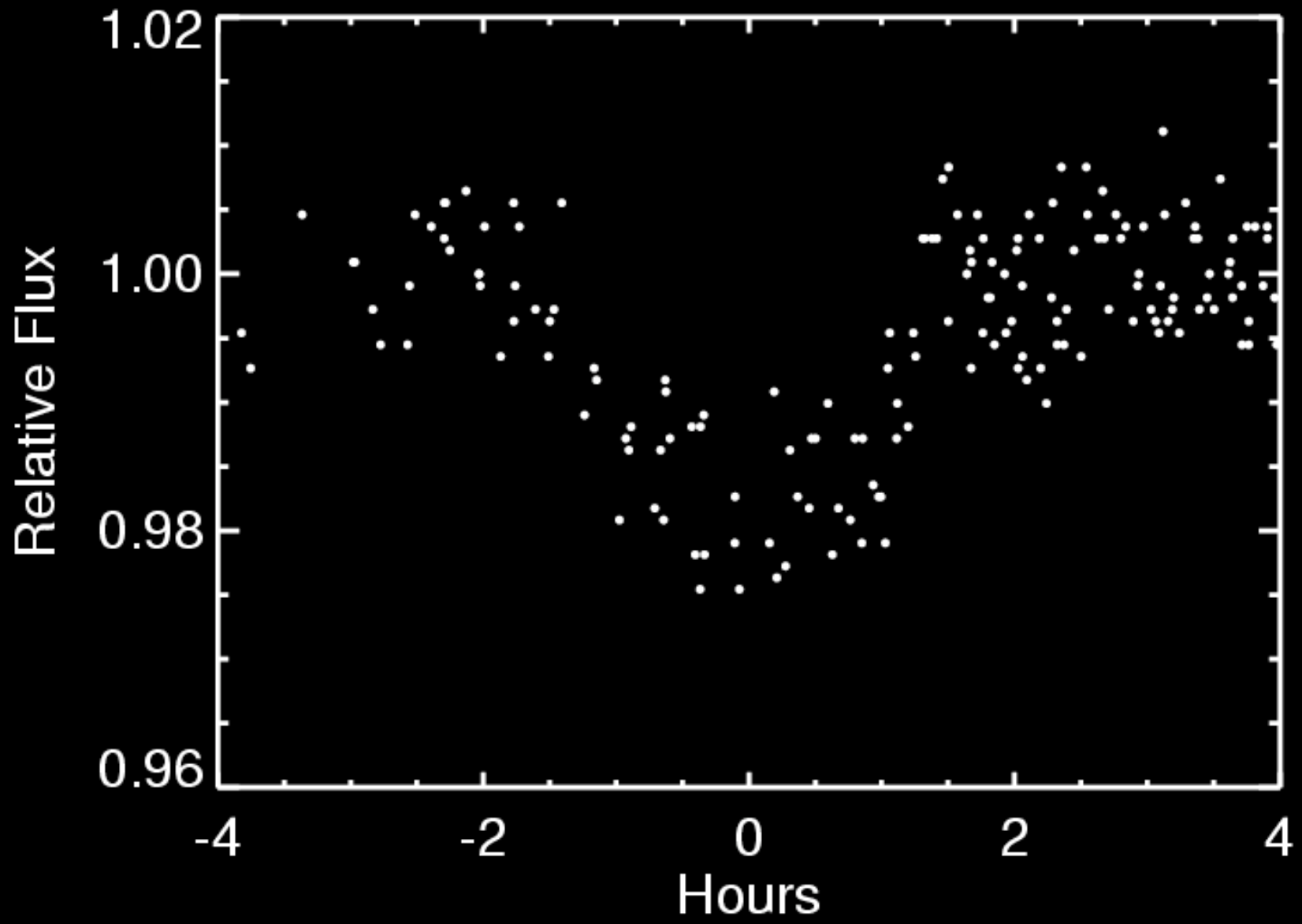
How do planets form?

How typical or unusual is the solar system?





Udalski et al. (the OGLE collaboration)



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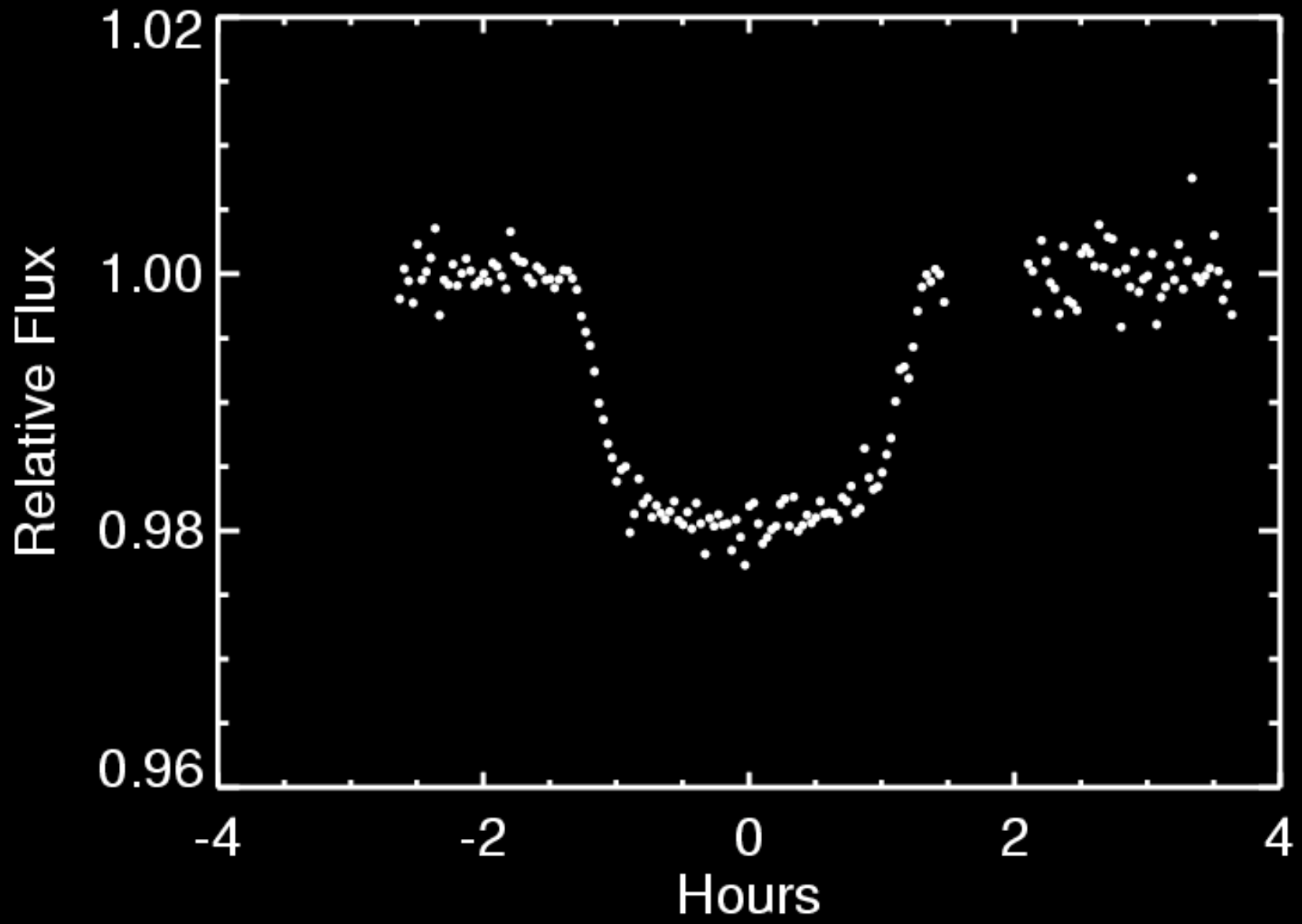
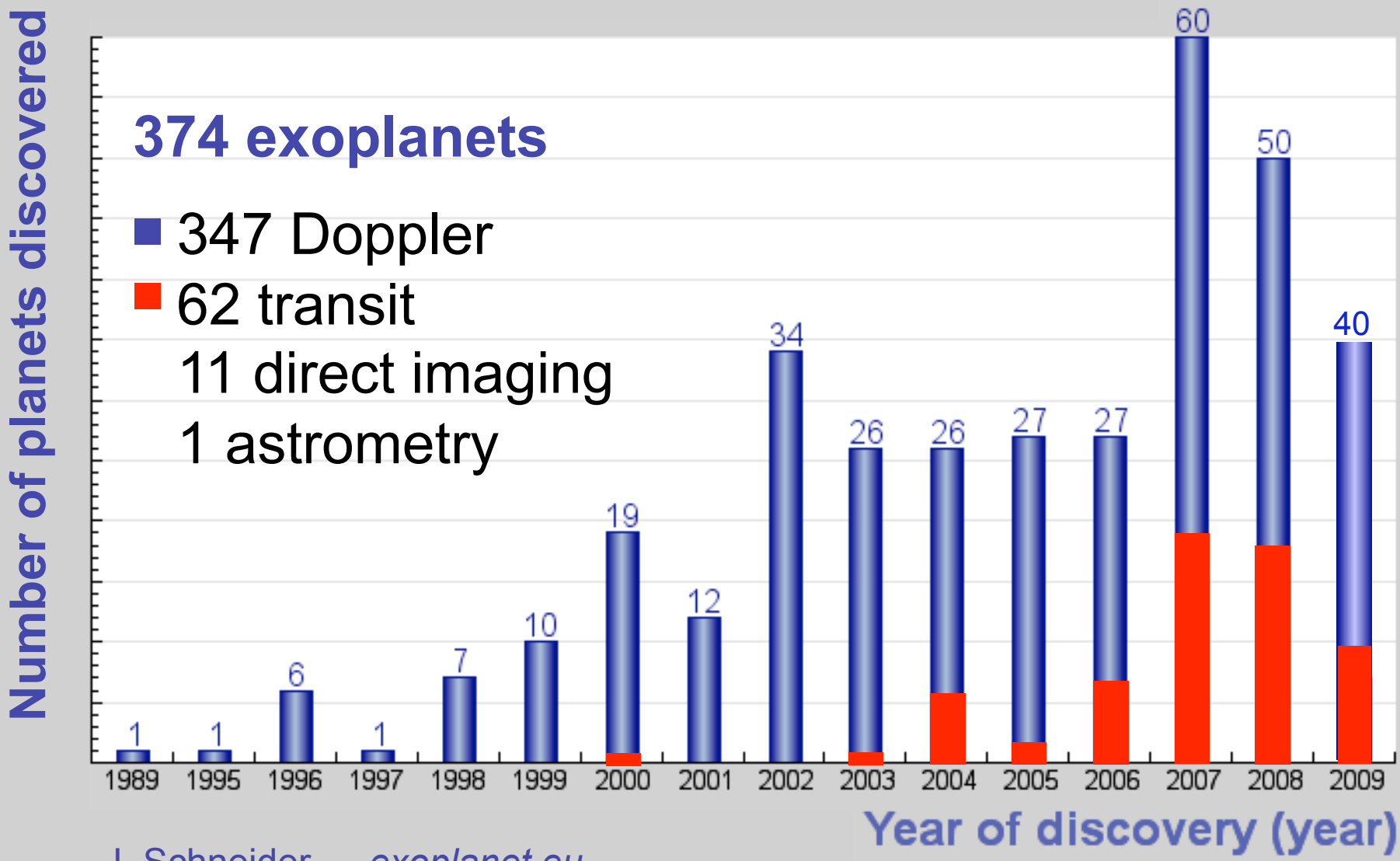
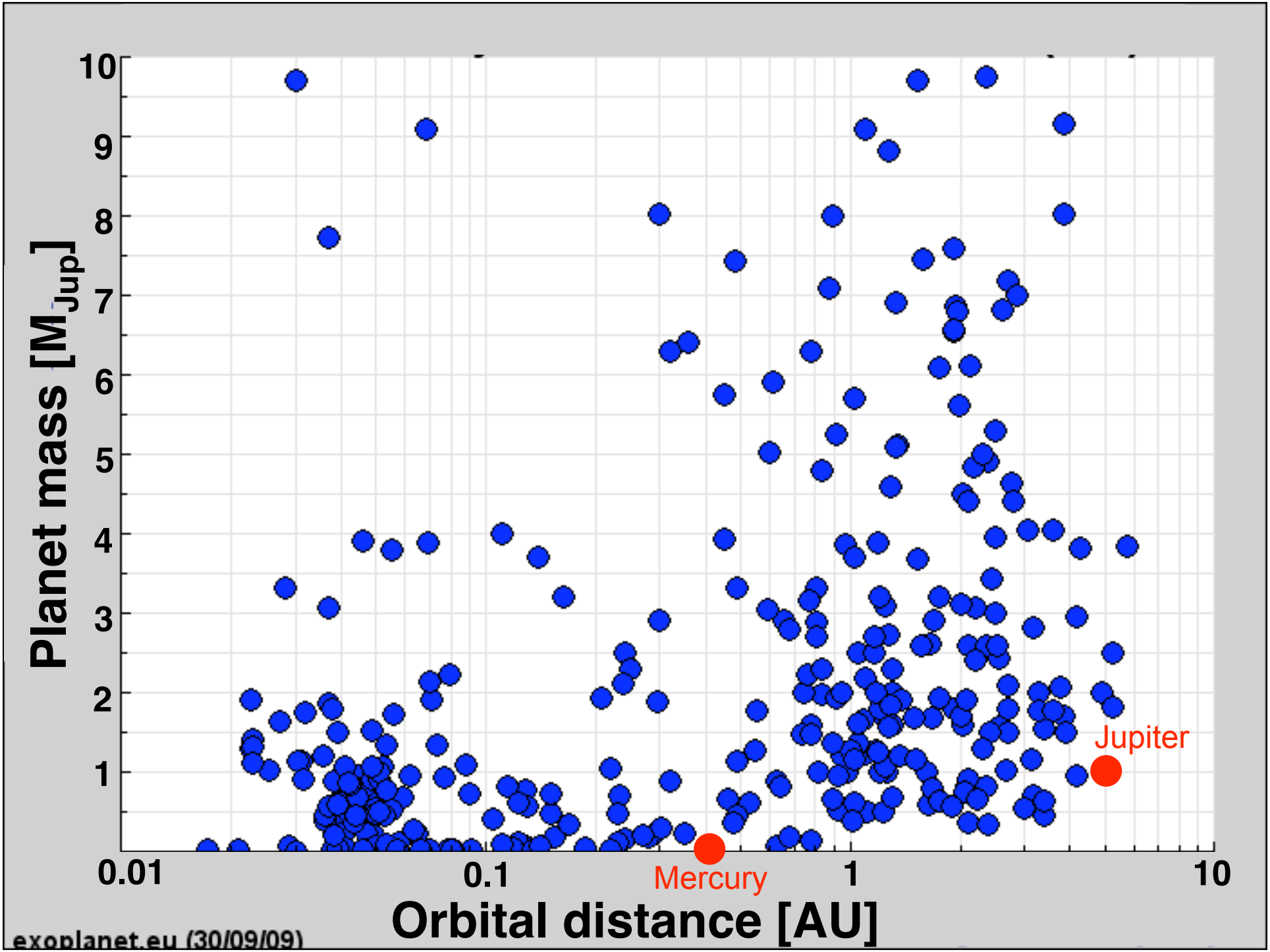


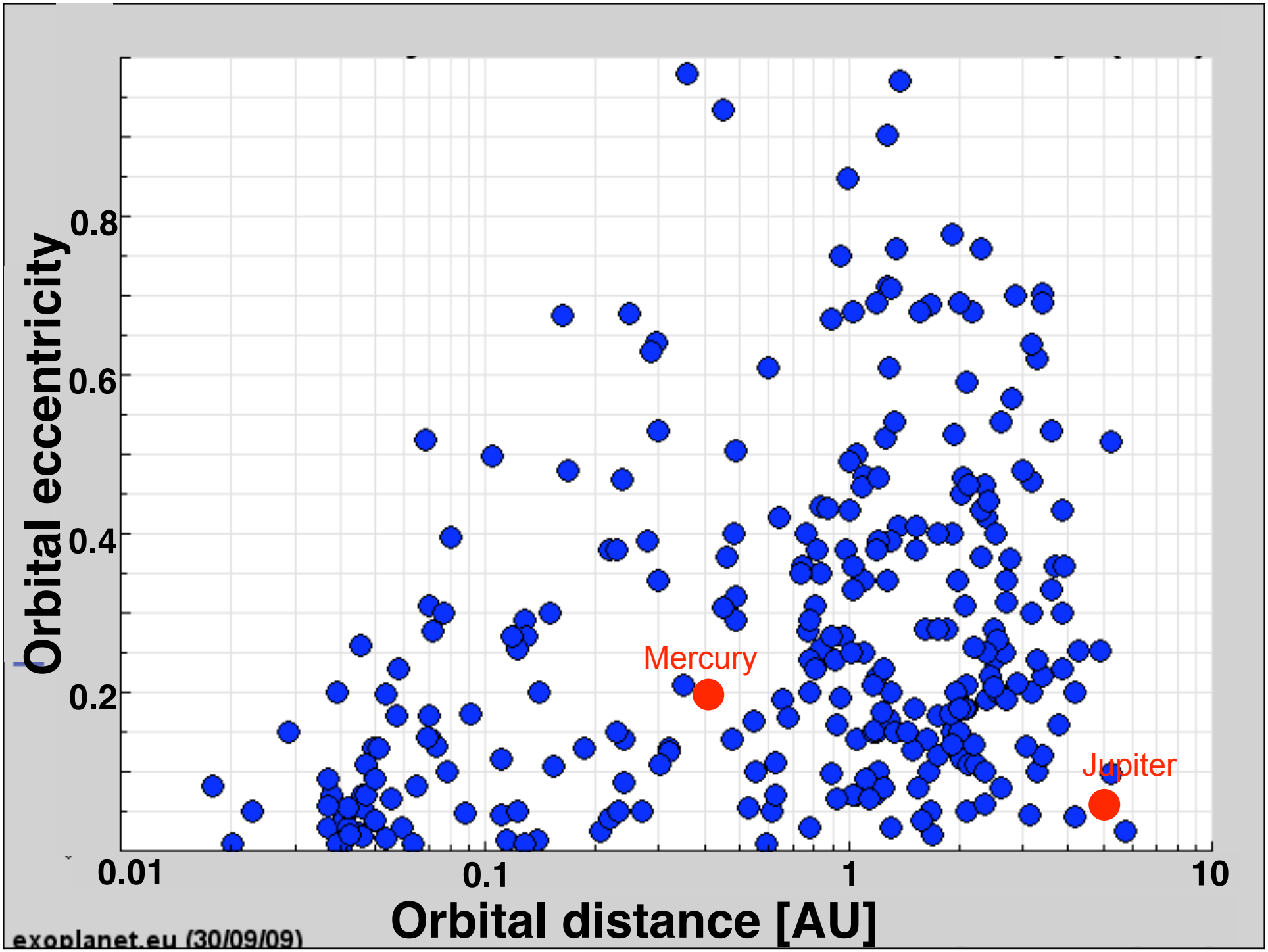
Winn, Holman, & Fuentes (2007)

Scorecard



J. Schneider — *exoplanet.eu*


Planet mass [M_Jup]
10
9
8
7
6
5
4
3
2
1
0.01
0.1
1
10
Mercury
Jupiter
Orbital distance [AU]
exoplanet.eu (30/09/09)


Orbital eccentricity
0.8
0.6
0.4
0.2
Mercury
Jupiter
0.01
0.1
1
10
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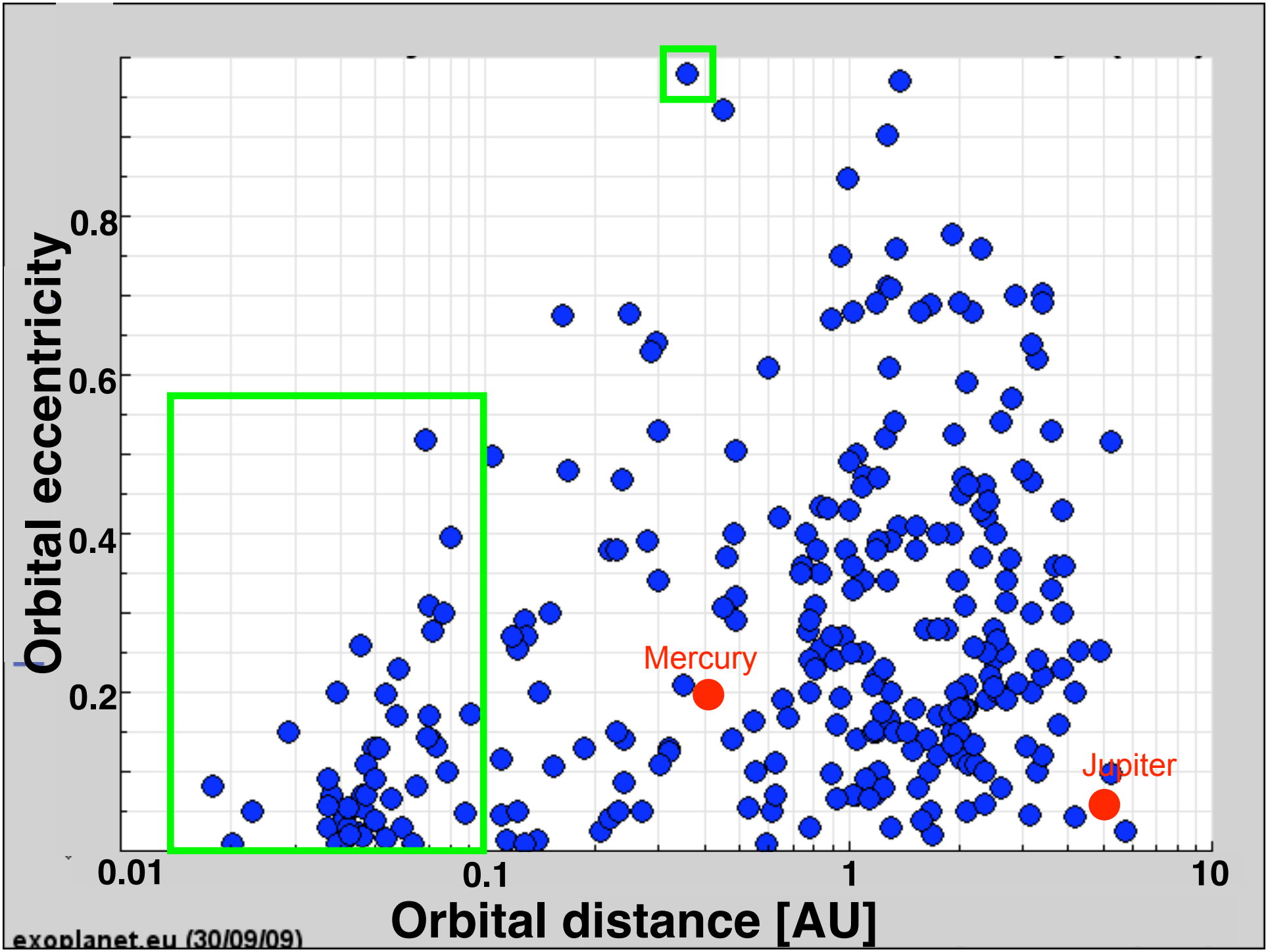
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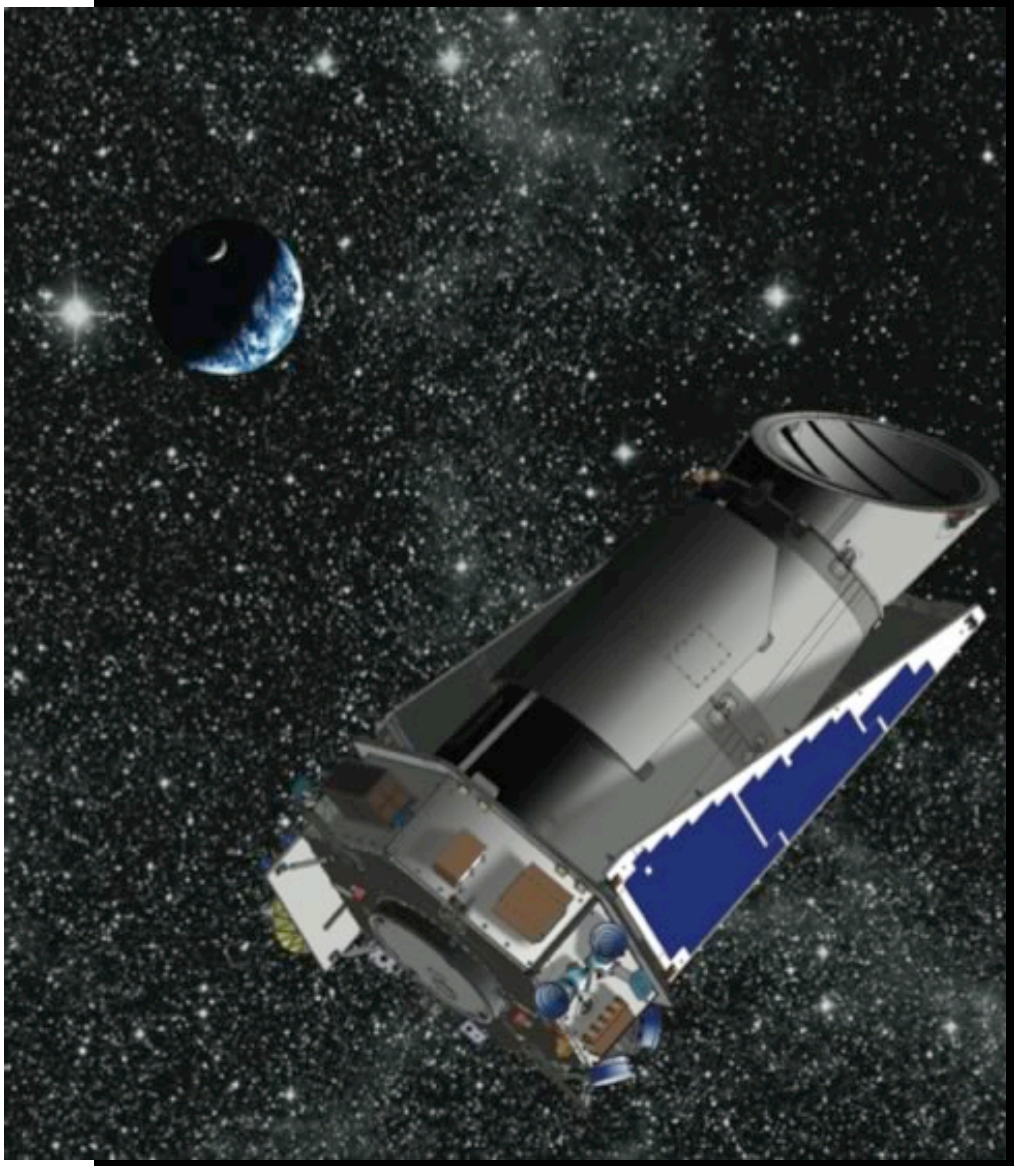
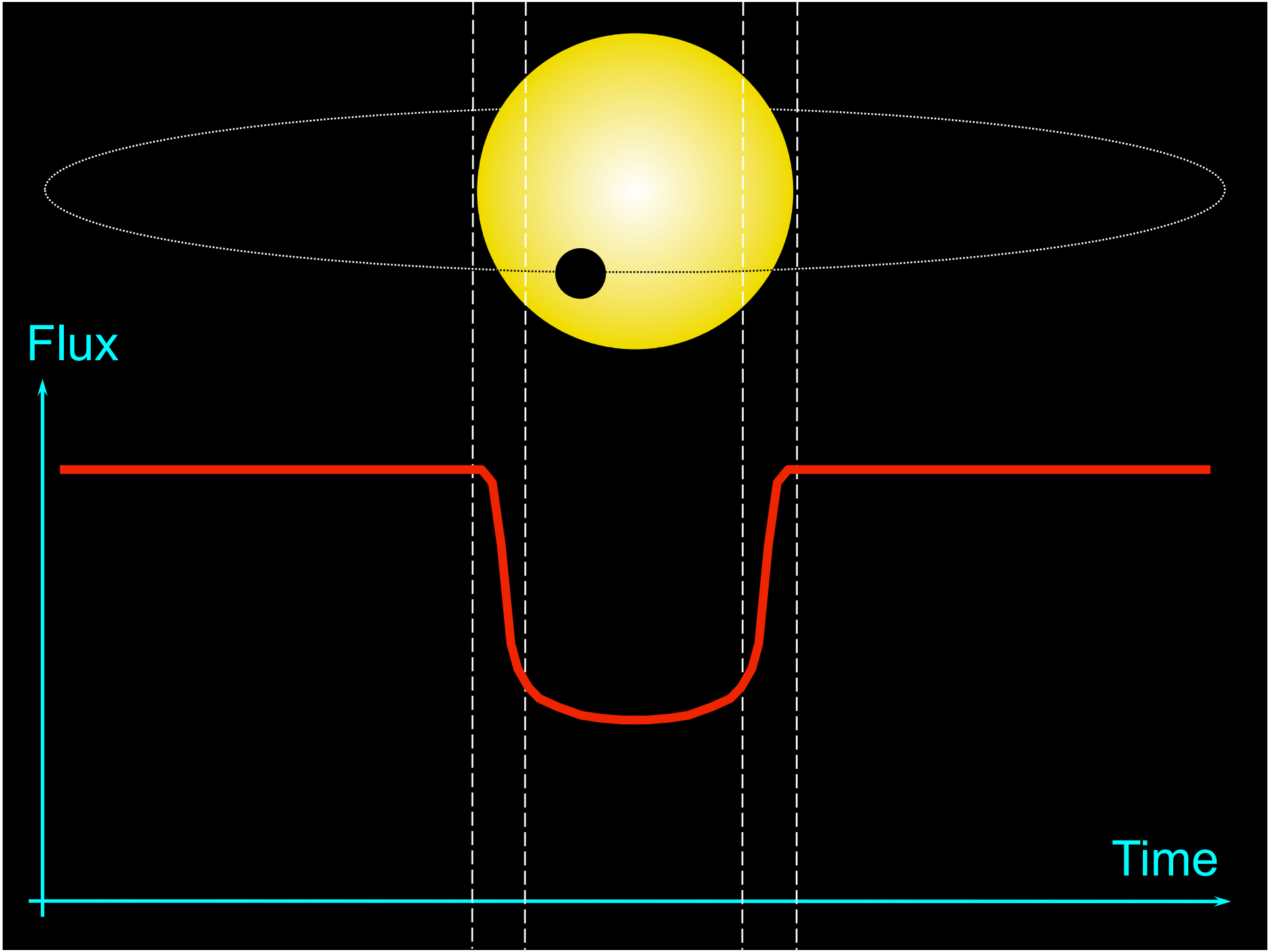


Kepler

- NASA Discovery mission
- Launched March 2009
- Earth-trailing orbit
- Monitor one field of 100,000 stars for 3.5 yr
- > 200 giant planets
- Many earthlike planets in the “habitable zone”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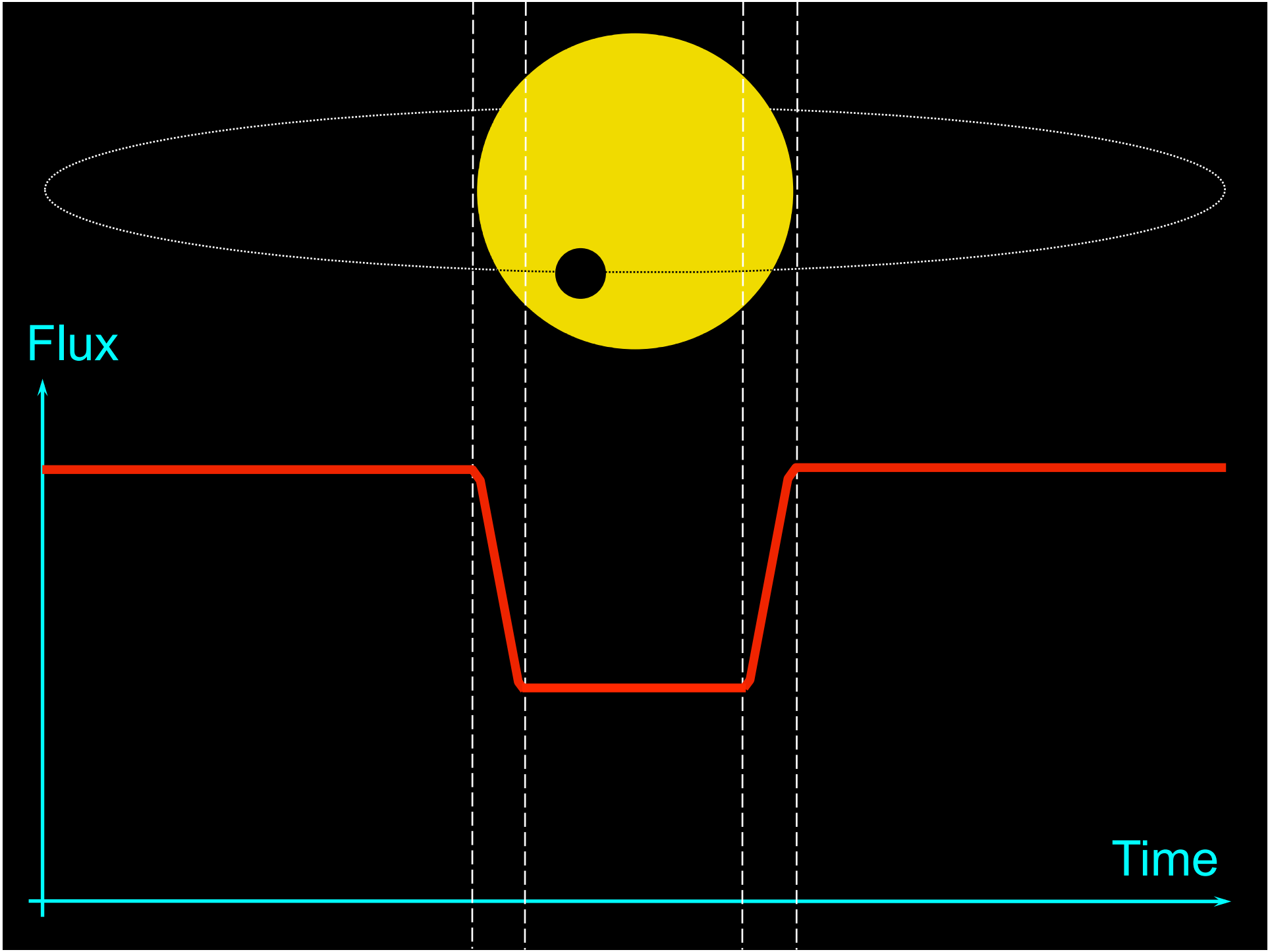
Time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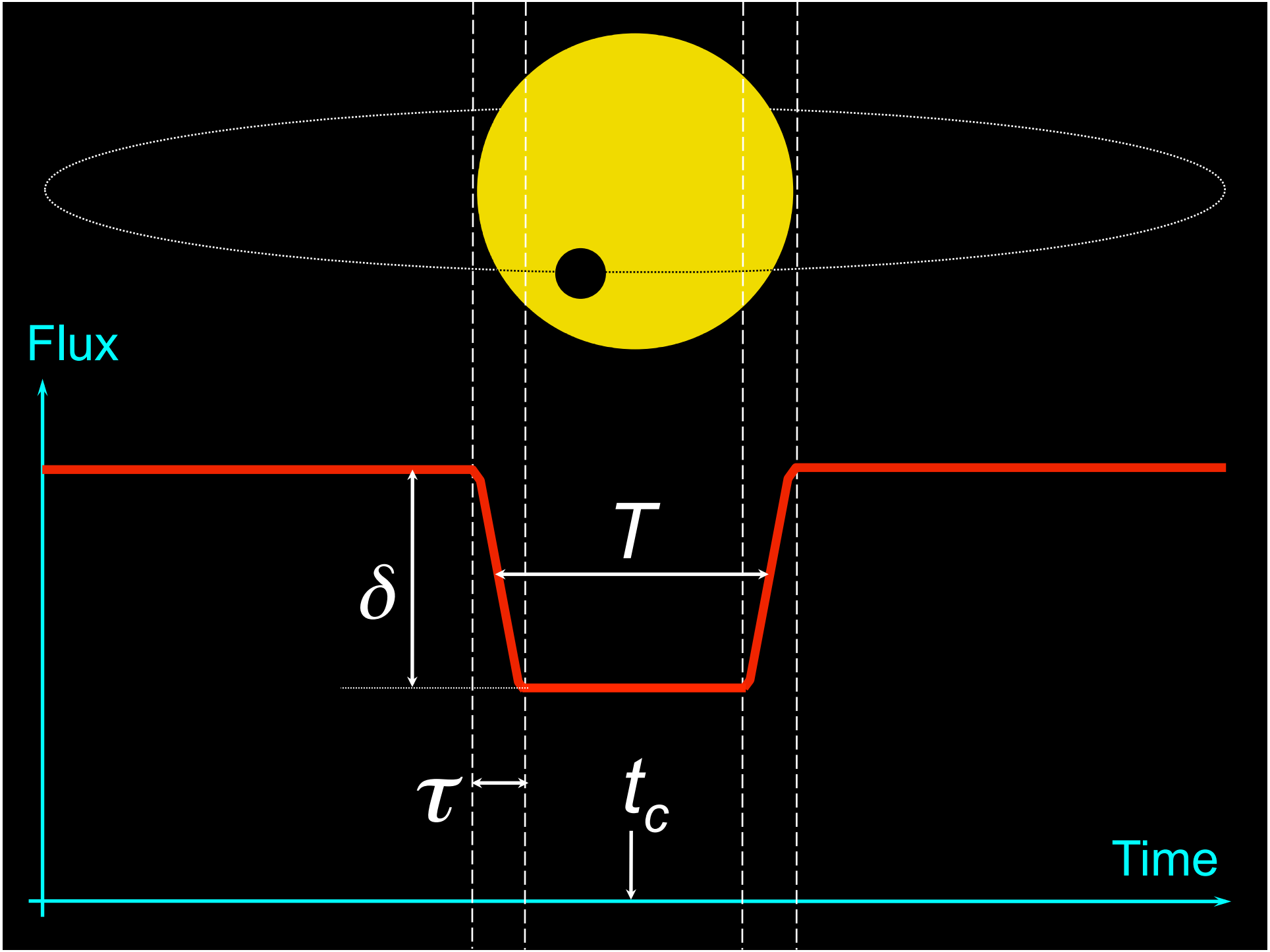
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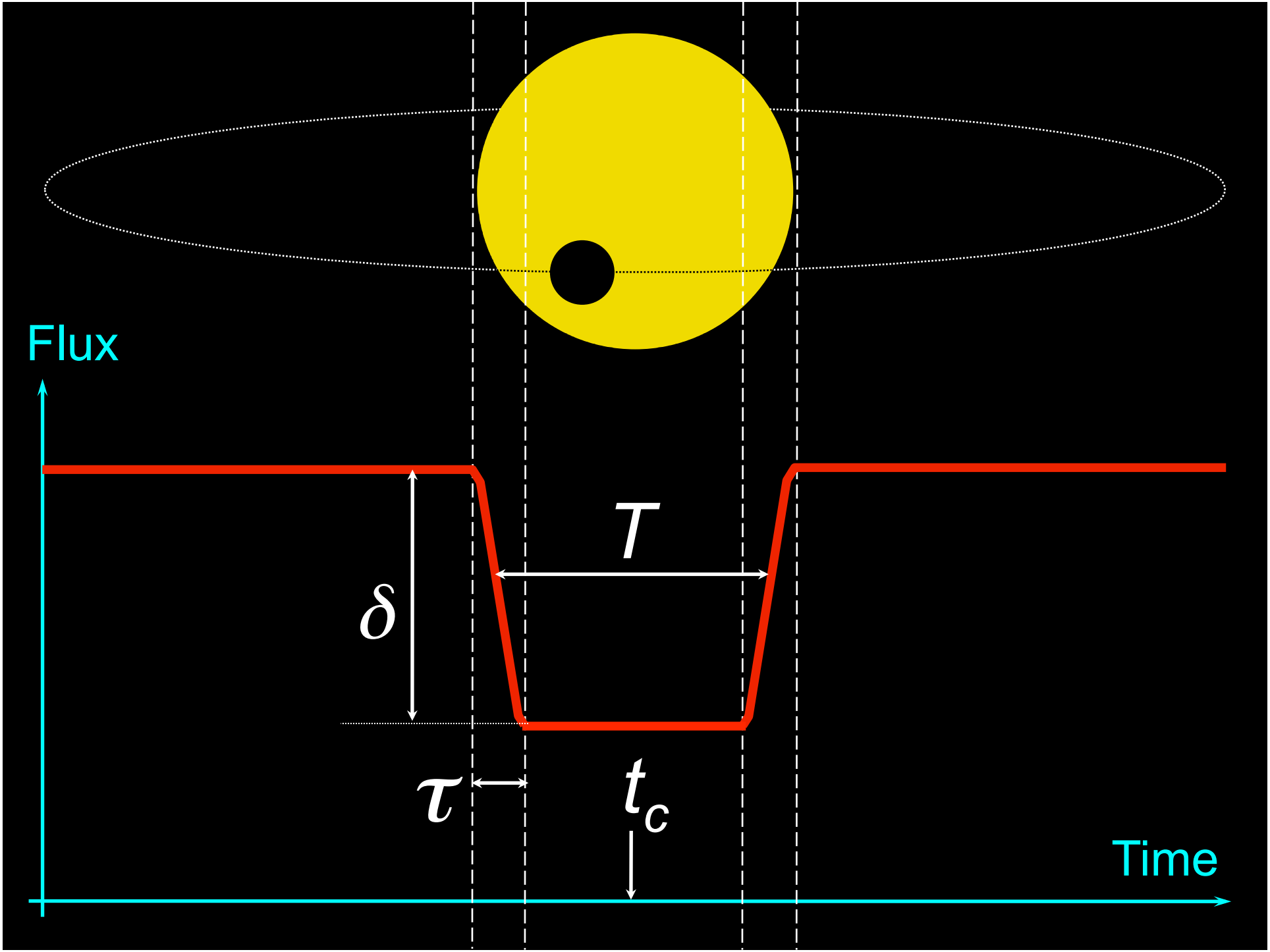
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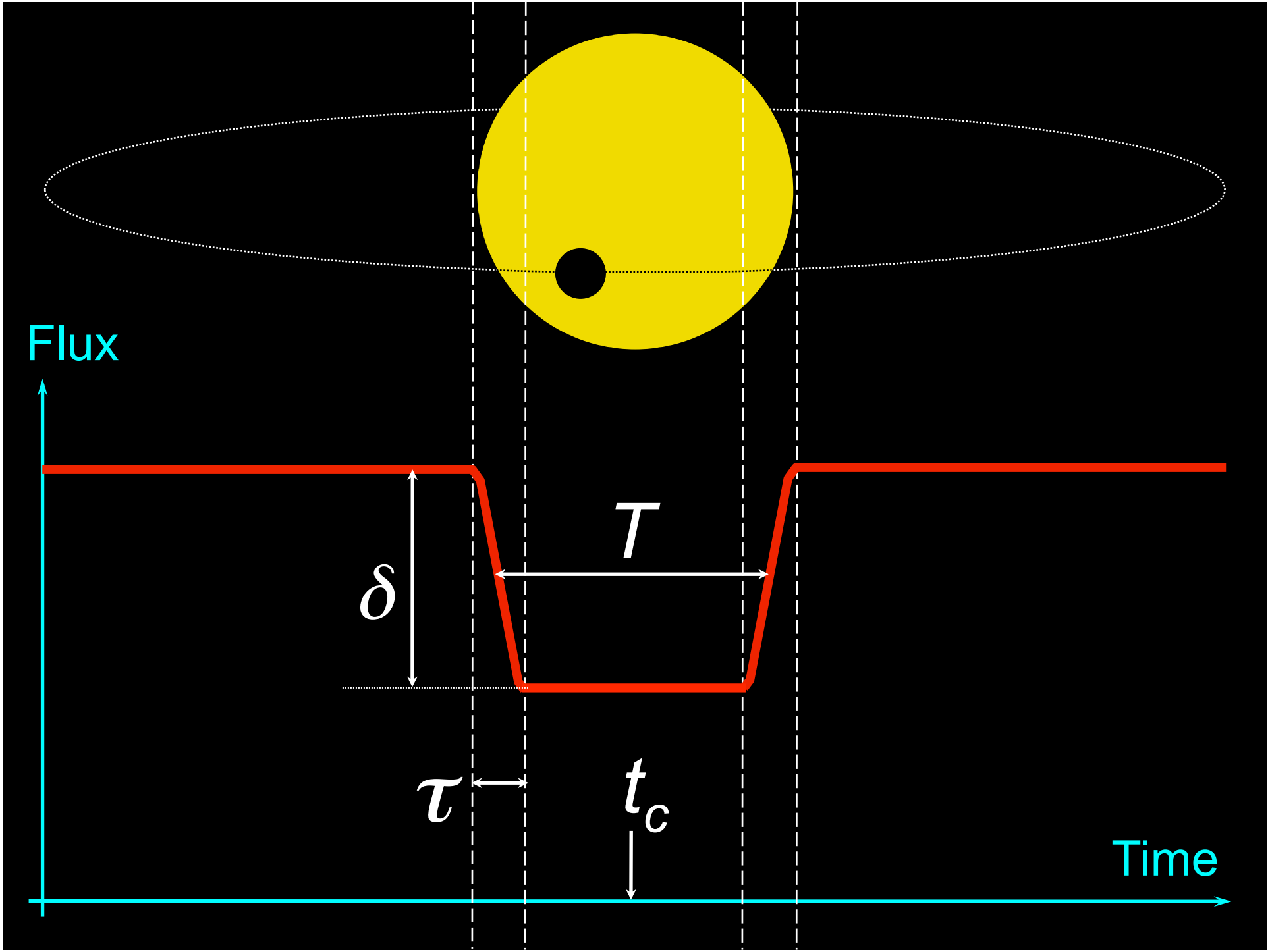
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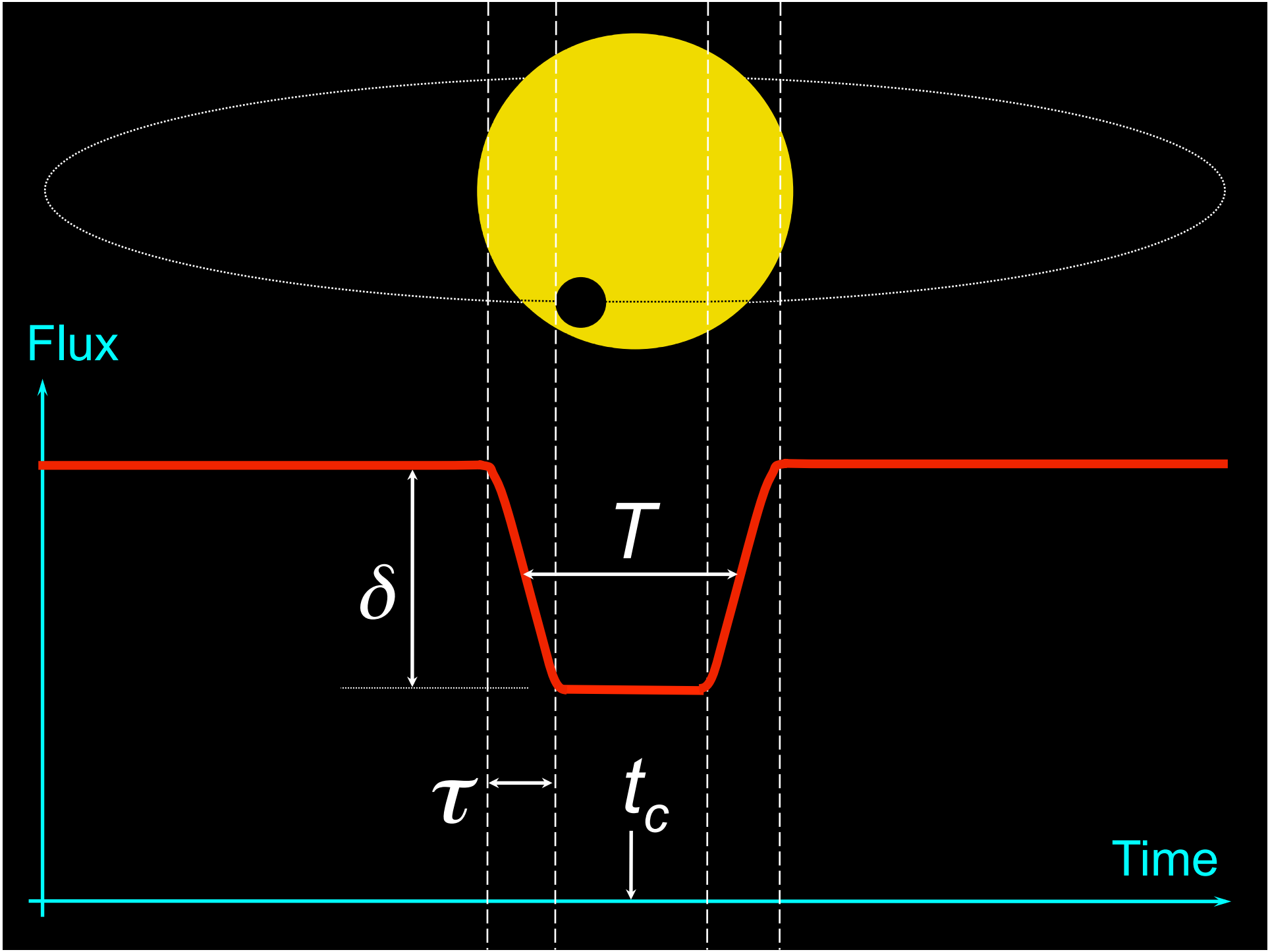
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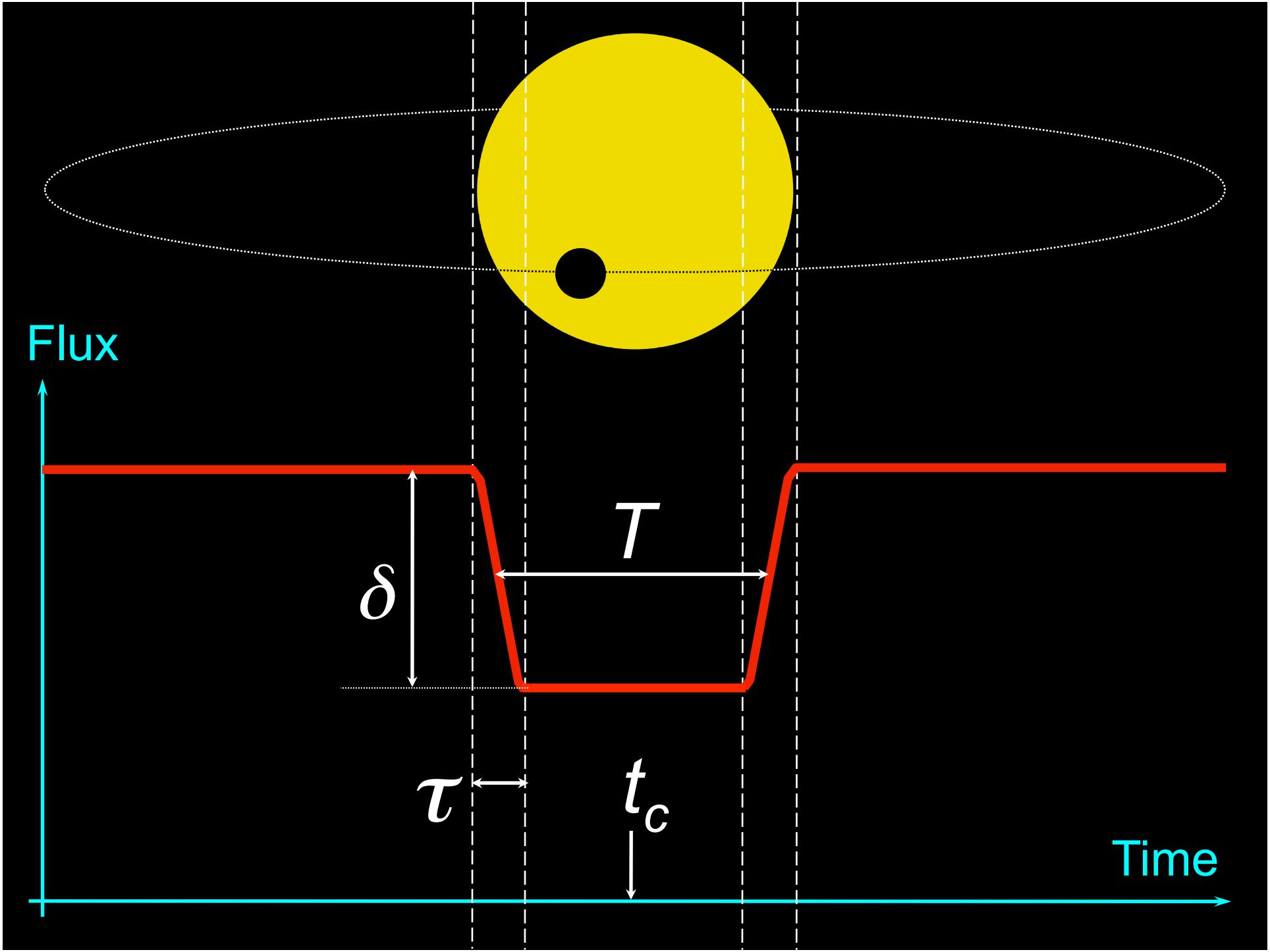
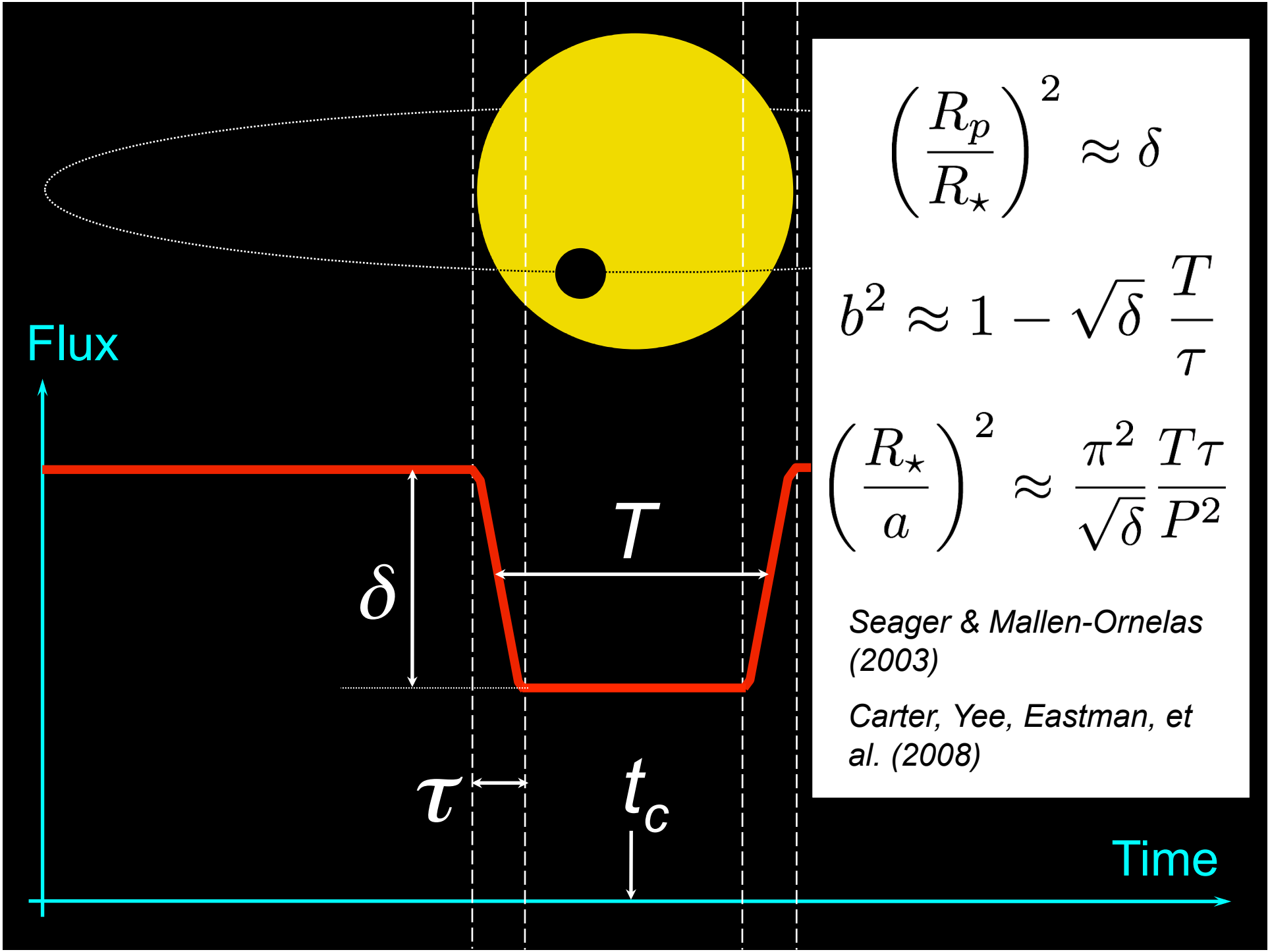
t_c
Time


Flux
δ
T
τ
t_c
Time



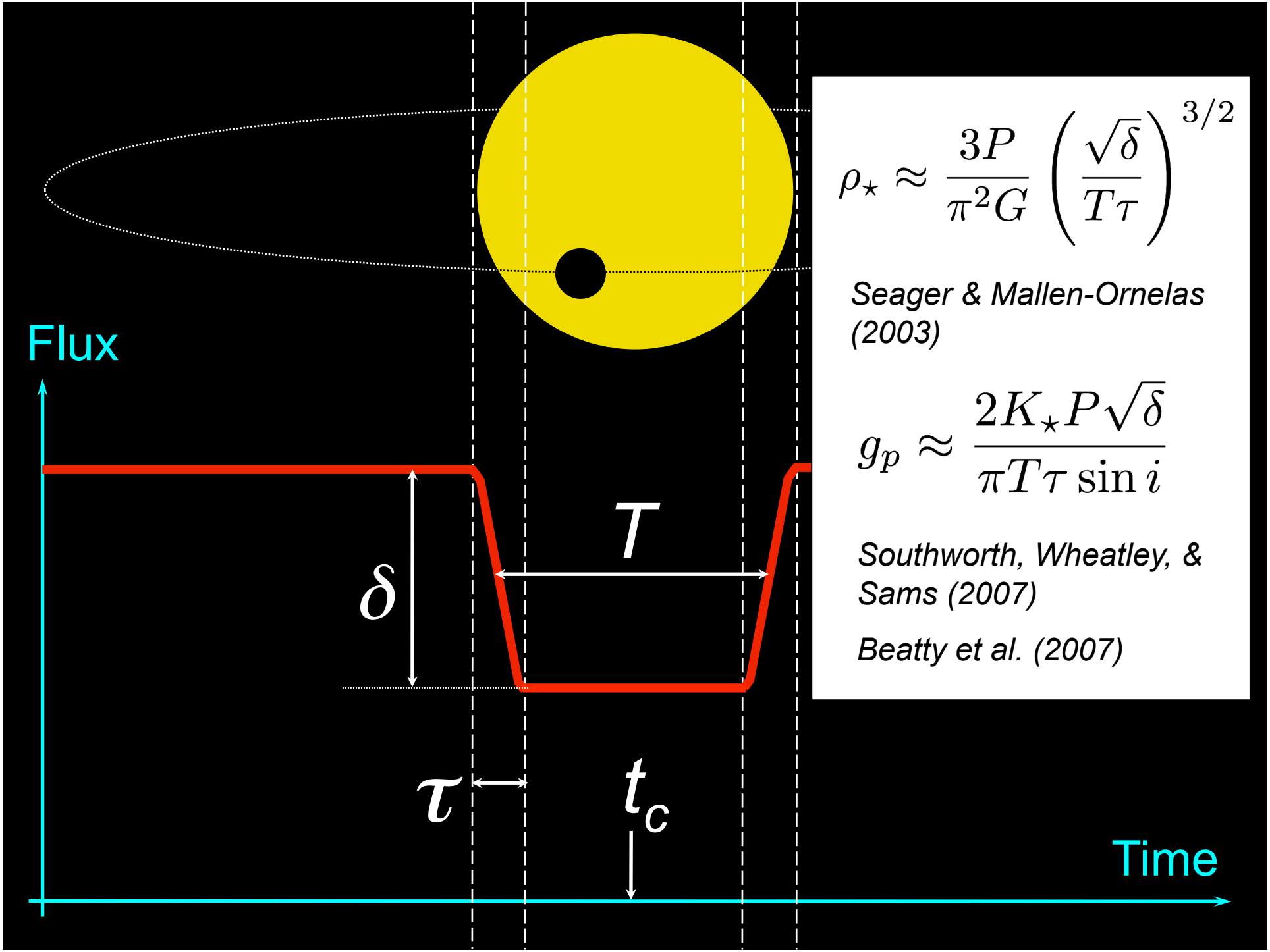
$$\left(\frac{R_p}{R_\star}\right)^2 \approx \delta$$

$$b^2 \approx 1 - \sqrt{\delta}\,\frac{T}{\tau}$$

$$\left(\frac{R_\star}{a}\right)^2 \approx \frac{\pi^2}{\sqrt{\delta}}\frac{T\tau}{P^2}$$

Seager & Mallen-Ornelas (2003)

Carter, Yee, Eastman, et al. (2008)



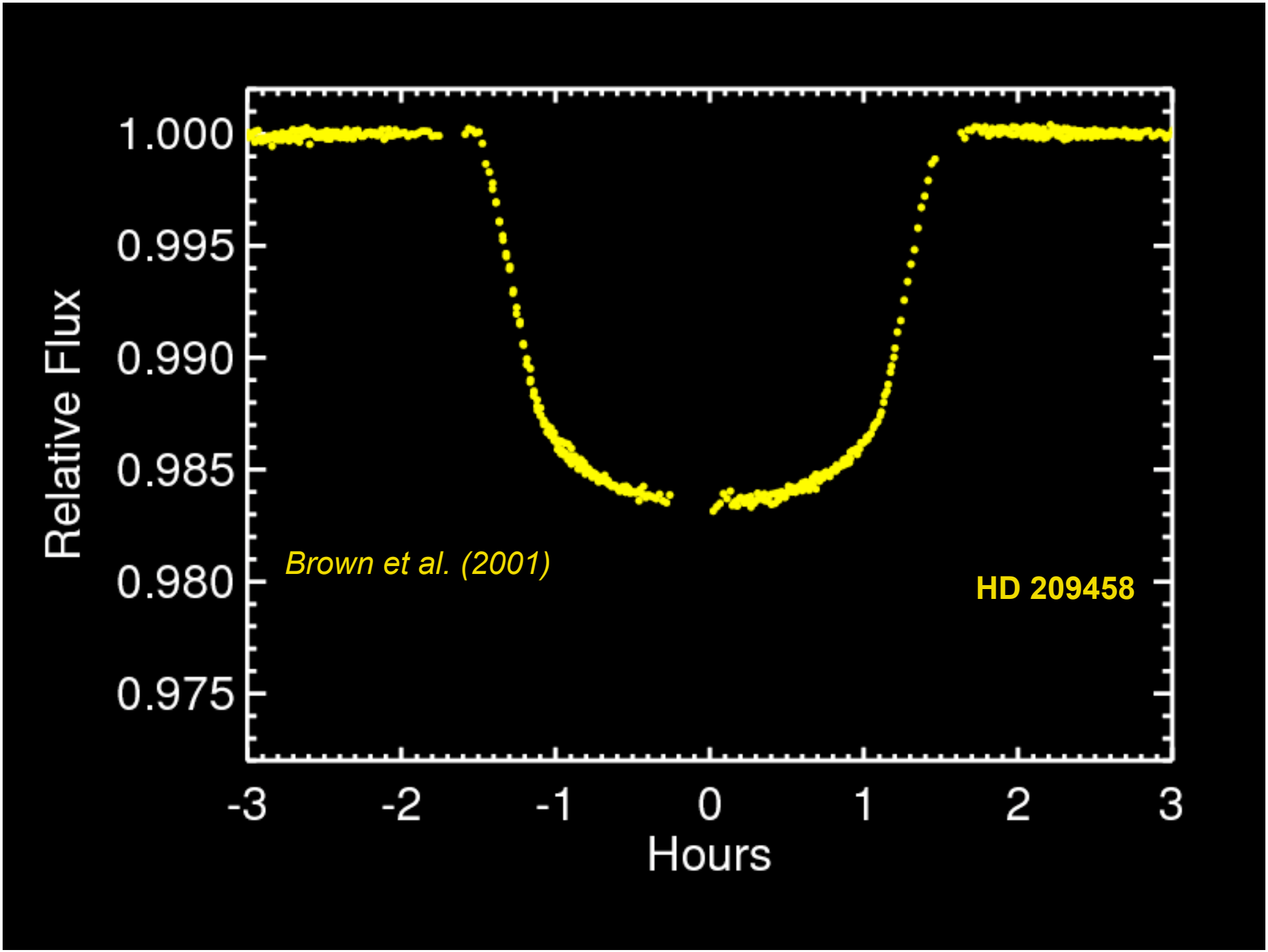
$$\rho_\star \approx \frac{3P}{\pi^2 G}\left(\frac{\sqrt{\delta}}{T\tau}\right)^{3/2}$$

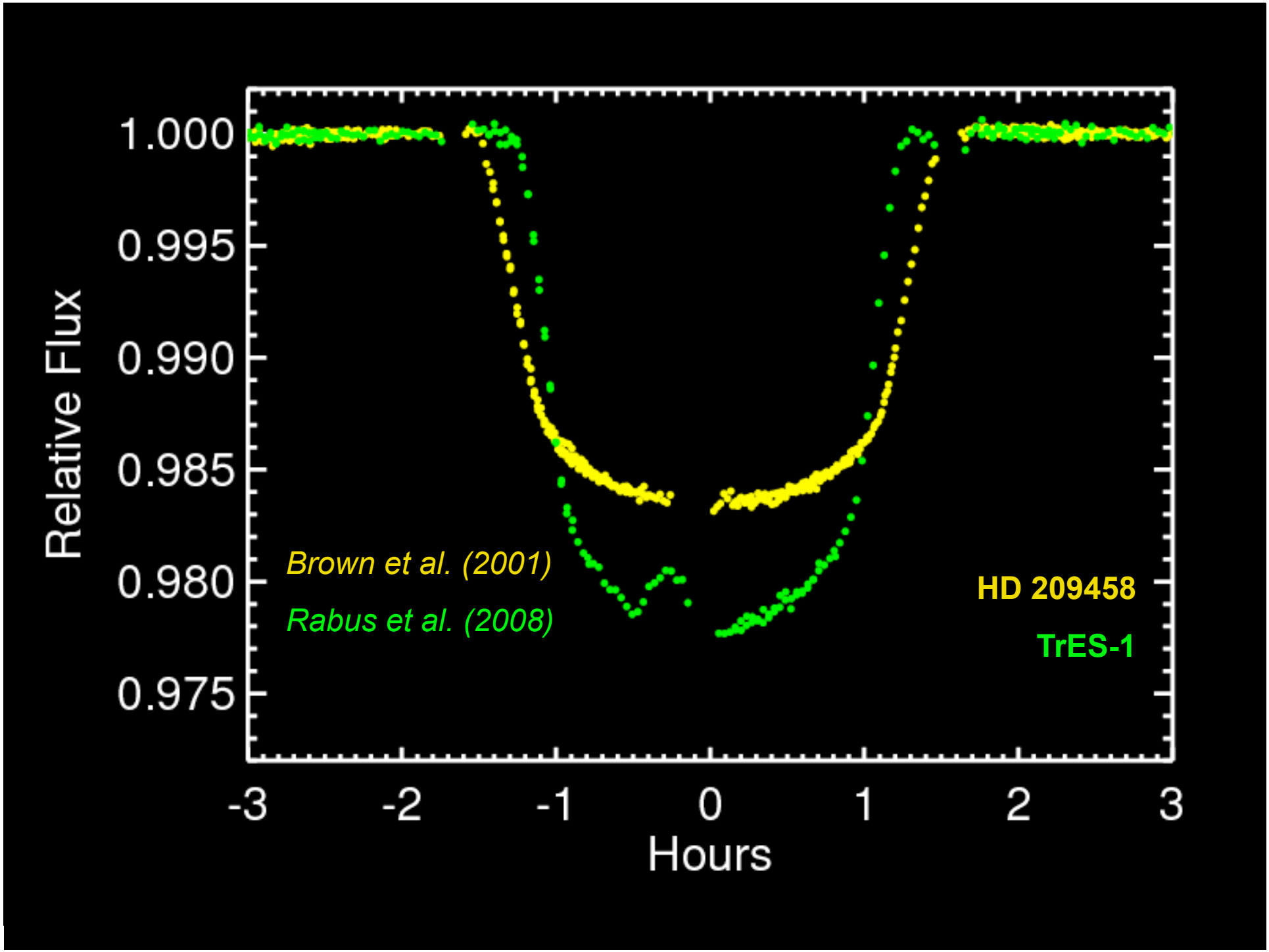
Seager & Mallen-Ornelas (2003)

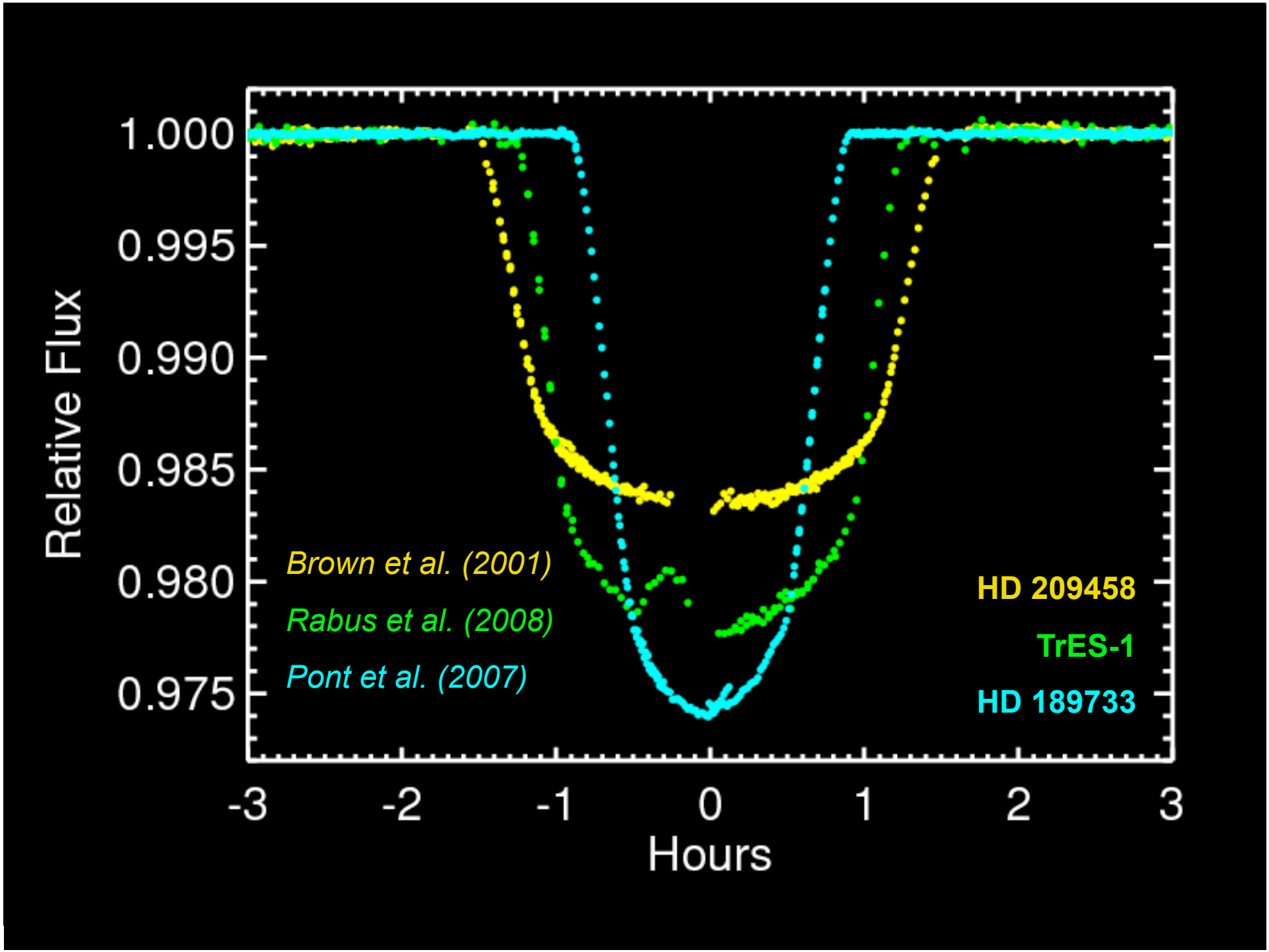
$$g_p \approx \frac{2K_\star P\sqrt{\delta}}{\pi T\tau \sin i}$$

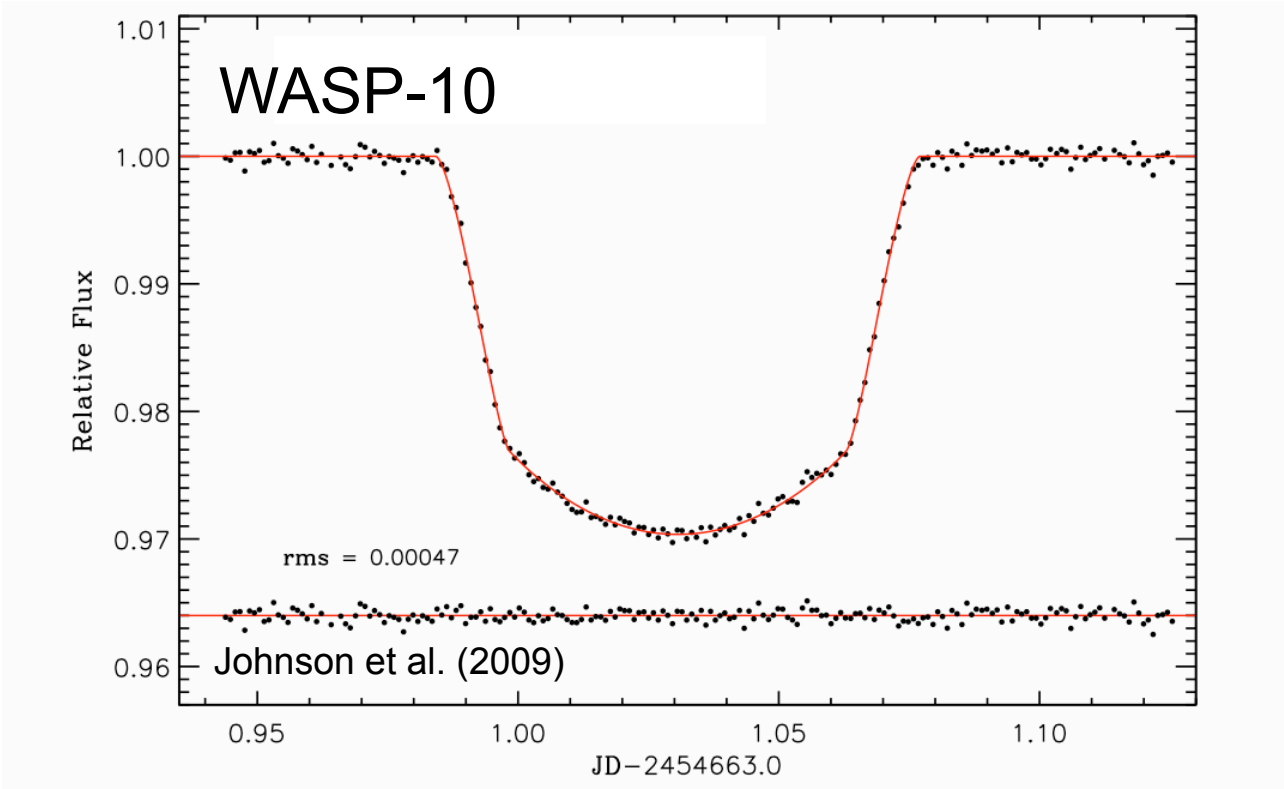
Southworth, Wheatley, & Sams (2007)

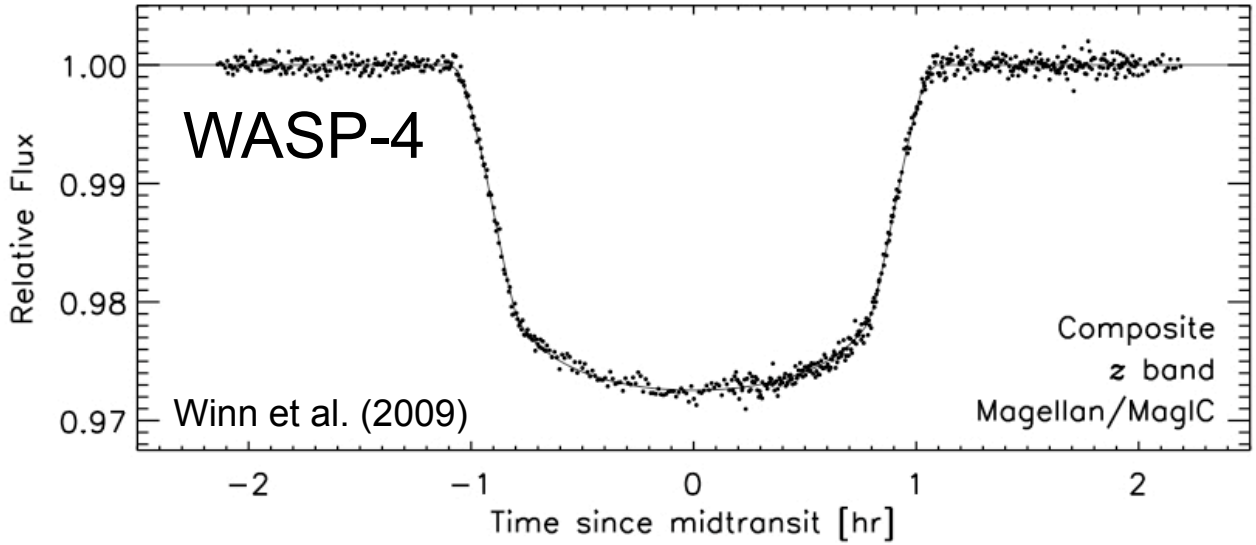
Beatty et al. (2007)

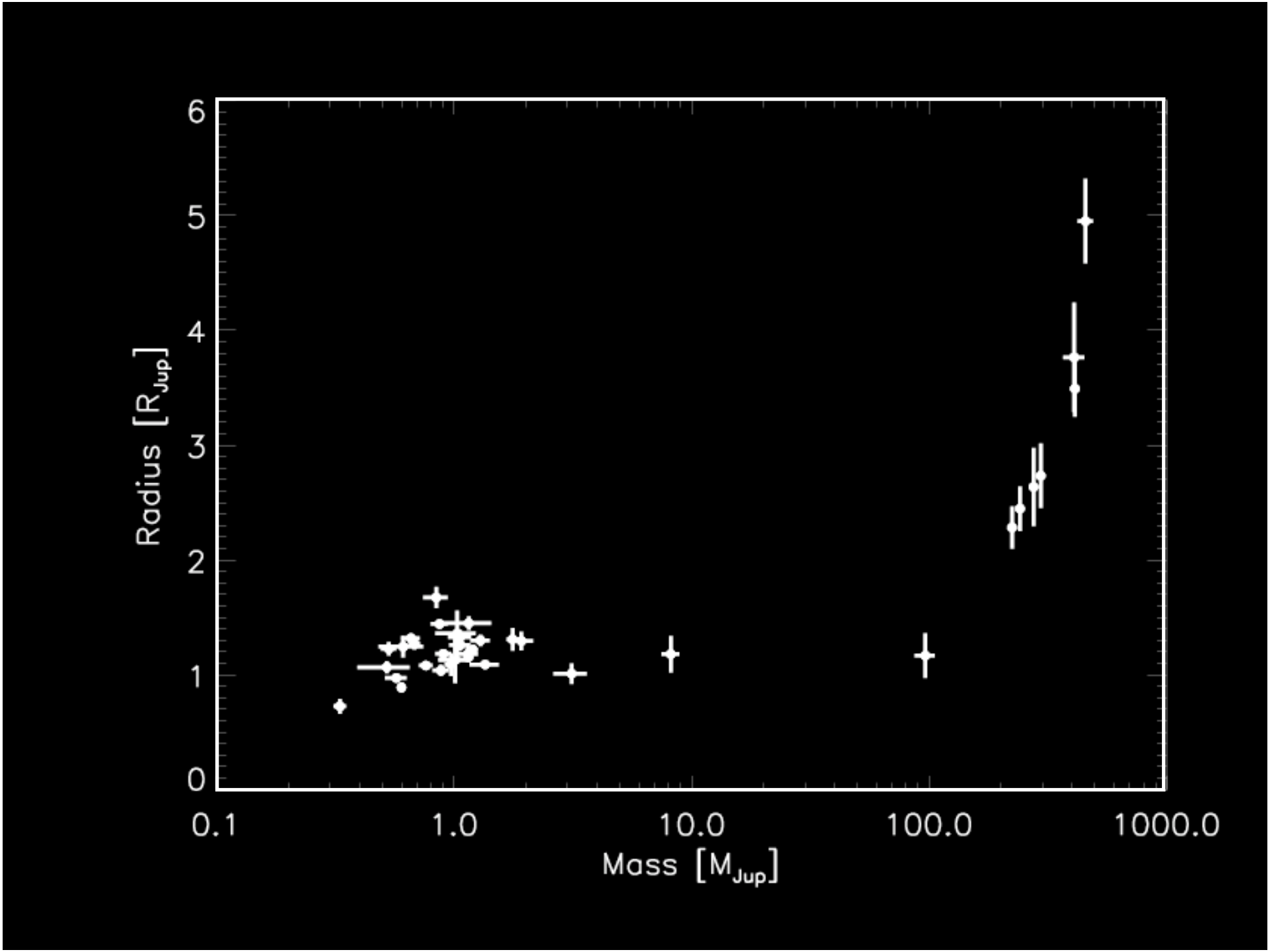

1.000
0.995
0.990
0.985
0.980
0.975
Relative Flux
-3
-2
-1
0
1
2
3
Hours
Brown et al. (2001)
HD 209458

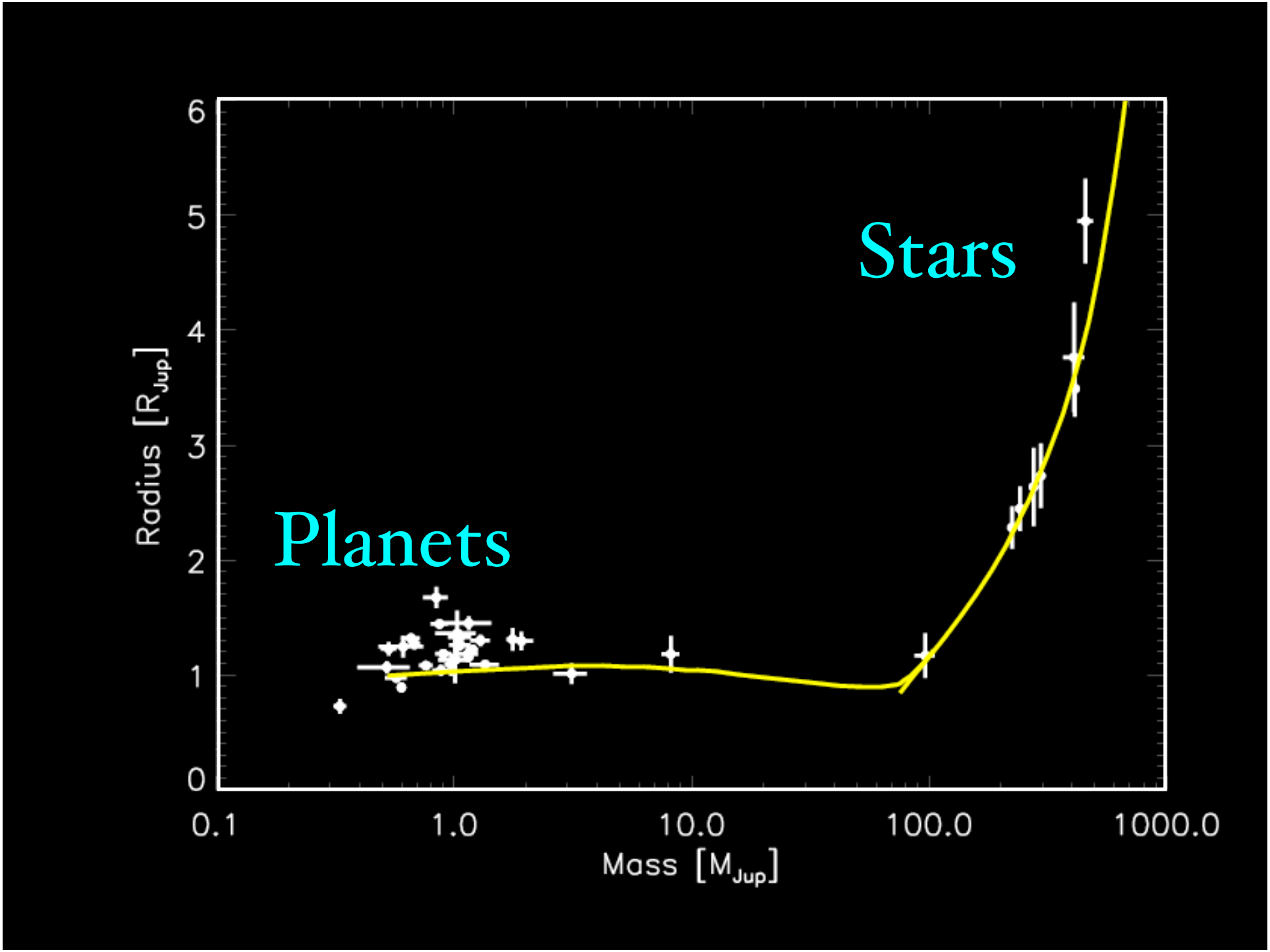

1.000
0.995
0.990
0.985
0.980
0.975
Relative Flux
-3
-2
-1
0
1
2
3
Hours
Brown et al. (2001)
Rabus et al. (2008)
HD 209458
TrES-1

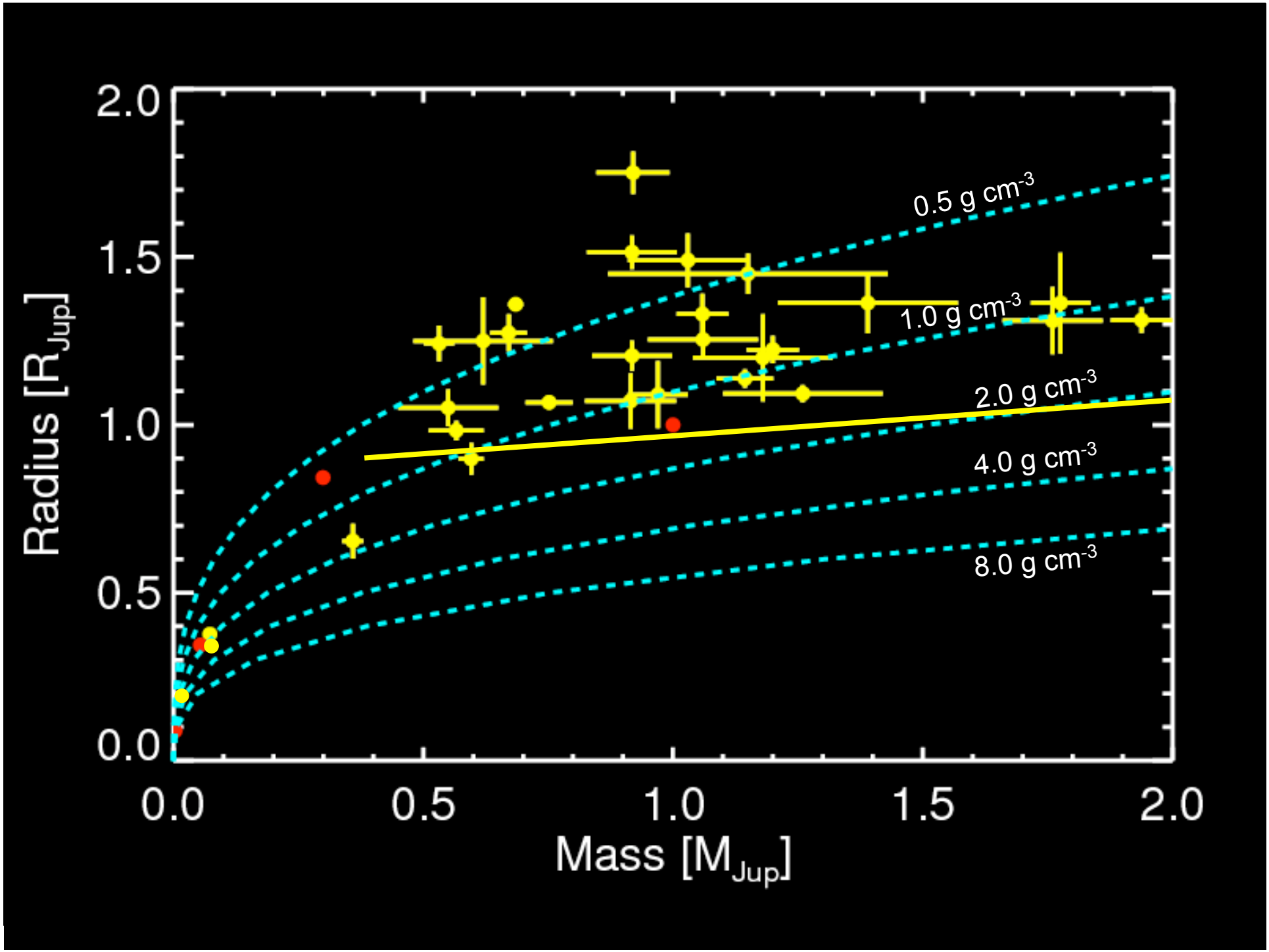

1.000
0.995
0.990
0.985
0.980
0.975
Relative Flux
-3
-2
-1
0
1
2
3
Hours
Brown et al. (2001)
Rabus et al. (2008)
Pont et al. (2007)
HD 209458
TrES-1
HD 189733

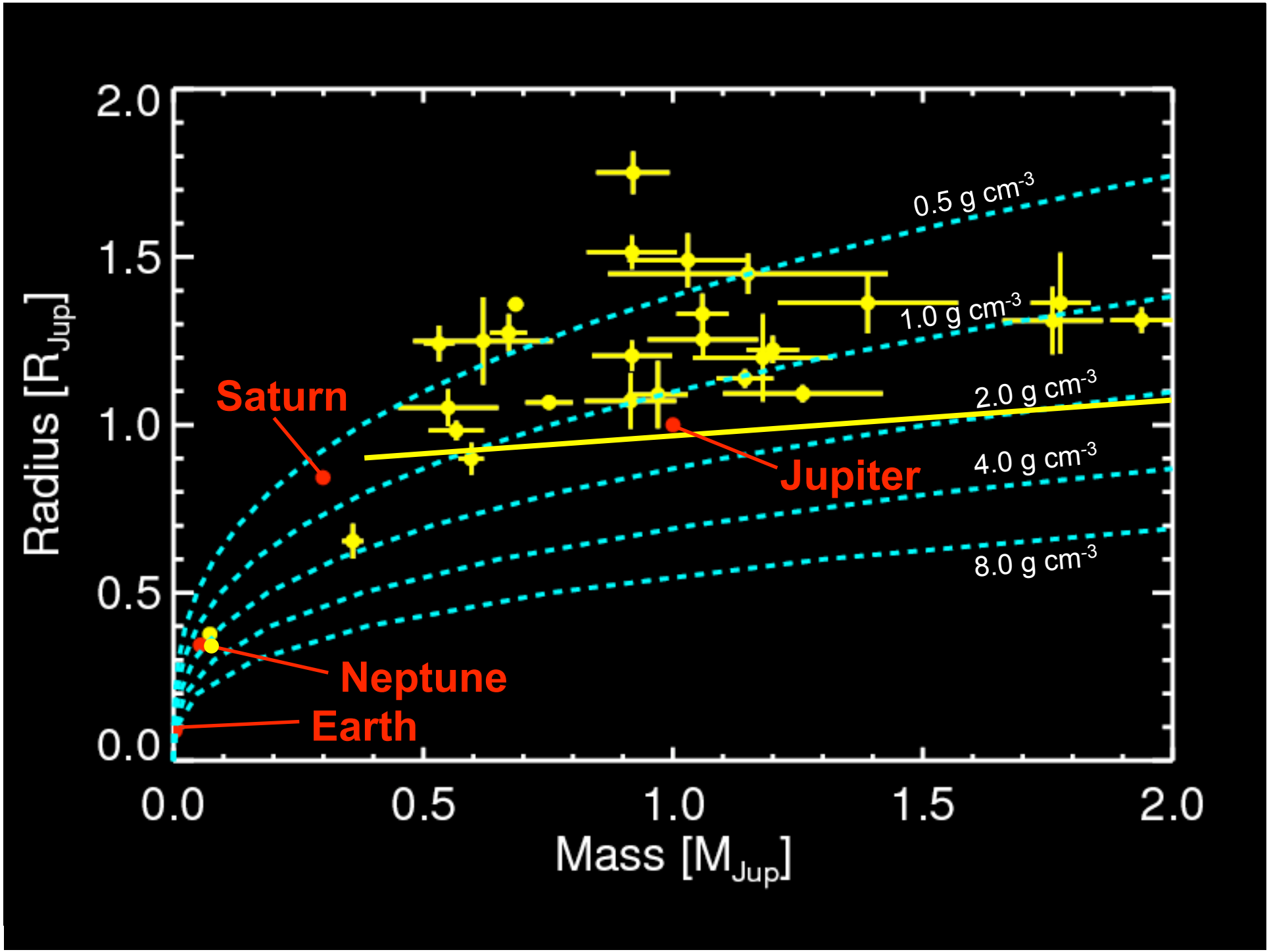

WASP-10
Relative Flux
1.01
1.00
0.99
0.98
0.97
0.96
rms = 0.00047
Johnson et al. (2009)
0.95
1.00
1.05
1.10
JD−2454663.0

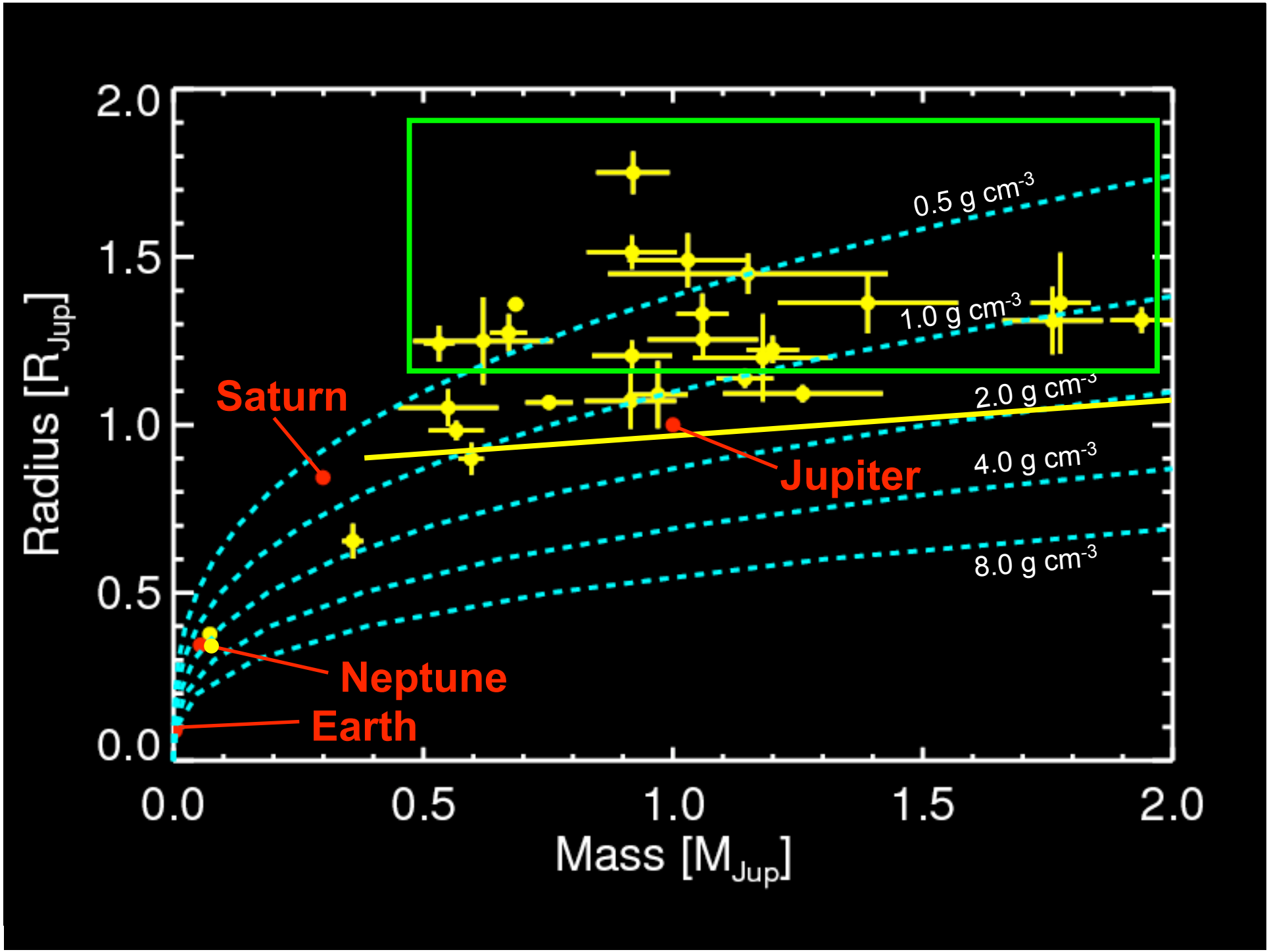

WASP-4
Relative Flux
1.00
0.99
0.98
0.97
Winn et al. (2009)
Composite
z band
Magellan/MagIC
−2
−1
0
1
2
Time since midtransit [hr]


Radius [R_Jup]
Mass [M_Jup]
0
1
2
3
4
5
6
0.1
1.0
10.0
100.0
1000.0


Stars
Planets
Radius [R_Jup]
Mass [M_Jup]
0
1
2
3
4
5
6
0.1
1.0
10.0
100.0
1000.0

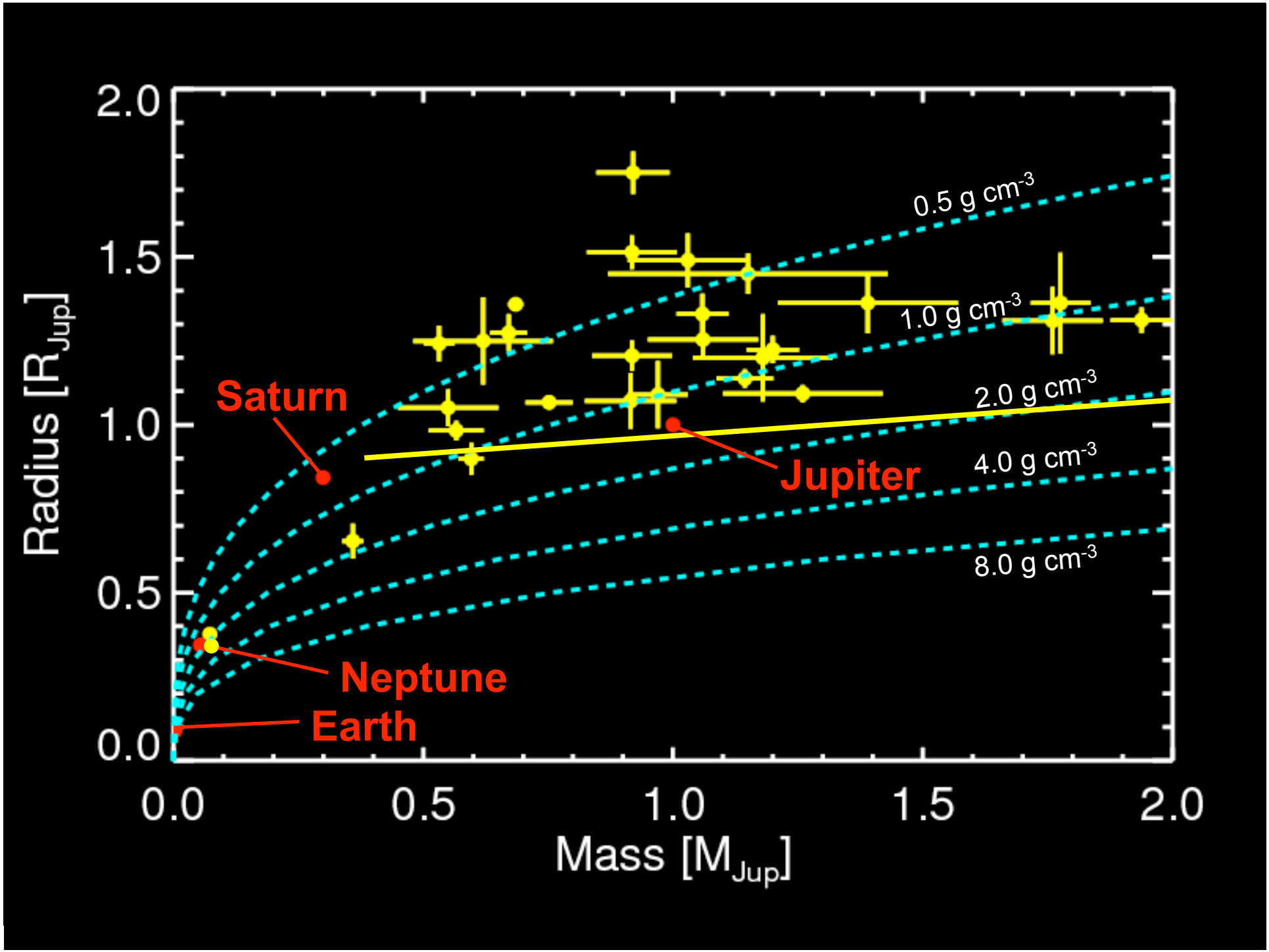

2.0
1.5
1.0
0.5
0.0
Radius [R_Jup]
0.0
0.5
1.0
1.5
2.0
Mass [M_Jup]
0.5 g cm-3
1.0 g cm-3
2.0 g cm-3
4.0 g cm-3
8.0 g cm-3

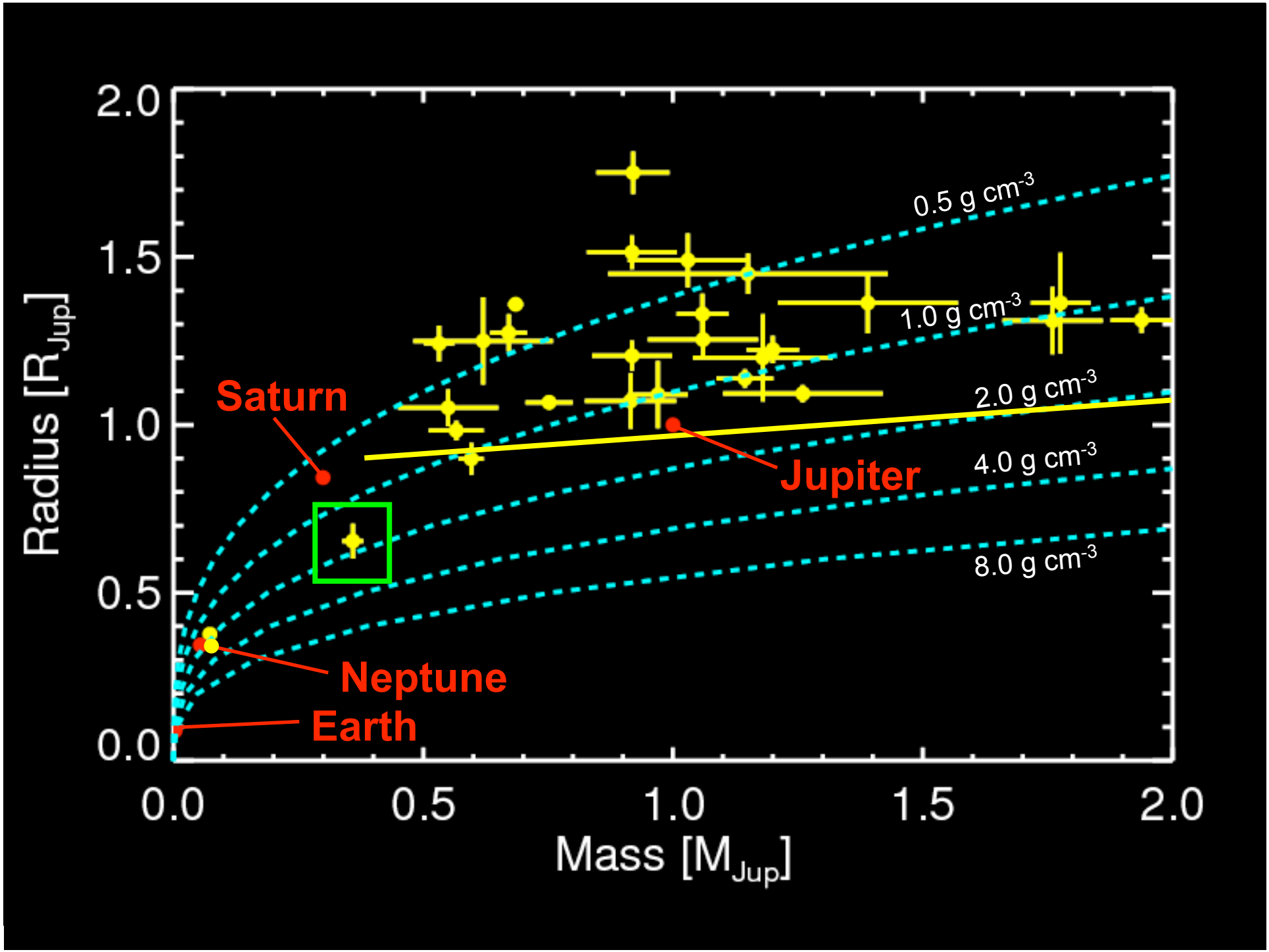

2.0
1.5
1.0
0.5
0.0
Radius [R_Jup]
0.0
0.5
1.0
1.5
2.0
Mass [M_Jup]
0.5 g cm-3
1.0 g cm-3
2.0 g cm-3
4.0 g cm-3
8.0 g cm-3
Saturn
Jupiter
Neptune
Earth


2.0
1.5
1.0
0.5
0.0
Radius [R_Jup]
0.0
0.5
1.0
1.5
2.0
Mass [M_Jup]
0.5 g cm-3
1.0 g cm-3
2.0 g cm-3
4.0 g cm-3
8.0 g cm-3
Saturn
Jupiter
Neptune
Earth

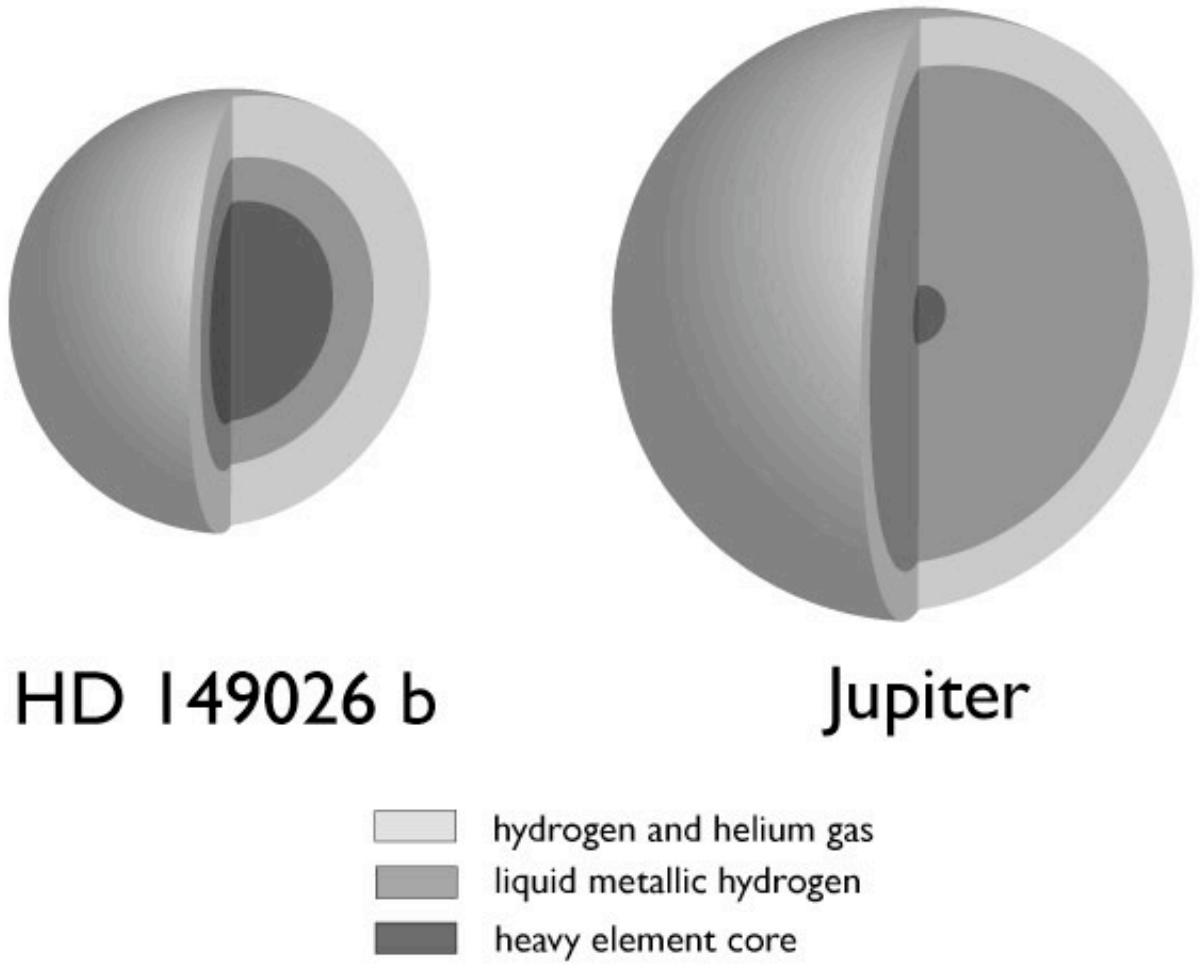
The “bloated” planets

- **Early migration** (*Burrows et al. 2000*)
- **Insolation-driven, deeply penetrating gravity waves** (*Showman & Guillot 2002*)
- **Eccentricity tides** (*Bodenheimer et al. 2001, 2003; Liu et al. 2008, Pont 2009, Ibgui & Burrows 2009*)
- **Obliquity tides** (*Winn & Holman 2005, ruled out by Fabrycky et al. 2007 and Levrard et al. 2007*)
- **Thermal tides** (*Arras & Socrates 2009, disputed by Goodman 2009*)
- **High atmospheric opacity** (*Burrows et al. 2007*)
- **Inhibited convection of planetary interior** (*Chabrier & Baraffe 2007*)


2.0
1.5
1.0
0.5
0.0
Radius [R_Jup]
0.0
0.5
1.0
1.5
2.0
Mass [M_Jup]
0.5 g cm-3
1.0 g cm-3
2.0 g cm-3
4.0 g cm-3
8.0 g cm-3
Saturn
Jupiter
Neptune
Earth

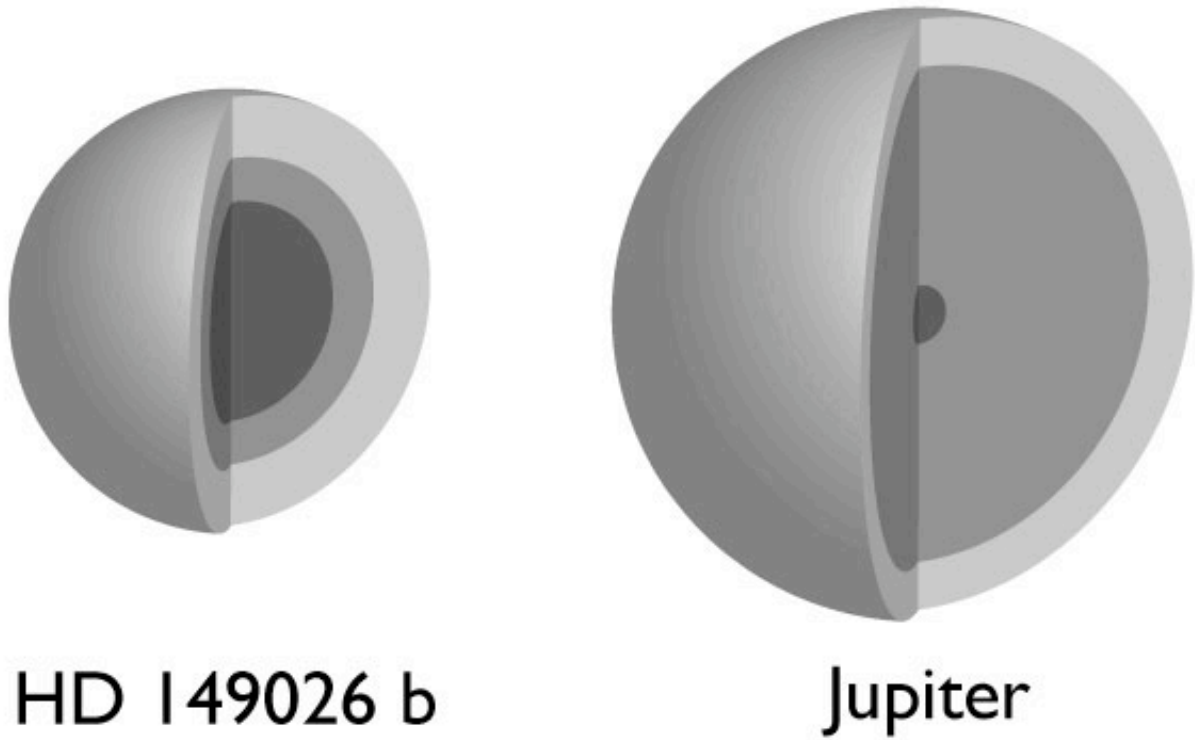

2.0
1.5
1.0
0.5
0.0
Radius [R_Jup]
0.0
0.5
1.0
1.5
2.0
Mass [M_Jup]
0.5 g cm-3
1.0 g cm-3
2.0 g cm-3
4.0 g cm-3
8.0 g cm-3
Saturn
Jupiter
Neptune
Earth

The "super-Neptune" HD 149026



G. Laughlin

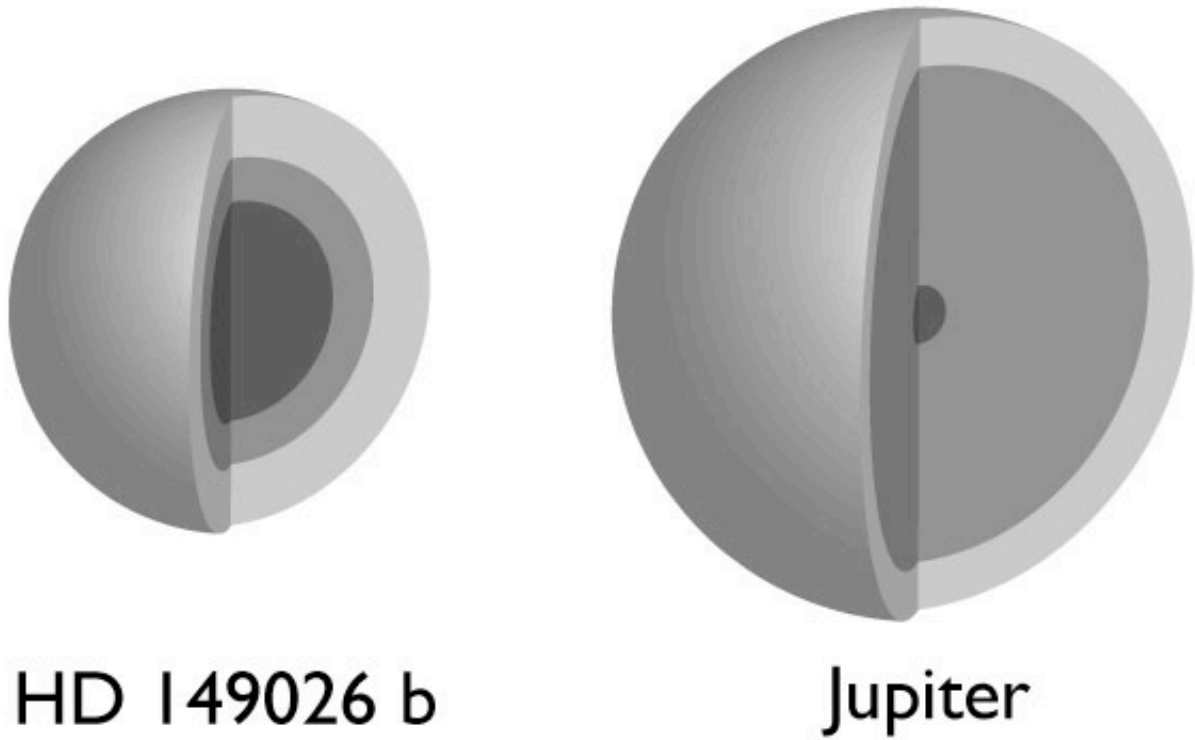
The "super-Neptune" HD 149026



Why did the core not accrete gas efficiently?

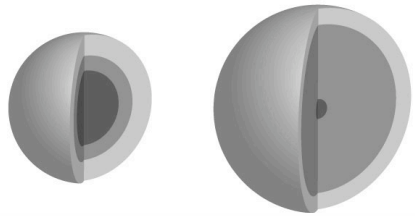
G. Laughlin

The "super-Neptune" HD 149026



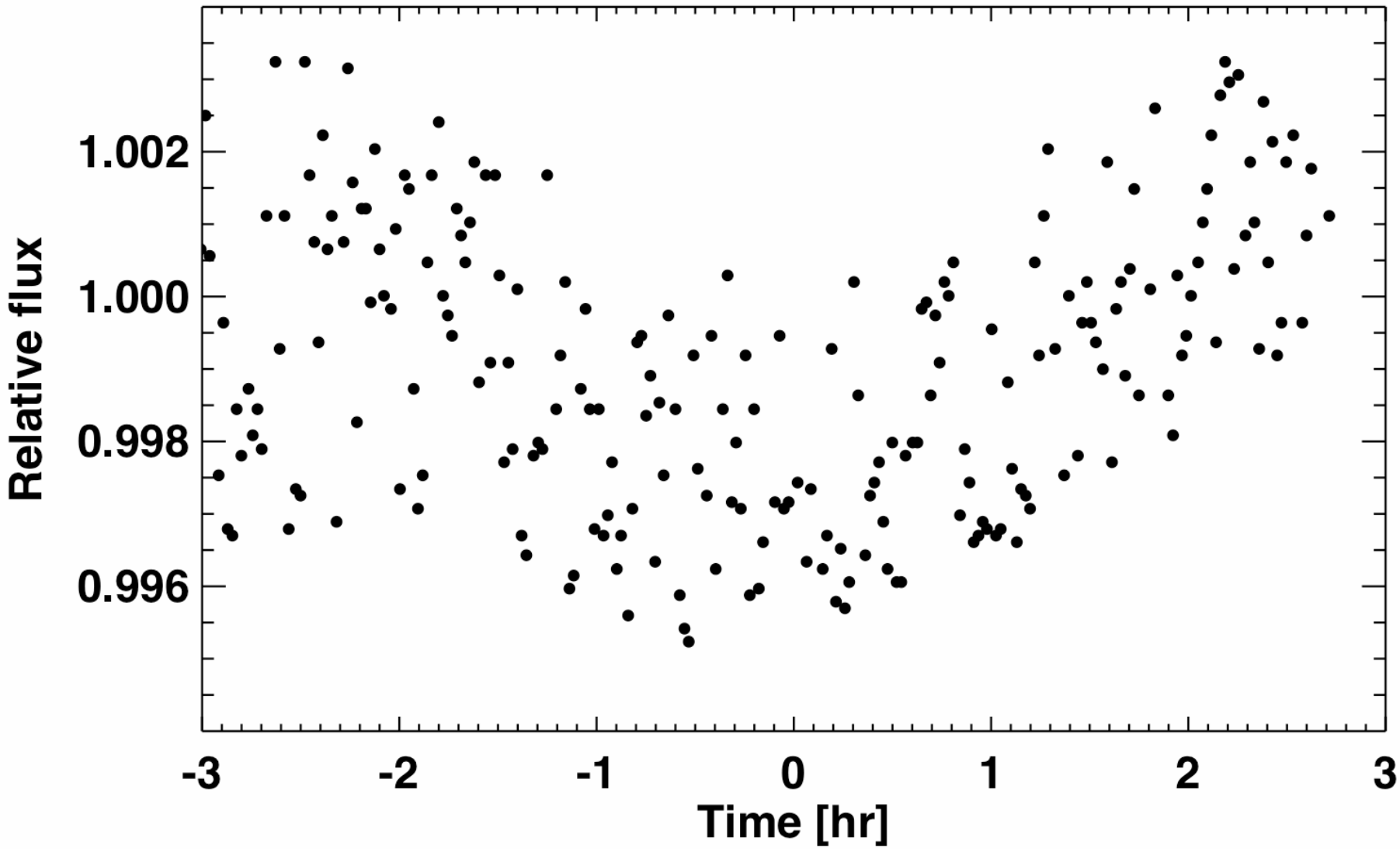
Why did the core not accrete gas efficiently?
Or, if it did, what happened to the gas?

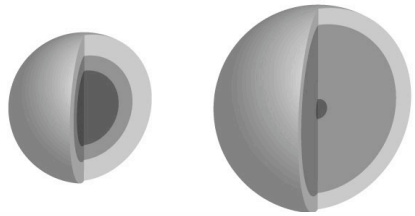
G. Laughlin



The super-Neptune HD 149026b

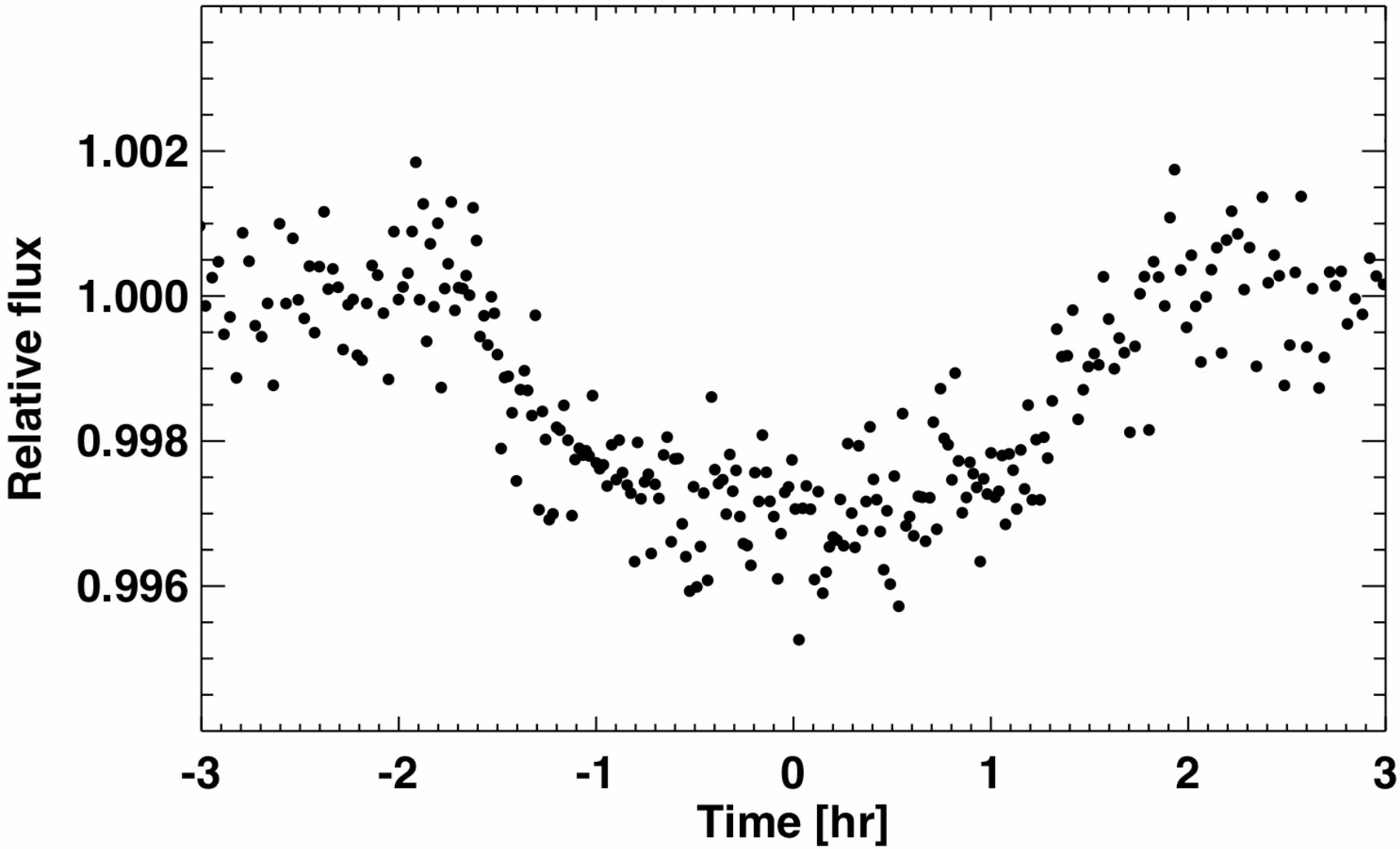
Discovery photometry: Sato et al. (2005)

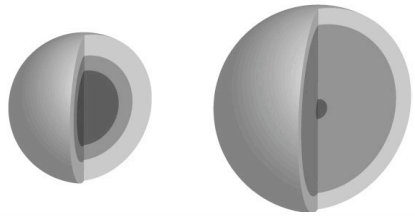




The super-Neptune HD 149026b

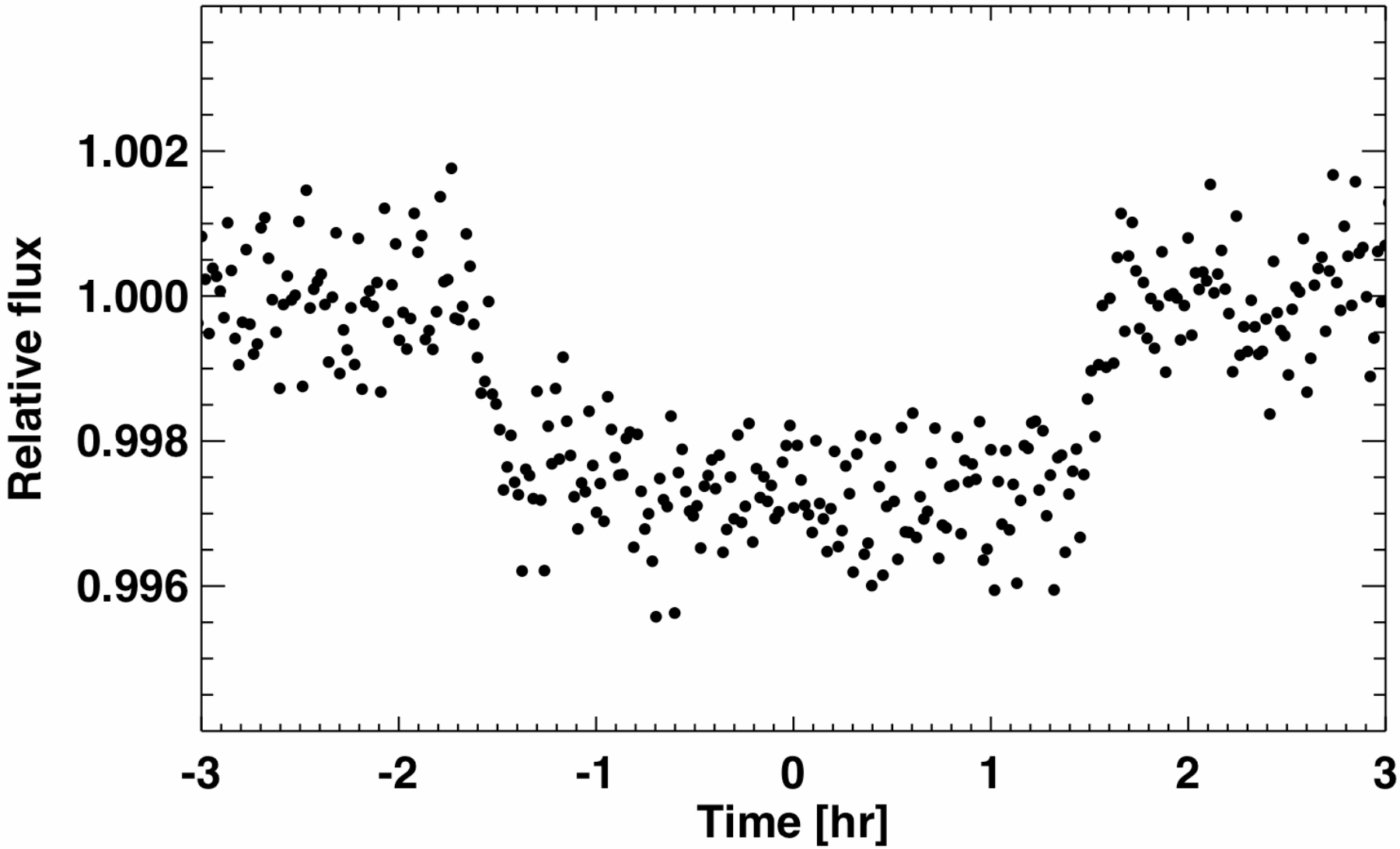
Follow-up photometry: Winn et al. (2008)

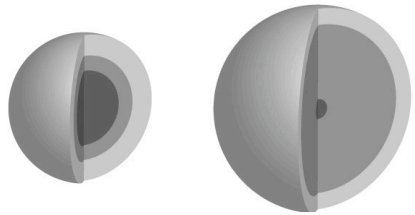




The super-Neptune HD 149026b

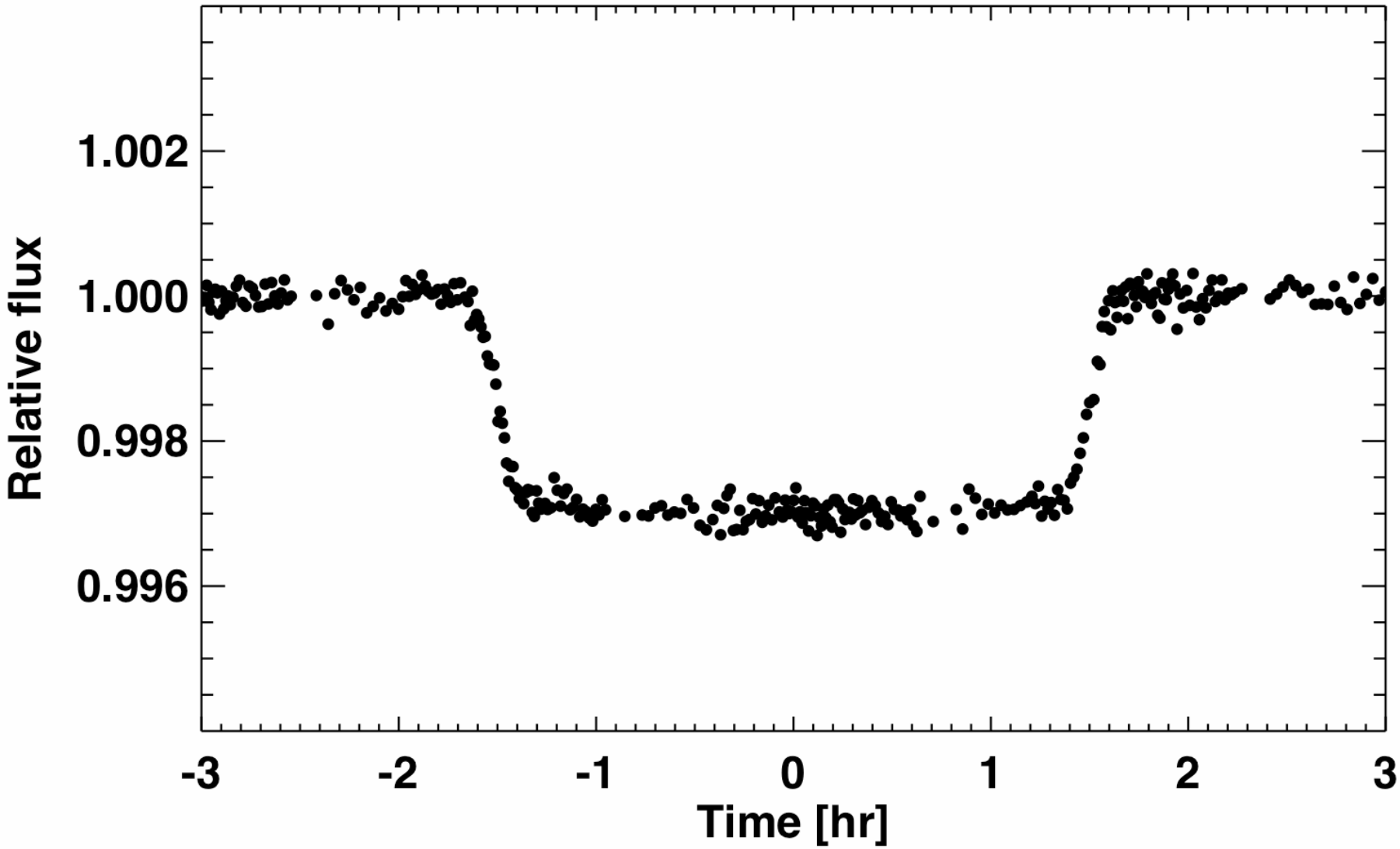
Spitzer photometry: Nutzman et al. (2008)

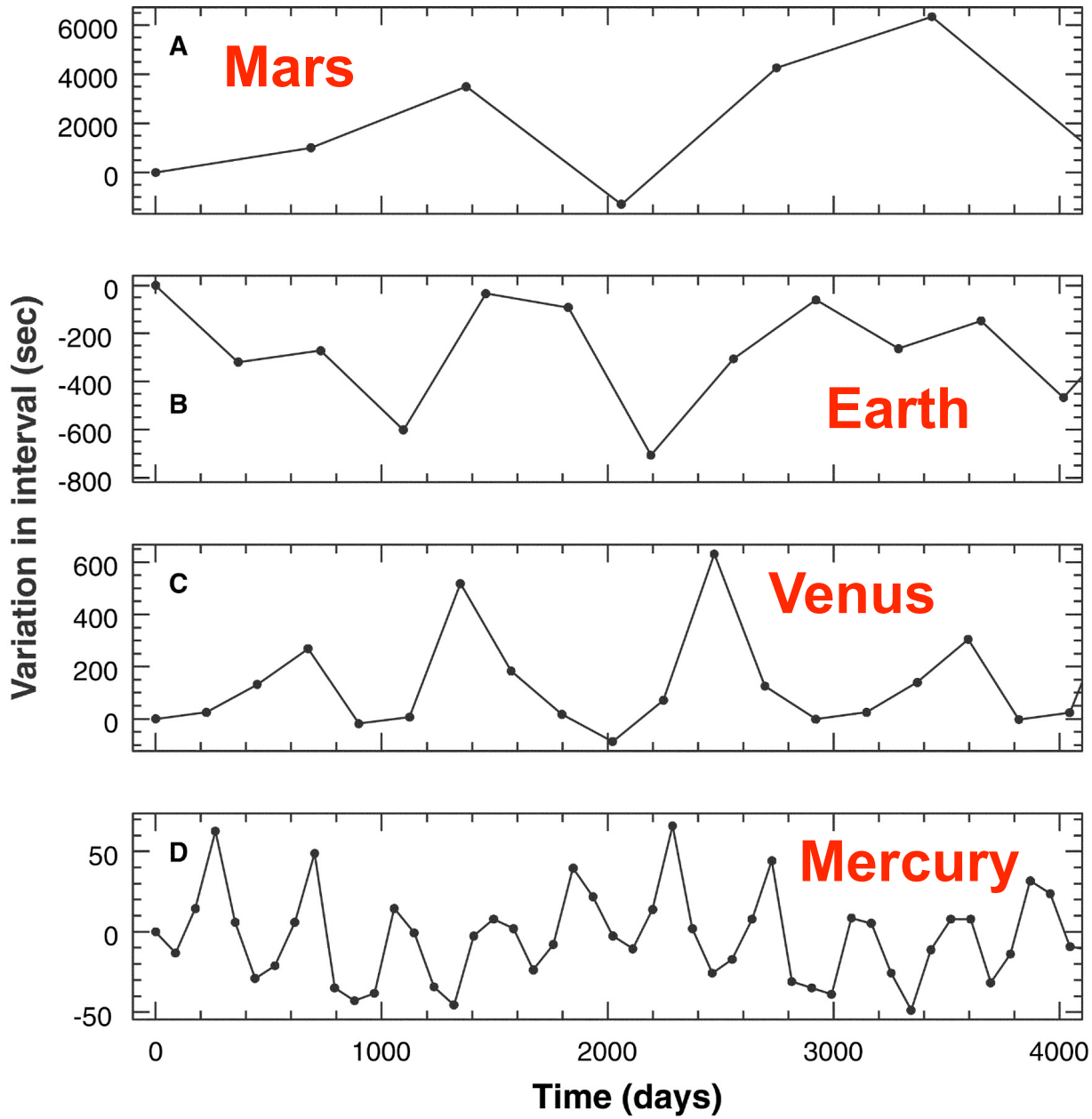




The super-Neptune HD 149026b

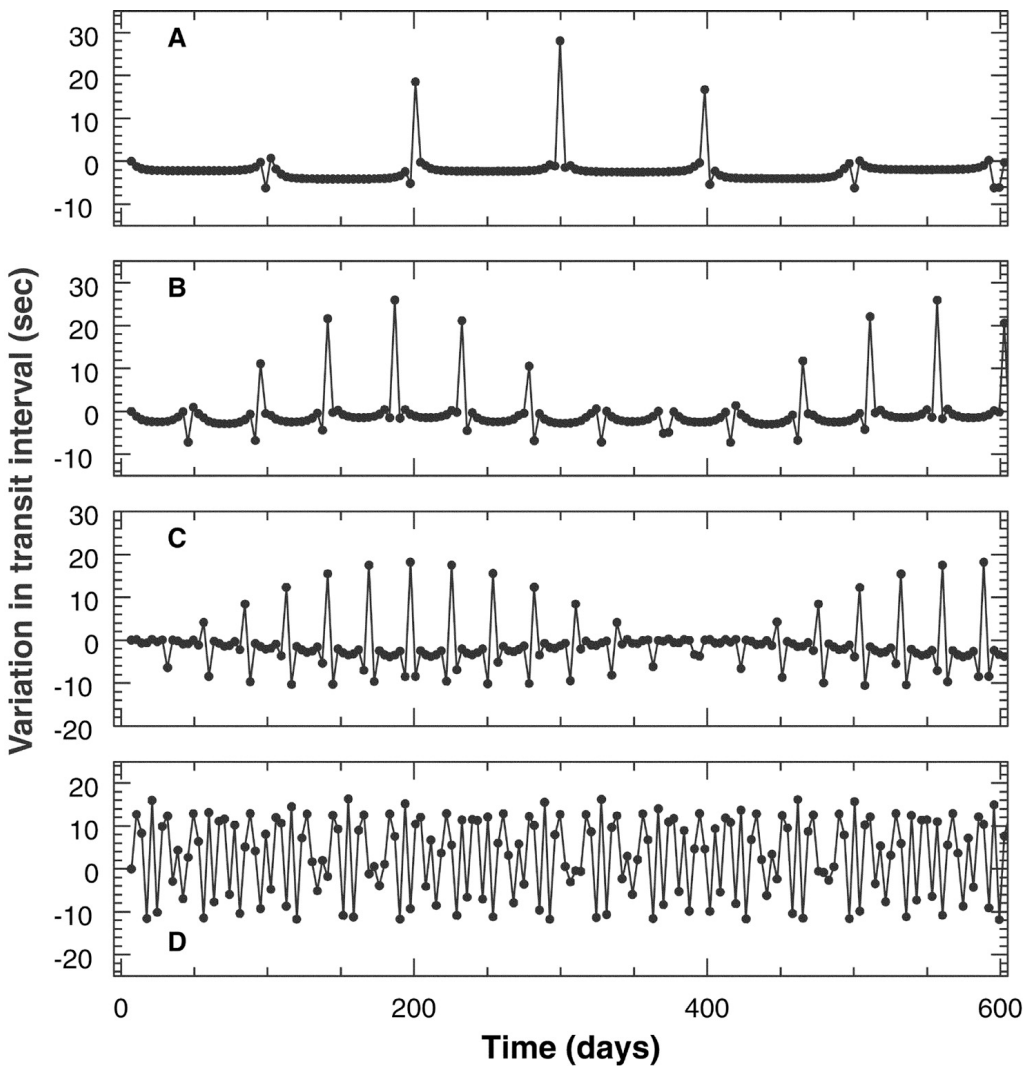
HST photometry: Carter et al. (2009)





Holman & Murray (2005); see also Agol et al. (2005)

Transit timing variations



HD 209458, assuming a second Jovian planet with…

P = 96 days, *e* = 0.7

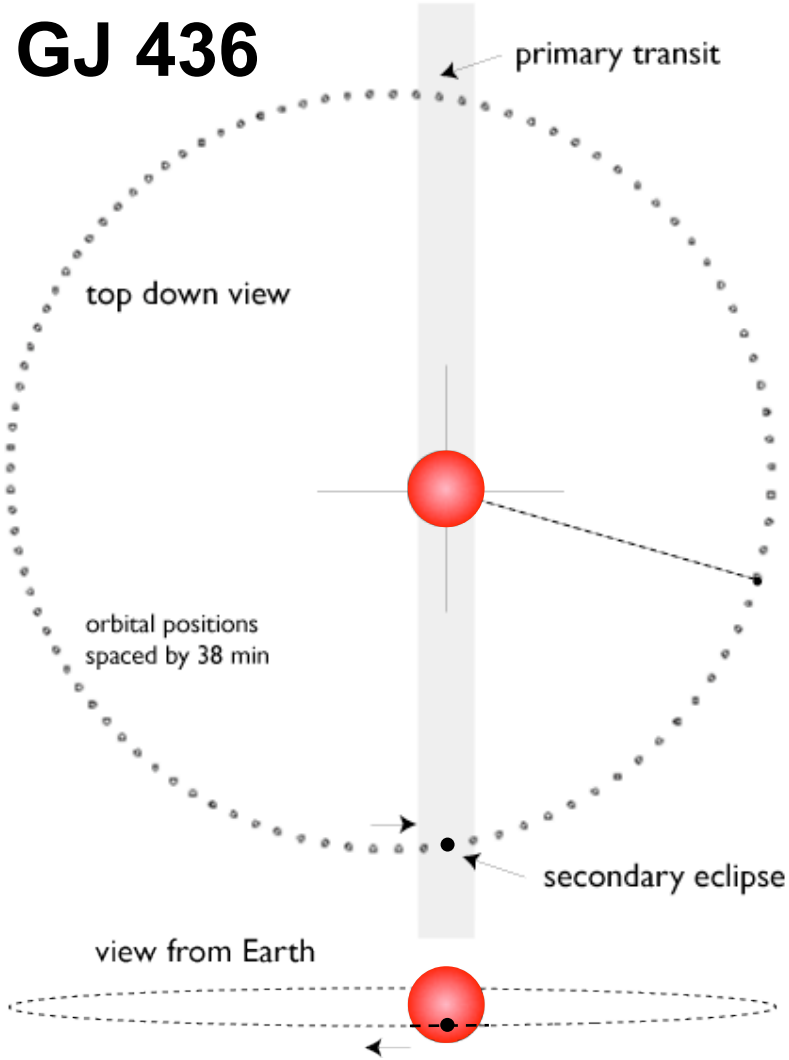
P = 46 days, *e* = 0.5

P = 28 days, *e* = 0.3

P = 19 days, *e* = 0.1

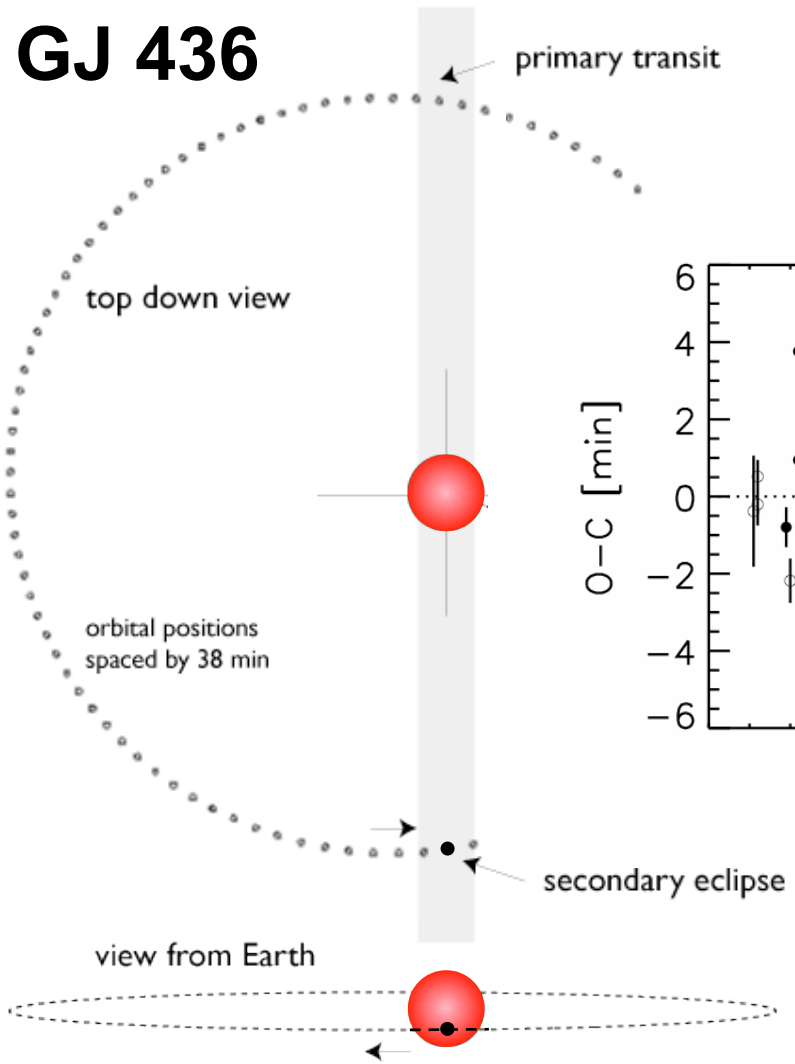
Holman & Murray (2005); see also Agol et al. (2005)

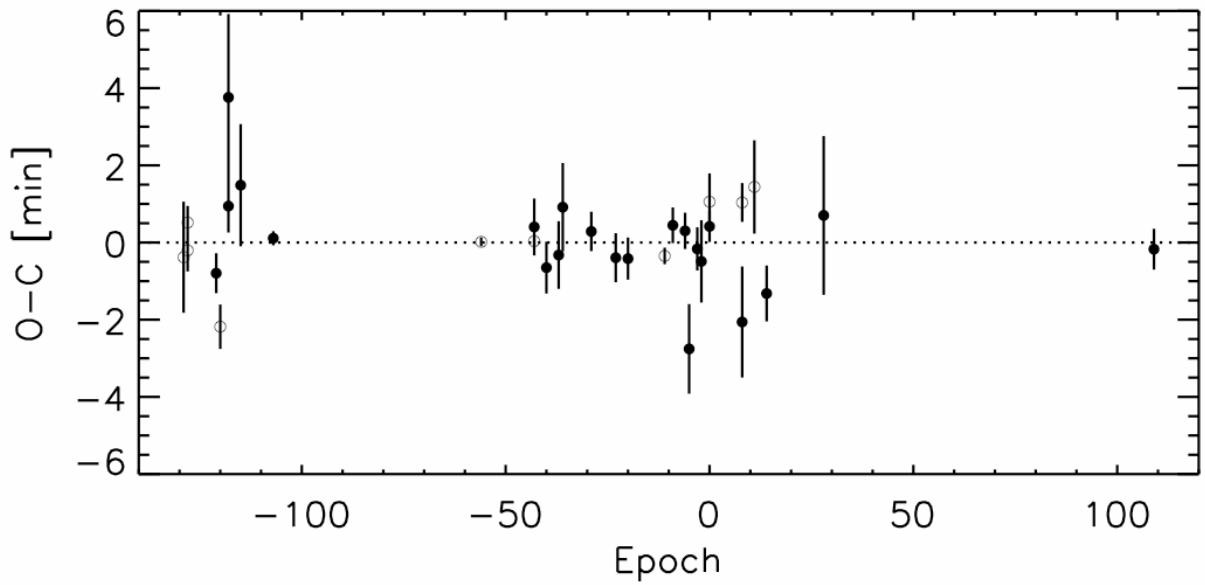
Transit timing variations



G. Laughlin

Transit timing variations



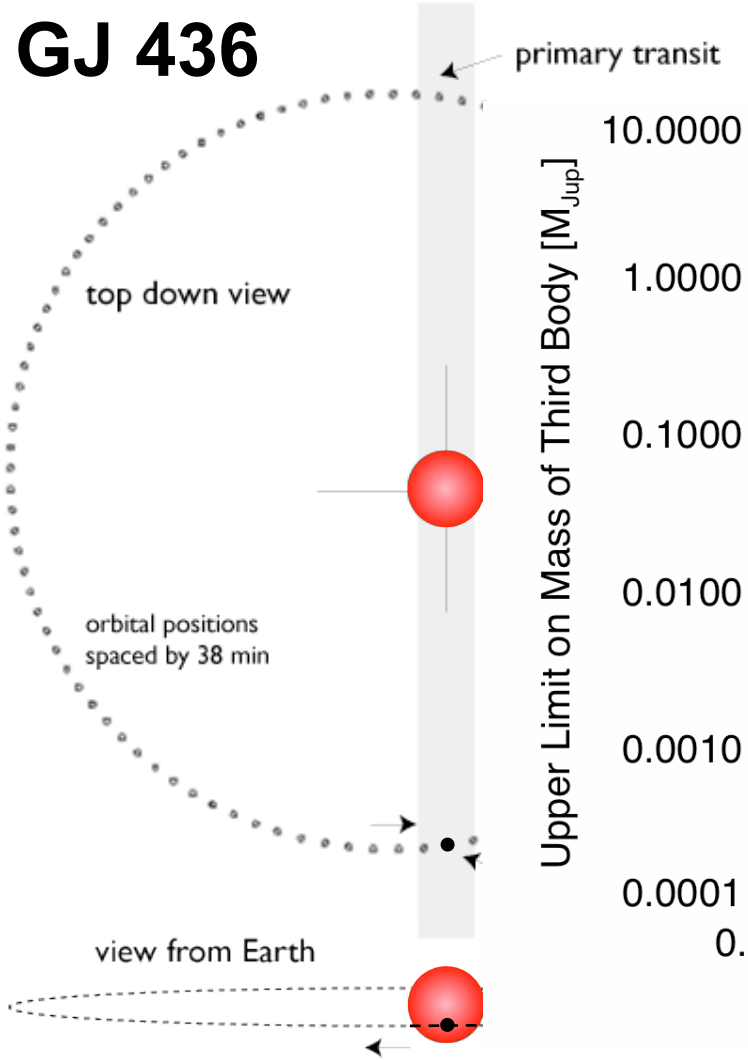


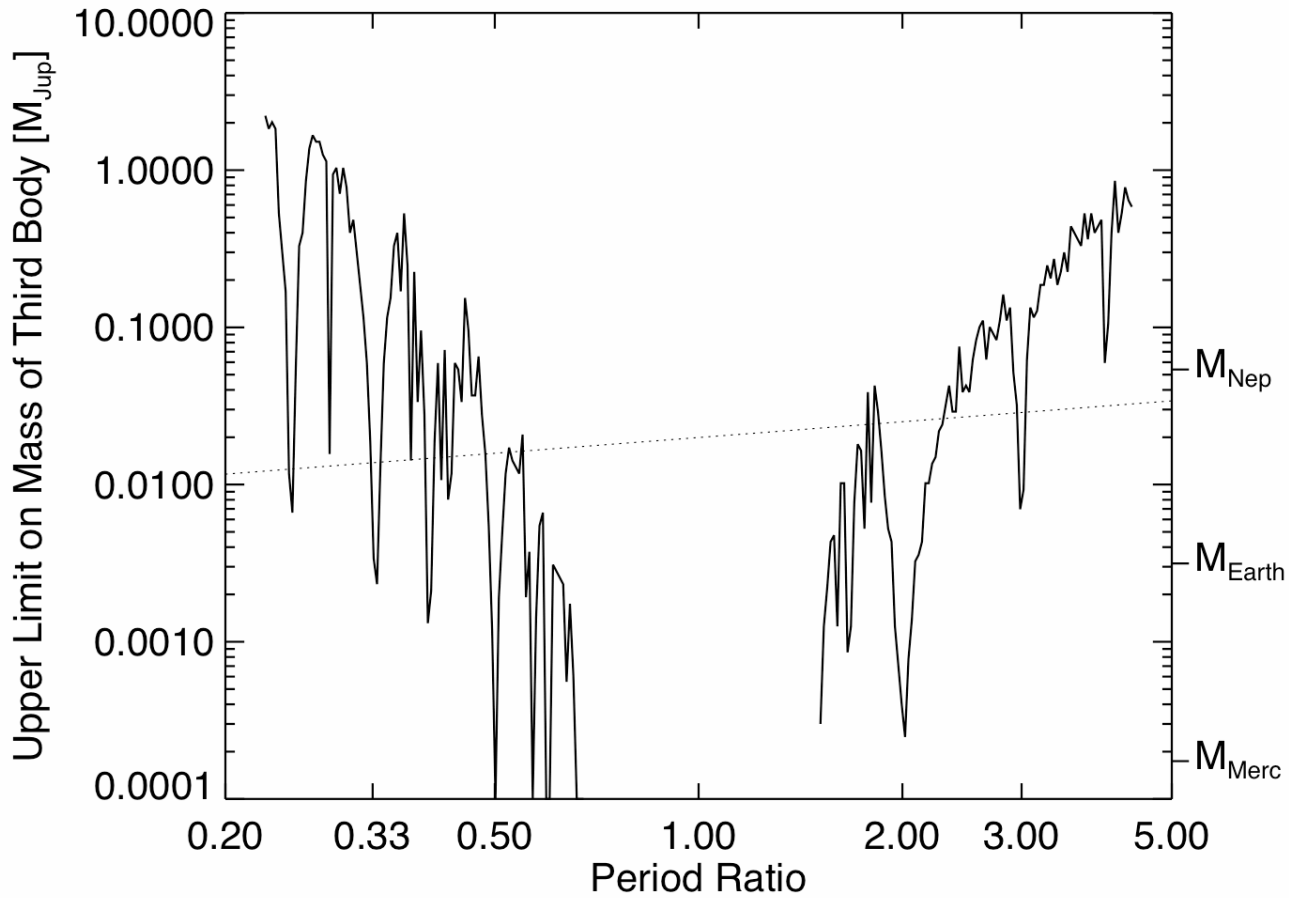
G. Laughlin

Holman, Winn, Fabrycky, et al., in prep.

Transit timing variations

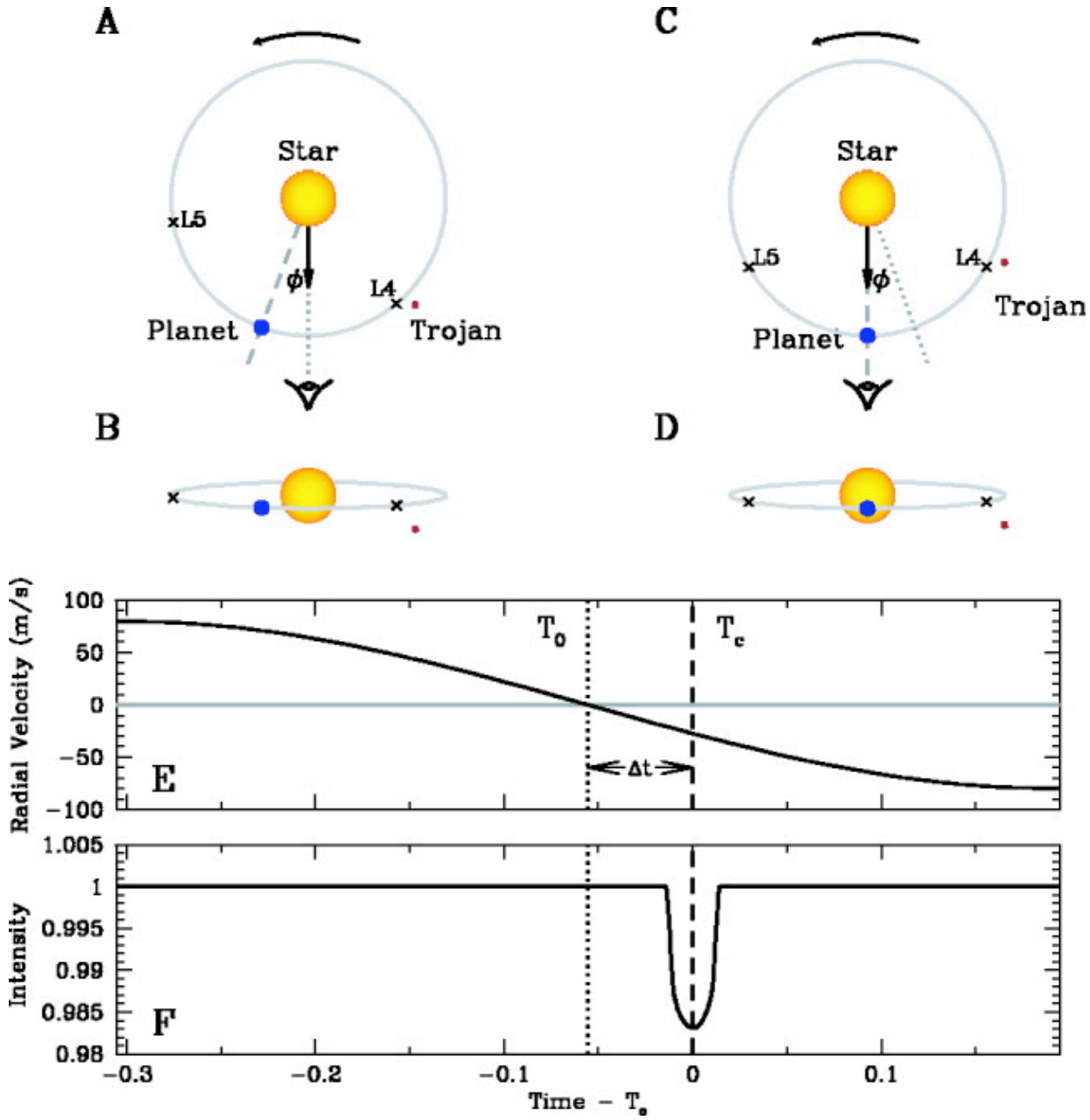
GJ 436



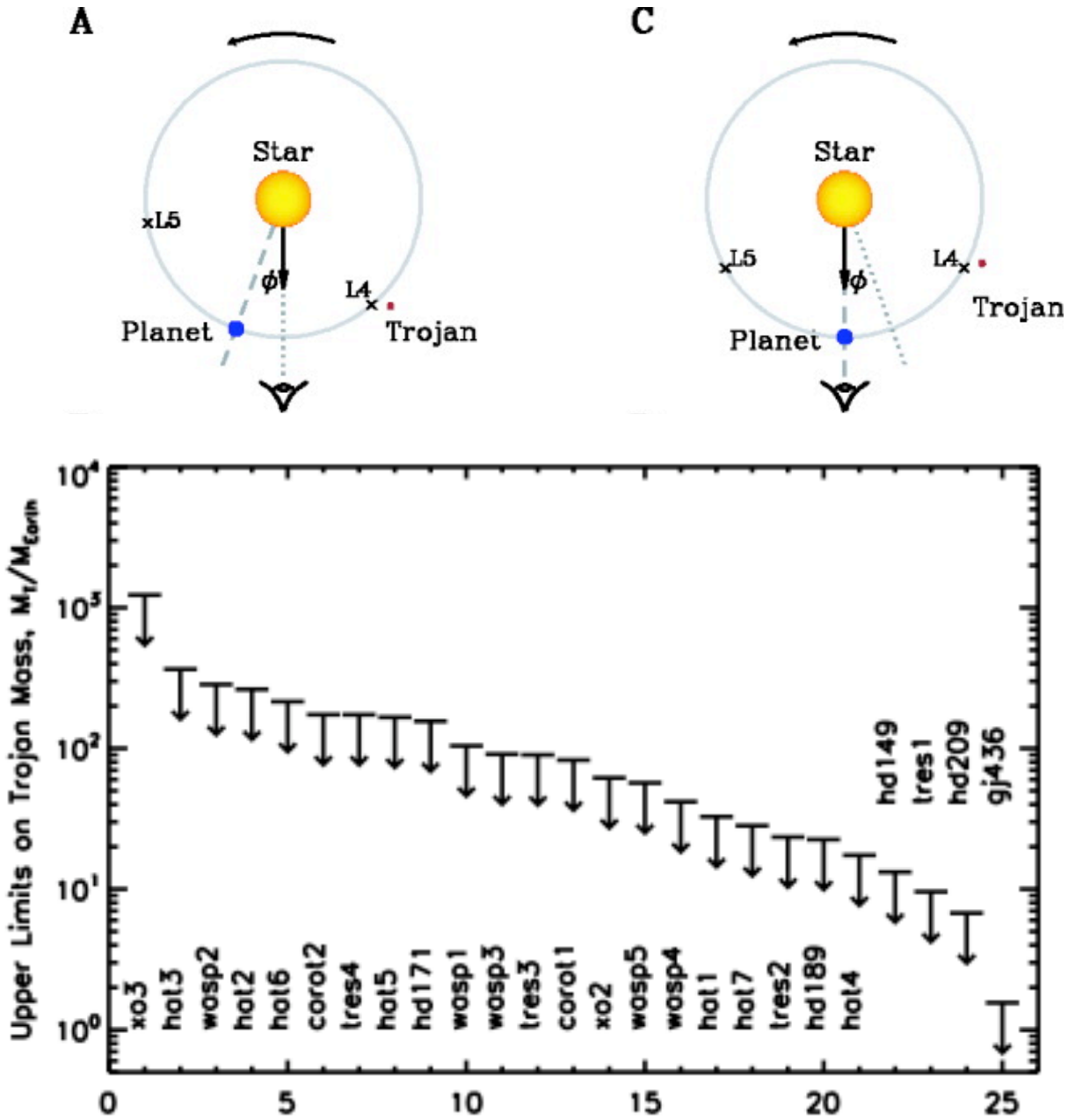


G. Laughlin

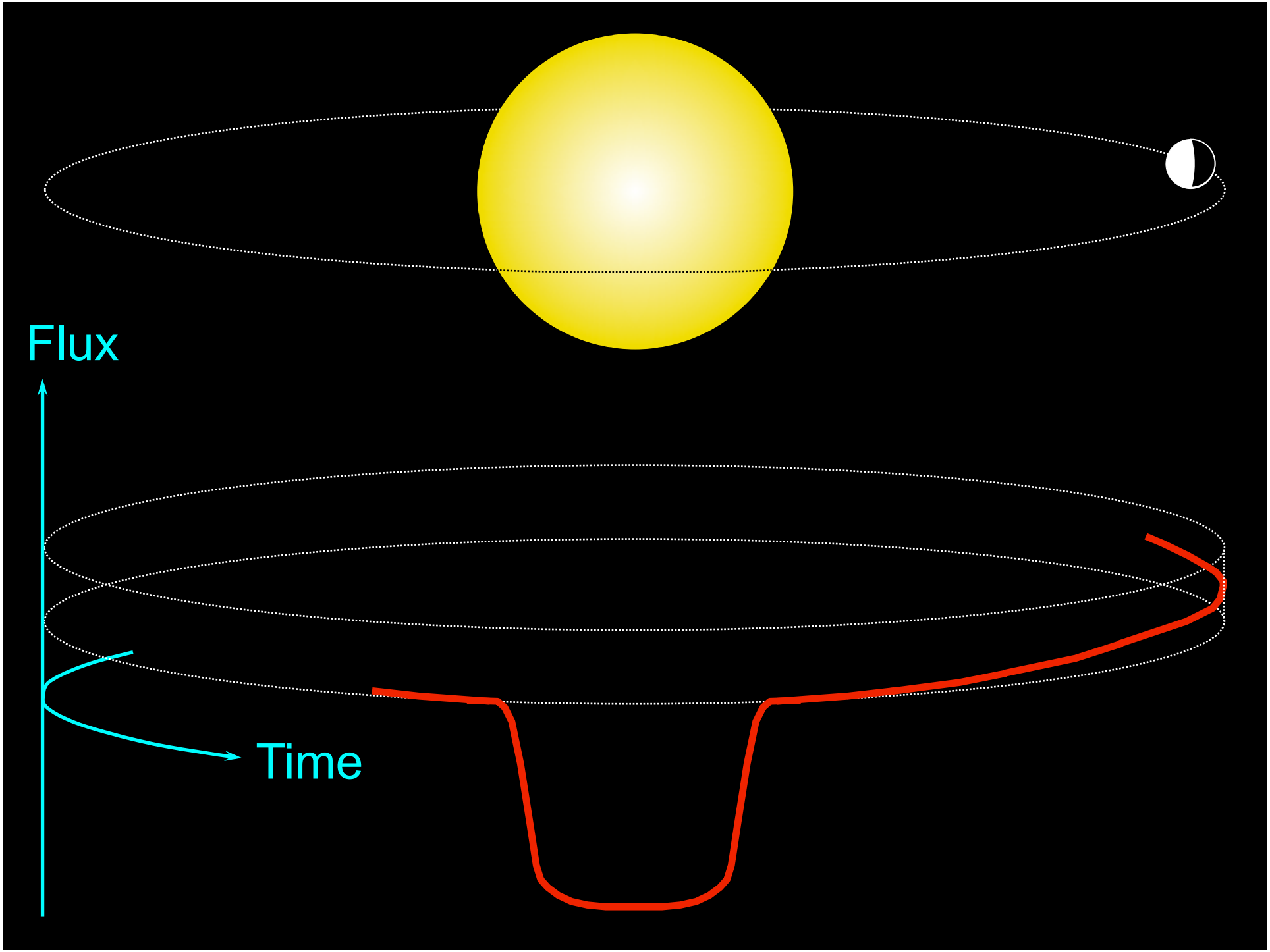
Holman, Winn, Fabrycky, et al., in prep.

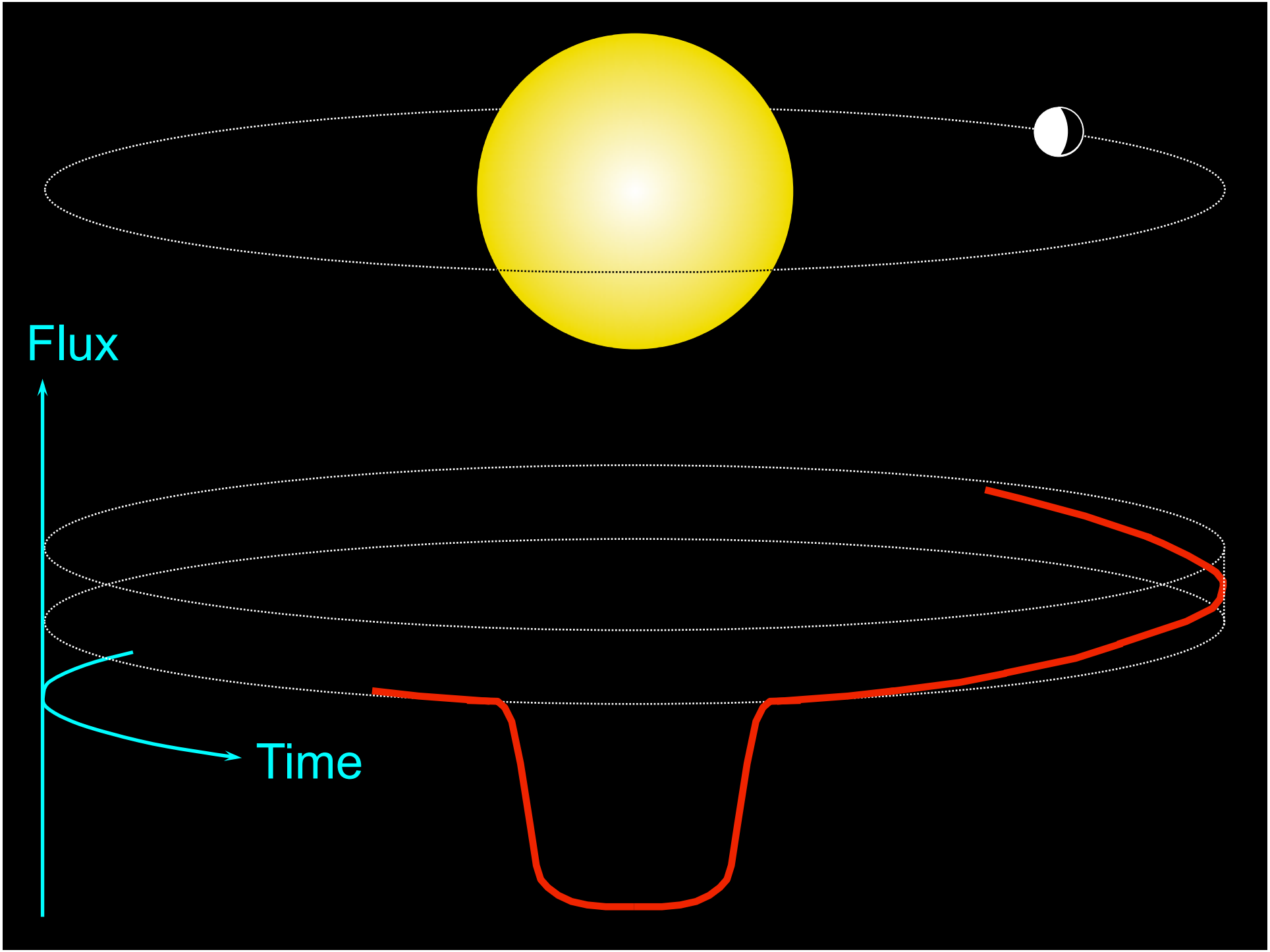


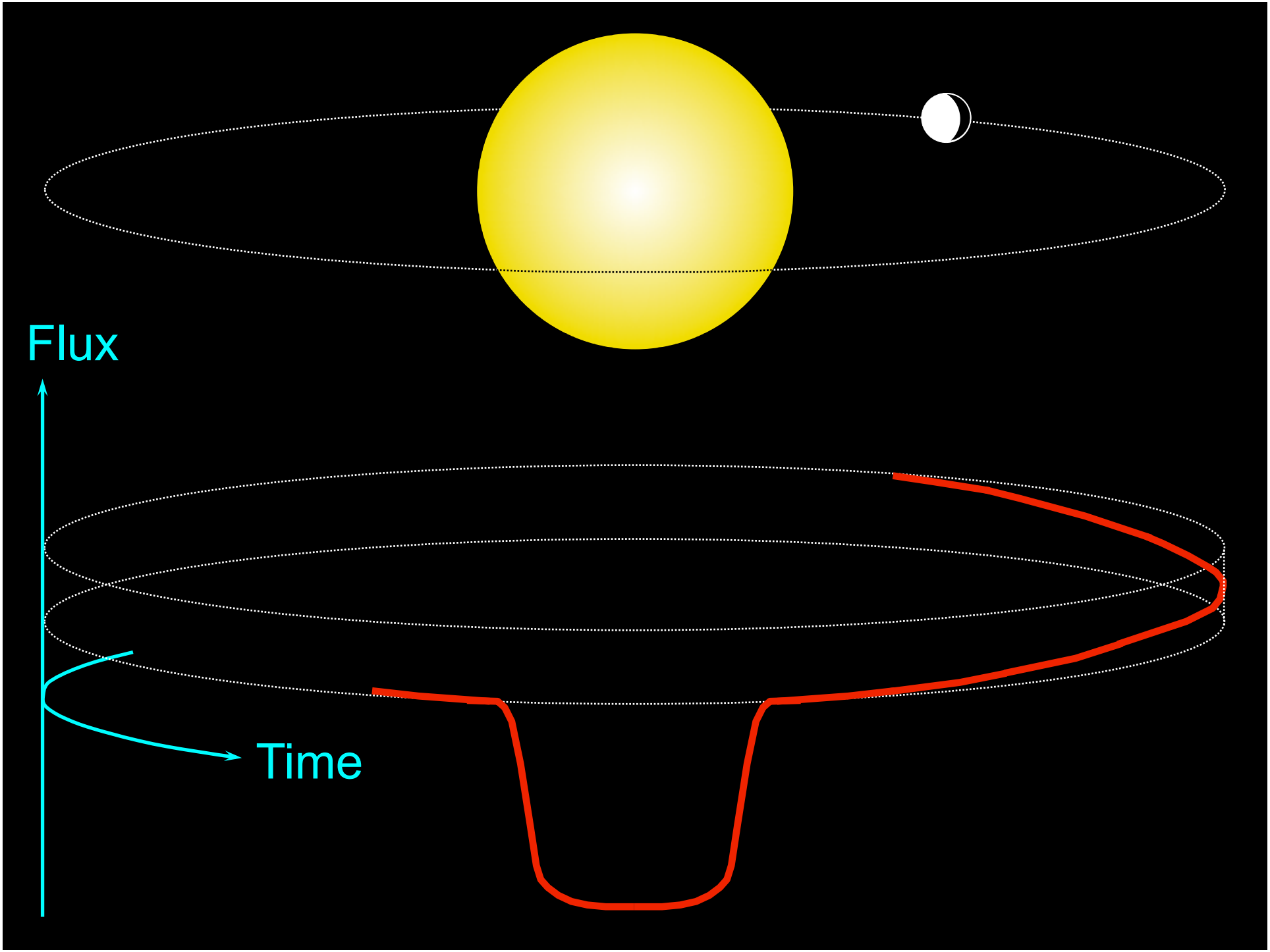
Ford & Gaudi (2006)

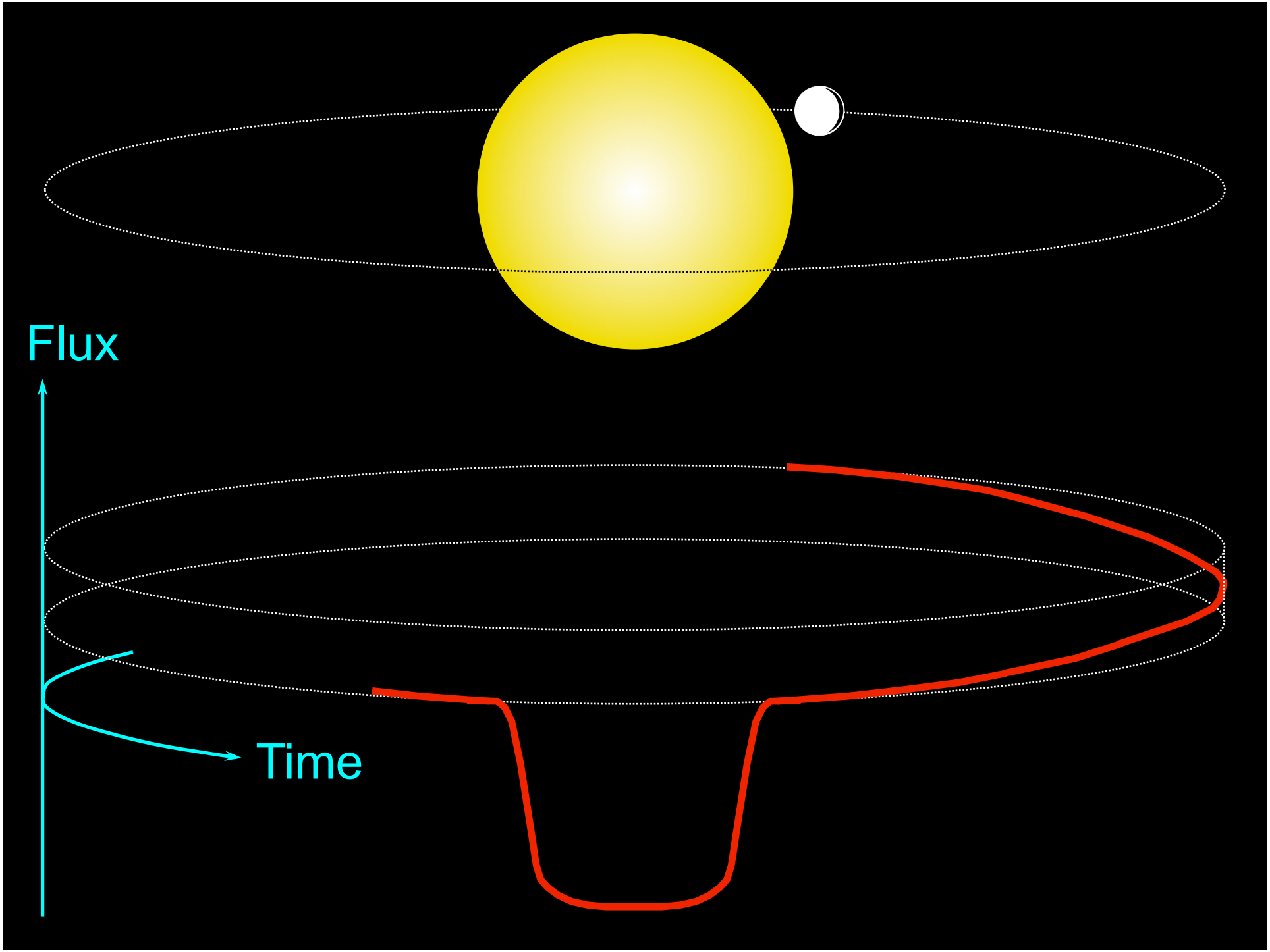


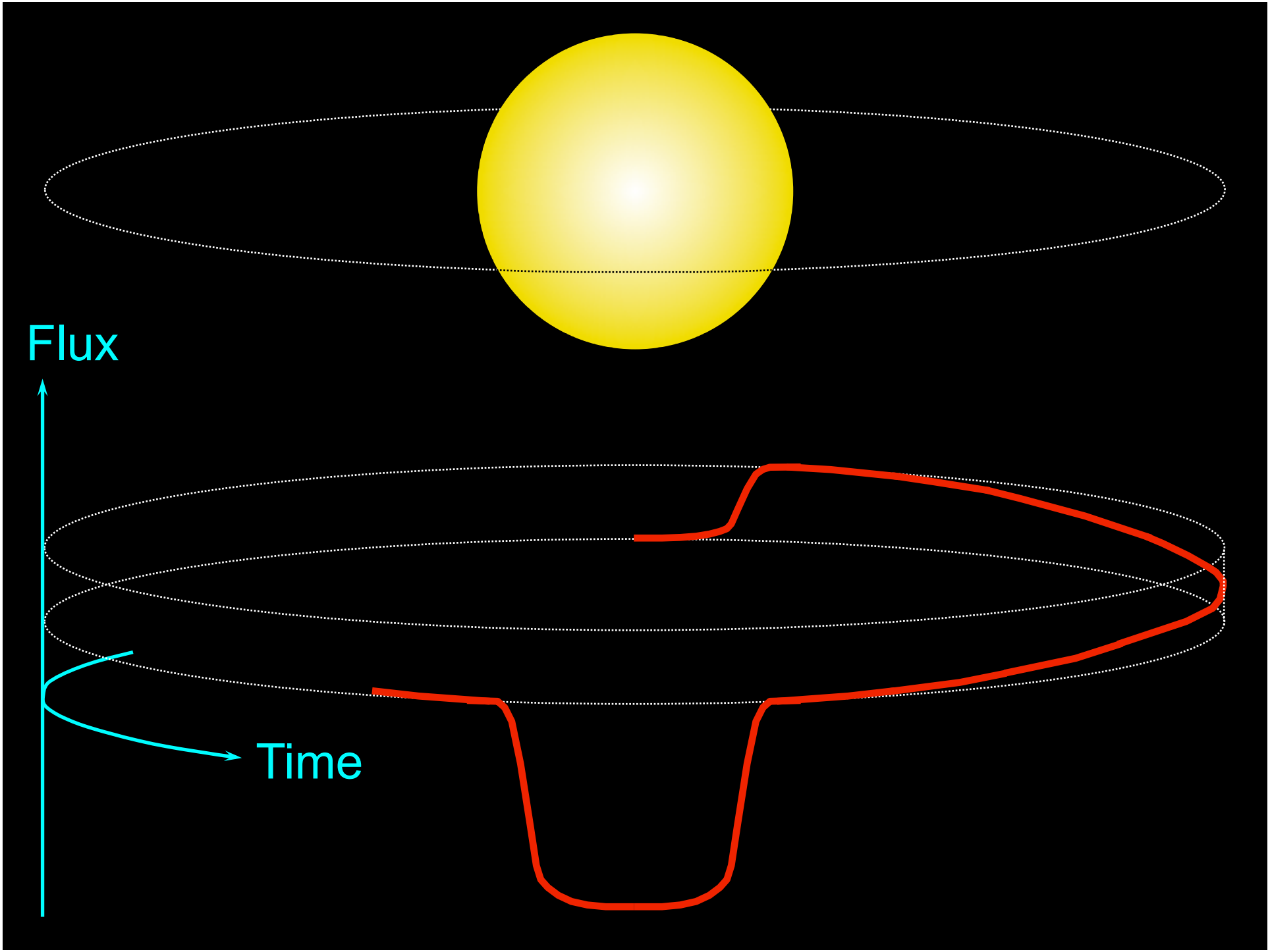
Madhusudhan & Winn (2009)

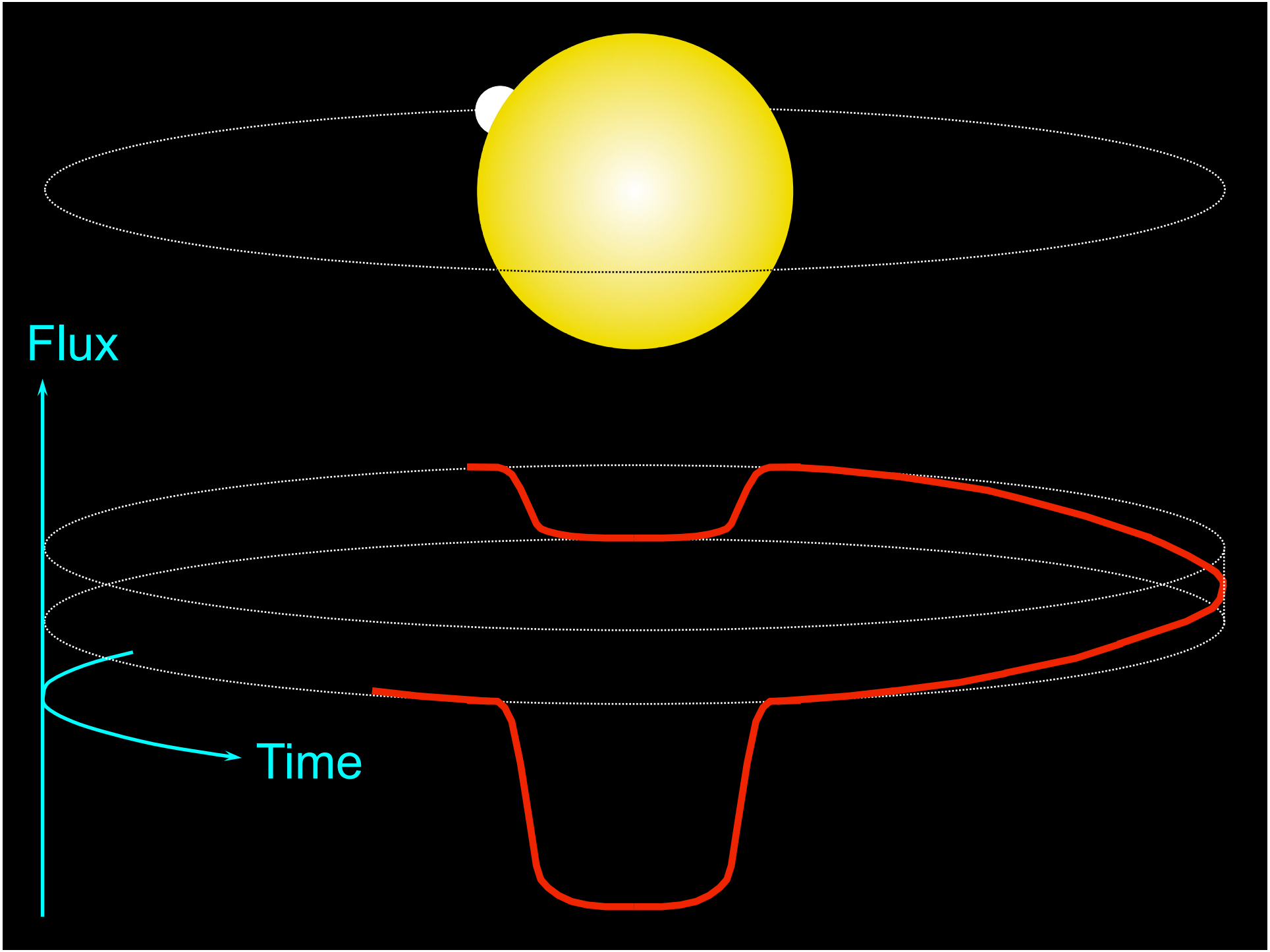

Flux
Time

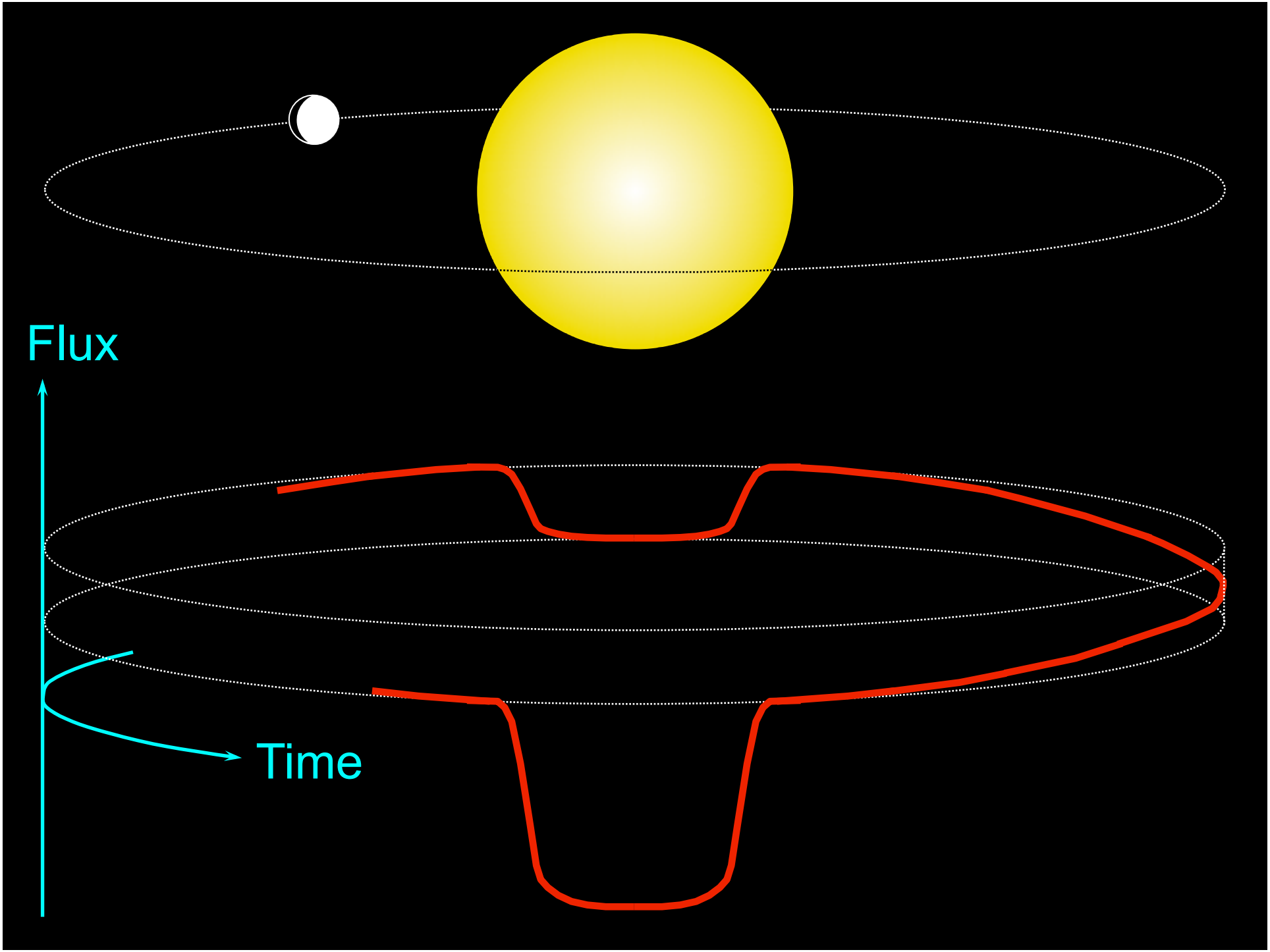

Flux
Time

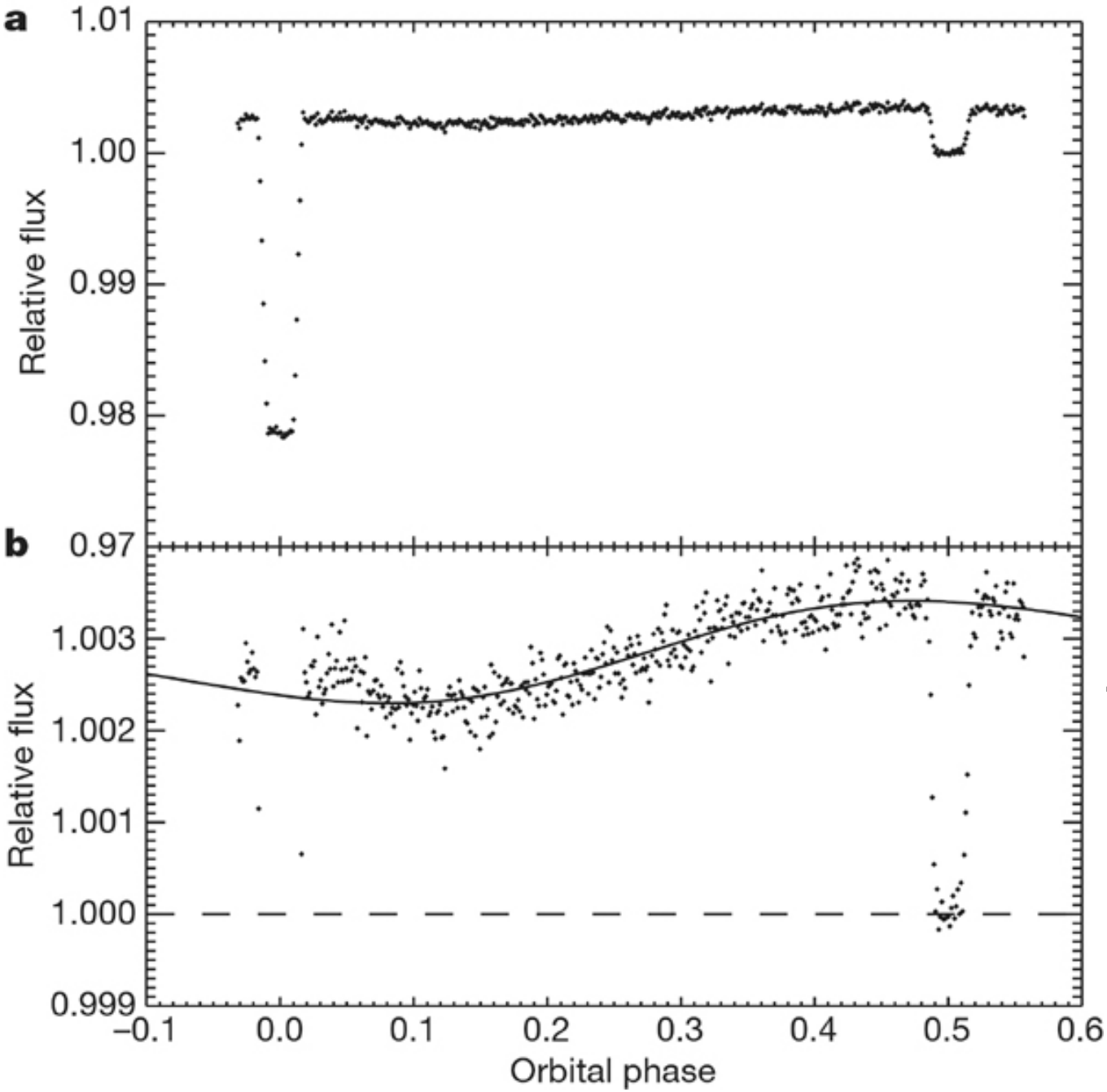

Flux
Time


Flux
Time


Flux
Time


Flux
Time


Flux
Time

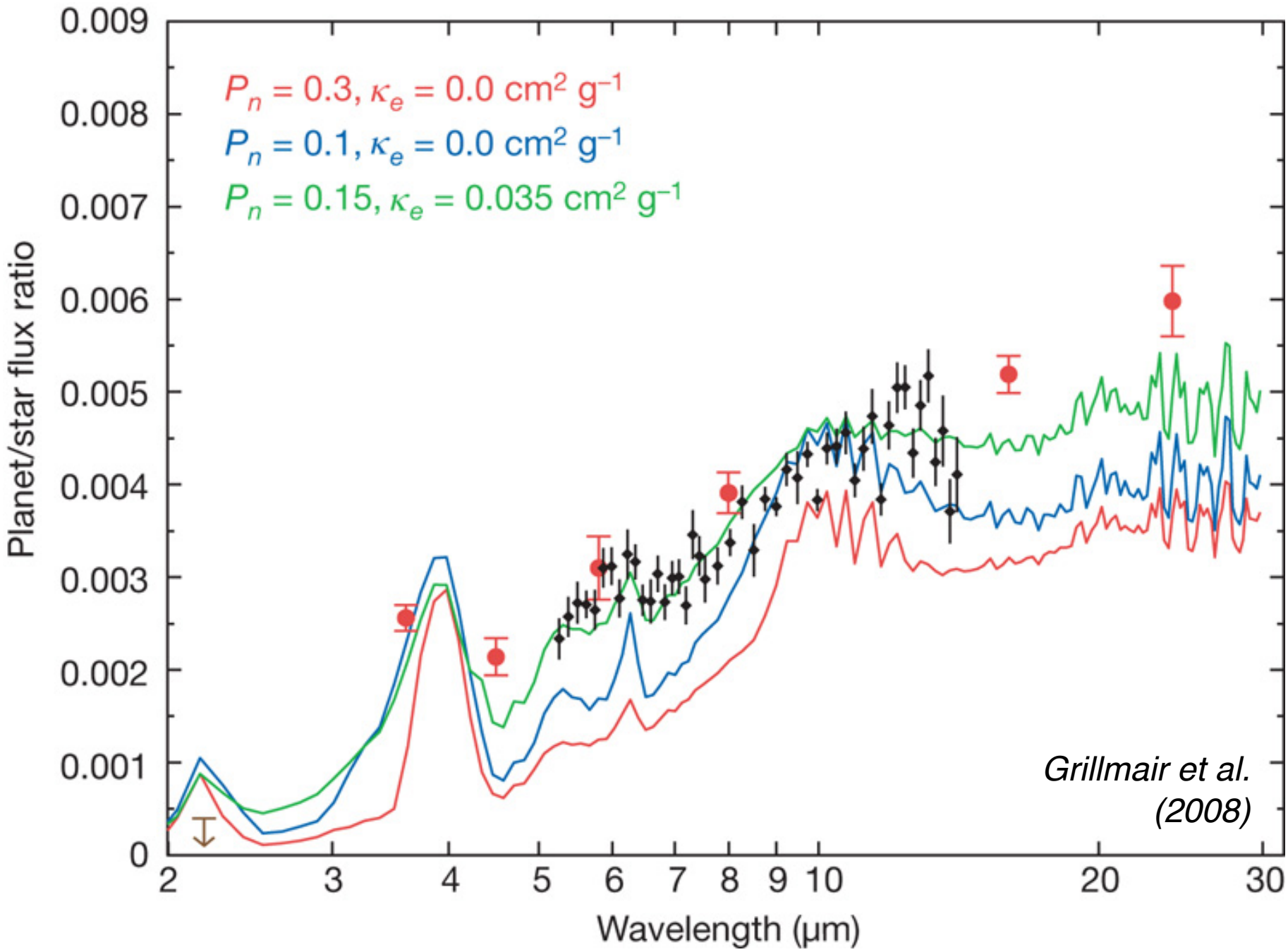


Transit and occultation of HD 189733

T_{min} = 973 K

T_{max} = 1212 K

Knutson et al. (2007)


$P_n = 0.3, \kappa_e = 0.0$ cm^2 g^{-1}
$P_n = 0.1, \kappa_e = 0.0$ cm^2 g^{-1}
$P_n = 0.15, \kappa_e = 0.035$ cm^2 g^{-1}
Planet/star flux ratio
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
2
3
4
5
6
7
8
9
10
20
30
Wavelength (μm)
Grillmair et al. (2008)

Exoplanetary spin-orbit alignment

Exoplanetary spin-orbit alignment

- Solar obliquity is 7° — how common or unusual is this?

Exoplanetary spin-orbit alignment

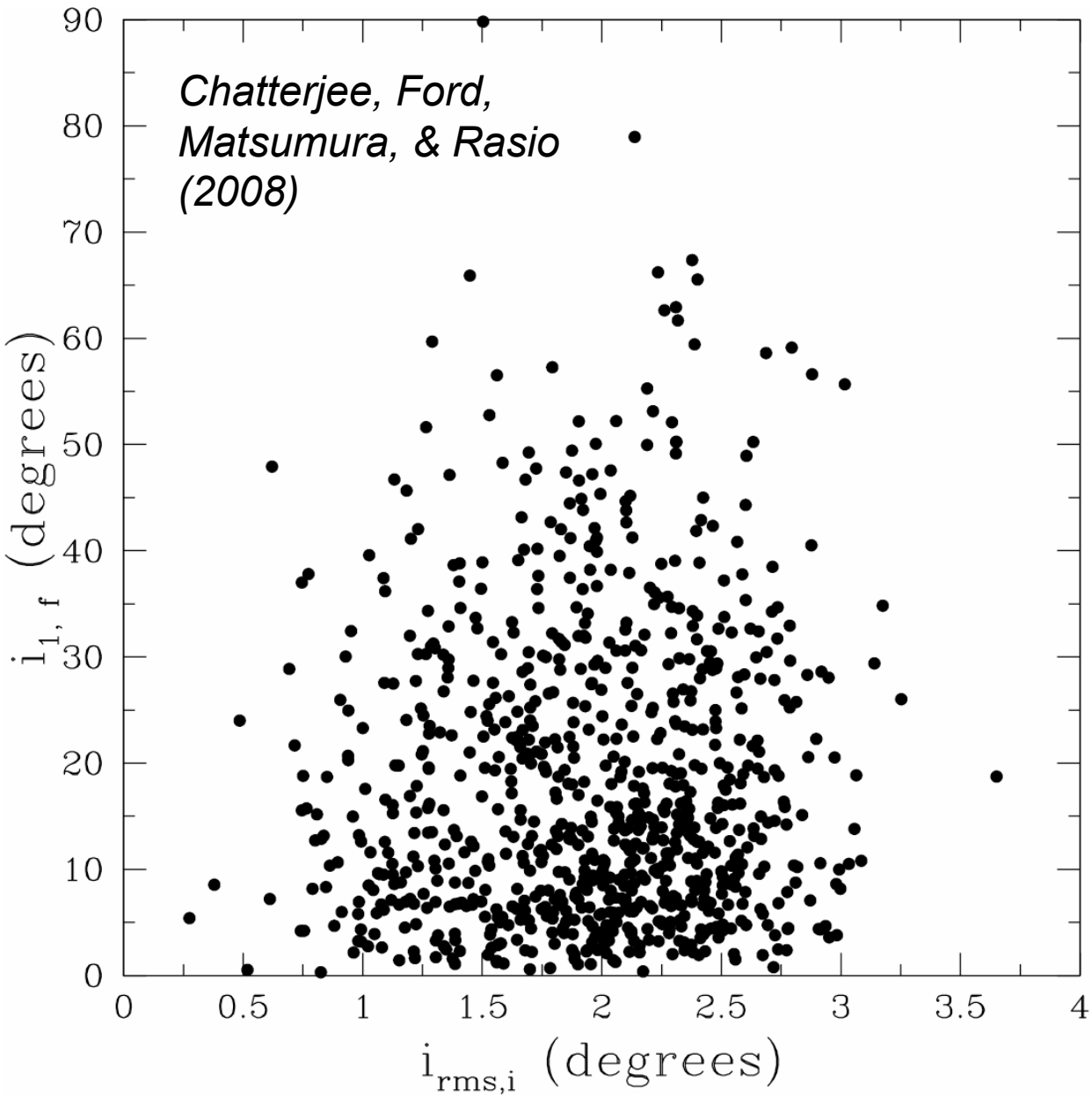
- Solar obliquity is 7° — how common or unusual is this?
- Specific reasons to expect misalignment:

Exoplanetary spin-orbit alignment

- Solar obliquity is 7° — how common or unusual is this?
- Specific reasons to expect misalignment:
  - Whatever perturbs eccentricities may also perturb inclinations

Exoplanetary spin-orbit alignment

- Solar obliquity is 7° — how common or unusual is this?
- Specific reasons to expect misalignment:
  - Whatever perturbs eccentricities may also perturb inclinations
  - Migration (gas-disk torque *vs.* planet-planet scattering, Kozai oscillations)

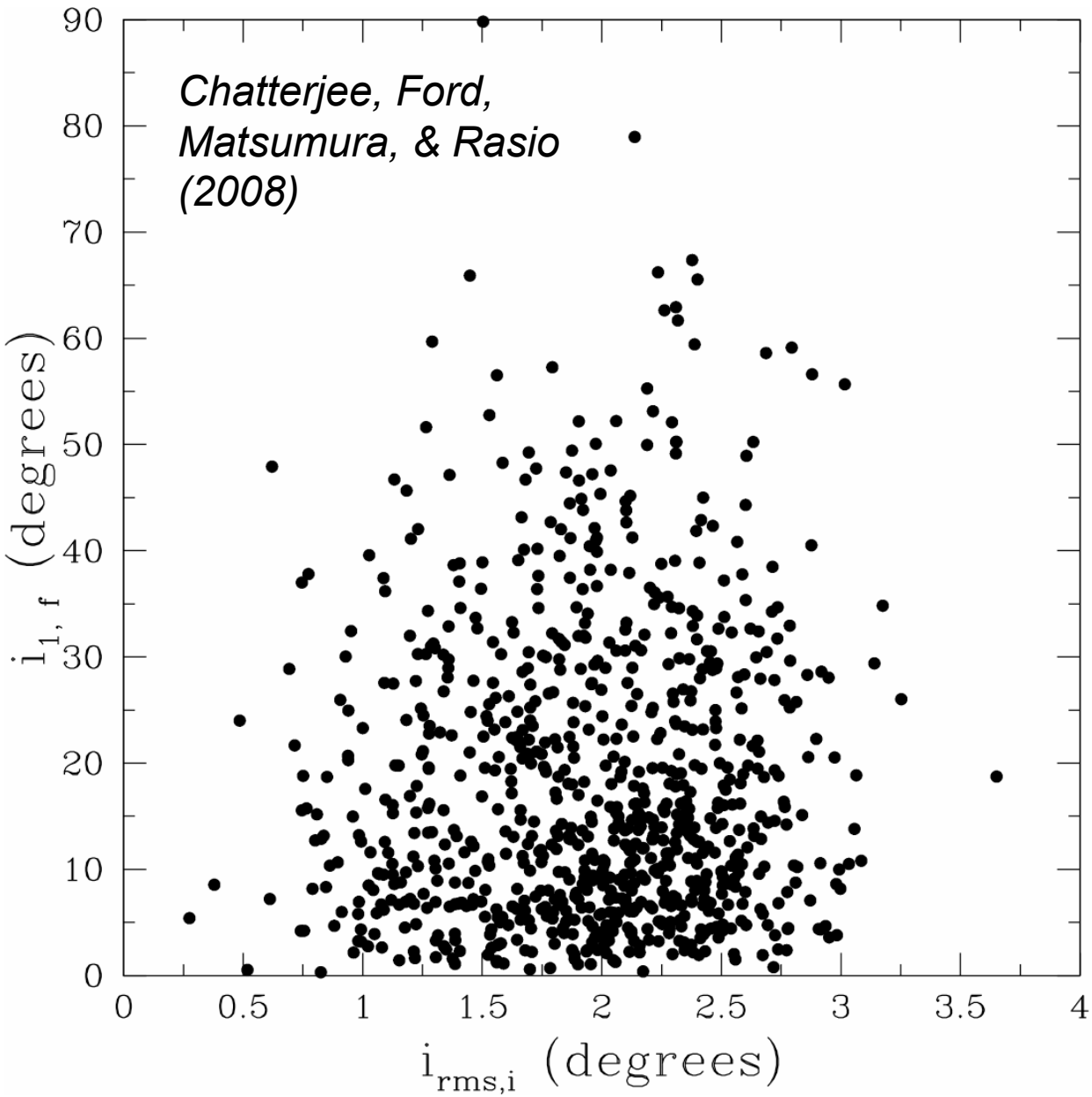


Planet-planet scattering scenarios

Rasio & Ford (1996)

Weidenschilling & Marzari (1996)

Lin & Ida (1997)

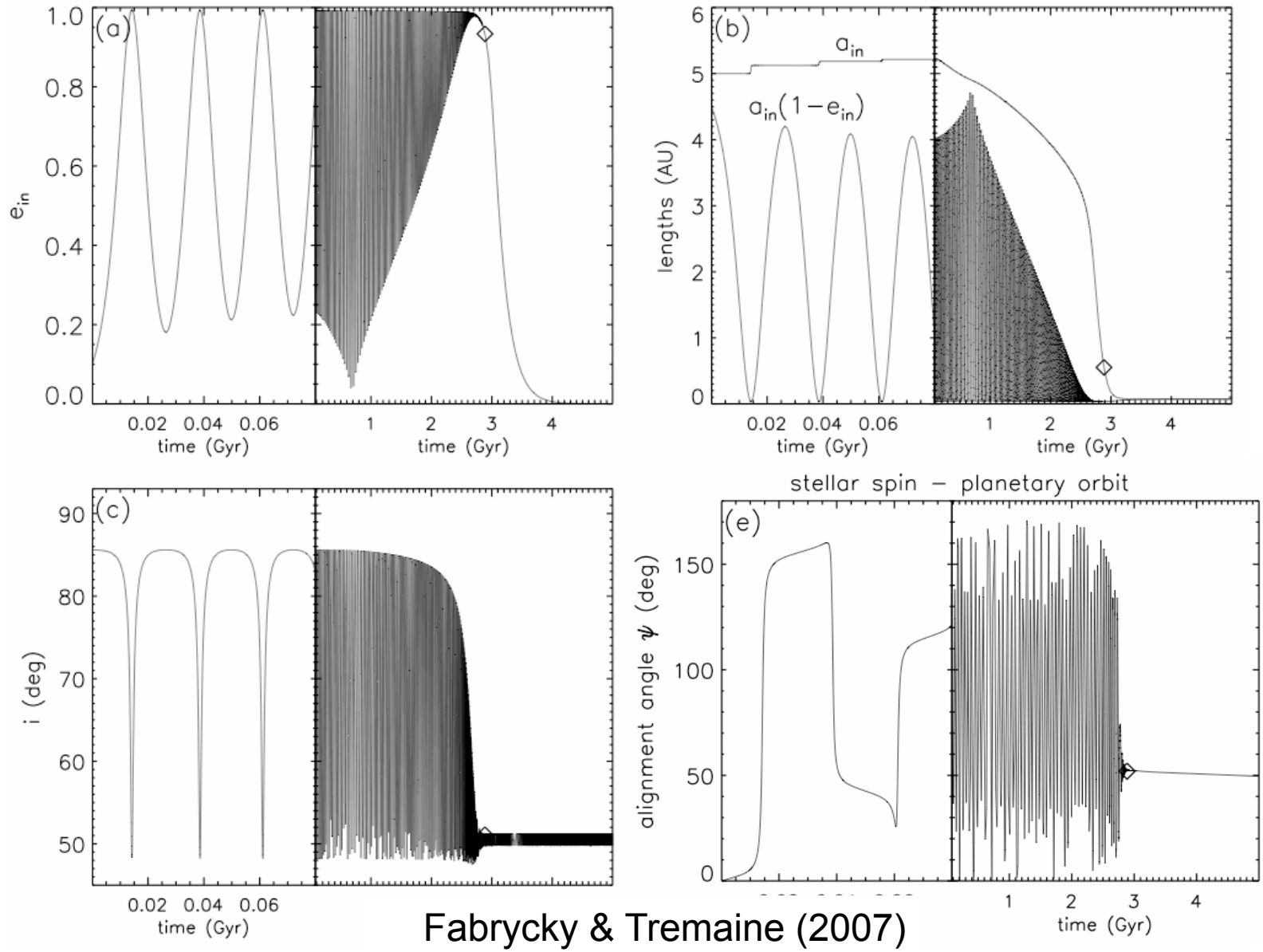


Planet-planet scattering scenarios produce a broad range of final inclinations

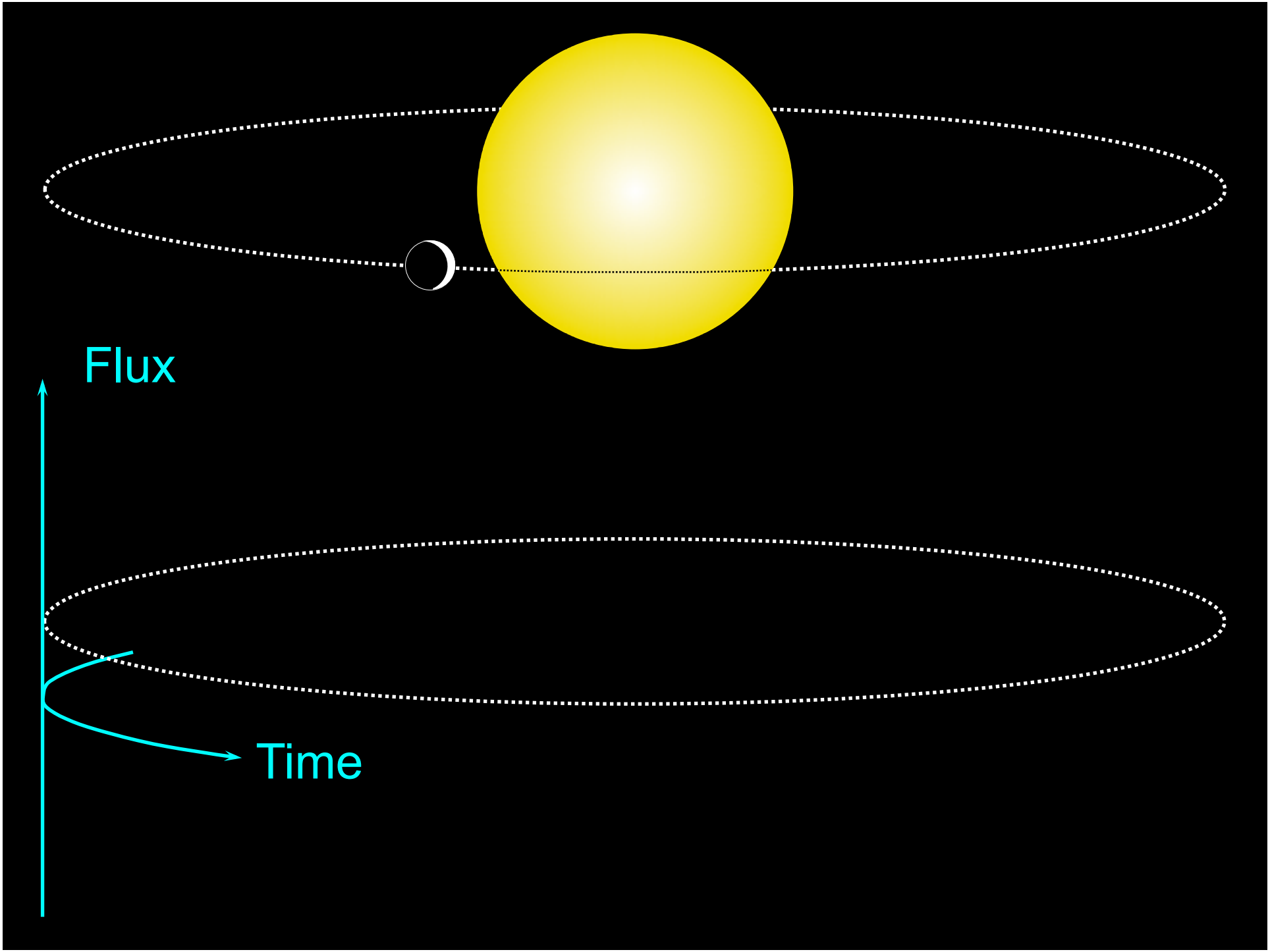
See also Yu & Tremaine (2001), Nagasawa et al. (2008), Juric & Tremaine (2008)

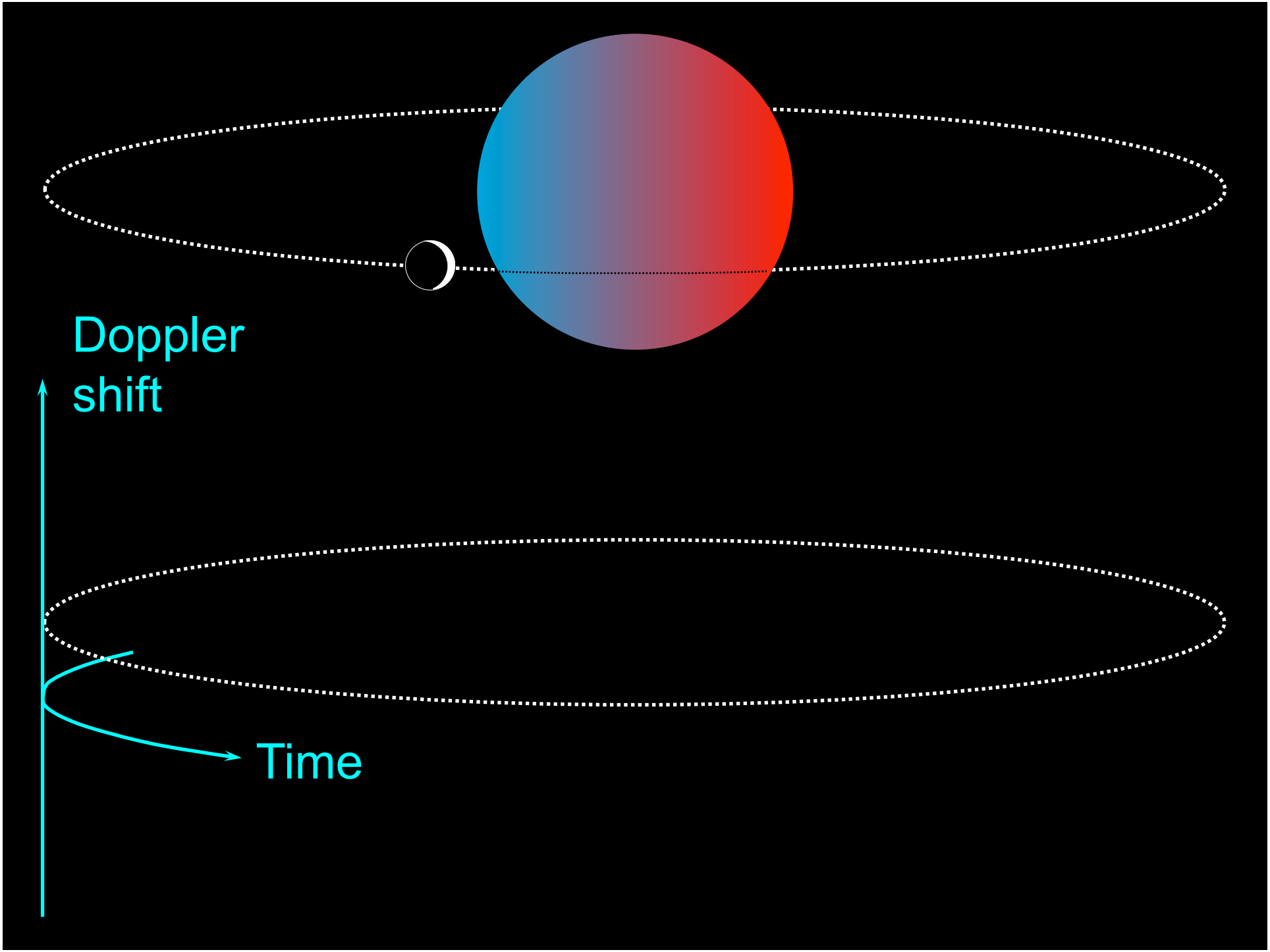
Kozai-cycle scenarios

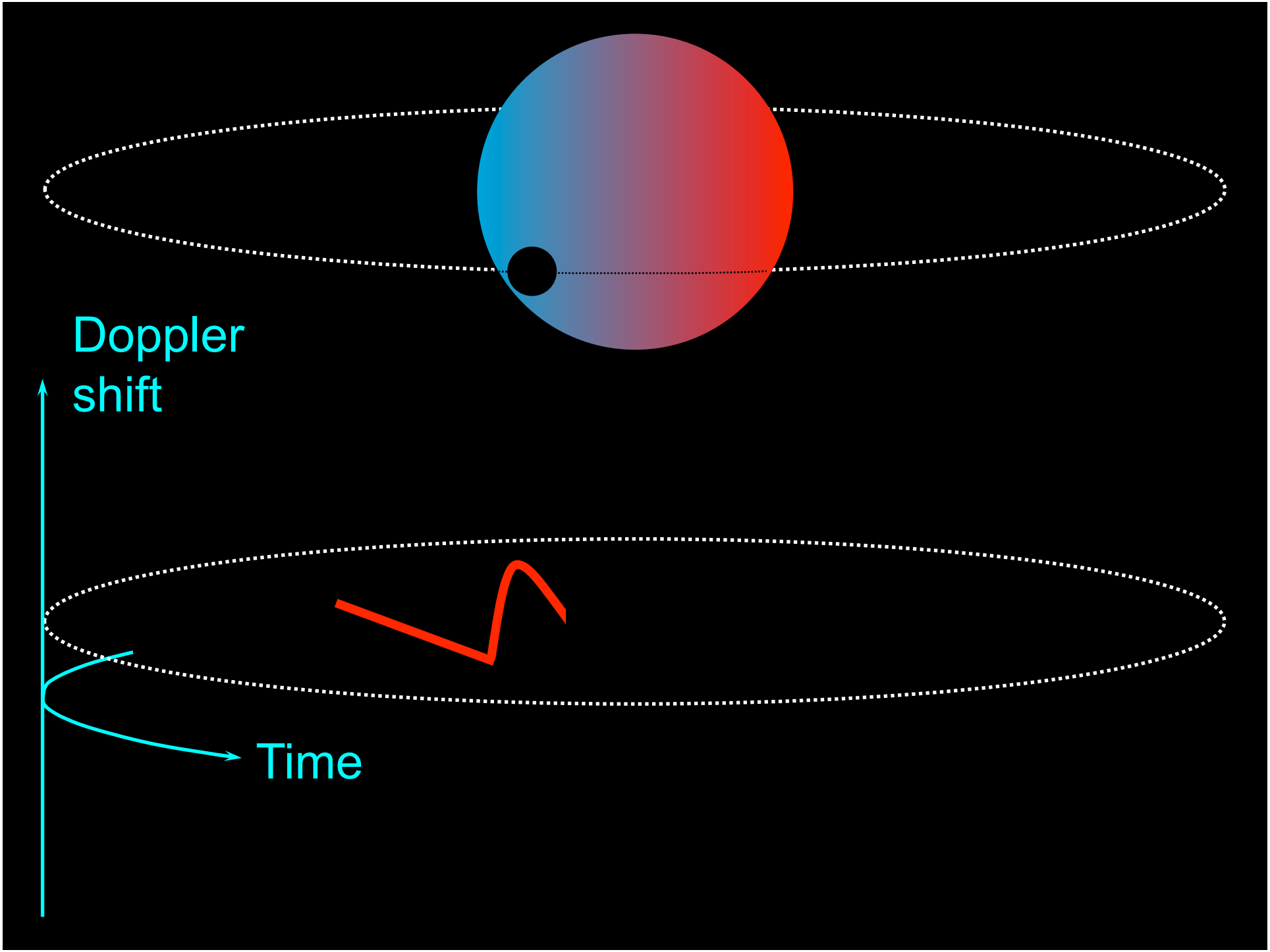
$$H = \sqrt{1 - e^2}\ \cos i$$

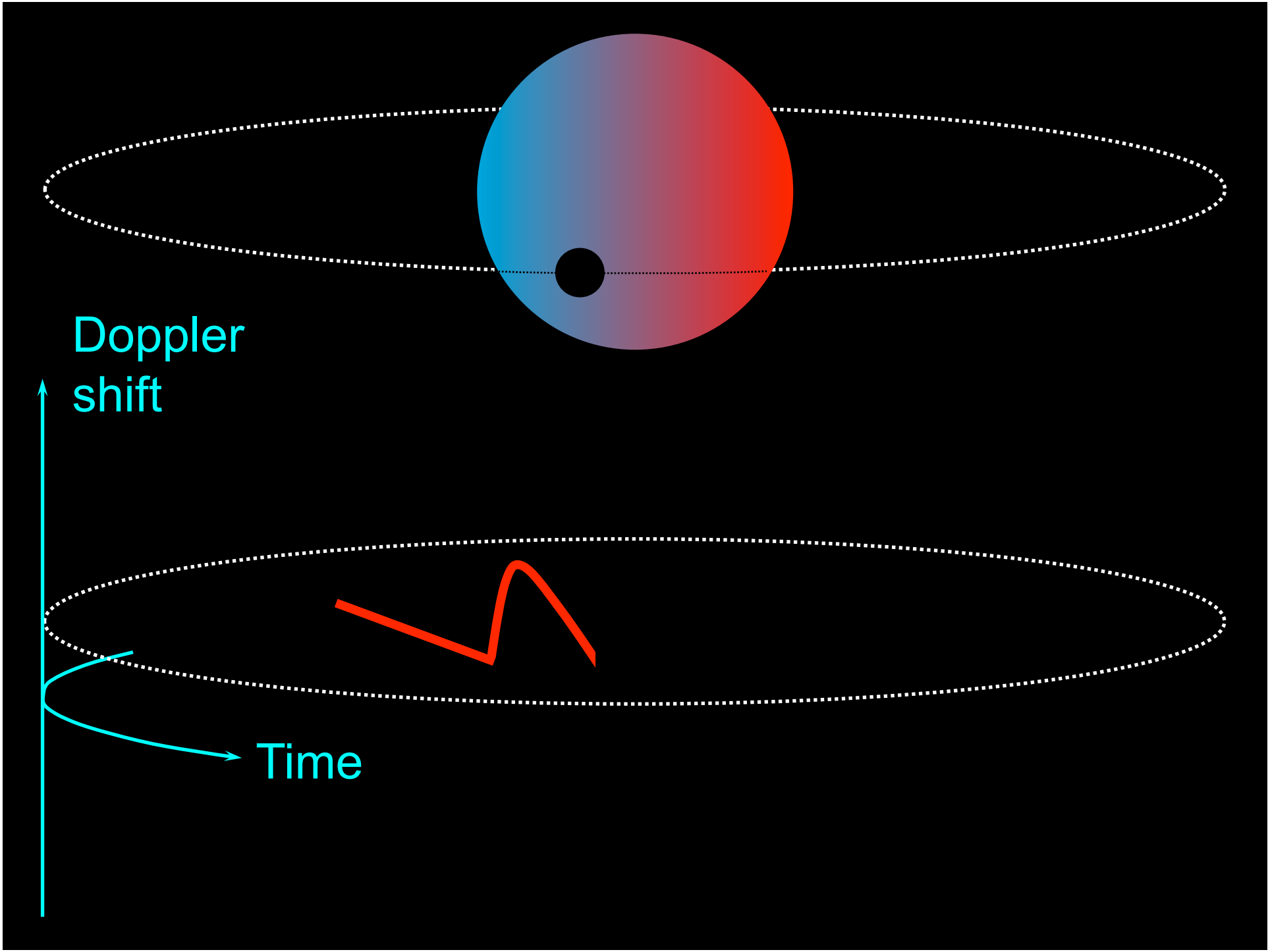


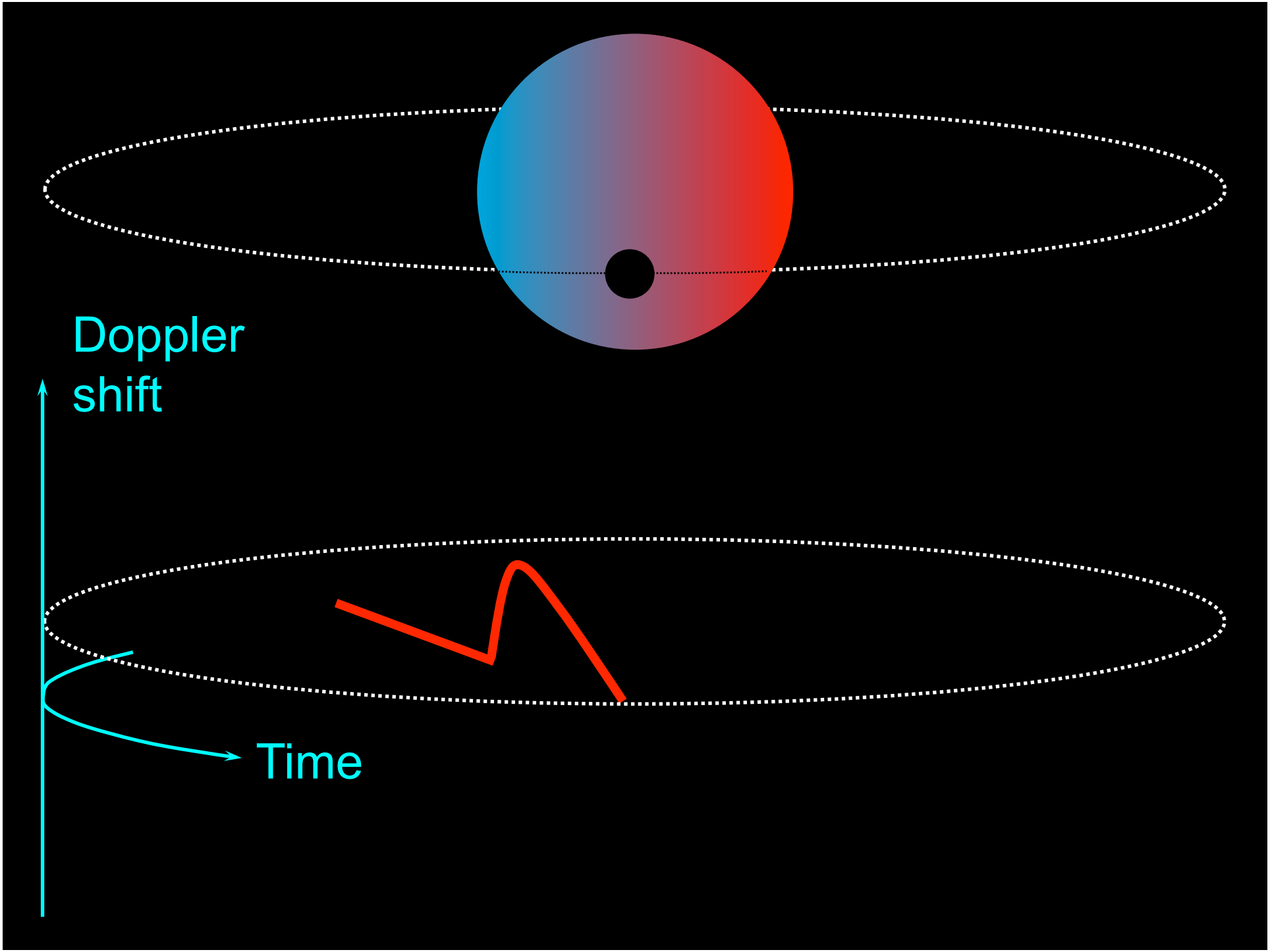
Fabrycky & Tremaine (2007)

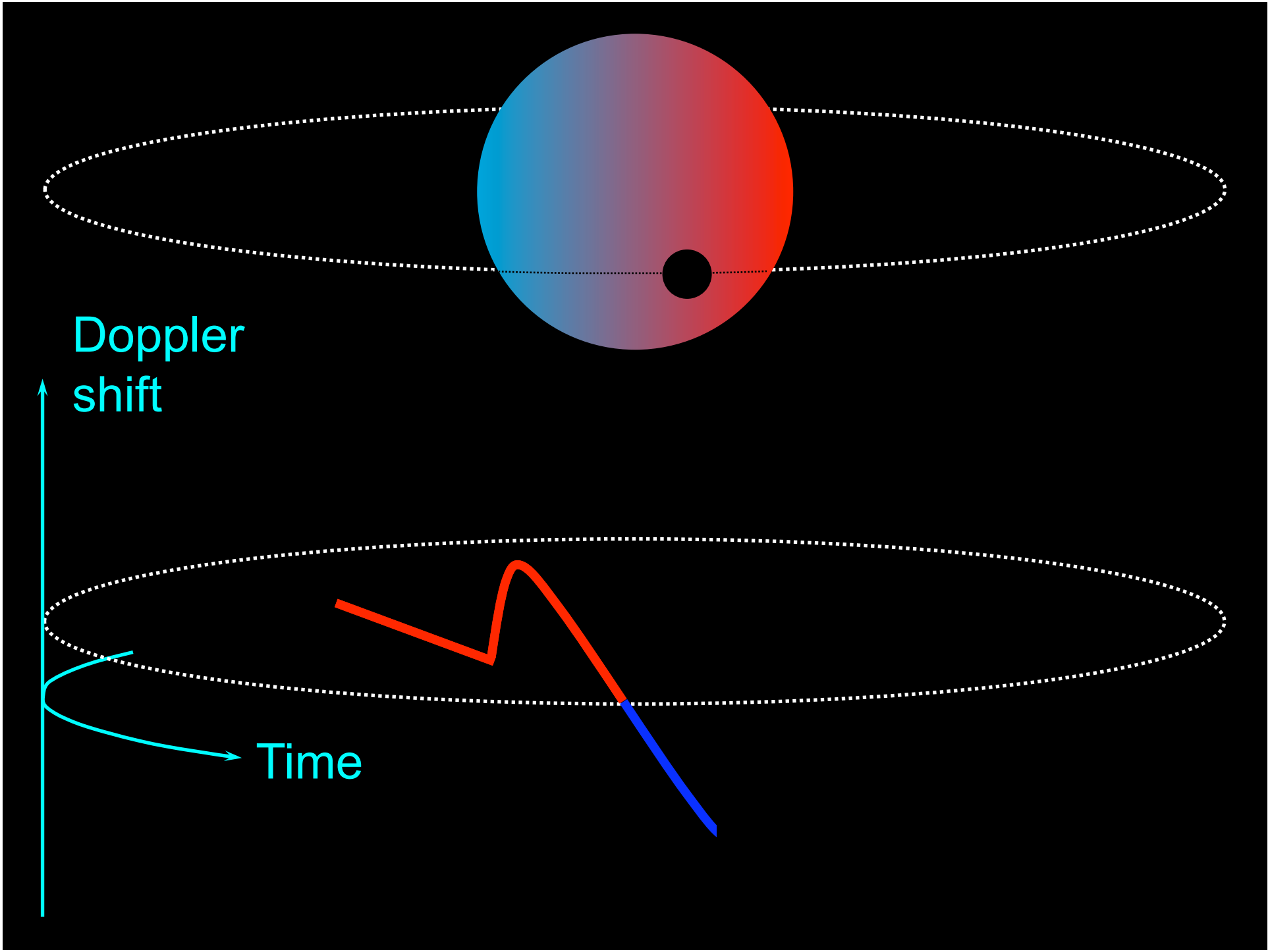

Flux
Time

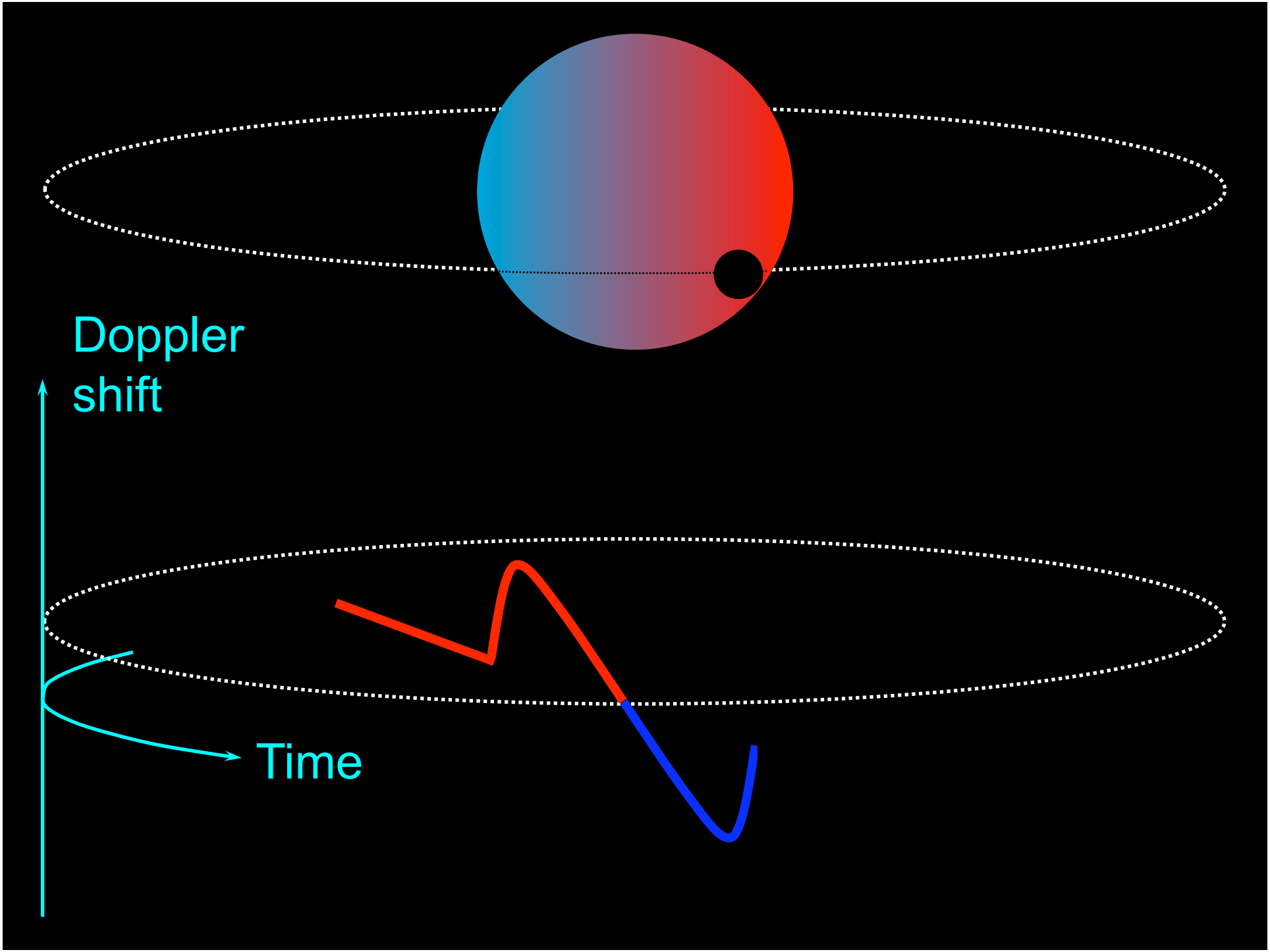

Doppler
shift
Time

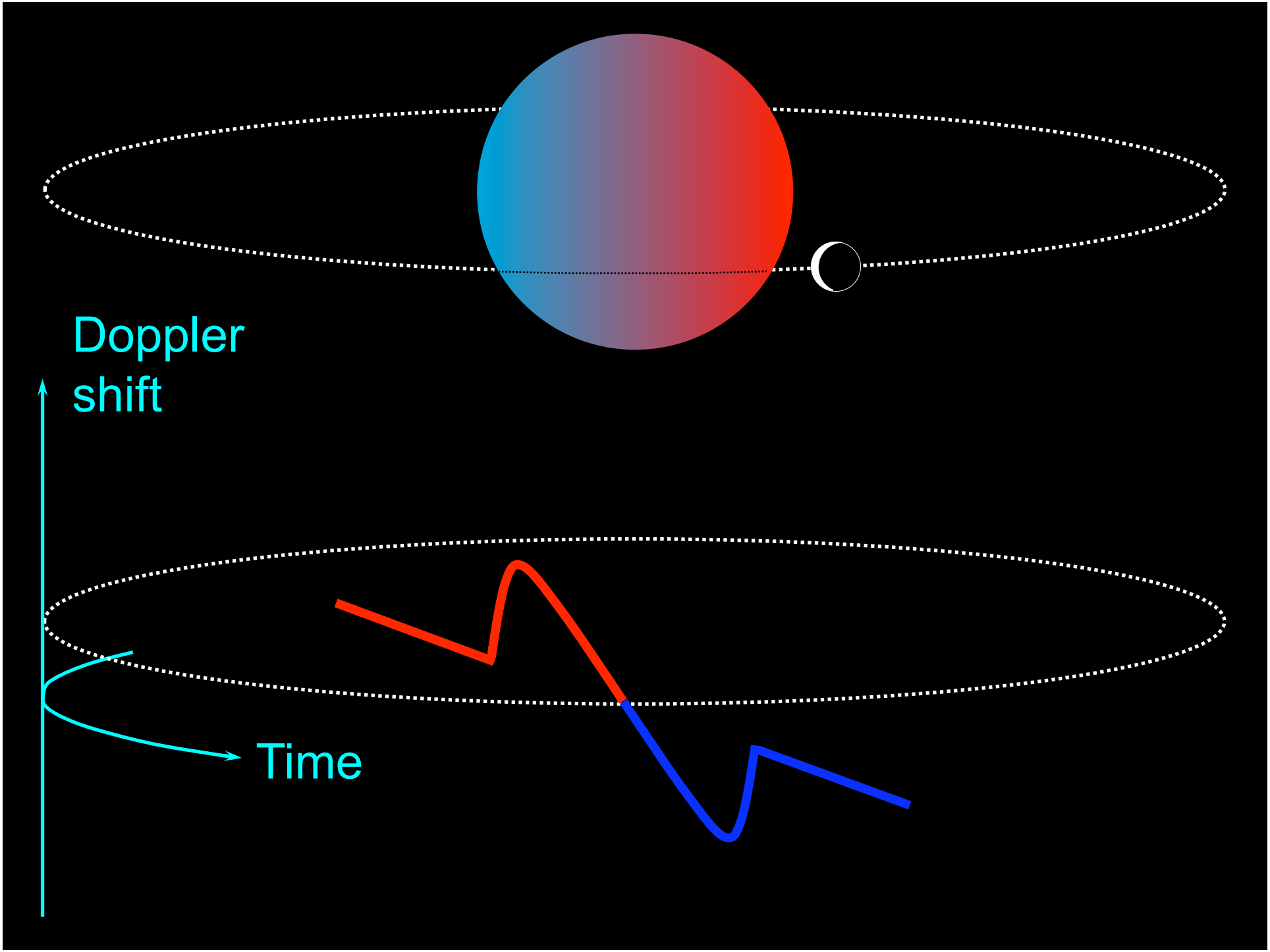

Doppler
shift
Time

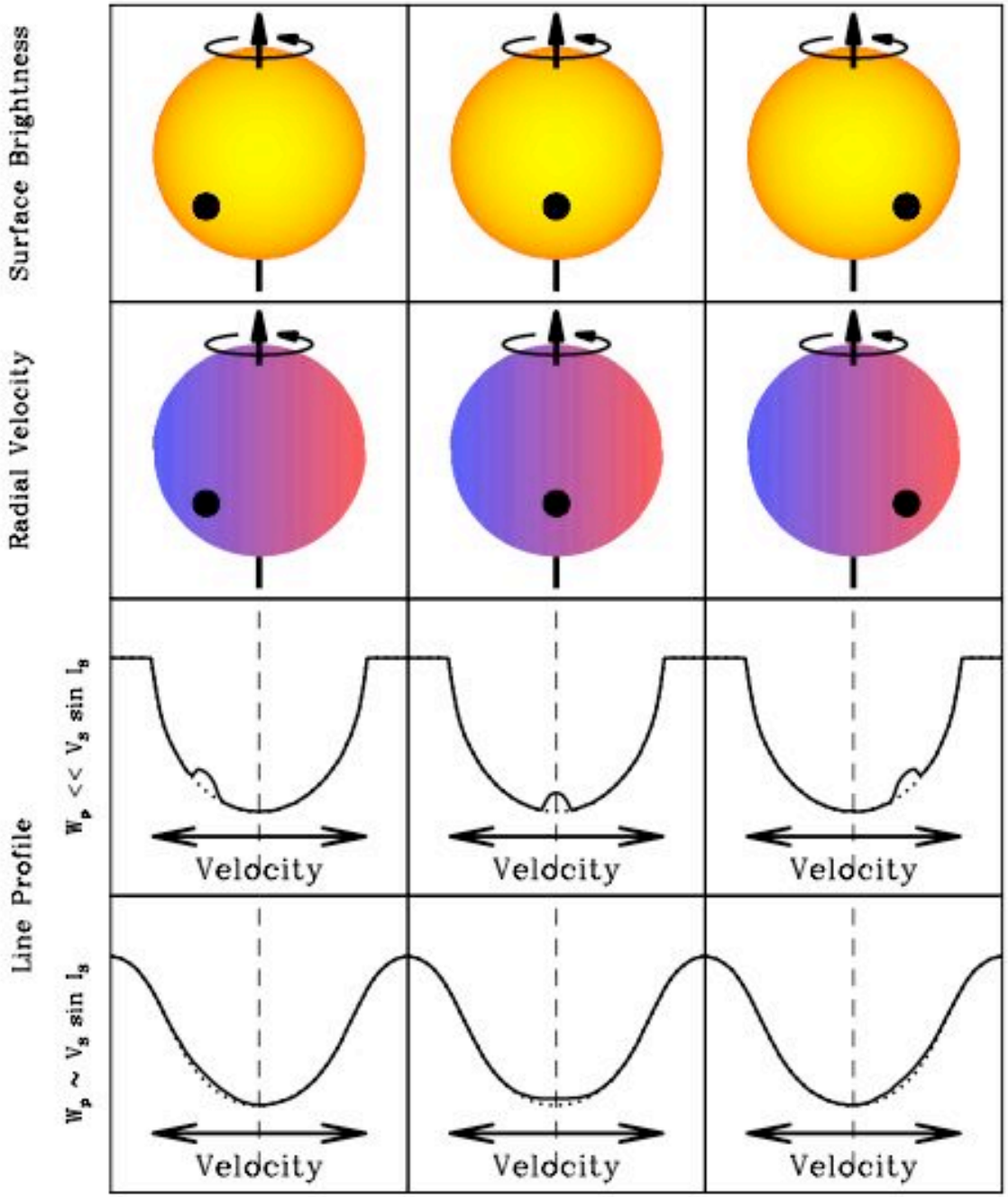

Doppler
shift
Time


Doppler
shift
Time


Doppler
shift
Time

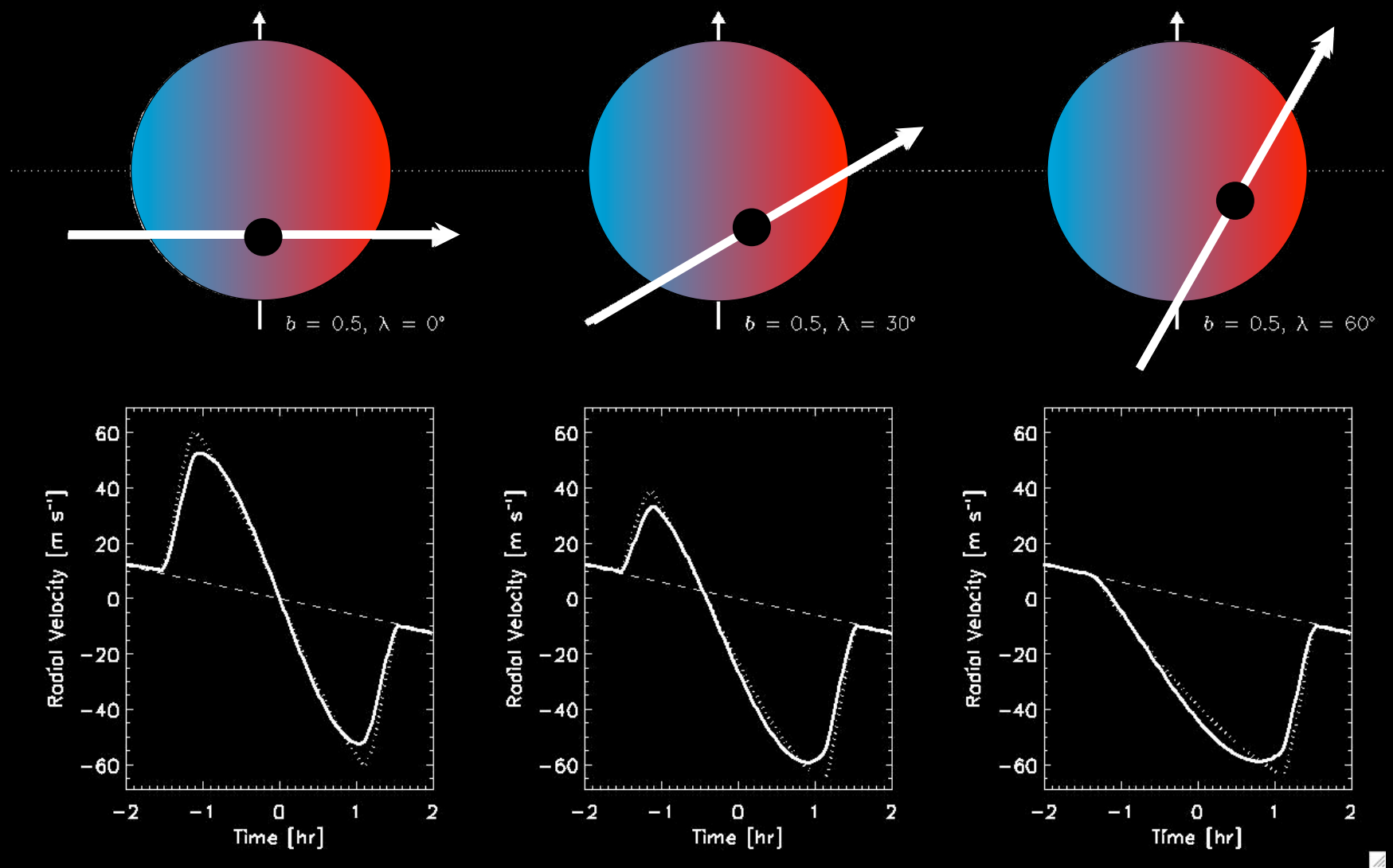

Doppler
shift
Time


Doppler
shift
Time



Gaudi & Winn (2007)

Measuring spin-orbit alignment

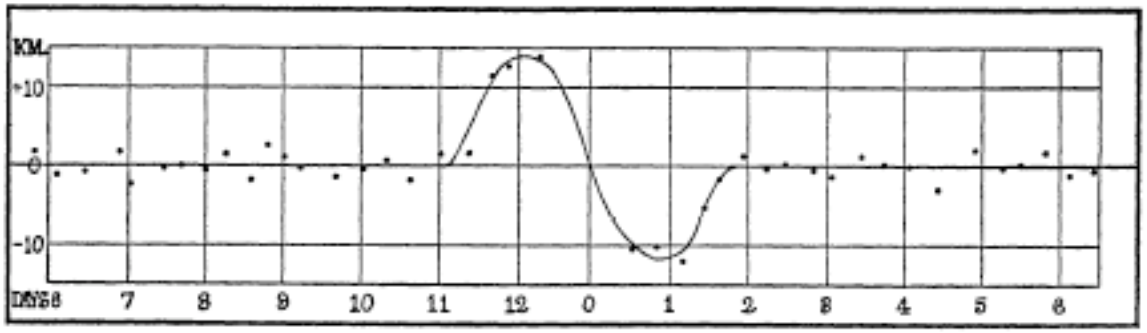


Ohta, Taruya, & Suto 2005; Gaudi & Winn 2007

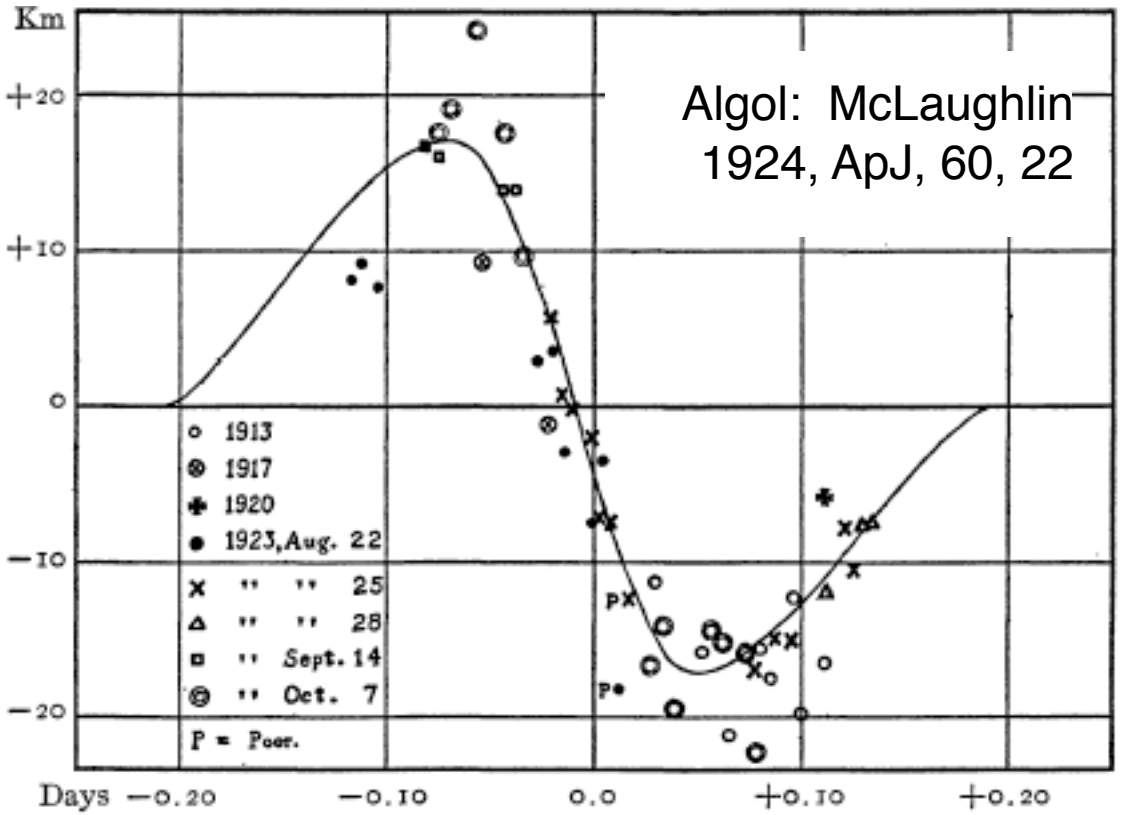
theorized by J. R. Holt (1893)
observed by F. Schlesinger (1909)

The Holt-Schlesinger effect

The Rossiter-McLaughlin effect



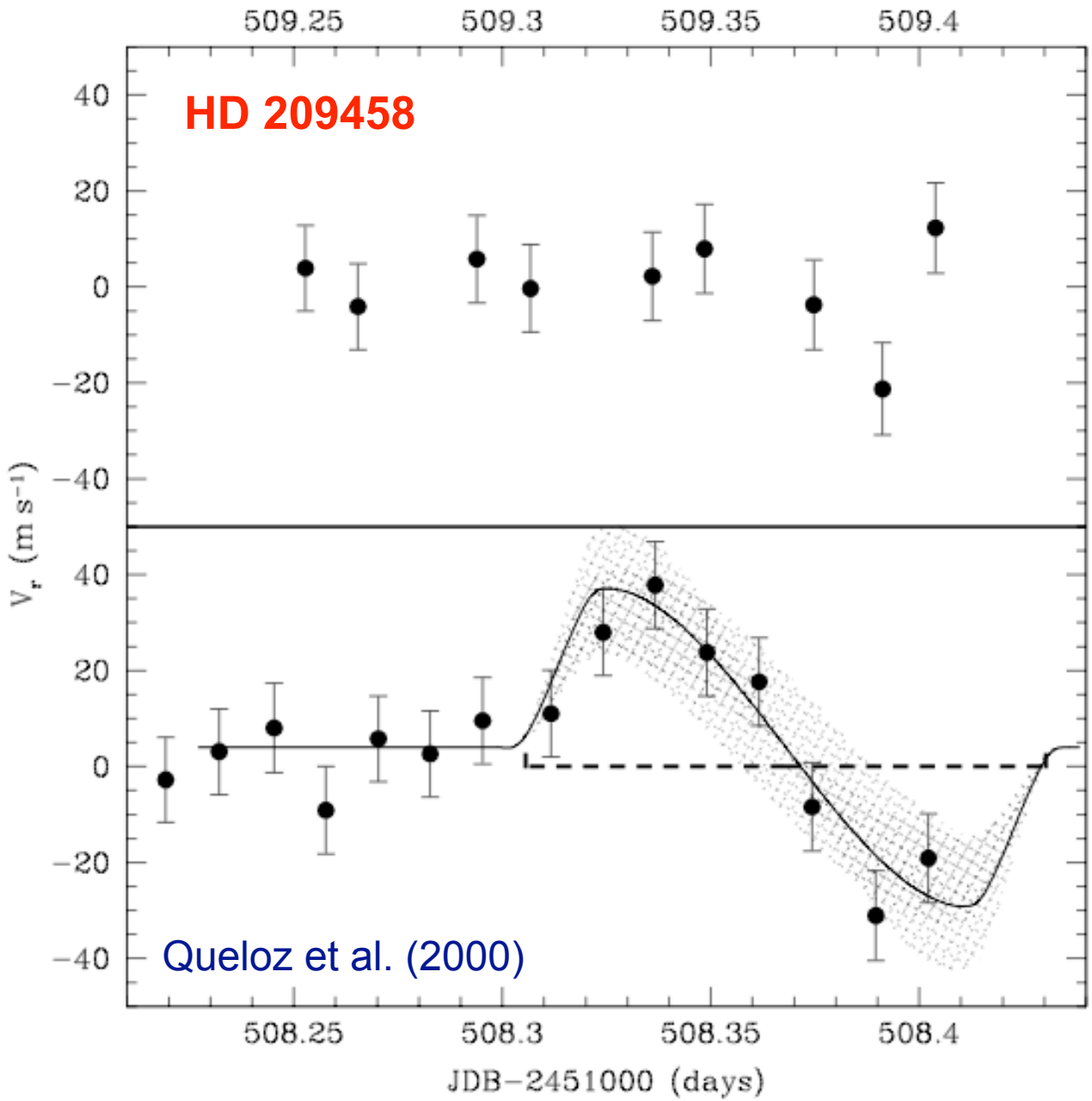
β Lyrae: Rossiter 1924, ApJ, 60, 15

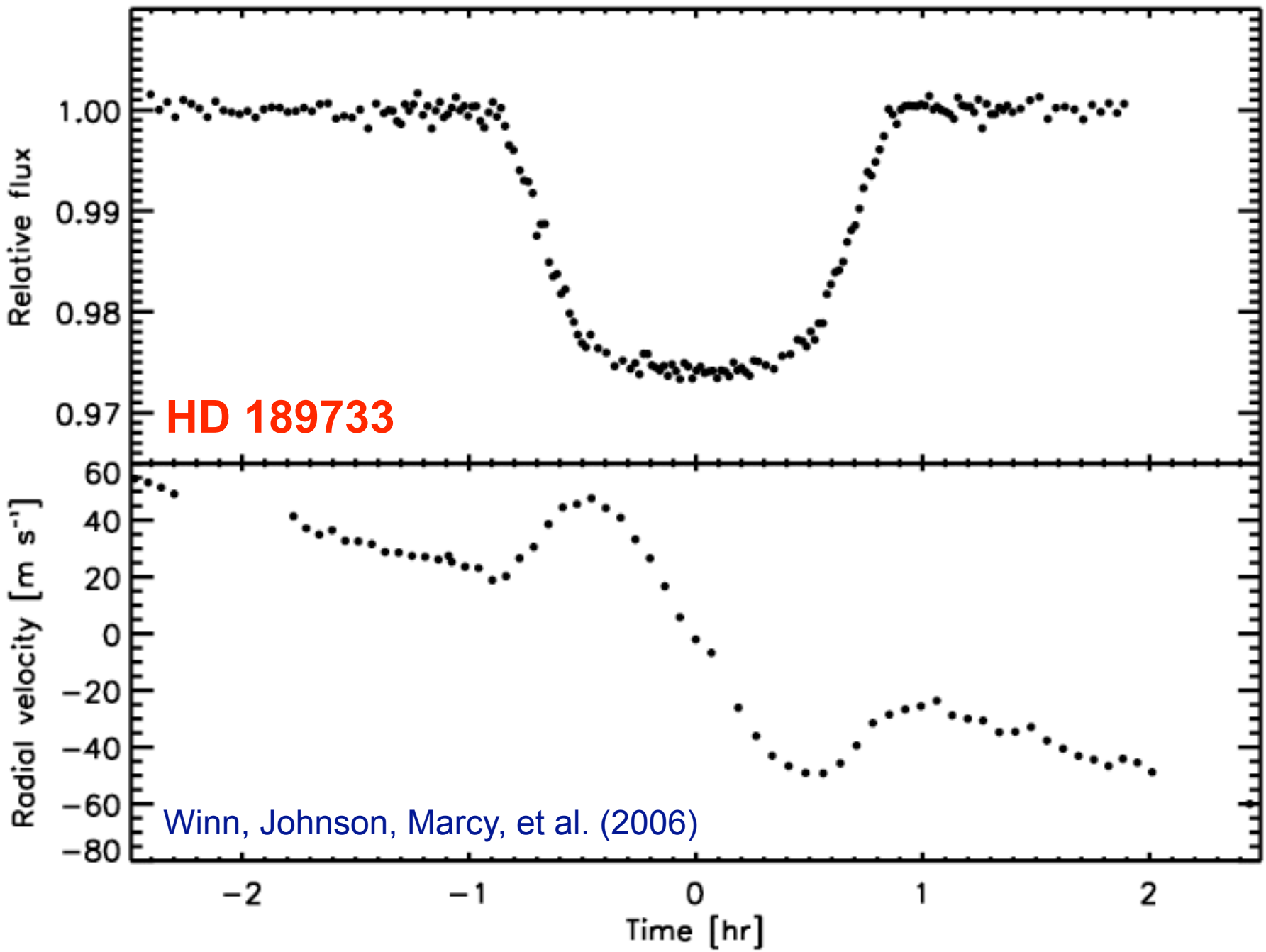


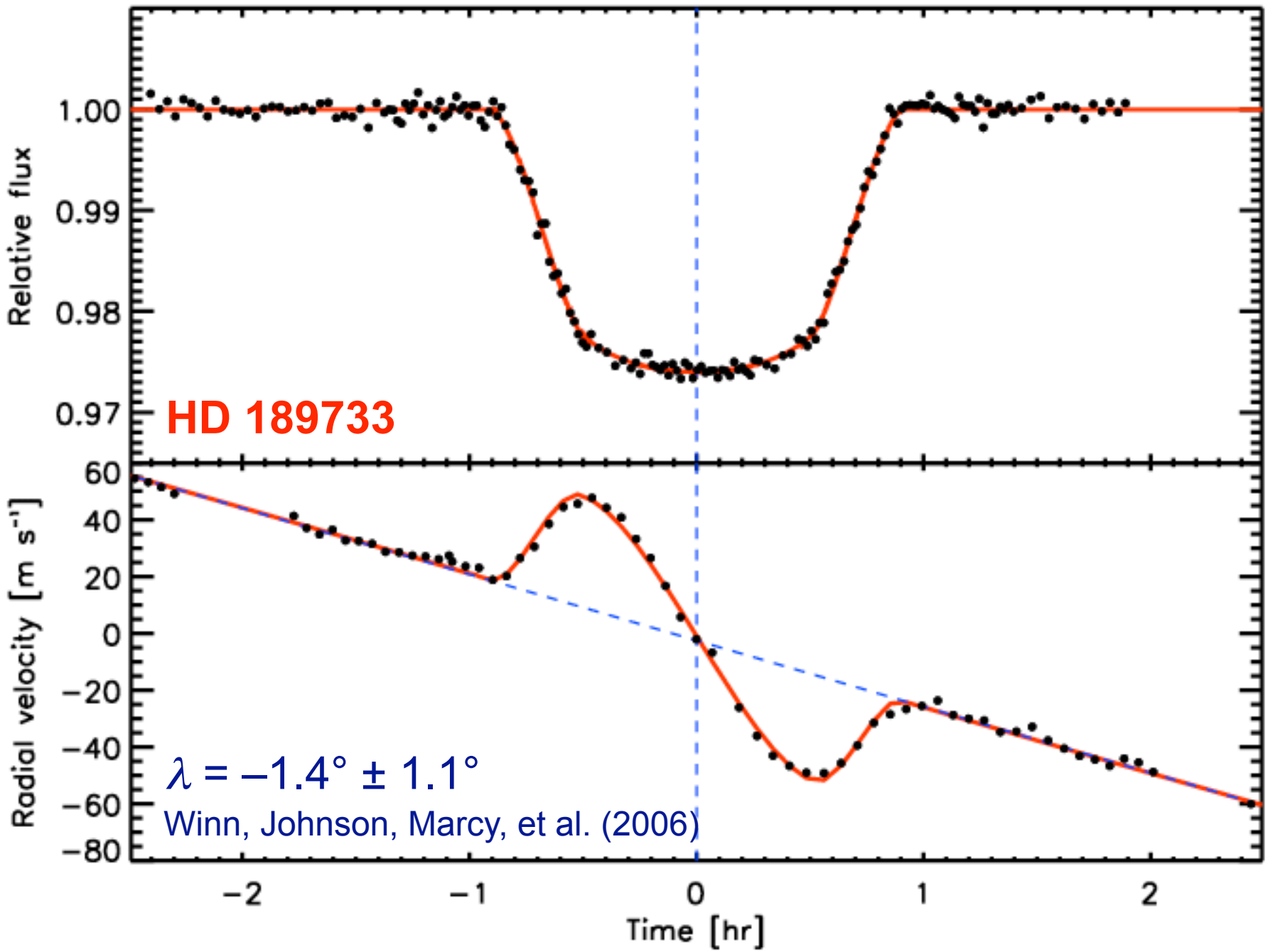
Algol: McLaughlin 1924, ApJ, 60, 22

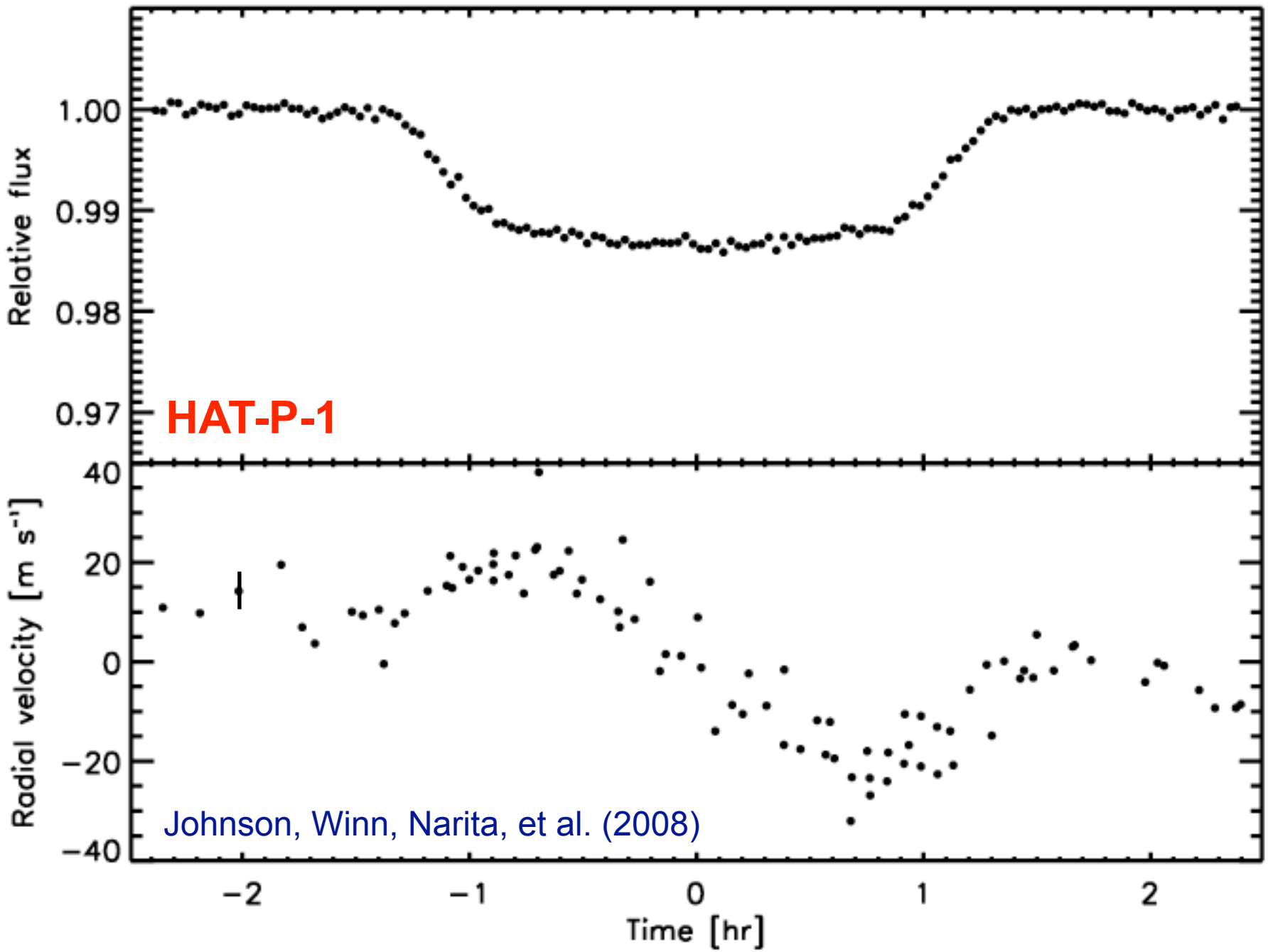
FIG. 1.—Curve of the rotational effect in Algol

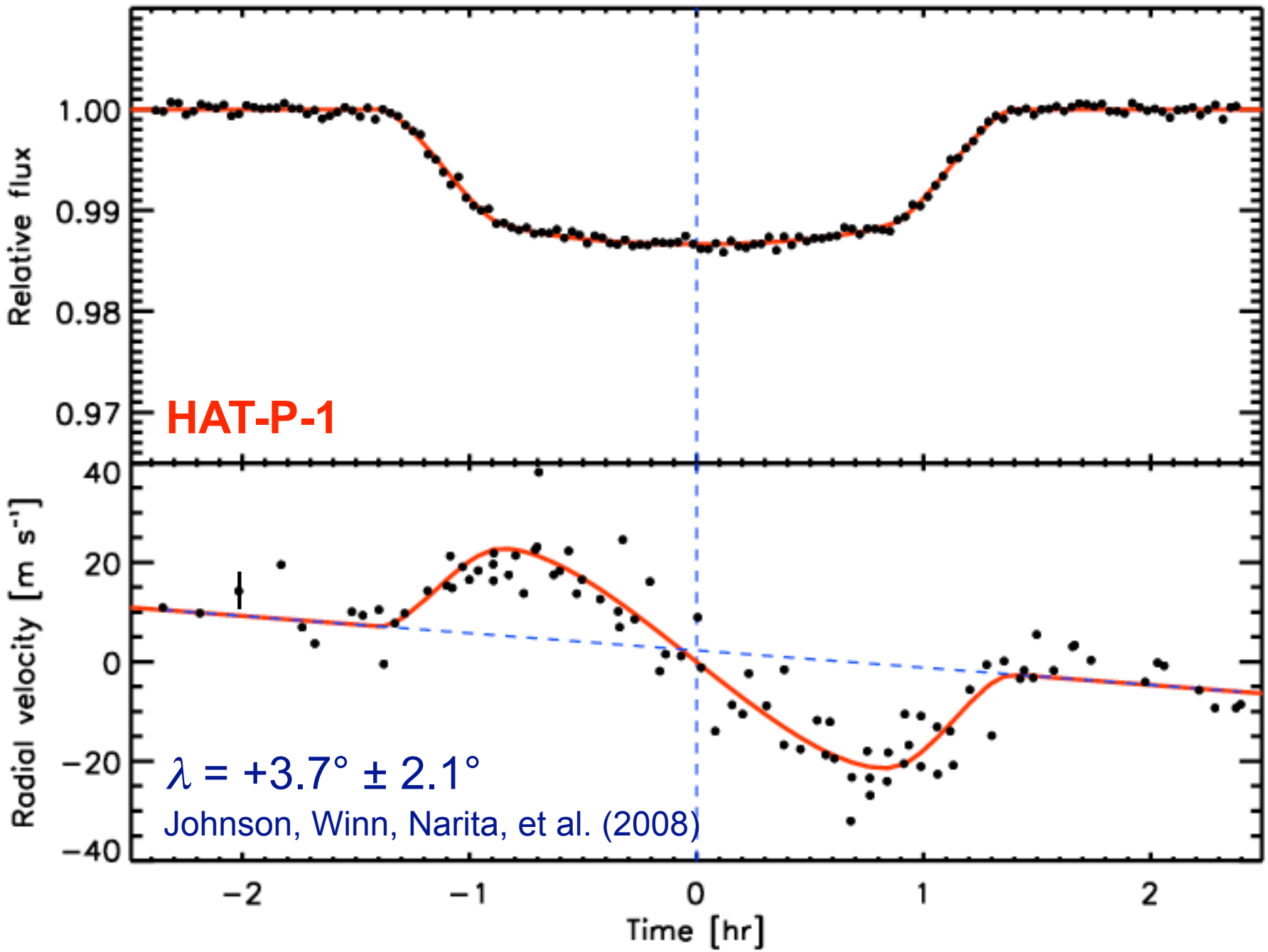
R. A. Rossiter (1896-1977)

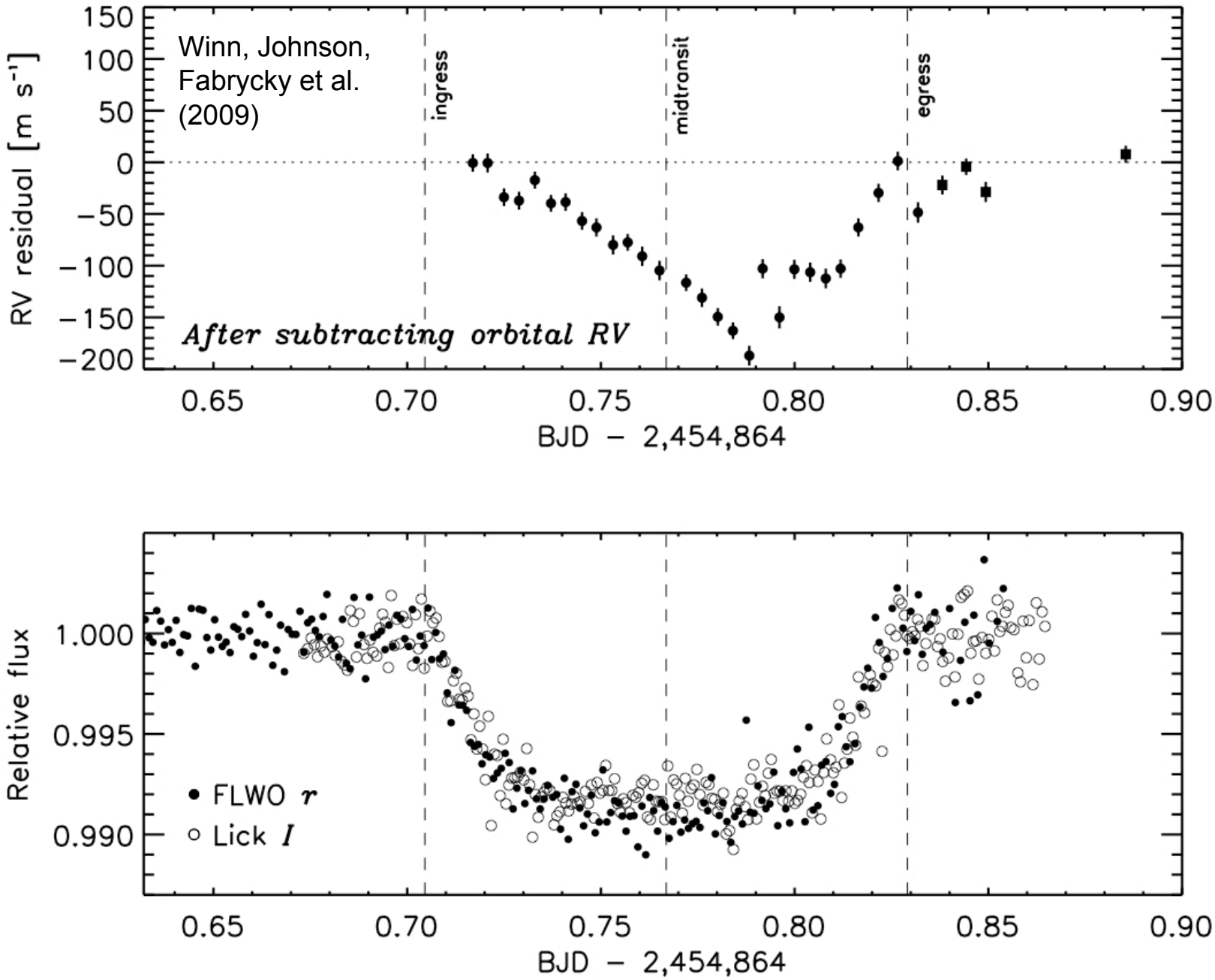

HD 209458
Queloz et al. (2000)
509.25
509.3
509.35
509.4
508.25
508.3
508.35
508.4
40
20
0
−20
−40
V_r (m s^{-1})
JDB−2451000 (days)

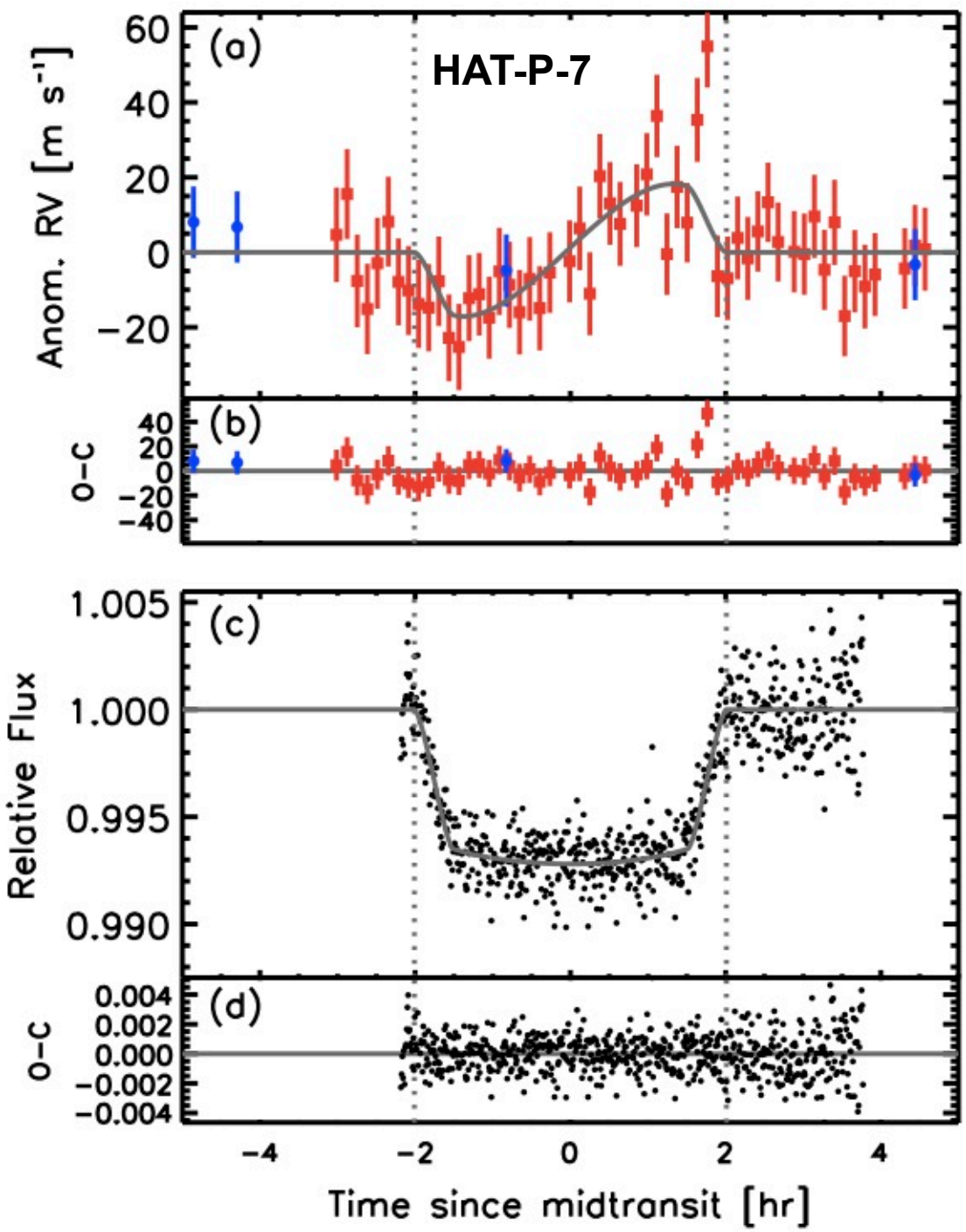

HD 189733
Winn, Johnson, Marcy, et al. (2006)
Relative flux
1.00
0.99
0.98
0.97
Radial velocity [m s⁻¹]
60
40
20
0
−20
−40
−60
−80
−2
−1
0
1
2
Time [hr]


HD 189733
Relative flux
1.00
0.99
0.98
0.97
Radial velocity [m s⁻¹]
60
40
20
0
−20
−40
−60
−80
λ = −1.4° ± 1.1°
Winn, Johnson, Marcy, et al. (2006)
−2
−1
0
1
2
Time [hr]


HAT-P-1
Relative flux
1.00
0.99
0.98
0.97
Radial velocity [m s⁻¹]
40
20
0
−20
−40
Johnson, Winn, Narita, et al. (2008)
−2
−1
0
1
2
Time [hr]


HAT-P-1
Relative flux
1.00
0.99
0.98
0.97
Radial velocity [m s⁻¹]
40
20
0
−20
−40
λ = +3.7° ± 2.1°
Johnson, Winn, Narita, et al. (2008)
−2
−1
0
1
2
Time [hr]

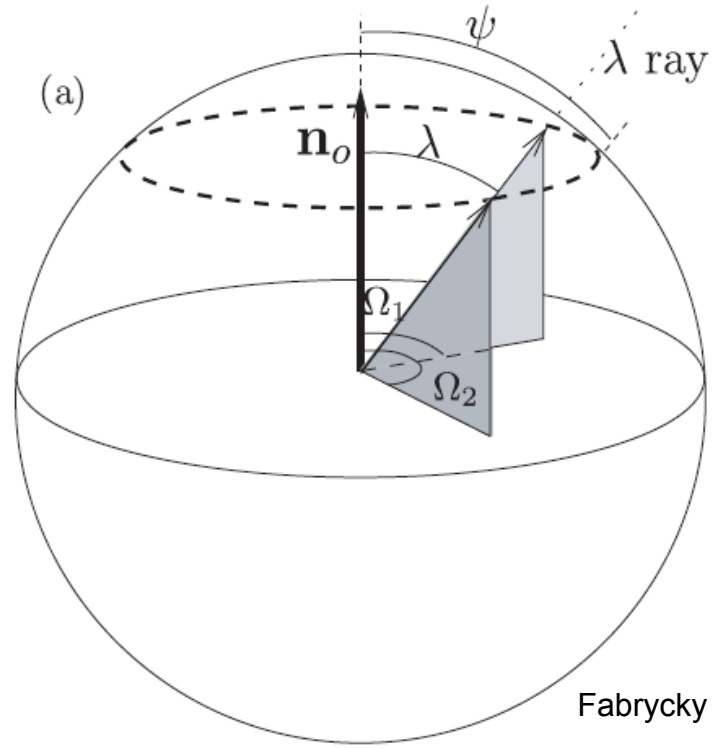

Winn, Johnson, Fabrycky et al. (2009)
ingress
midtransit
egress
After subtracting orbital RV
RV residual [m s⁻¹]
150
100
50
0
−50
−100
−150
−200
0.65
0.70
0.75
0.80
0.85
0.90
BJD − 2,454,864
Relative flux
1.000
0.995
0.990
FLWO r
Lick I
0.65
0.70
0.75
0.80
0.85
0.90
BJD − 2,454,864



Winn, Johnson, Albrecht et al. (2009)

See also Narita, Sato, Hirano, & Tamura (2009)

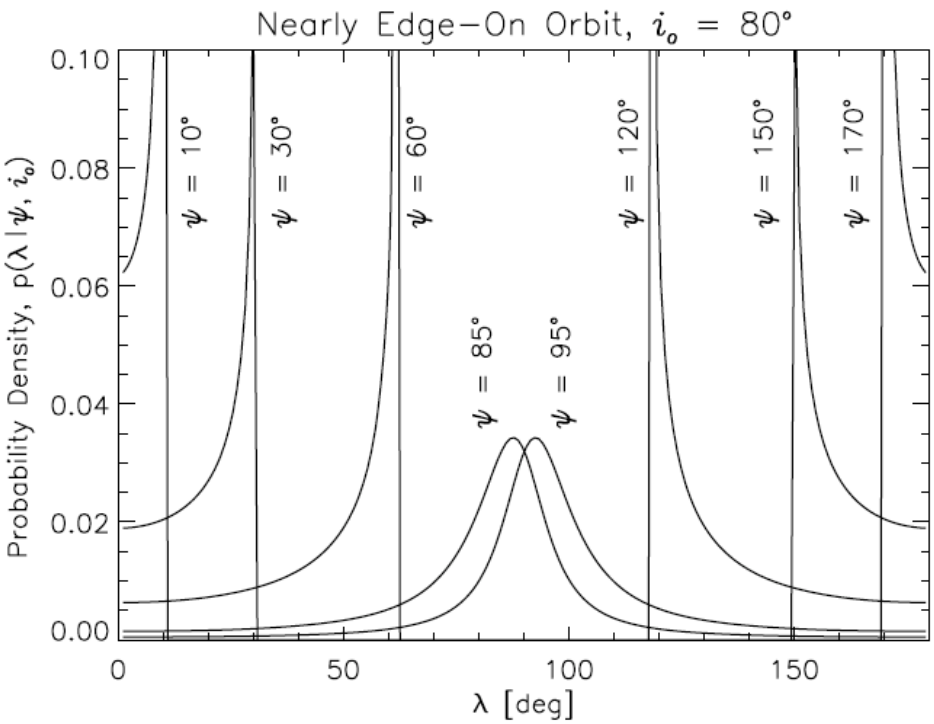
The pesky sky projection

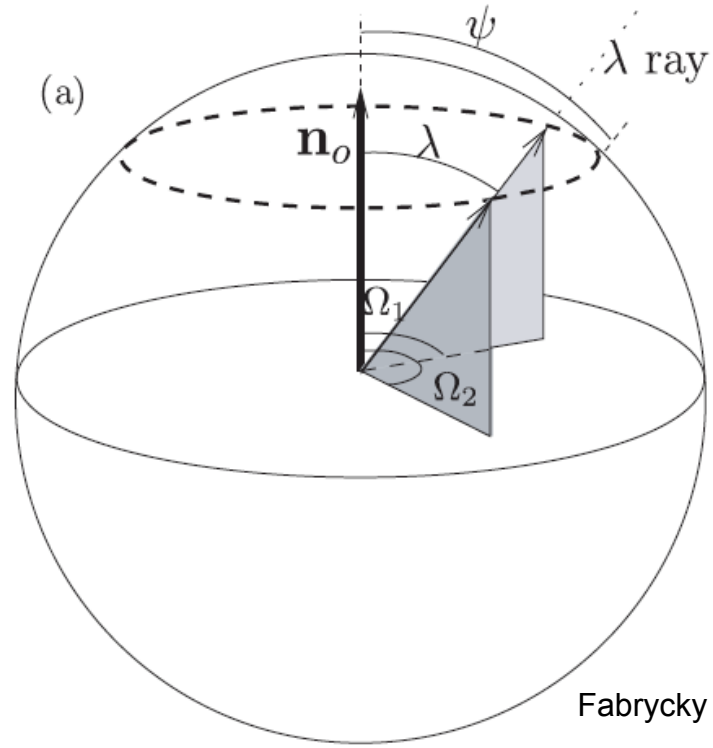


Fabrycky & Winn (2009)

$$\cos\psi = \cos i_s \cos i_o + \sin i_s \sin i_o \cos\lambda$$

The pesky sky projection



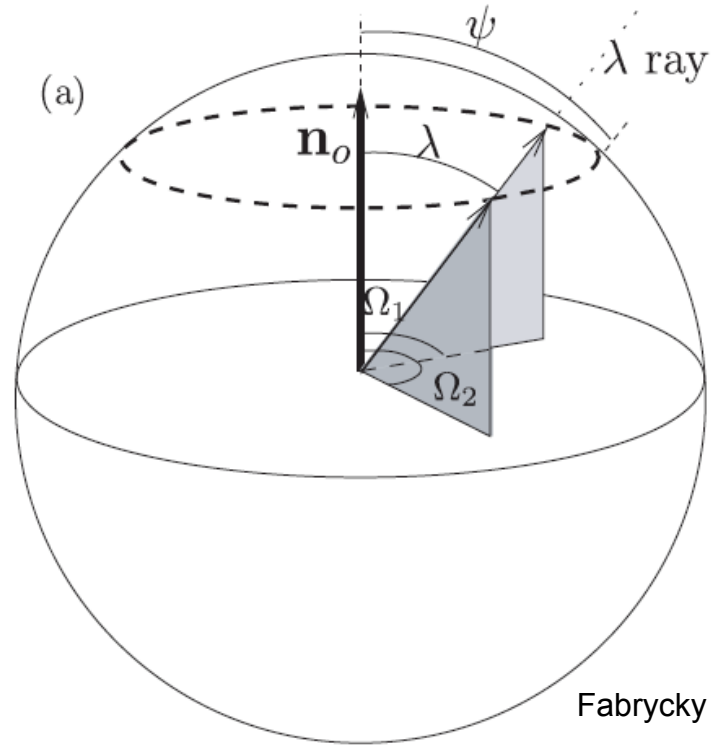


Fabrycky & Winn (2009)

$$\cos\psi = \cos i_s \cos i_o + \sin i_s \sin i_o \cos\lambda$$

$$\Pr(\lambda \mid \psi, i_o = \pi/2) = \frac{2}{\pi} \frac{\cos\psi}{\cos\lambda\sqrt{\cos^2\lambda - \cos^2\psi}}$$

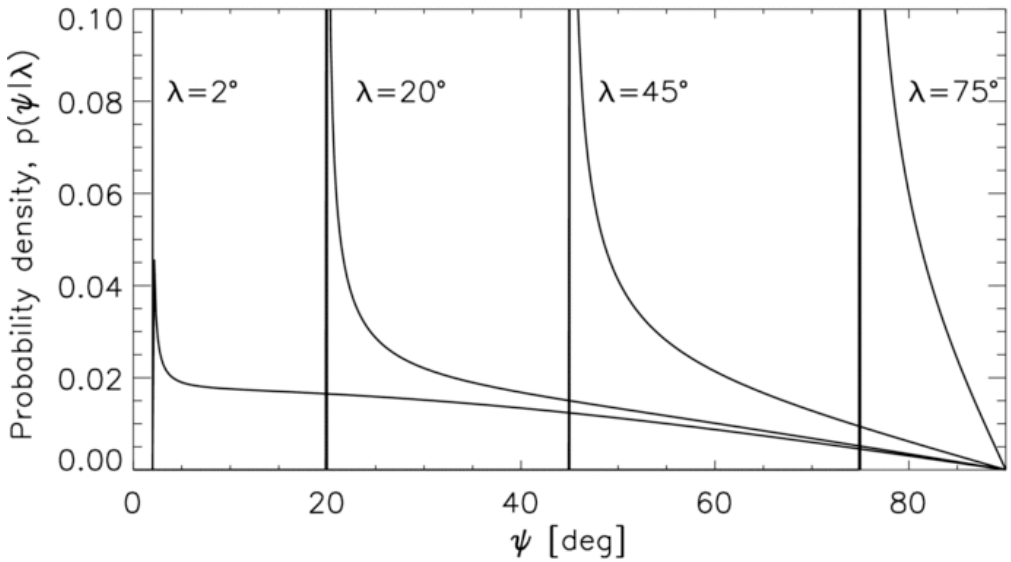
The pesky sky projection

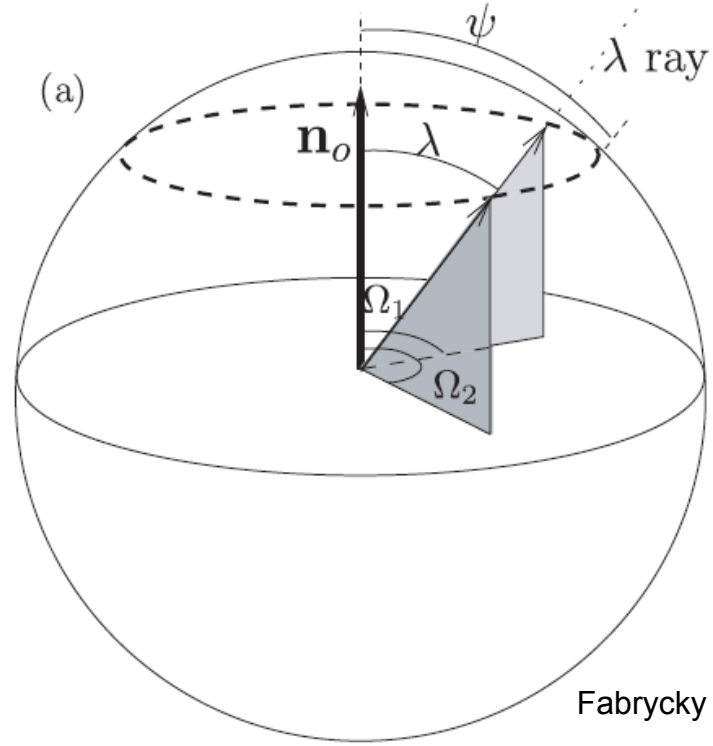


Fabrycky & Winn (2009)

$$\Pr(\psi \mid \lambda) \propto \Pr(\lambda \mid \psi)\, \Pr(\psi)$$

The pesky sky projection



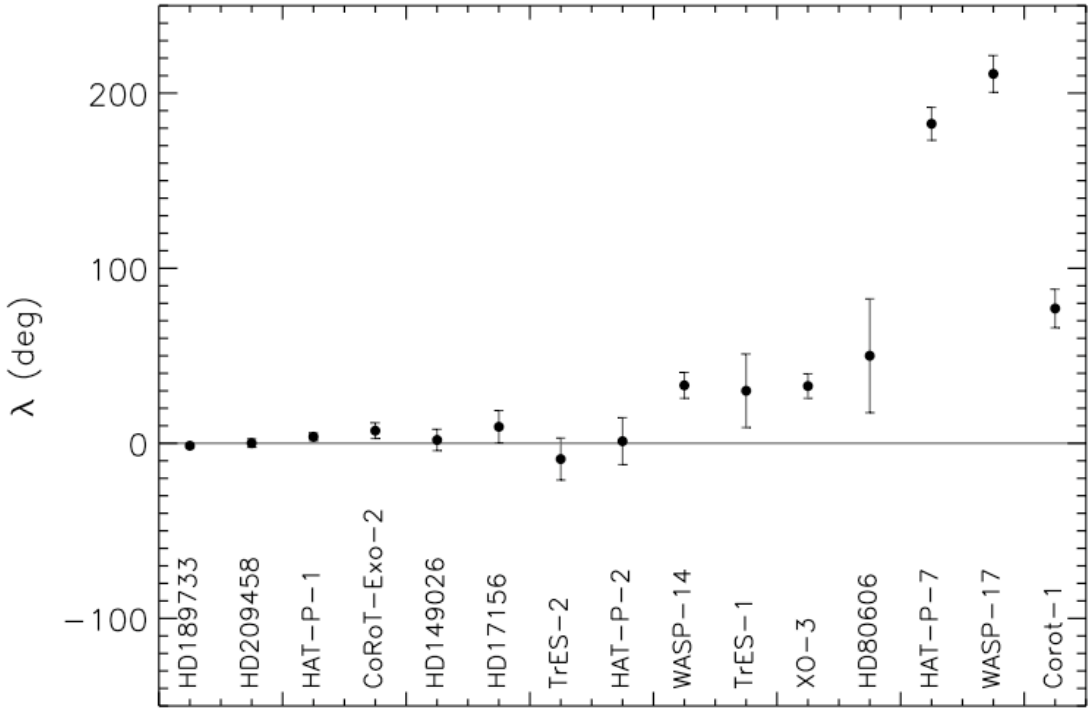


Fabrycky & Winn (2009)

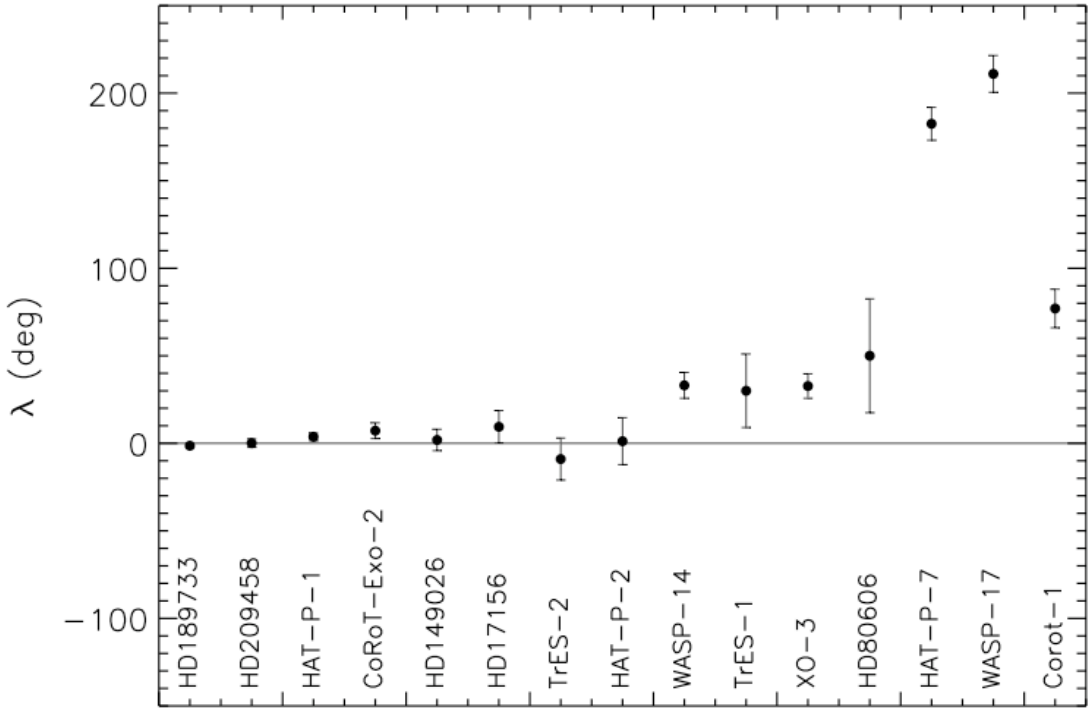
$$\Pr(\psi \mid \lambda) \propto \Pr(\lambda \mid \psi)\, \Pr(\psi)$$

$$\frac{1}{2}\sin\psi$$

Ensemble results

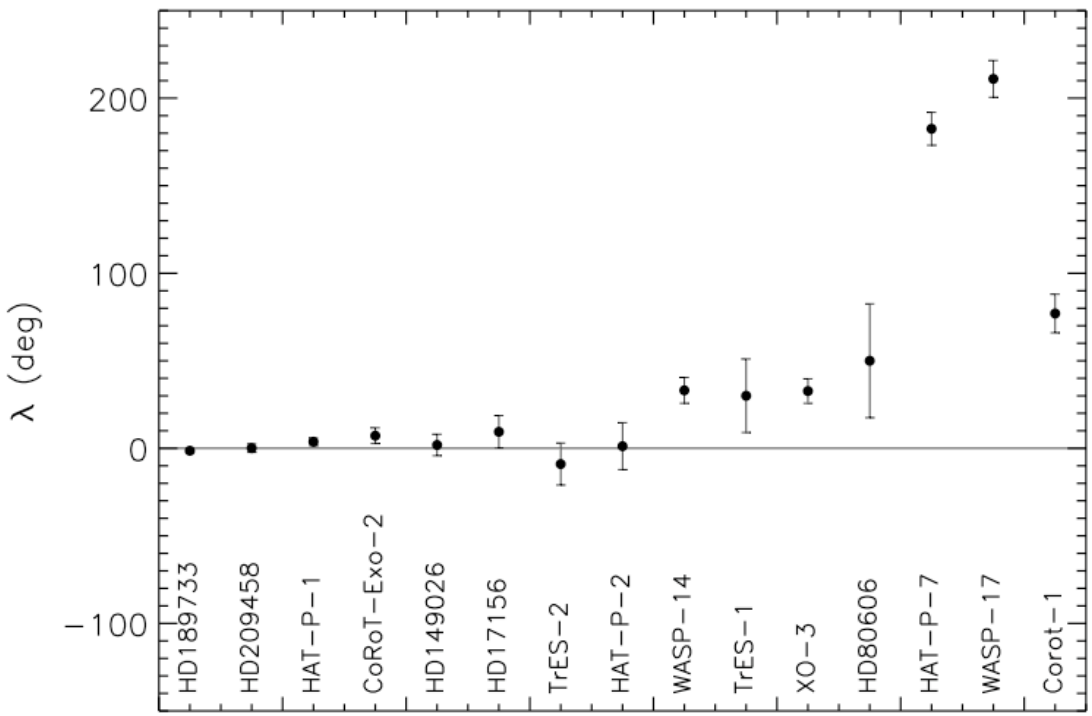


Ensemble results



$$p(\mathbf{a}|\text{data}) \propto p(\text{data}|\mathbf{a})p(\mathbf{a})$$

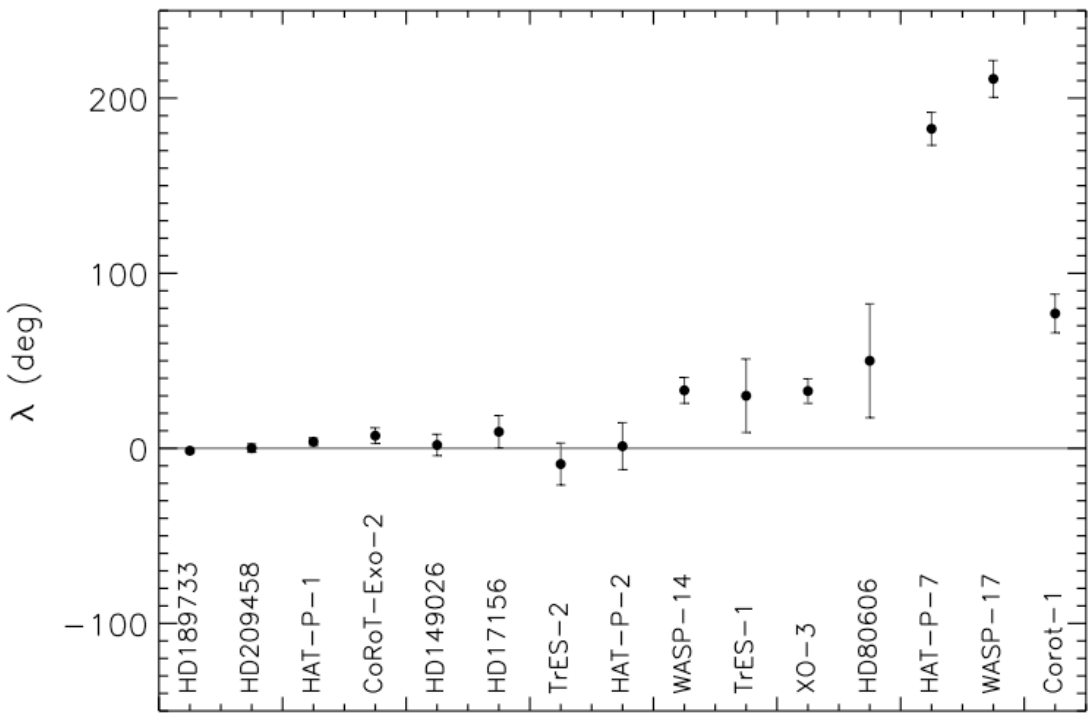
Ensemble results



$$p(\mathbf{a}|\text{data}) \propto p(\text{data}|\mathbf{a})p(\mathbf{a})$$

$$p(\text{data}|\mathbf{a}) = \prod_{k=1}^{N_s} \int_{-\pi}^{+\pi} p_{\text{obs},k}(\lambda) p'(\lambda | i_{p,k}, \mathbf{a}) d\lambda$$

Ensemble results

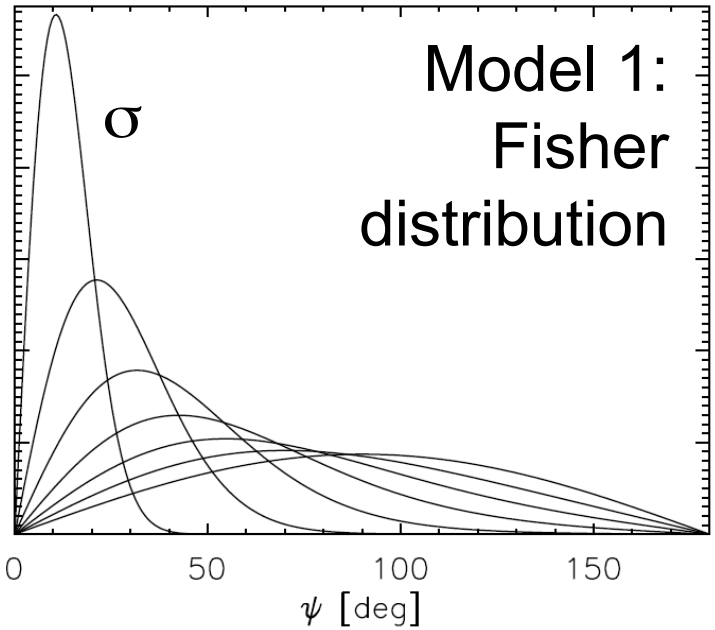


$$p(\mathbf{a}|\text{data}) \propto p(\text{data}|\mathbf{a})p(\mathbf{a})$$

$$p(\text{data}|\mathbf{a}) = \prod_{k=1}^{N_s} \int_{-\pi}^{+\pi} p_{\text{obs},k}(\lambda) p'(\lambda | i_{p,k}, \mathbf{a}) d\lambda$$

$$E \equiv \int p(\text{data}|\mathbf{a})p(\mathbf{a})d\mathbf{a}$$

Ensemble results

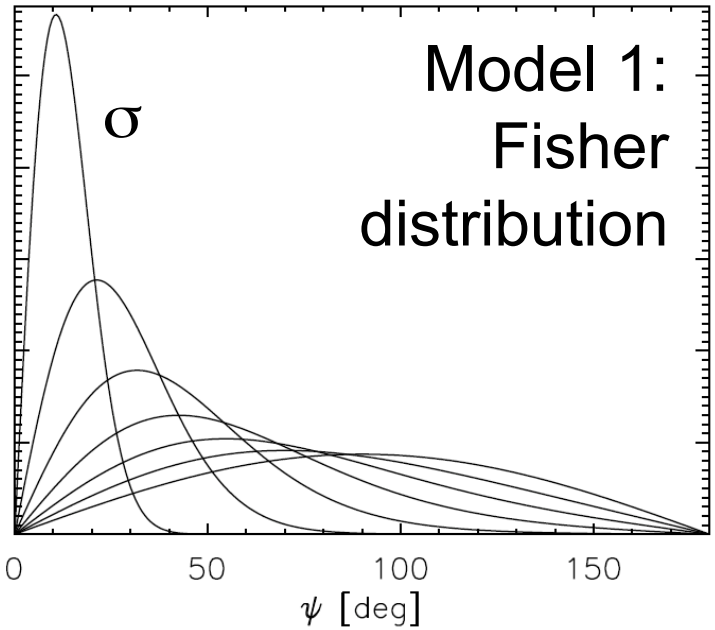


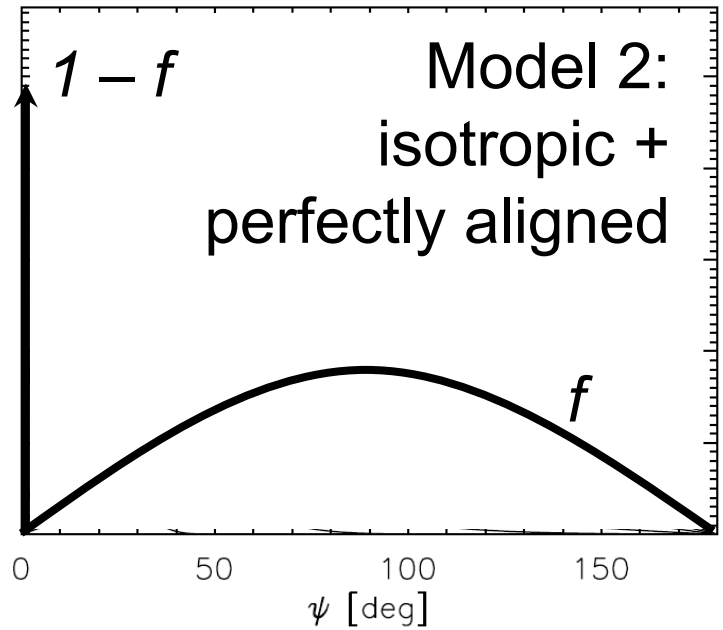
$$\mathrm{Pr_F}(\psi \mid \kappa) = \frac{\kappa}{2\sinh\kappa}\exp(\kappa\cos\psi)\sin\psi$$

$$\kappa \to \infty : \ \mathrm{Pr_R}(\psi \mid \sigma) = \frac{\psi}{\sigma^2}\exp\left(-\frac{\psi^2}{2\sigma^2}\right)$$

$$p(\kappa) \propto (1+\kappa^2)^{-3/4}$$

Ensemble results



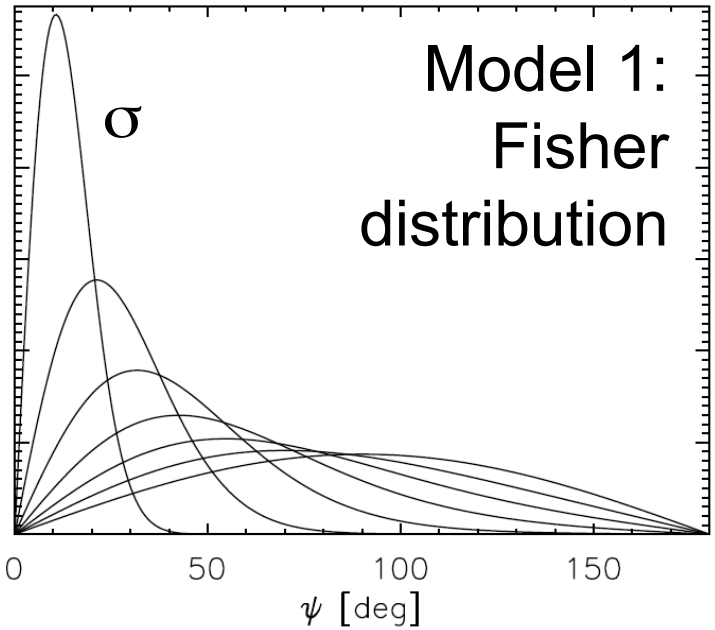


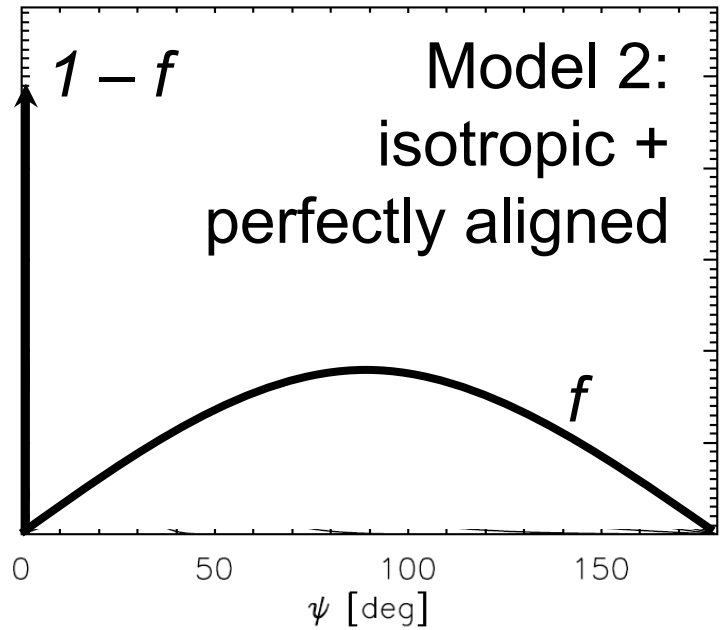
$$\mathrm{Pr_F}(\psi \mid \kappa) = \frac{\kappa}{2\sinh\kappa}\exp(\kappa\cos\psi)\sin\psi$$

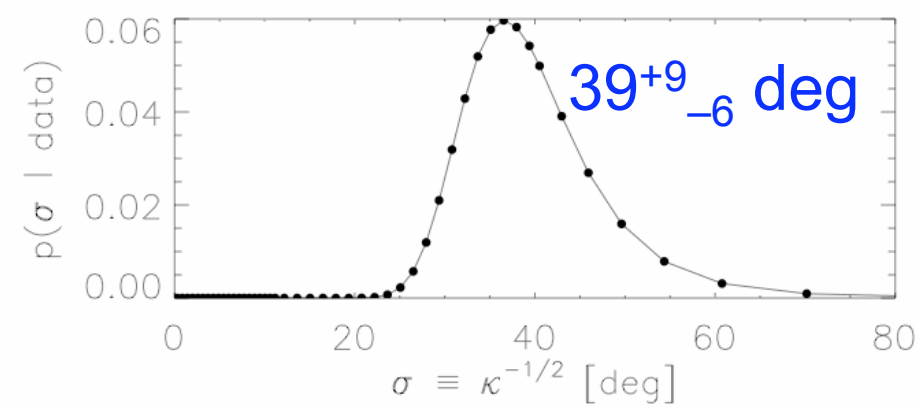
$$\kappa \to \infty : \ \mathrm{Pr_R}(\psi \mid \sigma) = \frac{\psi}{\sigma^2}\exp\left(-\frac{\psi^2}{2\sigma^2}\right)$$

$$p(\kappa) \propto (1+\kappa^2)^{-3/4}$$

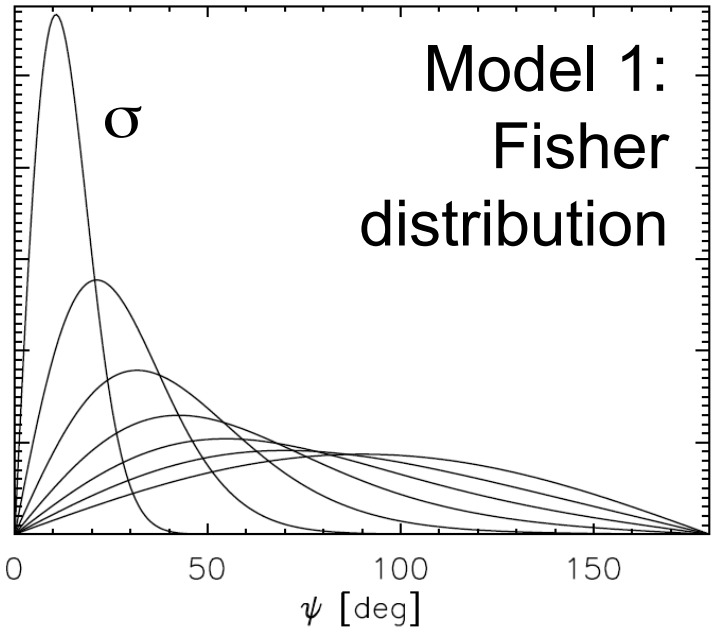
Ensemble results

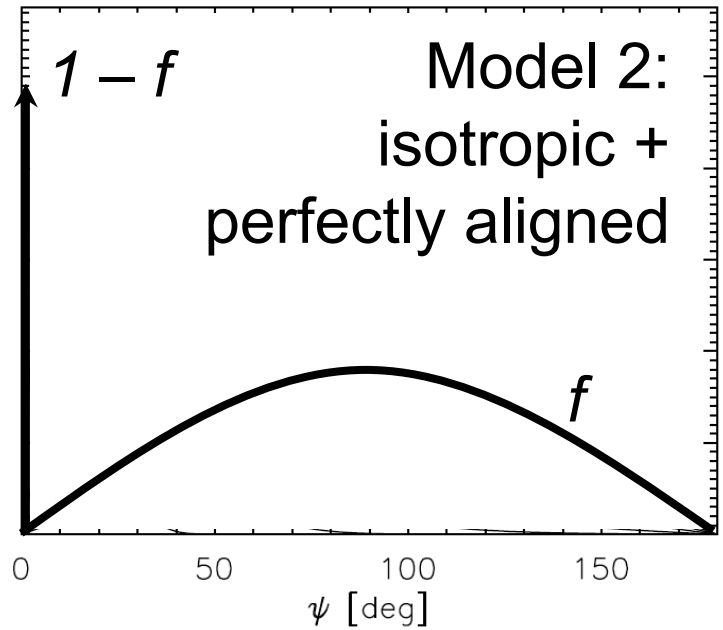


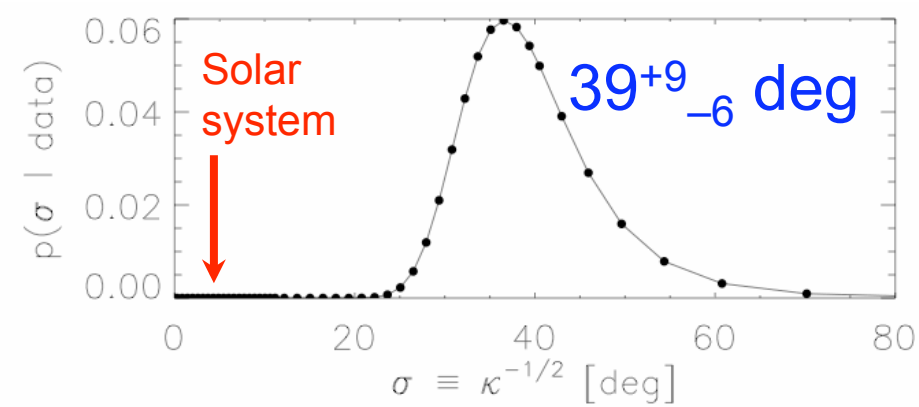




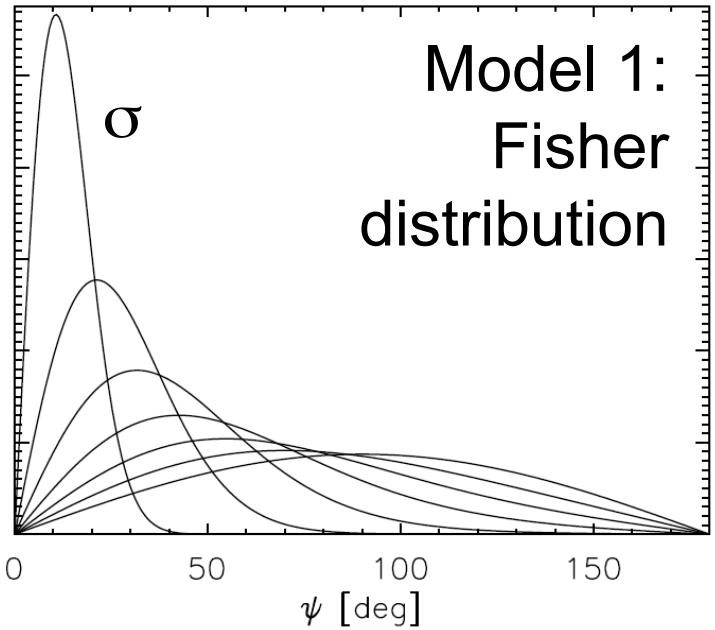
Ensemble results

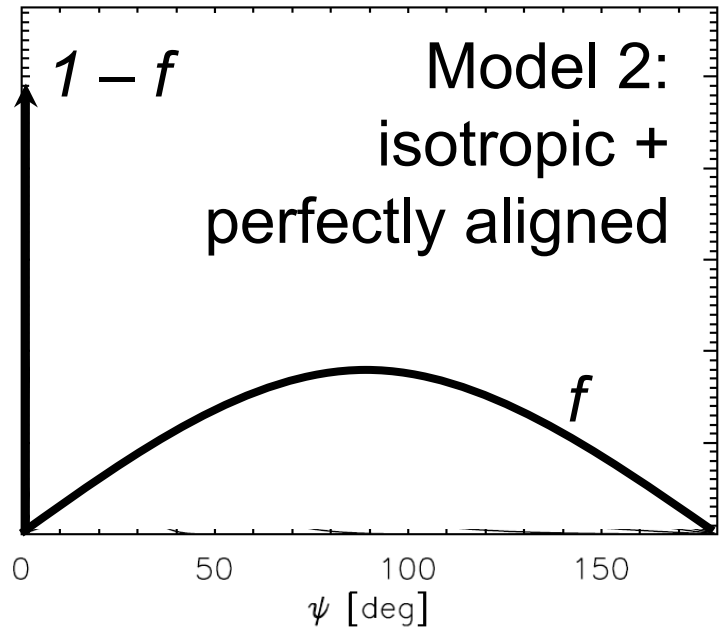


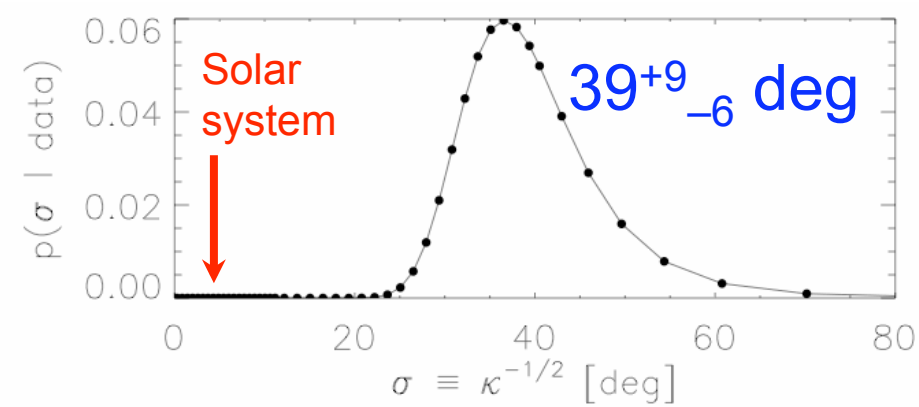


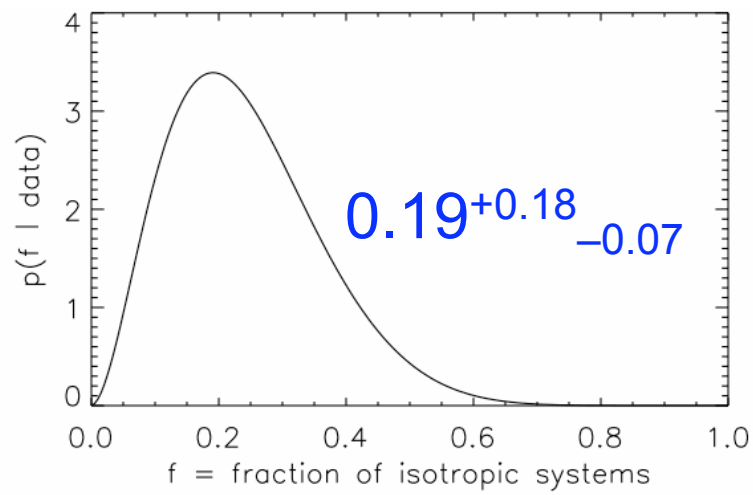


Ensemble results

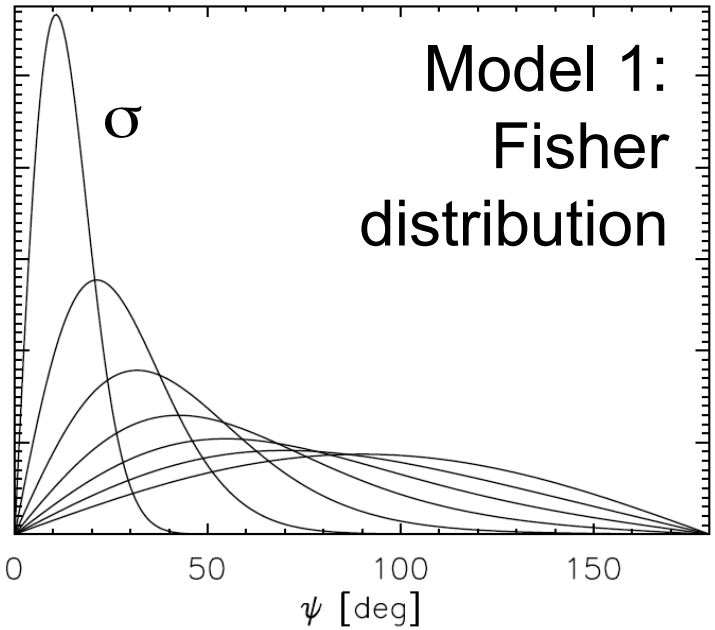


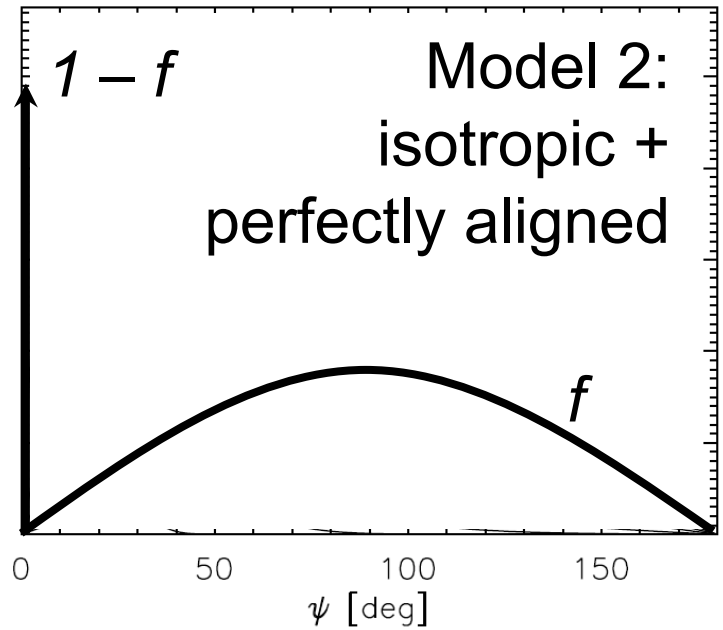


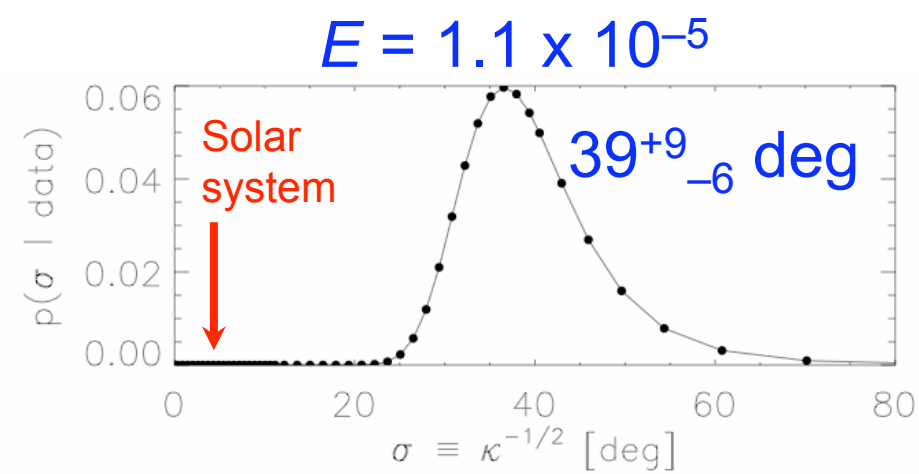


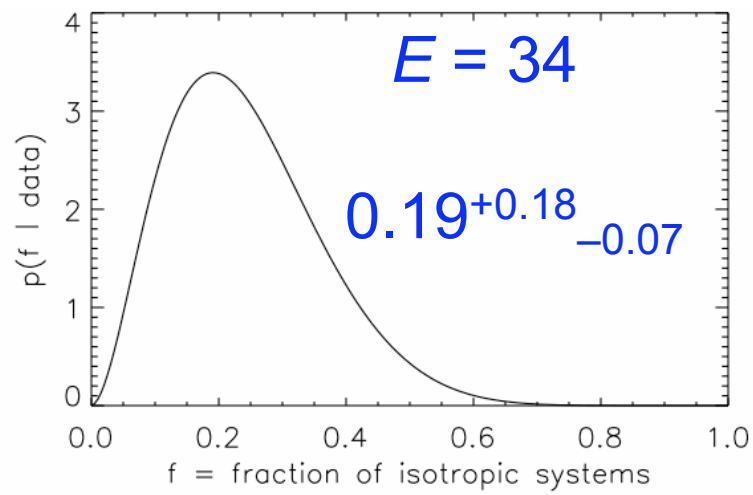


Ensemble results

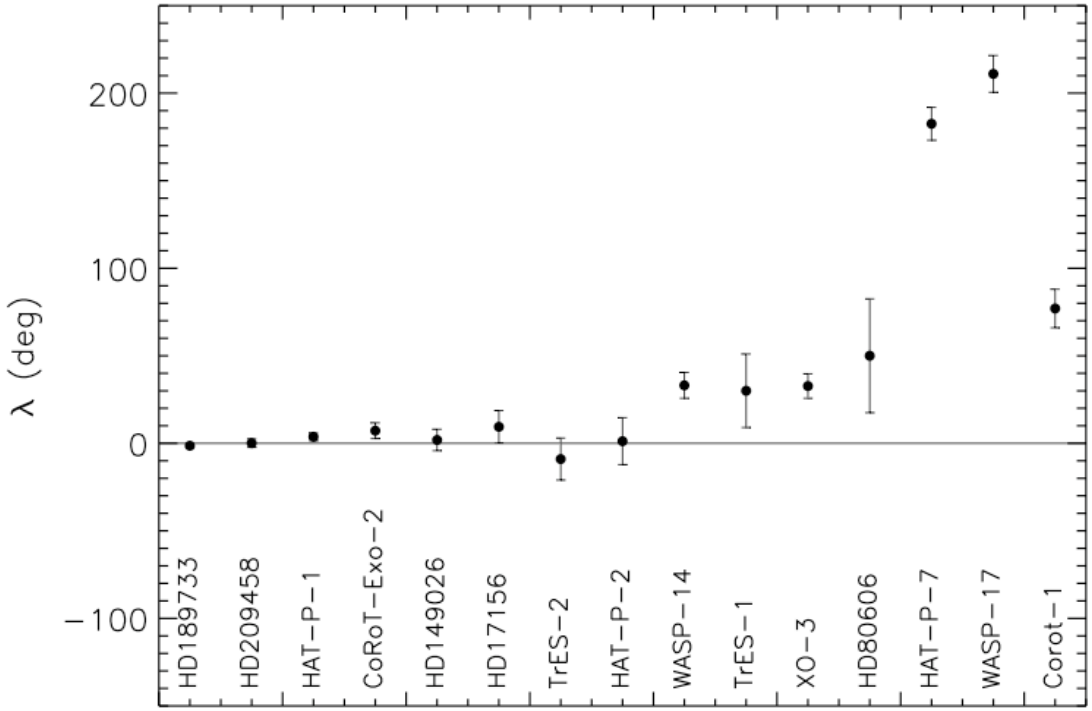




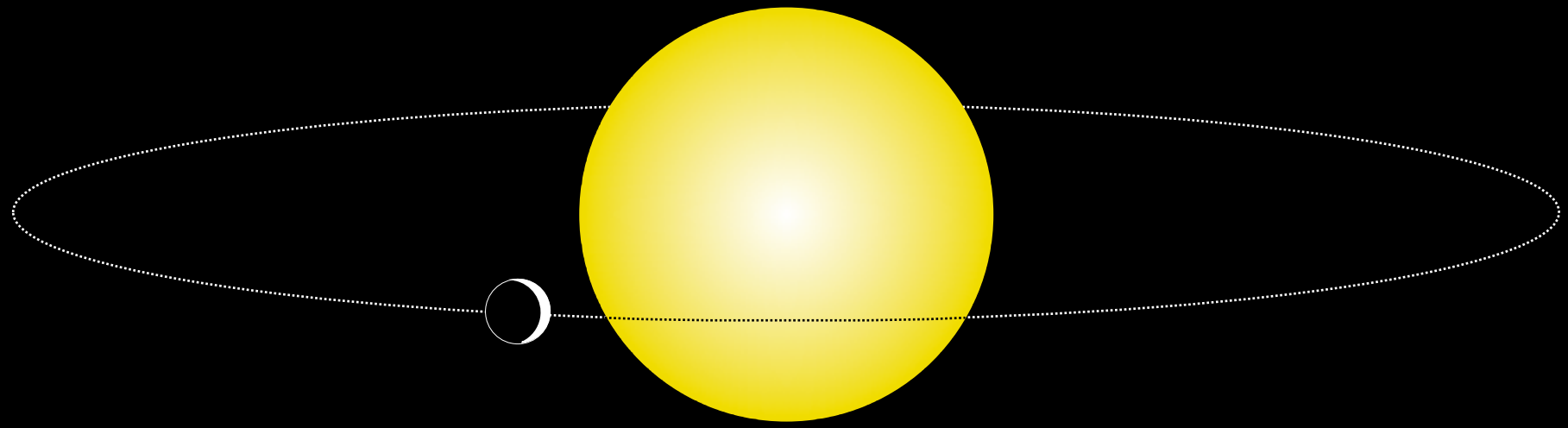




Ensemble results



Evidence for 2 different modes of planet migration



Parameter estimation from time-series data with correlated errors: a wavelet-based method

Josh Carter and Josh Winn
Massachusetts Institute of Technology

The “Horne problem”

The “Horne problem”

Expected discovery rate: >10 per month

— *K. Horne, 2002*

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Discovery rate up to 2005: 1.5 per year

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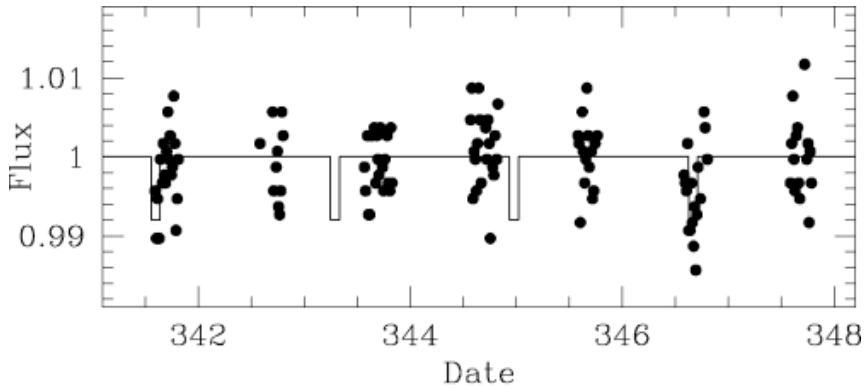
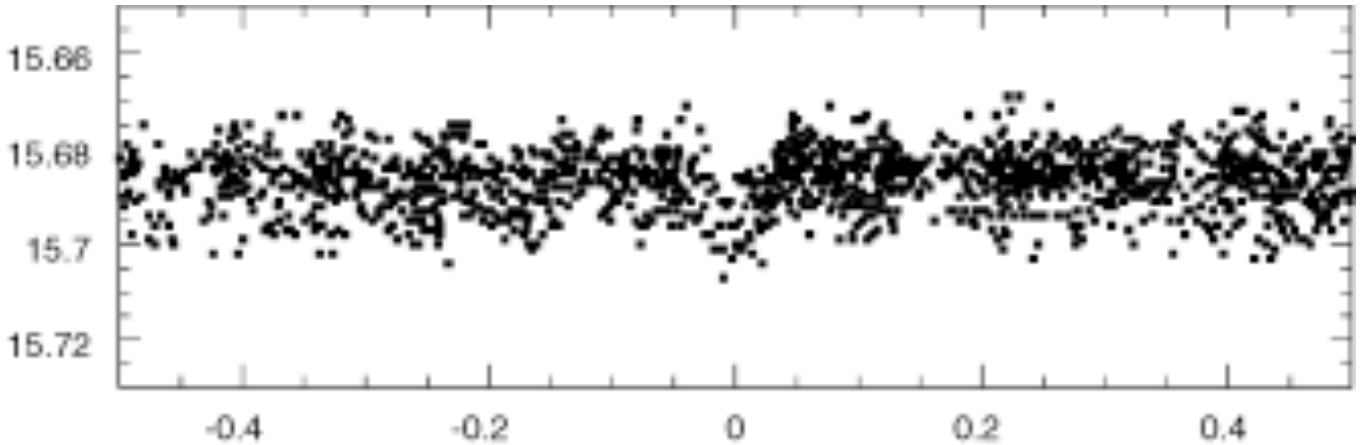
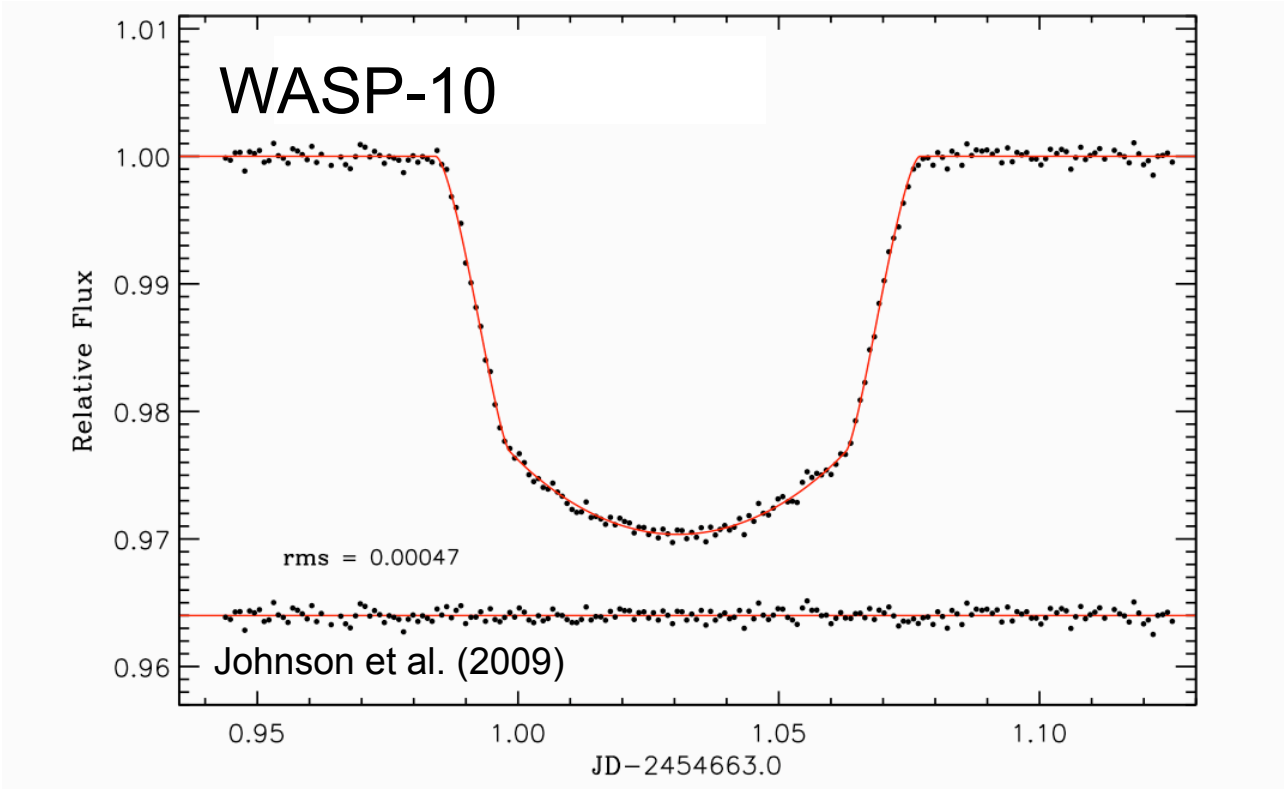
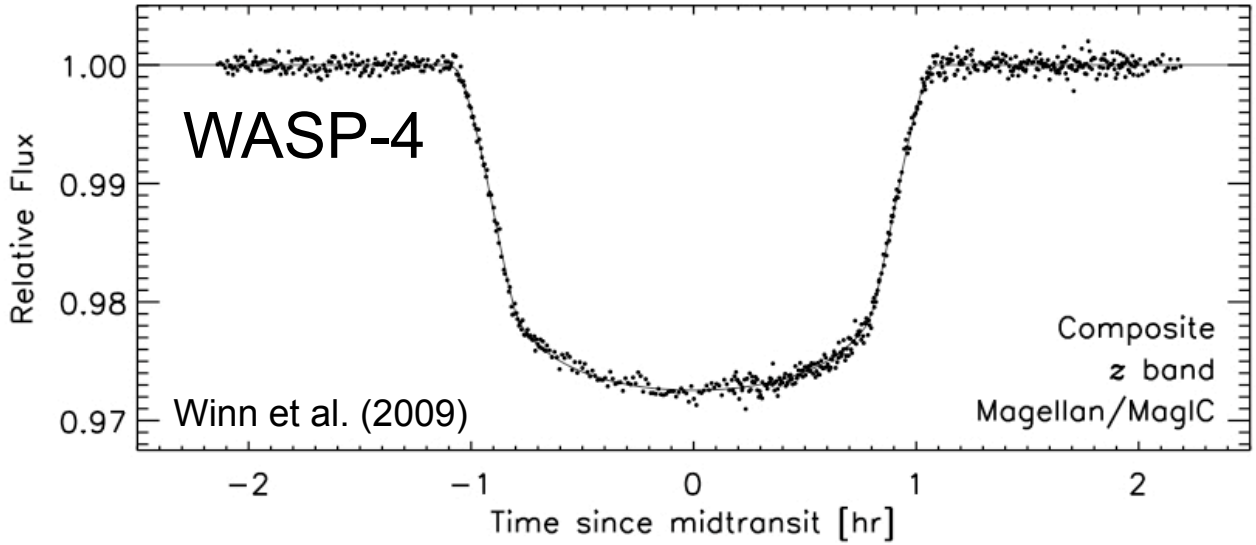
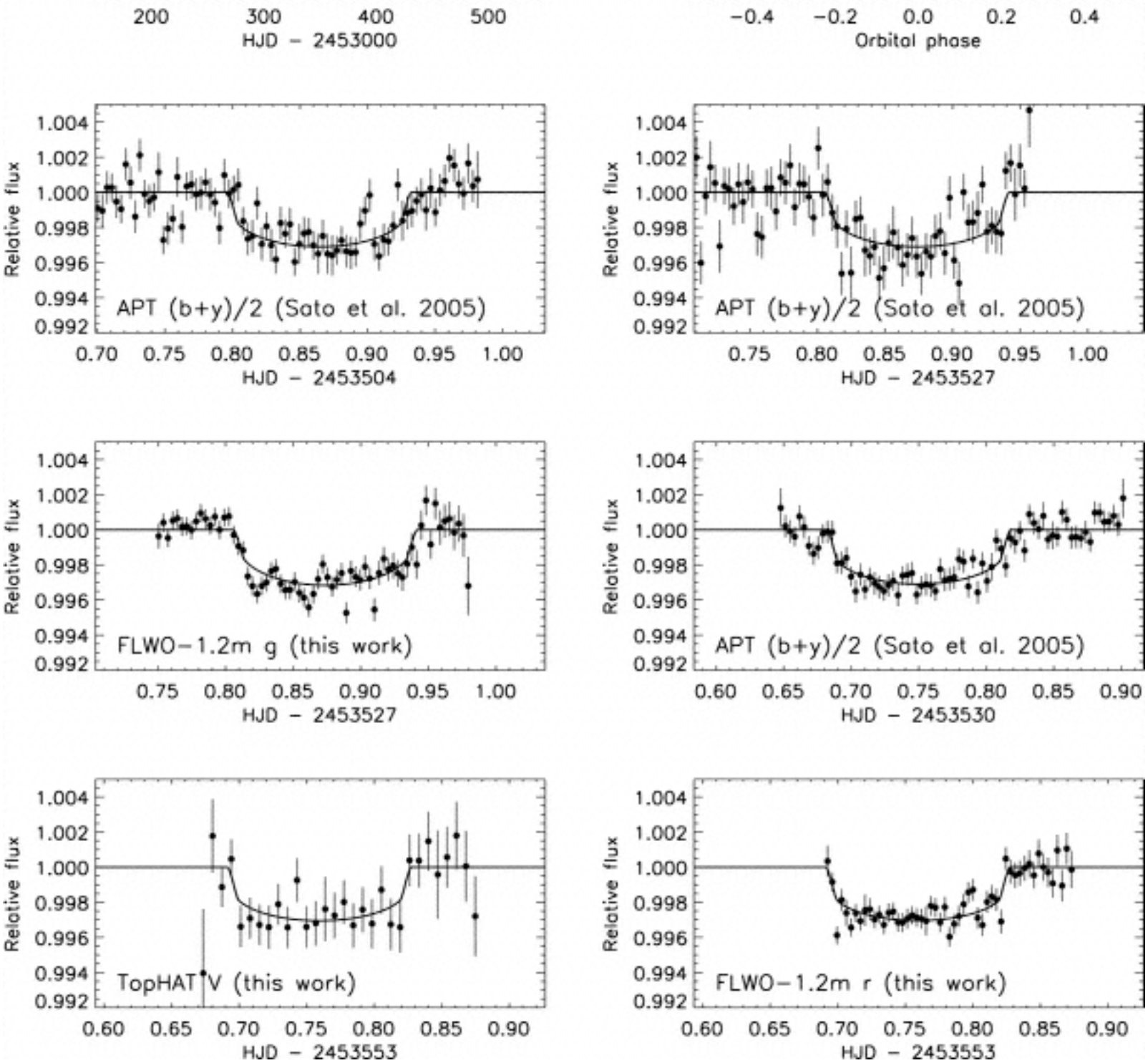


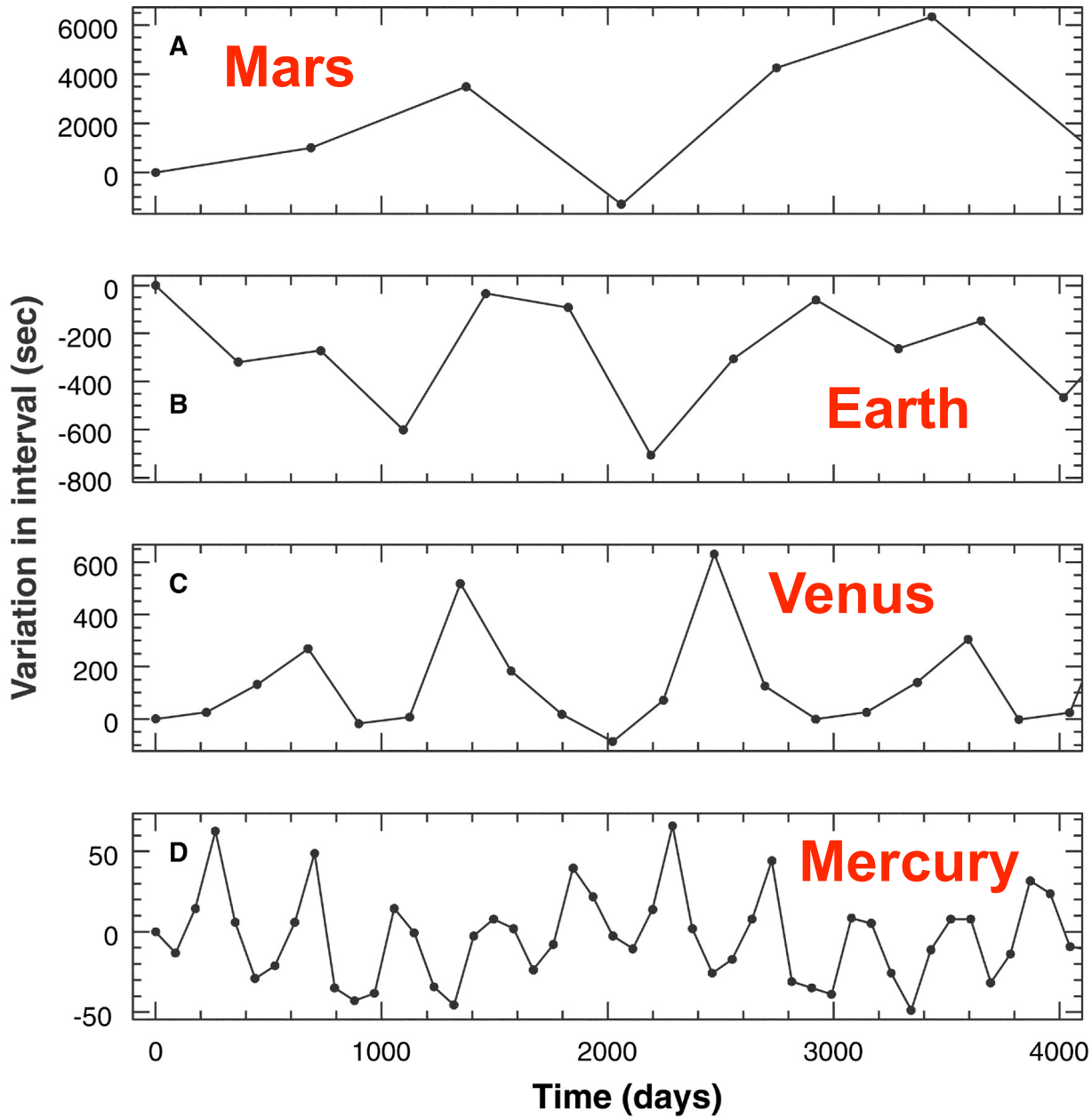


Pont et al. (2006)

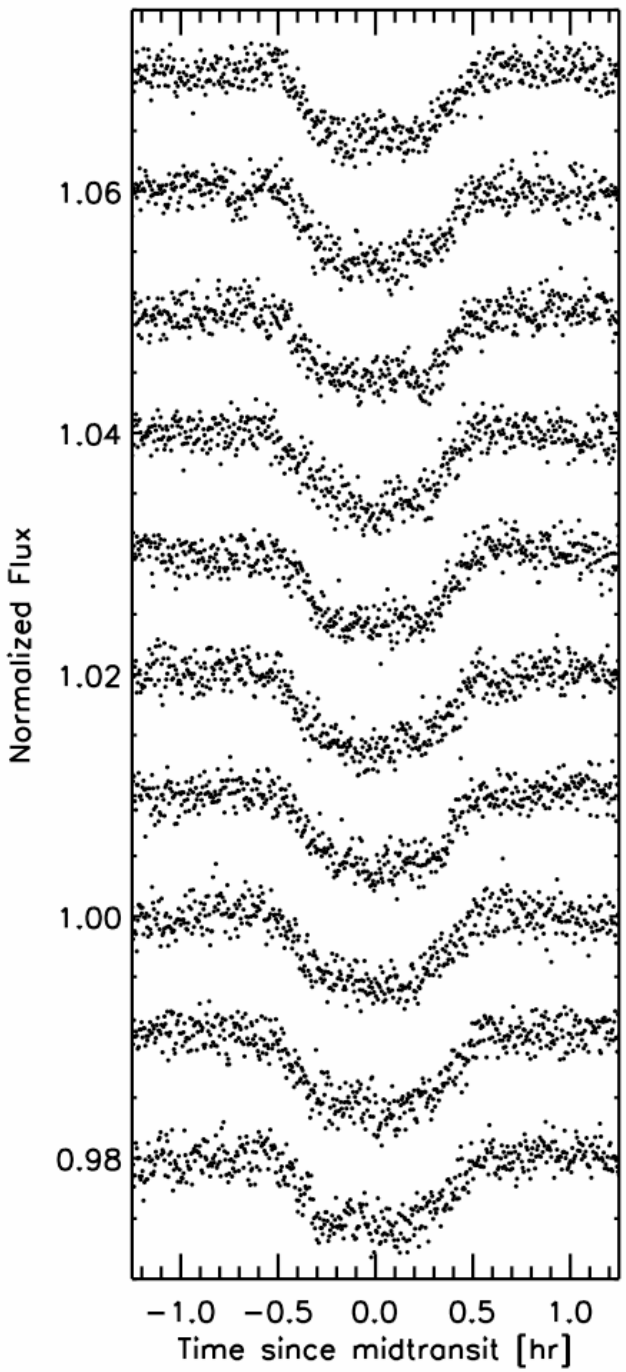

WASP-10
Relative Flux
1.01
1.00
0.99
0.98
0.97
0.96
rms = 0.00047
Johnson et al. (2009)
0.95
1.00
1.05
1.10
JD−2454663.0

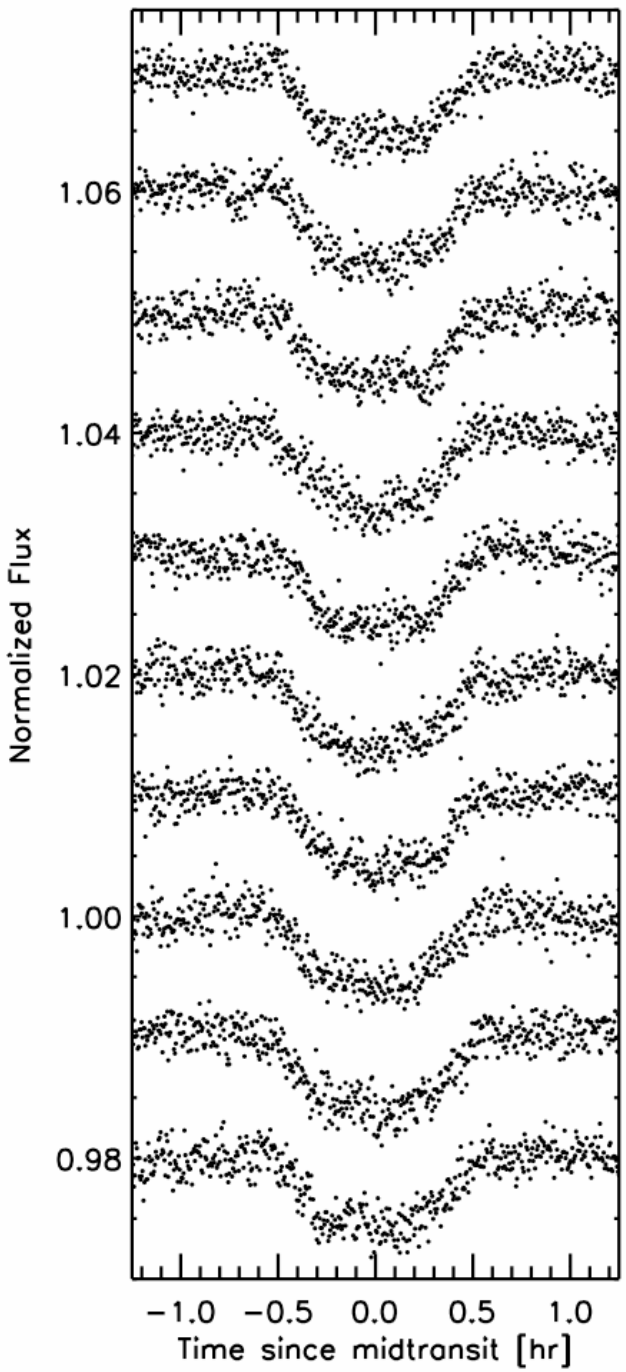

WASP-4
Relative Flux
1.00
0.99
0.98
0.97
Winn et al. (2009)
Composite
z band
Magellan/MagIC
−2
−1
0
1
2
Time since midtransit [hr]

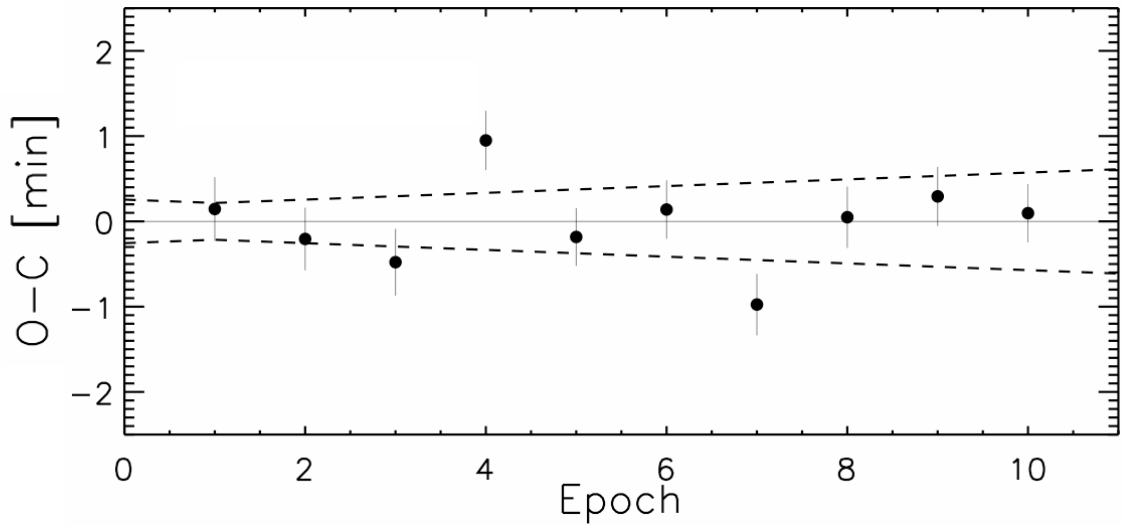

200 300 400 500
HJD − 2453000
−0.4 −0.2 0.0 0.2 0.4
Orbital phase
Relative flux
APT (b+y)/2 (Sato et al. 2005)
HJD − 2453504
APT (b+y)/2 (Sato et al. 2005)
HJD − 2453527
FLWO−1.2m g (this work)
HJD − 2453527
APT (b+y)/2 (Sato et al. 2005)
HJD − 2453530
TopHAT V (this work)
HJD − 2453553
FLWO−1.2m r (this work)
HJD − 2453553

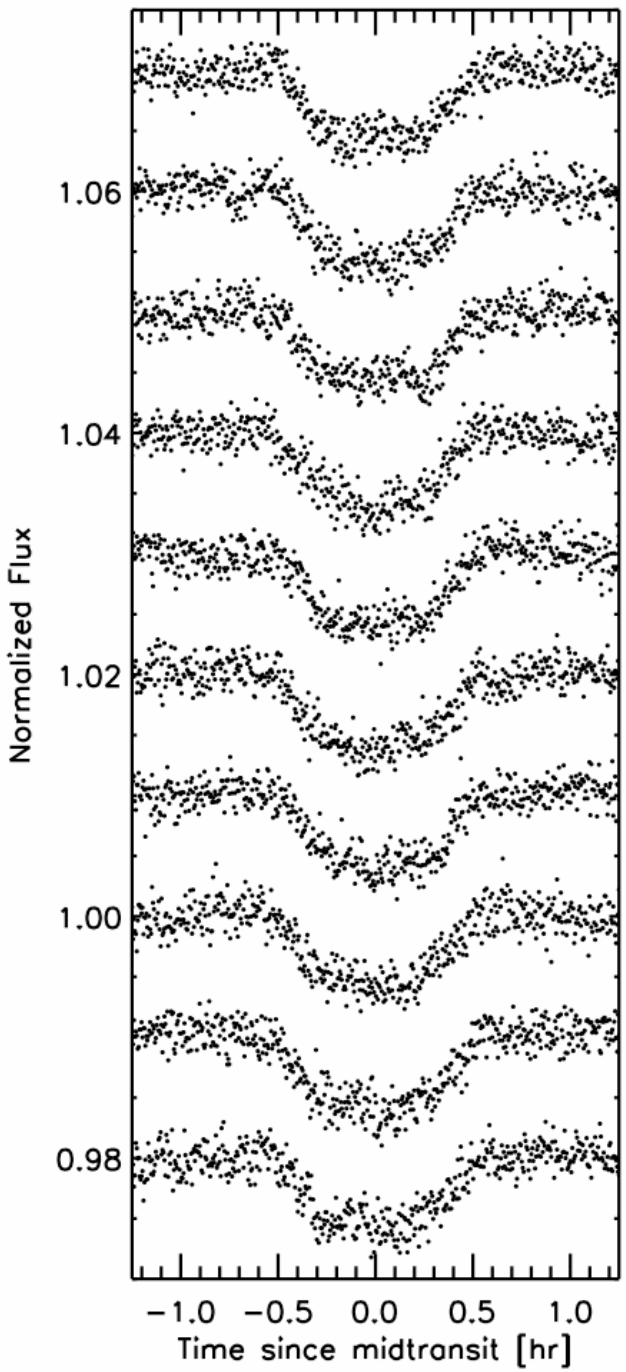


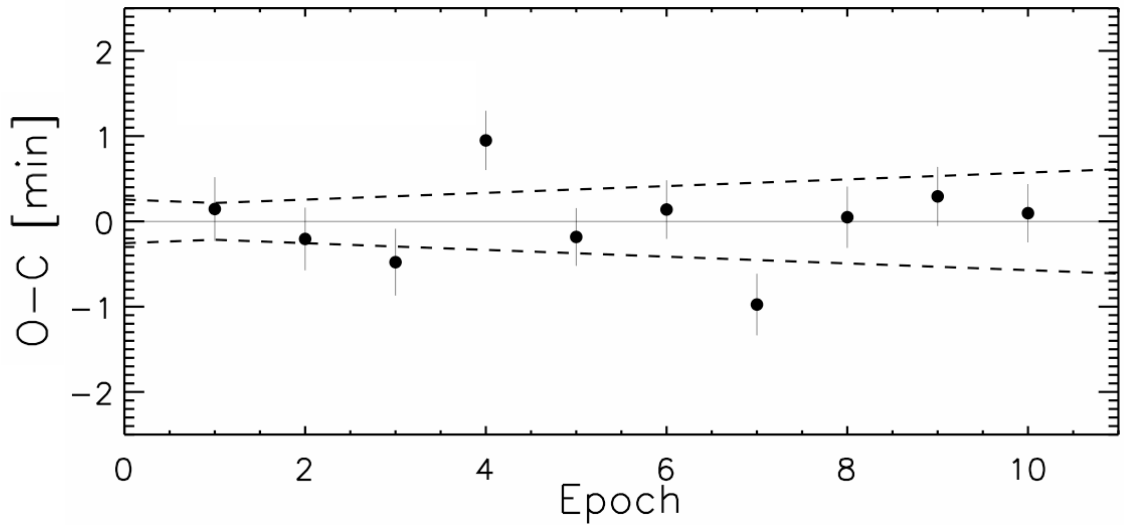
Holman & Murray (2005); see also Agol et al. (2005)


1.06
1.04
1.02
1.00
0.98
Normalized Flux
−1.0
−0.5
0.0
0.5
1.0
Time since midtransit [hr]


Normalized Flux
1.06
1.04
1.02
1.00
0.98
−1.0
−0.5
0.0
0.5
1.0
Time since midtransit [hr]


O−C [min]
2
1
0
−1
−2
0
2
4
6
8
10
Epoch





A constant period is ruled out with 98% confidence

How to cope with correlated errors

How to cope with correlated errors

$$\mathcal{L} \propto \exp(-\chi^2/2) \qquad \chi^2 = \sum_{i=1}^{N} \frac{r_i^2}{\hat{\sigma}^2}$$

Ignore them

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Ignore them

$$\chi^2 = \sum_{i=1}^{N} \frac{r_i^2}{\hat{\sigma}^2 + \hat{\sigma}_r^2} \qquad \hat{\sigma}_r^2 = \frac{\hat{\sigma}_n^2 - \hat{\sigma}_1^2/n}{1 - 1/n}$$

Time-averaging

$\hat{\sigma}_r$ = stddev of binned residuals

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Time-averaging

$\hat{\sigma}_r$ = stddev of binned residuals

$$\chi^2 = \sum_{i=1}^{N} \frac{r_i^2}{\hat{\sigma}^2}$$

Minimize for a collection of "permuted" light curves

Residual permutation (bootstrap)

$$\chi^2 = \sum_i^N \left(\frac{r_i}{\hat{\sigma}} \right)^2$$

Ignores correlated errors

$$\chi^2 = \sum_i^N \left(\frac{r_i}{\hat{\sigma}} \right)^2$$

Ignores correlated errors

$$\chi^2 = \sum_{i=1}^N \sum_{j=1}^N r_i (\hat{\Sigma}^{-1})_{ij} r_j$$

$$\chi^2 = \sum_i^N \left(\frac{r_i}{\hat{\sigma}}\right)^2$$

Ignores correlated errors

$$\chi^2 = \sum_{i=1}^N \sum_{j=1}^N r_i (\hat{\Sigma}^{-1})_{ij} r_j$$

Too slow

$$\chi^2 = \sum_i^N \left(\frac{r_i}{\hat{\sigma}}\right)^2$$

Ignores correlated errors

$$\chi^2 = \sum_{i=1}^N \sum_{j=1}^N r_i (\hat{\Sigma}^{-1})_{ij} r_j$$

Too slow

$$\chi^2 = \sum_{i=1}^N \sum_{l=-L}^L r_i (\hat{\Sigma}^{-1})_{i,i+l} r_{i+l}$$

$$\chi^2 = \sum_i^N \left(\frac{r_i}{\hat{\sigma}}\right)^2$$

Ignores correlated errors

$$\chi^2 = \sum_{i=1}^N \sum_{j=1}^N r_i (\hat{\Sigma}^{-1})_{ij} r_j$$

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$$\chi^2 = \sum_{i=1}^N \sum_{l=-L}^L r_i (\hat{\Sigma}^{-1})_{i,i+l} r_{i+l}$$

Still too slow

$$\chi^2 = \sum_i^N \left(\frac{r_i}{\hat{\sigma}}\right)^2$$

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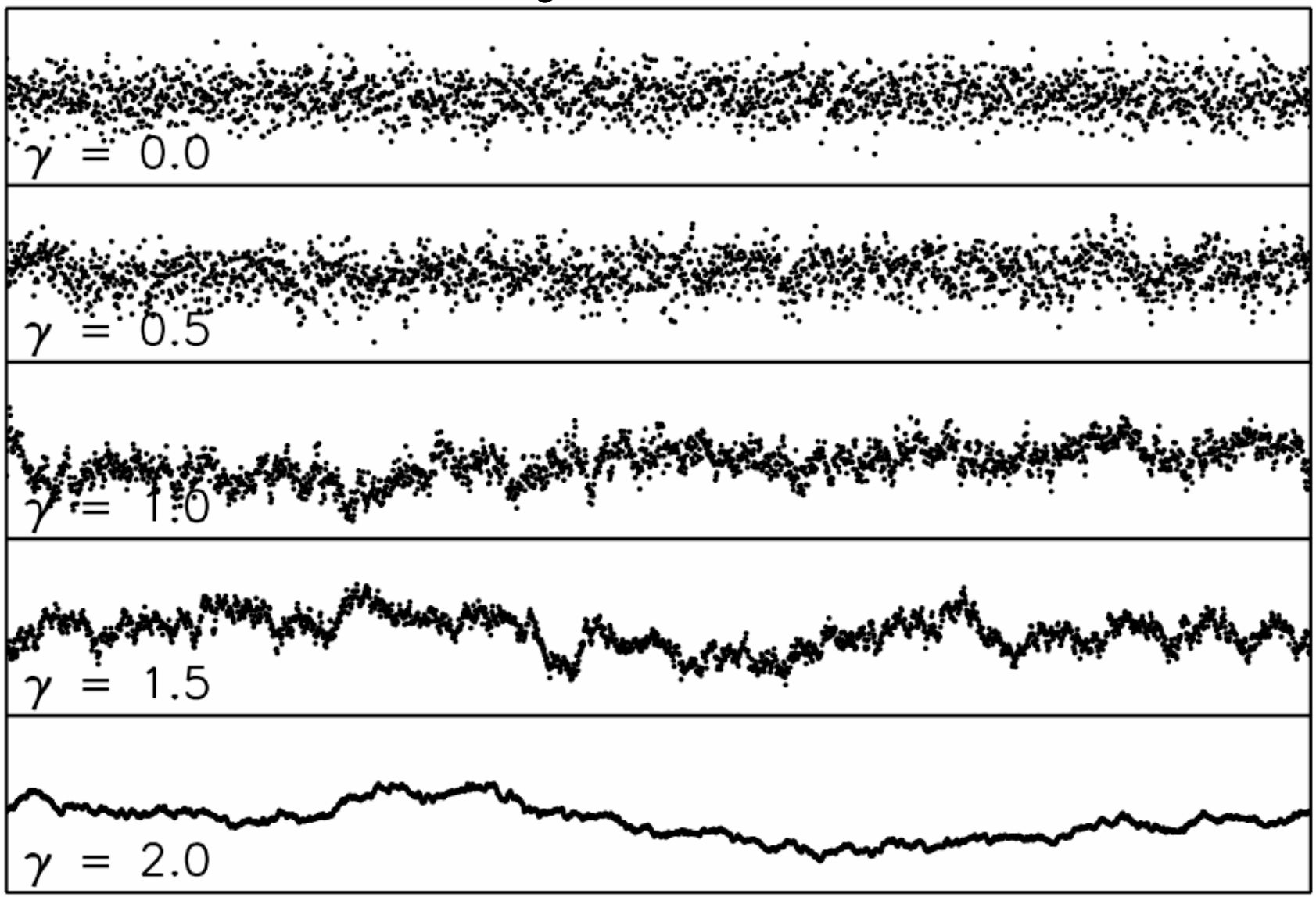
Too slow

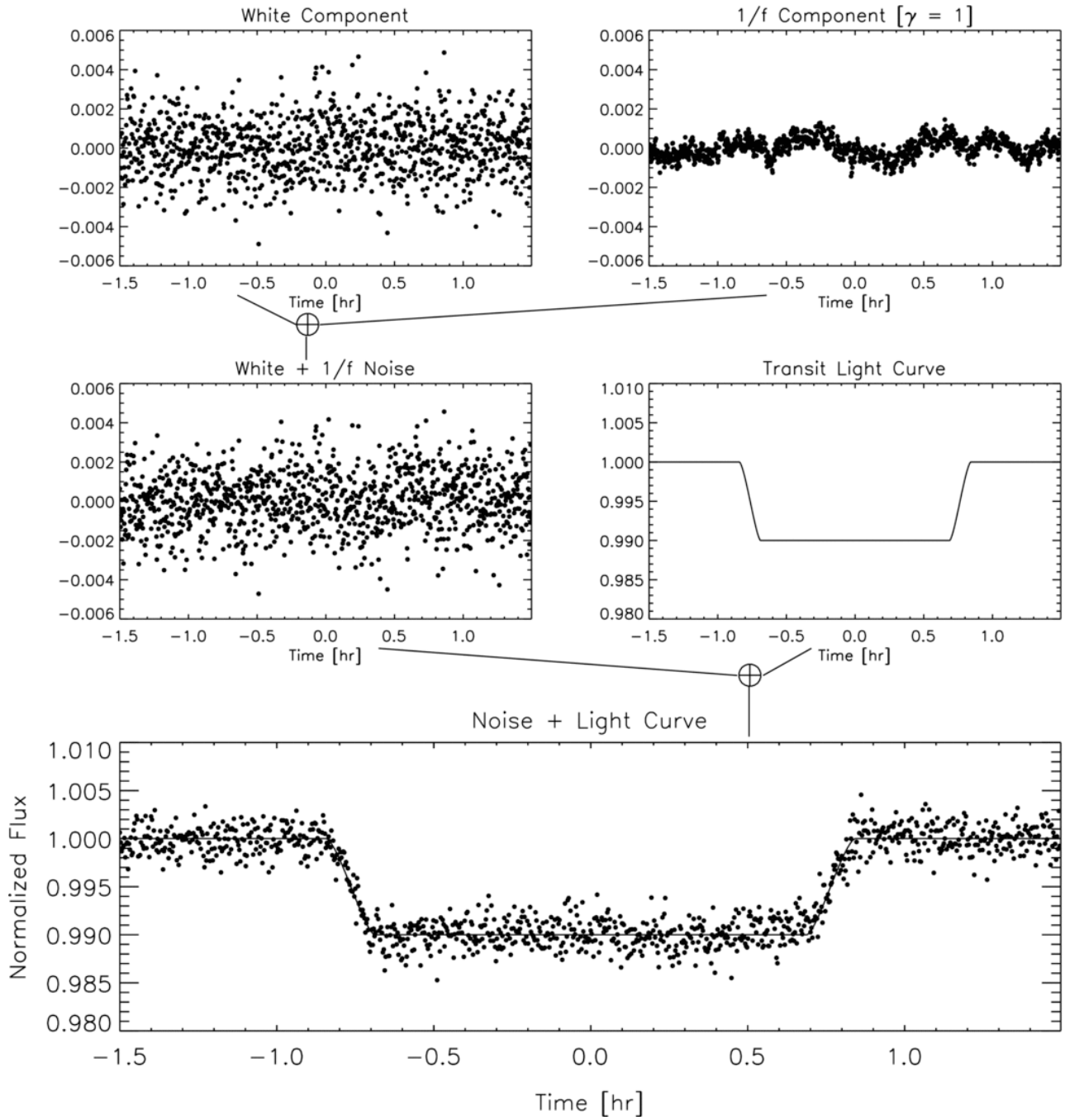
$$\chi^2 = \sum_{i=1}^N \sum_{l=-L}^L r_i (\hat{\Sigma}^{-1})_{i,i+l} r_{i+l}$$

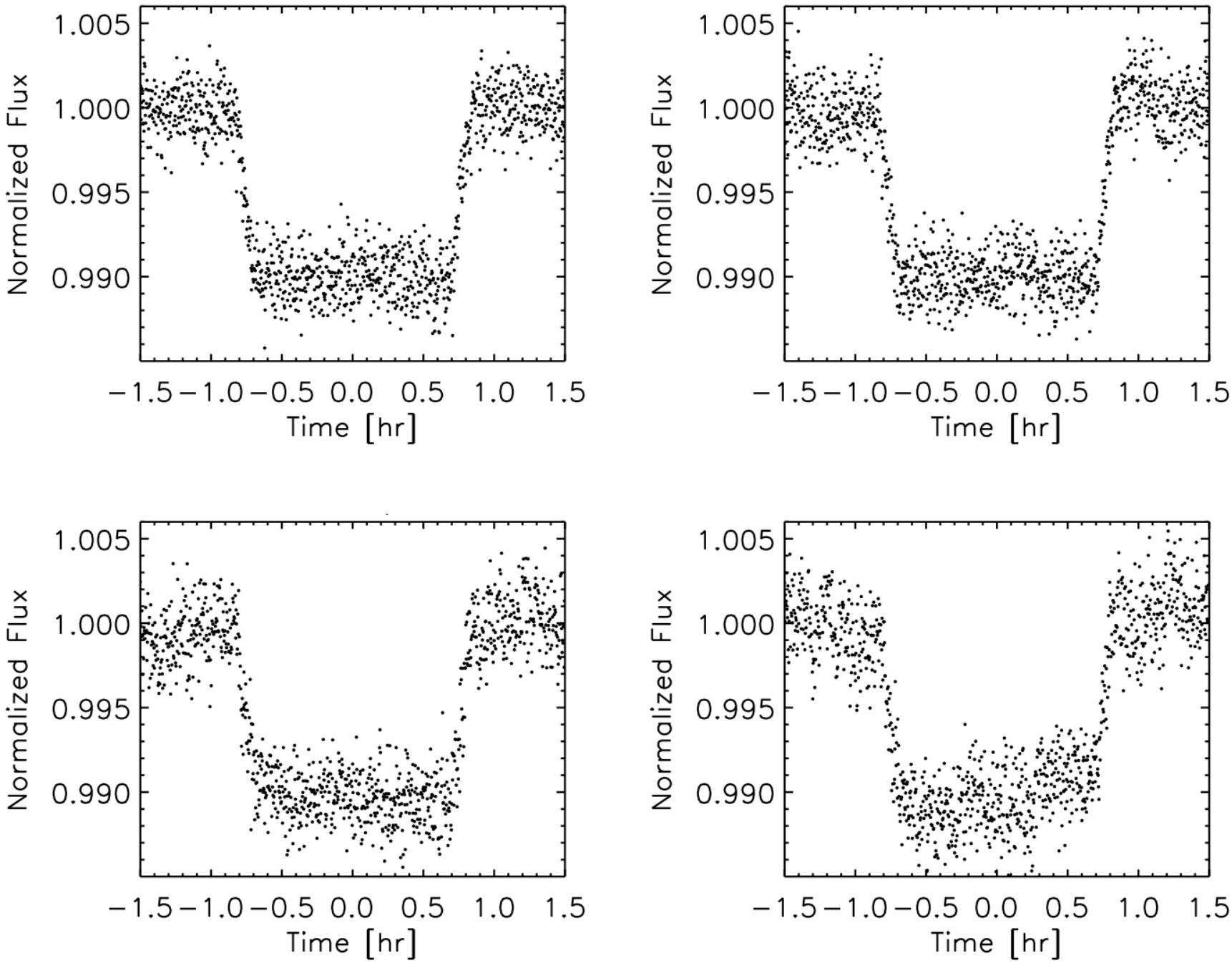
Still too slow

We need to diagonalize the covariance matrix

$1/f^{\gamma}$ noise




White Component
1/f Component [γ = 1]
White + 1/f Noise
Transit Light Curve
Noise + Light Curve
Normalized Flux
Time [hr]
0.006
0.004
0.002
0.000
−0.002
−0.004
−0.006
1.010
1.005
1.000
0.995
0.990
0.985
0.980
−1.5
−1.0
−0.5
0.0
0.5
1.0

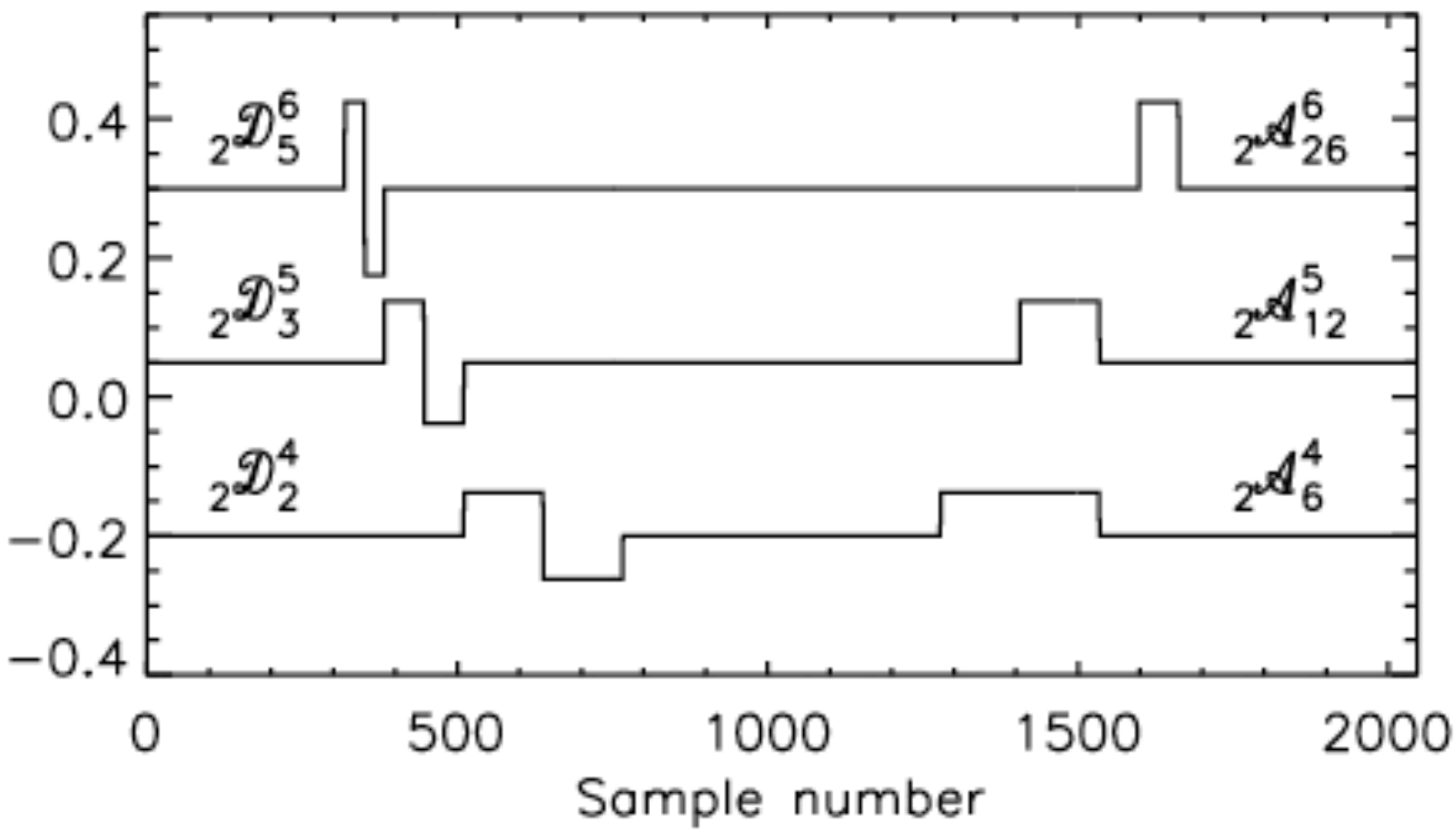

1.005
1.000
0.995
0.990
Normalized Flux
−1.5 −1.0 −0.5 0.0 0.5 1.0 1.5
Time [hr]
1.005
1.000
0.995
0.990
Normalized Flux
−1.5 −1.0 −0.5 0.0 0.5 1.0 1.5
Time [hr]
1.005
1.000
0.995
0.990
Normalized Flux
−1.5 −1.0 −0.5 0.0 0.5 1.0 1.5
Time [hr]
1.005
1.000
0.995
0.990
Normalized Flux
−1.5 −1.0 −0.5 0.0 0.5 1.0 1.5
Time [hr]

The *wavelet transform* is a near-diagonalizing operator for a covariance matrix describing white + $1/f^{\gamma}$ noise.

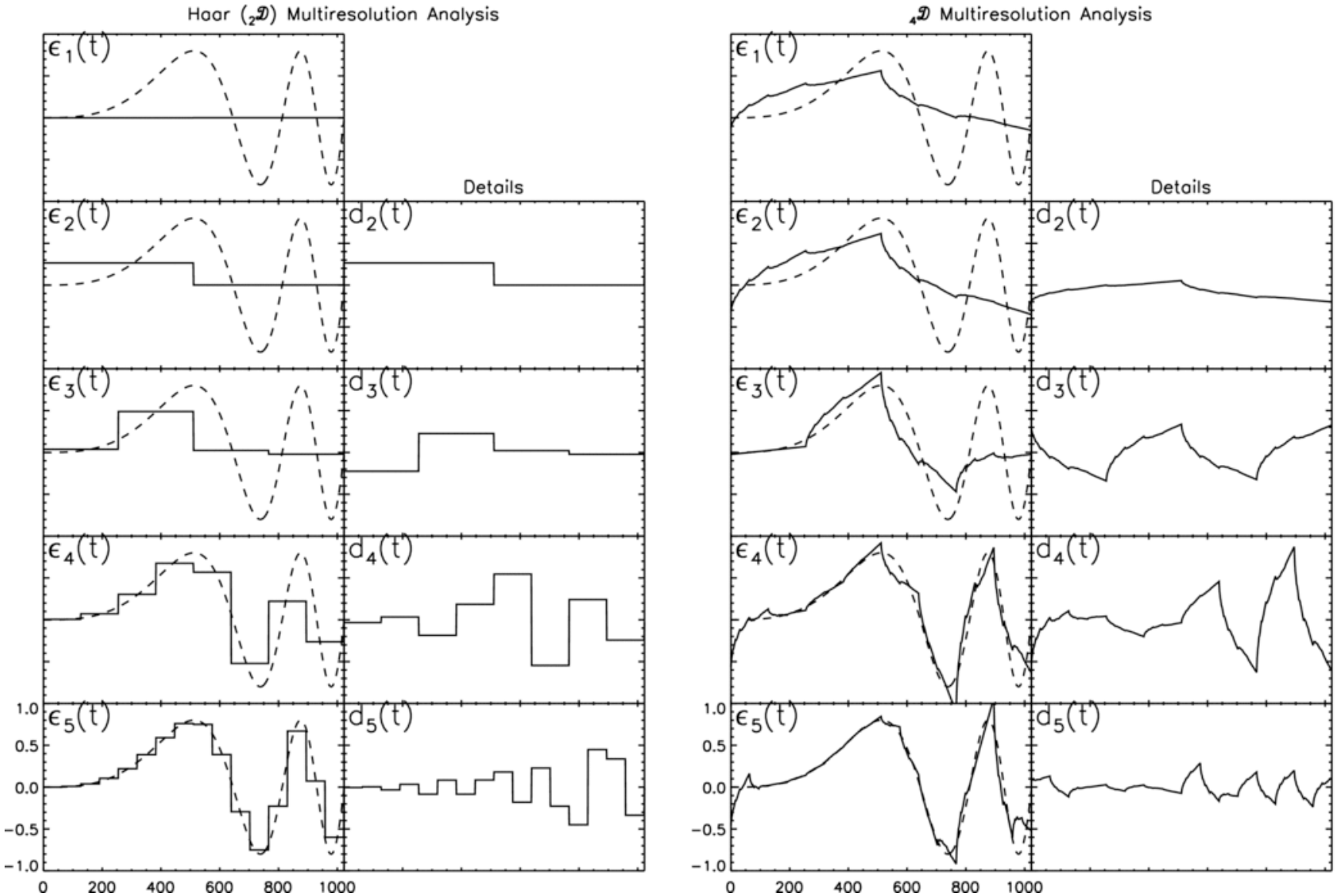
G. Wornell (1996) *Signal Processing with Fractals: A Wavelet-Based Approach* (Prentice-Hall)

The wavelet transform

The wavelet transform



The wavelet transform



The *wavelet transform* is a near-diagonalizing operator for a covariance matrix describing white + $1/f^{\gamma}$ noise.

G. Wornell (1996) *Signal Processing with Fractals: A Wavelet-Based Approach* (Prentice-Hall)

$(N = n_0\ 2^M)$

$$r(t_i) = \sum_{n=1}^{N_1} \bar{r}_n^1 \phi_n^1(t_i) + \sum_{m=2}^{M} \sum_{n=1}^{N_m} r_n^m \psi_n^m(t_i)$$

$$\left\langle r_n^m\ r_{n'}^{m'} \right\rangle \approx \left(\sigma_r^2\ 2^{-\gamma m} + \sigma_w^2\right) \delta_{m,m'}\ \delta_{n,n'}$$

$$\left\langle \bar{r}_n^1 \bar{r}_{n'}^1 \right\rangle \approx \left(\sigma_r^2\ 2^{-\gamma} g(\gamma) + \sigma_w^2\right) \delta_{n,n'}$$

$$\chi^2 = \sum_i^N \left(\frac{r_i}{\hat{\sigma}}\right)^2$$

Ignores correlated errors

$$\chi^2 = \sum_{i=1}^N \sum_{j=1}^N r_i (\hat{\Sigma}^{-1})_{ij} r_j$$

Too slow

$$\chi^2 = \sum_i^N \left(\frac{r_i}{\hat{\sigma}}\right)^2$$

Ignores correlated errors

$$\chi^2 = \sum_{i=1}^N \sum_{j=1}^N r_i (\hat{\Sigma}^{-1})_{ij} r_j$$

Too slow

$$\mathcal{L} = \left\{ \prod_{m=2}^{M} \prod_{n=1}^{n_0 2^{m-1}} \frac{1}{\sqrt{2\pi\sigma_W^2}} \exp\left(-\frac{\left(r_n^m\right)^2}{2\sigma_W^2}\right) \right\}$$
$$\times \left\{ \prod_{n=1}^{n_0} \frac{1}{\sqrt{2\pi\sigma_S^2}} \exp\left(-\frac{\left(\bar{r}_n^1\right)^2}{2\sigma_S^2}\right) \right\}$$

$$\sigma_W^2 = \sigma_r^2\, 2^{-\gamma m} + \sigma_w^2 \qquad \sigma_S^2 = \sigma_r^2\, 2^{-\gamma}\, g(\gamma) + \sigma_w^2$$

$$\chi^2 = \sum_i^N \left(\frac{r_i}{\hat{\sigma}}\right)^2$$

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Too slow

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It's fast!

$$\sigma_W^2 = \sigma_r^2\, 2^{-\gamma m} + \sigma_w^2 \qquad \sigma_S^2 = \sigma_r^2\, 2^{-\gamma}\, g(\gamma) + \sigma_w^2$$

$$\chi^2 = \sum_i^N \left(\frac{r_i}{\hat{\sigma}}\right)^2$$

Ignores correlated errors

$$\chi^2 = \sum_{i=1}^N \sum_{j=1}^N r_i (\hat{\Sigma}^{-1})_{ij} r_j$$

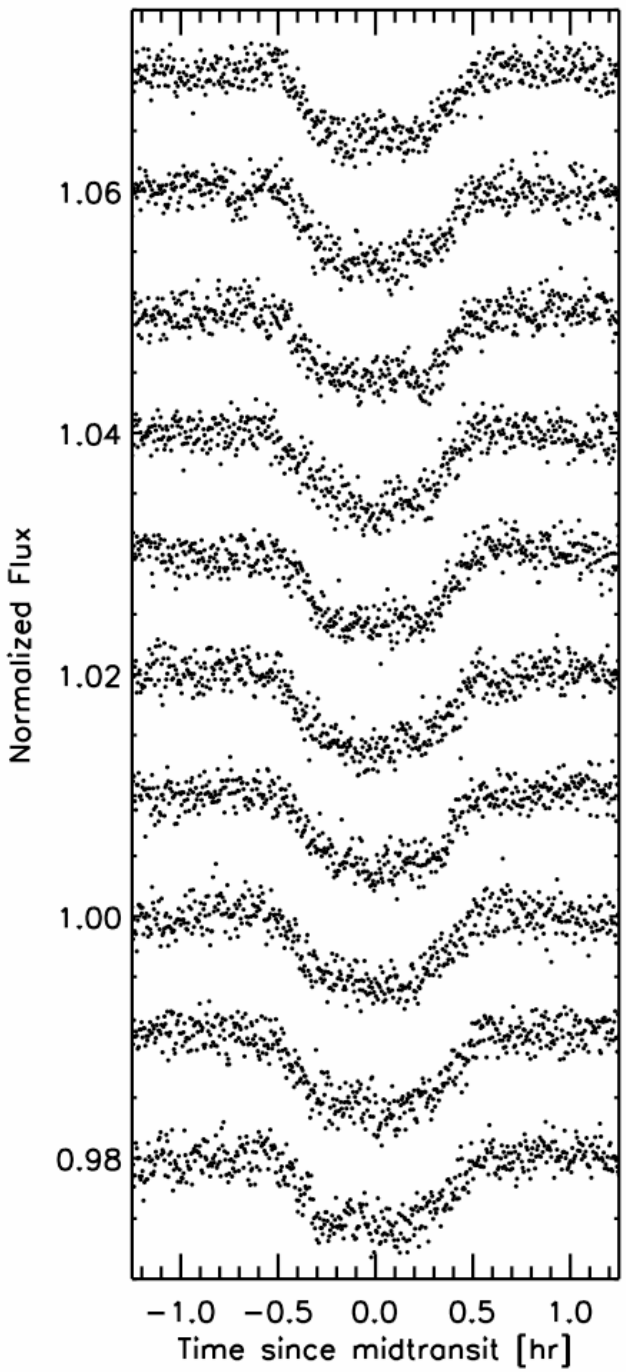
Too slow

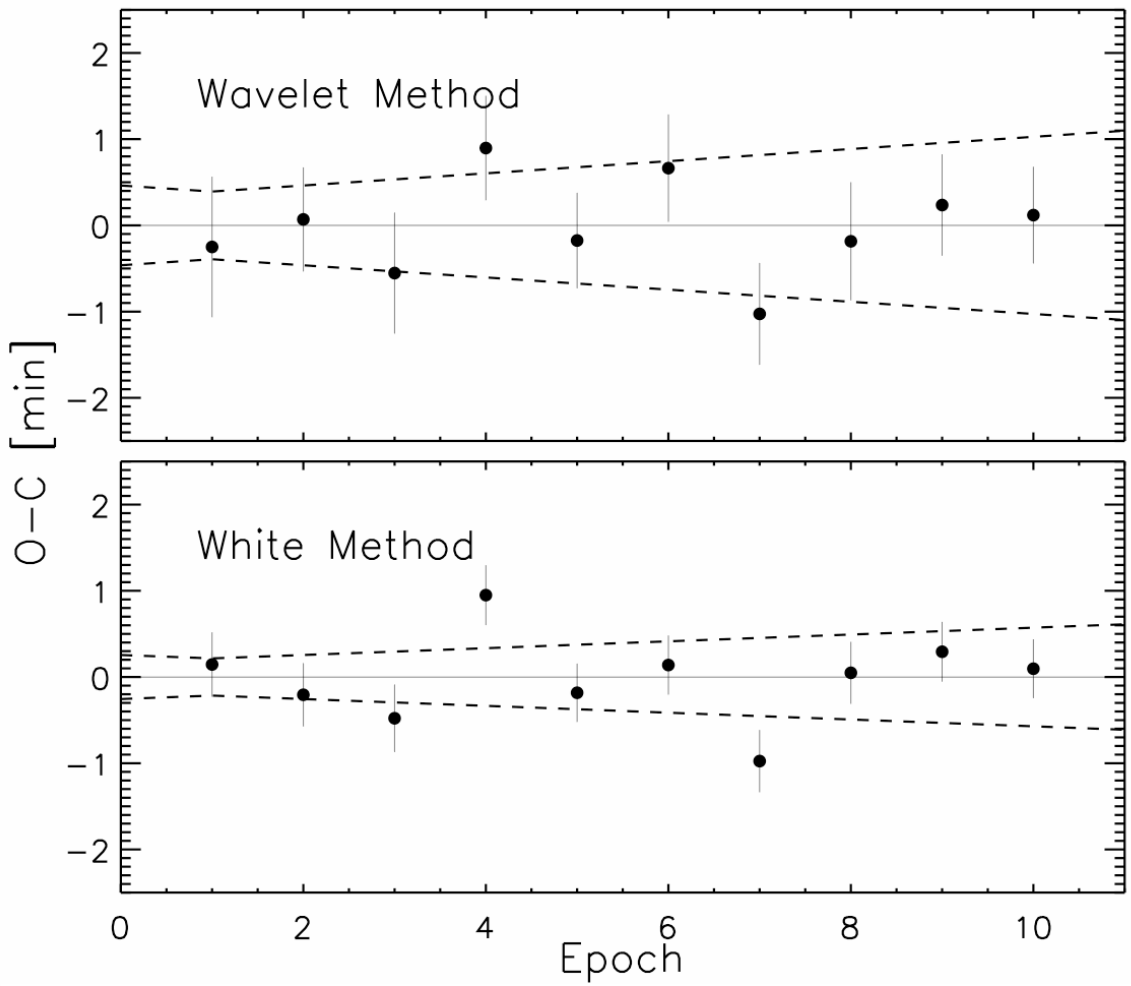
$$\mathcal{L} = \left\{ \prod_{m=2}^{M} \prod_{n=1}^{n_0 2^{m-1}} \frac{1}{\sqrt{2\pi\sigma_W^2}} \exp\left(-\frac{\left(r_n^m\right)^2}{2\sigma_W^2}\right) \right\} \times \left\{ \prod_{n=1}^{n_0} \frac{1}{\sqrt{2\pi\sigma_S^2}} \exp\left(-\frac{\left(\bar{r}_n^1\right)^2}{2\sigma_S^2}\right) \right\}$$

$$\sigma_W^2 = \sigma_r^2\, 2^{-\gamma m} + \sigma_w^2 \qquad \sigma_S^2 = \sigma_r^2\, 2^{-\gamma}\, g(\gamma) + \sigma_w^2$$

It's fast!

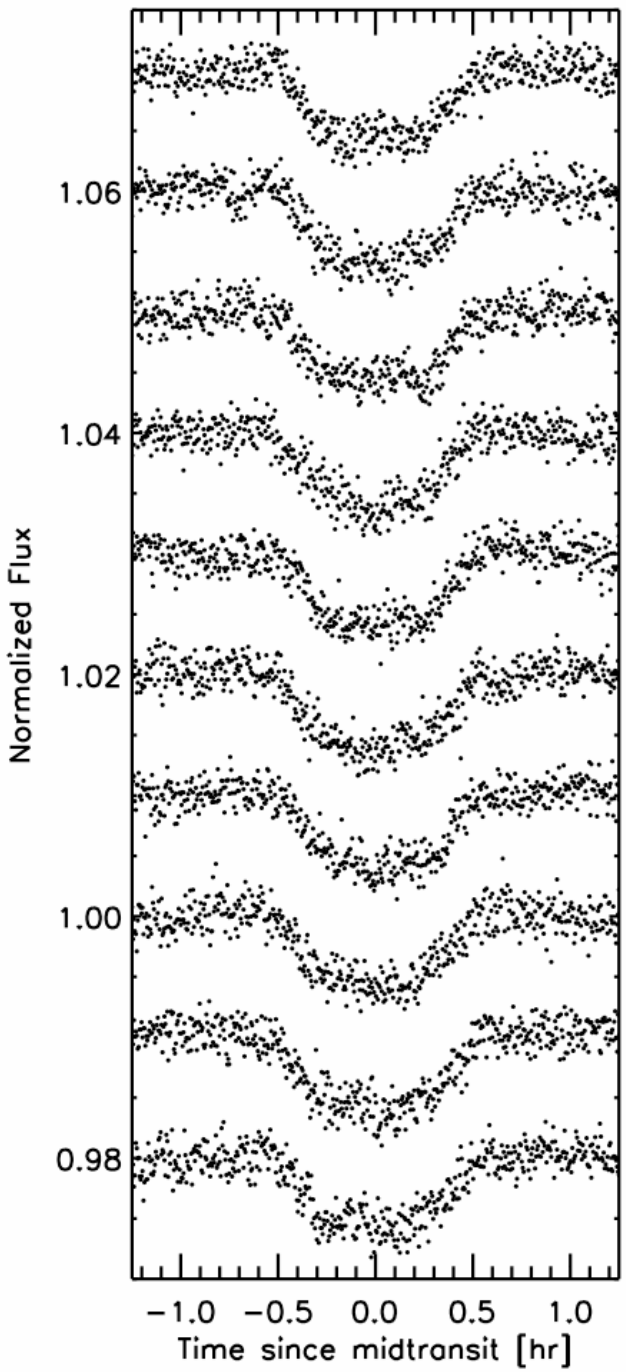
It works!

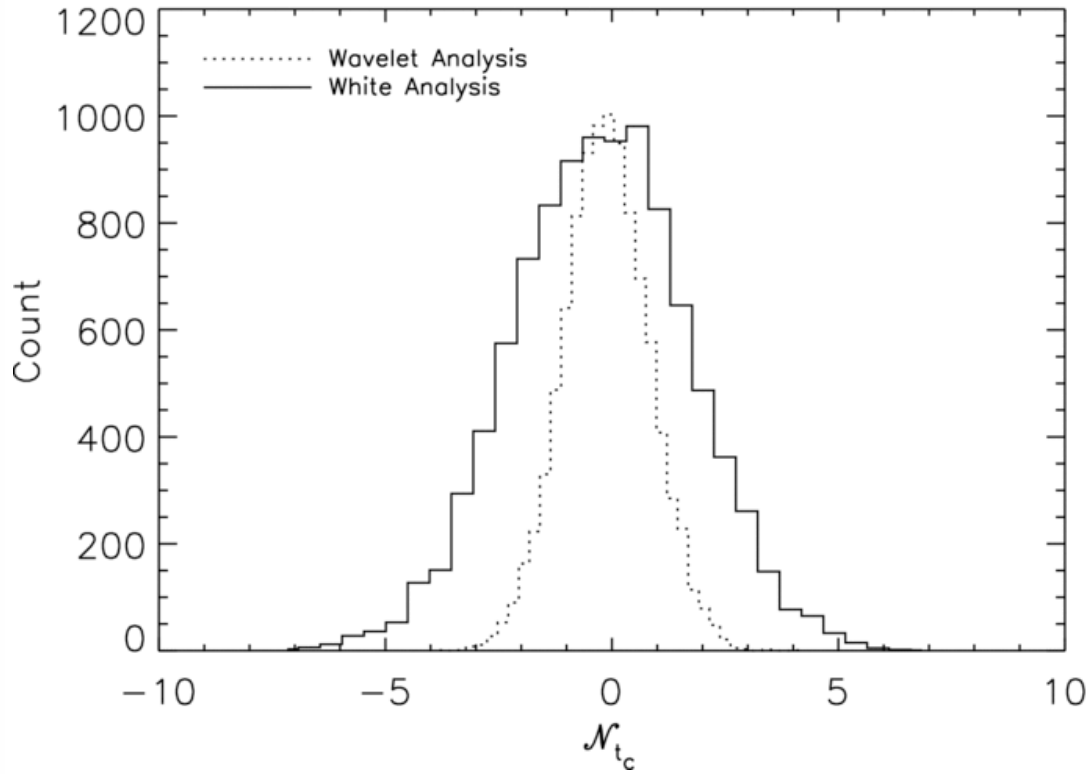




Constant period model is fine

$$\chi^2_{\rm dof} = 0.93$$

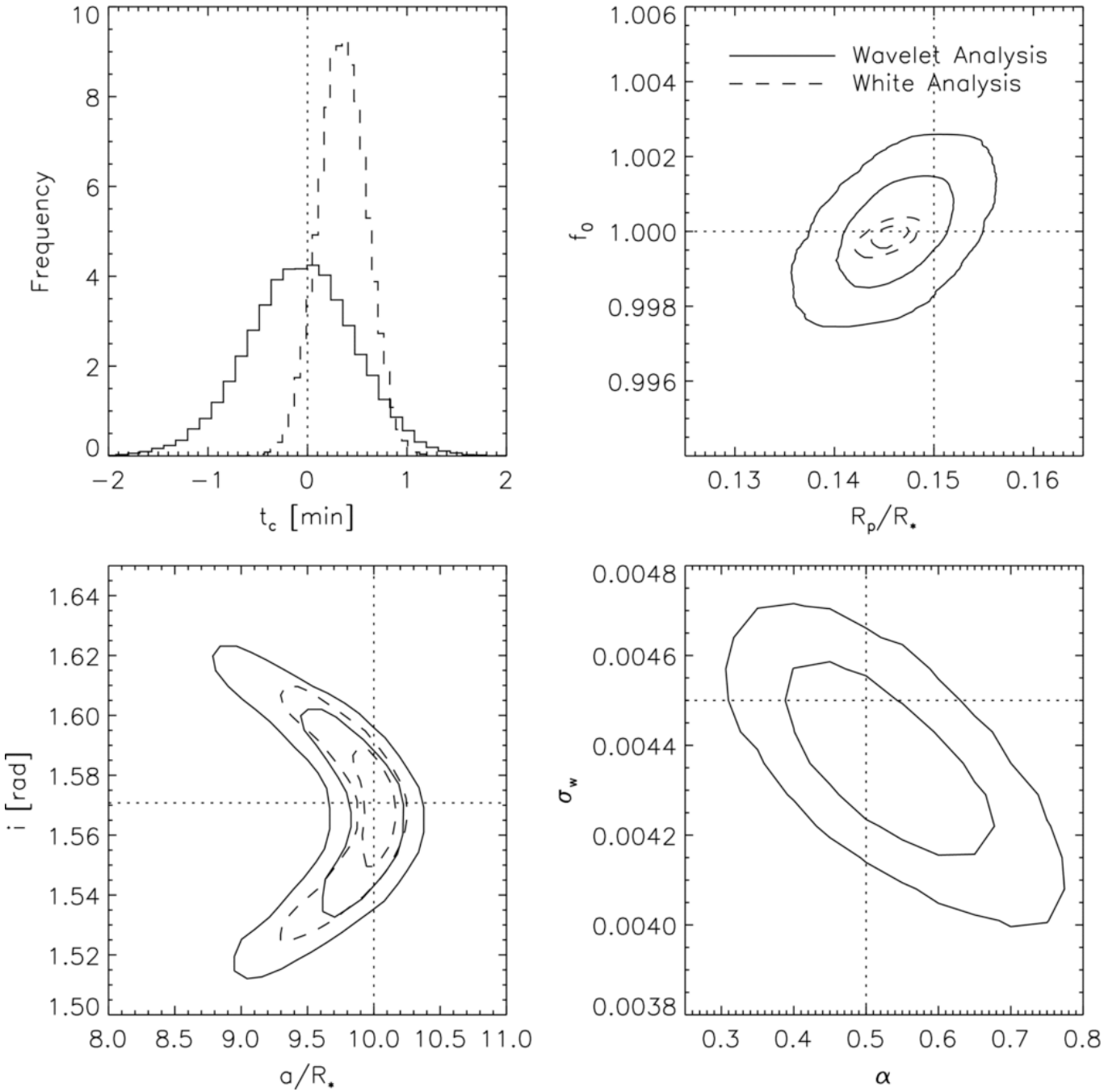




Compare to other methods using the "Number-of-sigma" statistic

Table 2
Estimates of t_c from Data with Unknown Noise Properties

Method	α	$\langle\hat{\sigma}_{t_c}\rangle$ (s)	$\langle\mathcal{N}\rangle$	$\sigma_{\mathcal{N}}$	Prob($\mathcal{N} > 1$) (%)	Prob(better)[a] (%)
White	0	4.0	−0.011	0.97	31	···
	1/3	4.2	+0.010	1.70	57	···
	2/3	4.9	+0.012	2.69	73	···
	1	5.8	+0.023	3.28	78	···
Wavelet	0	4.5	−0.009	0.90	26	50
	1/3	6.9	−0.003	1.03	33	56
	2/3	11.2	−0.005	1.07	35	57
	1	15.7	−0.007	1.09	36	57
Time-averaging	0	4.4	−0.006	0.88	26	50
	1/3	6.8	+0.009	1.15	36	50
	2/3	11.6	−0.012	1.24	40	50
	1	17.6	+0.007	1.21	38	50
Residual-permutation	0	3.5	−0.012	1.16	37	50
	1/3	6.6	+0.013	1.24	37	50
	2/3	11.8	−0.014	1.28	38	49
	1	17.3	+0.008	1.30	38	48


Frequency
t_c [min]
Wavelet Analysis
White Analysis
f_0
R_p/R_*
i [rad]
a/R_*
σ_w
α