The 6th Asian Winter school on String Theory @ Kusatsu.

2012 Jan 15, 16, 18, 19

Supersymmetric Compactification and Grand Unification

4 x [1hr 15min.]

by Taizan Watari
(IPMU)

Day 1 Landscape of $D=4$ $N=1$ SUSY compactification.

+ seminar on LHC. (by Prof. Tokushuku)

Day 2 SUSY GUT

Day 3 Particle Physics in Het/CY_3, IIA/CY_3 orientifold, M/G_2

Day 4 Particle Physics in F-theory or IIB

This is a note prepared for a winter school lecture. 4 x [1hr 15min.]

This note is far from being perfect, complete or self-contained, and the lecture that was delivered at the winter school was not precisely the same as what is written in this note, either.

Because it does not seem realistic for me to take time to complete this note in a near future, I decided to make this note in this very incomplete form publically available, thinking that it ~~is~~ will be better to have something than nothing.

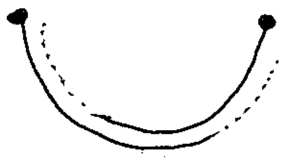
summer 2012.
Taizan Watari
(渡利 泰山)

$\boxed{\text{Day 1}}$ Landscape of $D=4$ $N=1$ SUSY Compactification.

String Duality

5 String Theories.

IIA on 10D IIB on 10D I Het $SO(32)$ Het $E_8 \times E_8$



IIA on S^1 (radius R) IIB on S^1 (radius R') on S^1.

map between moduli space ⇒ spectra match

(radius, dilaton) (dilaton, radius, $16 \times$ Wilson lines)

☆ make sense to claim that our vac. is in string theory ??

- coverage mutually.
- approximation, easiness of description
- discinction between IIA vs IIB vs ... not necessarily for

☆ many vacua w/ non-Abelian gauge sym & changed matter.

physics property we observe.

What is the goal of string pheno?

- existence proof?
- predictions? (possible?)
- understanding?

$\boxed{\text{“String pheno after String Revolution in 90's”}}$

M-theory & IIA

- $11\ \mathrm{SUGRA}/S^1 \Longleftrightarrow \mathrm{IIA}$.
$$\begin{cases} \left(\dfrac{R}{l_{11}}\right)^{3/2} \sim g_s \\ \left(R/l_{11}^3\right) \sim 1/l_s^2 \end{cases}$$

- 11D SUGRA / ALE space. (Taub-NUT) asymptotically Locally Euclidean $\Longleftrightarrow$ IIA w/ D6, O6.

A. Sen th/9707123

metric

$$ds^2 = U(\vec{x})^{-1}\left(d\tau + \omega_i(\vec{x})\,dx^i\right)^2 + U(\vec{x})\,dx^i dx^i$$

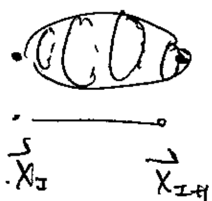
$$\begin{cases} U(\vec{x}) = \displaystyle\sum_{I=1}^{N} \frac{1}{|\vec{x}-\vec{x}_I|} \quad (+1?) \text{ ok.} & x_i = \vec{x} \in \mathbb{R}^3 \quad \tau \in [0, 4\pi] \\ \vec{\nabla}\times\vec{\omega} = -\vec{\nabla} U(\vec{x}) \end{cases}$$

S^1-fibration

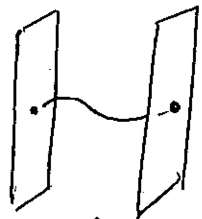
(shrinks at $\vec{x} = \vec{x}_I$)

$(N-1)$ 2-cycles (S^1-fiber) over $\vec{x} \in [\vec{x}_I, \vec{x}_{I+1}]$

Interaction form $\begin{bmatrix} -2 & 1 & & \\ 1 & -2 & \ddots & \\ & \ddots & \ddots & \\ & & & \end{bmatrix} = -C_{A_{N-1}}$



- $\vec{x} = \vec{x}_I \Rightarrow$ D6-brane.
- W-boson $\Leftarrow$ M2-brane on 2-cycles.
- Lie alg. $\Leftarrow$ topology sum of 2-cycles.



$\vec{x}_I$ closer... $\mathbb{C}^2/\mathbb{Z}_N$ singularity (A_{N-1} type)

$\Updownarrow$

SU(N) gauge group on D6-brane.

D_N type, & $E_{6,7,8}$-type also possible.

Het/T^3 — M/K3

(Het/T^4 — IIA/K3)

[10−3 = 7D] [11−4 = 7D] 16 SUSY charges.

massless spectrum below the KK scale

- 7D metric
- 7D 2-form B^{Het} $\longleftrightarrow$ [$C^{(3)}$ Hodge dual in 7D]
- 7D vector 16 Wilson line + $3_D \times 2_{g+B}$ KK $\longleftrightarrow$ 22 2-cycles w $C^{(3)}$
- 7D scalar. $3 \times 16 + 6 + 3 + 1$ $\longleftrightarrow$ $3 \times 19 + 1$ metric

Het/T^3 : full Wilson line $\Rightarrow$ [SO(32) or $E_8 \times E_8$ $\Longrightarrow$ $U(1)^{16}$]

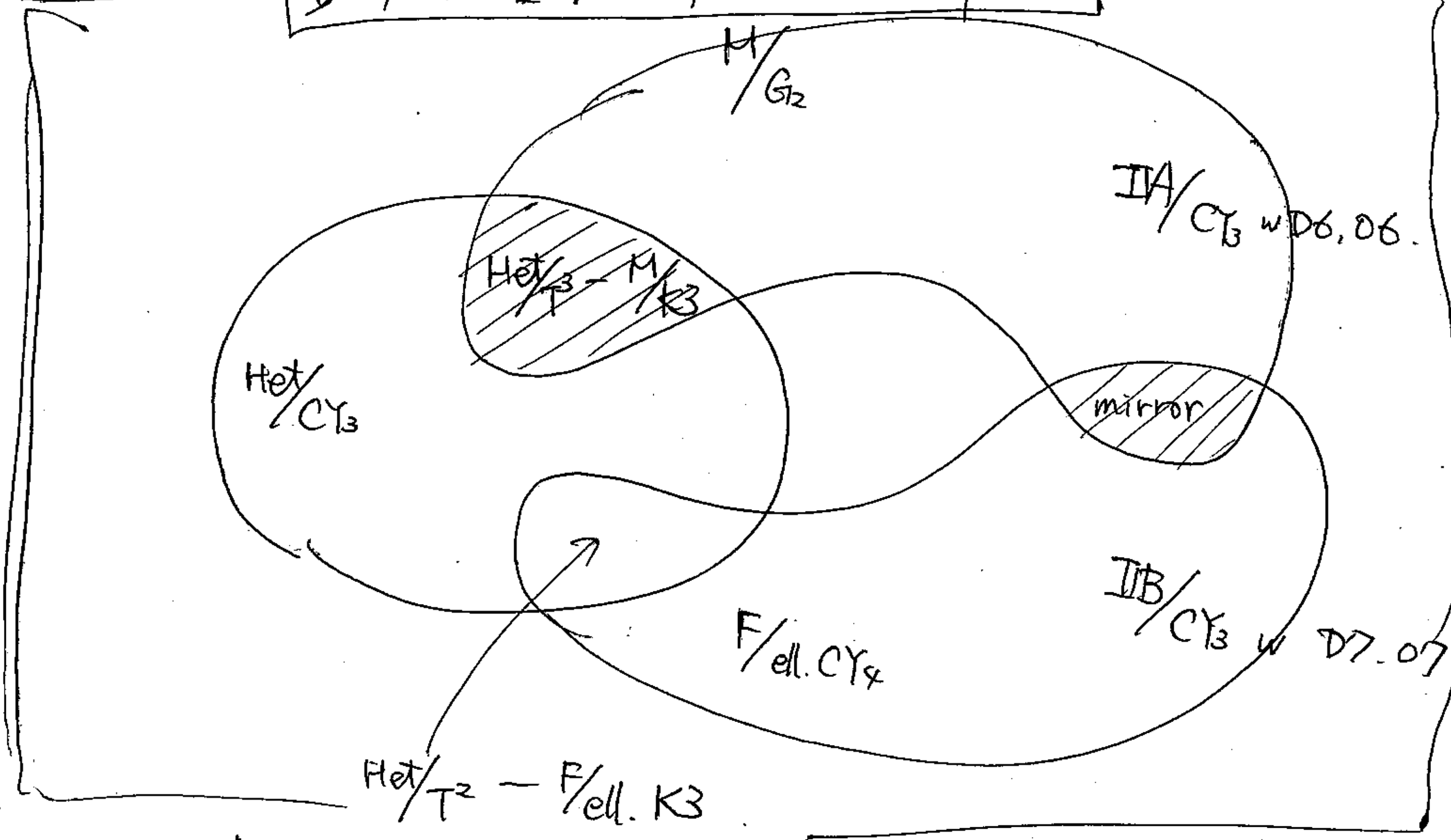
enhanced gauge sym. at special pts in moduli space.

M/K3 : enhanced gauge sym. when 2-cycles collapse.

Het/T^2 — F/ellip. K3.

at 8D

D=4 N=1 SUSY Landscape?



☆ $\boxed{\mathrm{Het}/CY_3} \supset \boxed{\mathrm{Het}/T^3\text{-fib. } CY_3 = M/K3\text{-fib. } G_2\text{-hol}} \subset \boxed{M/G_2\text{-hol.}}$

☆ similar relation between Het & F/or IIB.

Non-geometric Phase

CY_3 on world sheet by linear σ-model.

eg quintic $\quad \phi_i \quad (i=1\sim5) \quad +1$ charge
$\quad P \quad -5$ charge. $\quad$ under $U(1)$

$$\begin{cases} \text{D-term} & \sum_i |\phi_i|^2 - 5\,|P| - \underset{\uparrow\ \text{FI parameter.}}{r} = 0 \\ \text{superpotential} & W = P\cdot \underset{\hookrightarrow\ \text{homog. deg } 5.}{F^{(5)}(\phi_i)} \end{cases}$$

$$\Rightarrow \begin{cases} r \gg 0 \ (\text{large vol}) & \Rightarrow \underset{CY_3}{\underbrace{(F(\phi_i)=0)}} \subset \mathbb{P}^4 \\ r \ll 0. & W = \langle P\rangle . F^{(5)}(\phi_i) \quad w/ \ \langle\phi_i\rangle = 0. \\ & (\text{non-geometric}) \end{cases}$$

Q. Are all non-geometric CFT understood as some limit of geometric CFT?

Orbifolds. / fractional branes

☆ toroidal orbifold:

resolution of singularity $\Rightarrow$ smooth CY.

(particular choice of Kähler moduli)

(particular choice of ~~CY~~ CY topology)

twisted sector field vev $\cong$ resolution

— Het orbifold

particular limit of vector bdle moduli.

— IIB D3-brane at an orbifold singularity (fractional brane)

e.g. D3-brane at $\mathbb{C}^3/\mathbb{Z}_3$ orbifold singl.

3-types. (orbifold conditions)

interpretation

$\mathbb{C}^3/\mathbb{Z}_3$ blow-up $\Rightarrow$ $\mathbb{P}^2$ appear
$\quad\cup$
$\quad\mathbb{P}^1$.

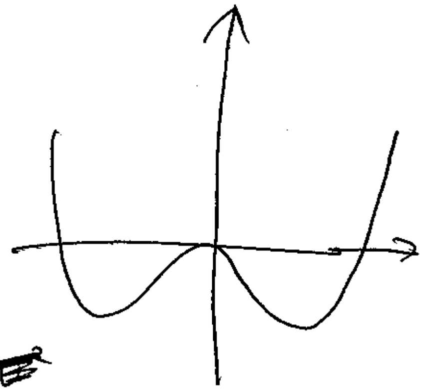
$$\text{3-types} \Rightarrow \begin{cases} D7 + \overline{D5} + \frac{1}{2} D3 \\ 2\,\overline{D7} + D5 + \frac{1}{2} D3 \\ D7 \end{cases}$$

stable config.

Higgs boson mass

In the SM

$$V(\phi) = -\mu^2 (\phi^\dagger \phi) + \frac{\lambda}{2} (\phi^\dagger \phi)^2.$$



- $\langle \phi \rangle^2 = \mu^2/\lambda \equiv v^2 = (174\ \text{GeV})^2$ = known.
- $m_h^2 = 2\lambda v^2$

λ ↳ not known. any value

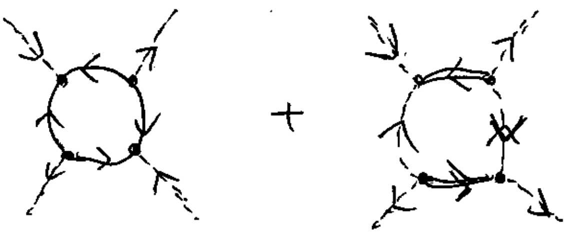
In SUSY SM

$$V_{D\text{-term}} = \frac{g_L^2 + g_Y^2}{8} \left(|\phi_u|^2 - |\phi_d|^2 \right)^2.$$

$$\lambda \simeq \frac{g_L^2 + g_Y^2}{4} \Rightarrow m_{h^0}^2 \simeq m_Z^2$$ 91 GeV

very light!?

$$V_{\cancel{SUSY}\ \text{1-loop}} = \frac{3}{8\pi^2} \lambda_t^4 \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) |\phi_u|^4$$



$$m_{h^0}^2 \simeq m_Z^2 + \frac{3}{4\pi^2} \lambda_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right).$$

LHC

m_h^{SM} [130 GeV ~ 600 GeV] excluded!!

remaining $m_h \lesssim 130$ GeV light Higgs boson.

- SM w/ accidentally light Higgs?
- SUSY w/ $M_{\cancel{SUSY}} \approx$ TeV?
- something else?

Low-energy SUSY in string theory?

$$\frac{1}{2\pi\alpha'}\int d^2\sigma \; G_{MN}(X)(\partial X^M)(\bar\partial X^N) + \cdots$$

$\beta^{(1\text{-loop})}_{G_{MN}} \propto R_{MN}.$ $\Leftarrow$ [not about genous expansion. but non-lin. σ-model perturbation].

- Ricci-flat manifold.
- Ricci-flat Kähler manifold ($\Rightarrow C_1 = 0$. CY) $\Rightarrow$ low-energy SUSY.

beyond 1-loop ... ??

$\beta = 0$ on world sheet $\Longleftrightarrow$ equation of motion on target space.

if ... w/ low-energy SUSY

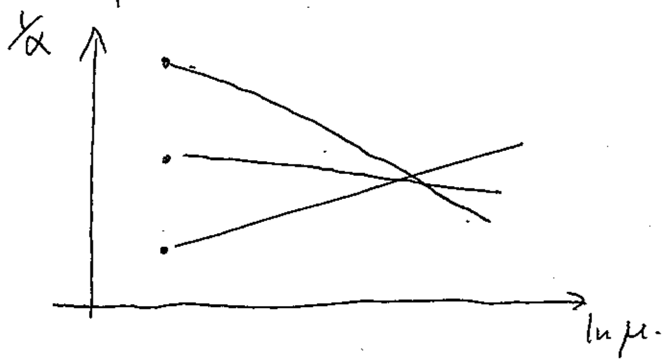
equation of motion $\longrightarrow$ BPS condition.

non-renormalization theorem

(no α' correction in superpotential)

SUSY GUT

Unification

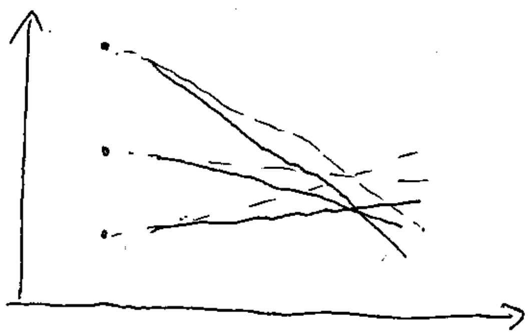


$$\frac{\partial(1/\alpha_a)}{\partial \ln\mu} = \frac{b_a}{2\pi} + \mathcal{O}(\alpha).$$

↑ 1-loop.

assume MSSM spectrum $\begin{cases} M_{GUT} \simeq 10^{16}\,\mathrm{GeV} \\ 1/\alpha_{GUT} \simeq 24 \sim 25 \end{cases}$

w/ extramatter .... Δb_a : same.



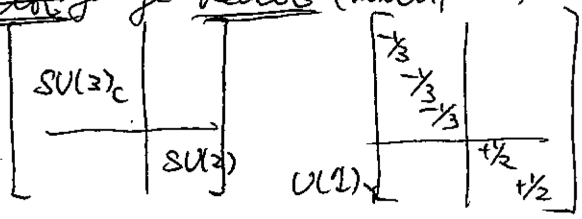
$\begin{cases} M_{GUT} : \text{same.} \\ 1/\alpha_{GUT} \Rightarrow \text{smaller.} \end{cases}$

(e.g. in gauge mediation)

New particles at GUT scale. [D-T splitting problem]

SU(5) (Georgi-Glashow)

~~vector~~ ~~gauge~~ vector (multiplet)



in SU(5).

off-diagonal : Higgsed. at the GUT scale. (symmetry breaking).

chiral matter

$$\left[\begin{array}{c|c} \varepsilon^{abc}(U^c)_c & Q_L^a \\ \hline & \varepsilon^{ij} e^c \end{array}\right] = \wedge^2 5 = 10. \qquad \left[\begin{array}{c} (d^c)_a \\ \hline (L_L)_i \end{array}\right] = \bar{5}$$

extra matter $\Leftarrow$ $\left[\begin{array}{c} \\ \hline H_u \end{array}\right] \subset H(5)$ $\quad \left[\begin{array}{c} \\ \hline \bar{H}_d \end{array}\right] \subset \bar{H}(\bar{5})$

(colored Higgs)

Doublet – Triplet splitting problem

colored Higgs mass.

$$\begin{bmatrix} H_c \\ H_u \end{bmatrix} = \begin{bmatrix} \bar{H}_{\bar{c}} \\ H_d \end{bmatrix}$$ fine tuning?

massless ??

w/o fine tuning

☆ ⇒ no $H(5) - \bar{H}(\bar{5})$ massterm.

⟹ other mass partner. $\begin{bmatrix} H_c \\ H_u \end{bmatrix} — R'_{epr.}$

$\langle \cancel{SU(5)} \rangle$. containg $SU(3)$-$\bar{3}$ ~~break~~

⟹. $\begin{bmatrix} H_c \\ H_u \end{bmatrix} = \begin{bmatrix} \bar{3} \\ \bar{2} \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} \bar{3} \\ \bar{2} \end{bmatrix} = \cdots$

[require infinite copies of $H(5)$ & $\bar{H}(\bar{5})$]

symmetry breaking in geometry

In. Het $E_8 \times E_8$ language... (mod string duality ...)

E_8-adj (248) ⟹ $(\text{adj}, 1) + (1, \text{adj.}) + [(5, \wedge^2 5) + (\wedge^2 5, \bar{5})]$ + h.c.

$E_8 \times \cancel{E_8} \supset SU(5)_{STr} \times SU(5)_{GUT}$

gaugino in $(\wedge^2 5, \bar{5})$ ⇛ $\psi(y)\lambda(x)$ separation of variables.

$\not{D}\psi = 0$ on CY_3 ⇛ λ massless on 4D.

$SU(5)_{STr} - (\wedge^2 5)$ repr. bdle.

⇓

$[SU(5)_{STr} \times U(1)_Y] - (\wedge^2 5) \otimes \left(L_Y^{1/3} \text{ or } L_Y^{-1/2}\right)$

differant bg ⇛ differant spectrum

$\boxed{\text{KK-scale} = \text{GUT scale} < \text{string scale} < \text{Planck scale.}}$

geometric phase

use the known value of $\left(M_{pl}.\ \alpha_{GUT},\ M_{GUT}=M_{KK}\right)$

Het $\quad \dfrac{\left(1/2\pi\sqrt{\alpha'}\right)}{M_{KK}} \sim 6.$

IIB/F $\quad \left[\dfrac{vol\,(7\text{-brane})}{g_s(2\pi\sqrt{\alpha'})^4}\right] \sim \left[\dfrac{1}{\alpha_{GUT}} \sim 24\right]$

$$\begin{cases} L_Y : \text{ flat bdle (connection)} & \text{w/. } \pi_1 \not\equiv \{1\} \\ & \text{Wilson line.} \\ L_Y : \ c_1(L_Y) \neq 0. & \end{cases}$$

different physics

flavor str. and neutrino

quark sector $\quad \Delta W = \lambda^{(u)}_{ij} Q_i U^c_j H_u + \lambda^{(d)}_{ij} Q_i D^c_j H_d$

lepton sector $\quad \Delta W = \lambda^{(e)}_{ij} L_i E^c_j H_d + \begin{cases} \lambda^{(\nu)}_{ij} L_i N^c_j H_u \\ \dfrac{\kappa^{(\nu)}_{ij}}{M} L_i L_j H_u H_u \end{cases}$

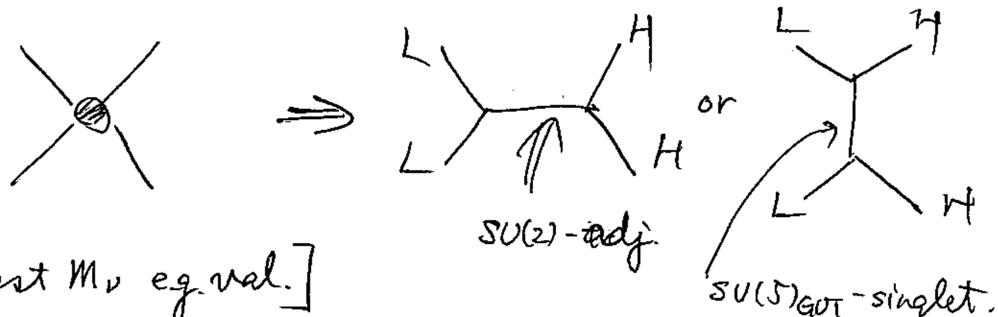
- $\lambda^{(u)}_t$ larger than expected.
- ν-oscillation.

Dirac $\nu \to \lambda^{(\nu)}_{ij} L_i N^c_j H_u$, $\lambda^{(\nu)} \lesssim 10^{-12}$.

Majorana $\nu \to \dfrac{\kappa^{(\nu)}_{ij}}{M} L_i L_j H_u H_u$

$\left[\begin{array}{l} \Delta m^2_{atm} \simeq (2\text{-}3) \times 10^{-3}\,eV^2. \\ \quad \text{larger than expected.} \\ \text{mixing angles!!} \\ \text{large.} \end{array}\right.$ $\quad [\theta_{\mu\tau}\ (\text{atm}).\ \ \theta_{e\mu}\ (\text{solar})\ \&^{\text{now}}\ \underline{\underline{\theta_{e\tau}}}]$

$\dfrac{\kappa}{M} LLHH$



$\sqrt{\Delta m^2} \le$ [largest M_ν eg. val.]

$\Downarrow$

κ/M bounded below

$\to M$ upper bound.

below 10^{15} GeV.

M: mass of right-handed ν. (see-saw mechanism)

something has to happen (e.g. RH ν appear)

below $\underline{M_{GUT}}$.

<u>mass eigenvalues & mixing angles</u>

$$\lambda^{(u)}_{ij} Q_i U^c_j H_u \quad + \quad \lambda^{(d)}_{ij} Q_i D^c_j H_d .$$

$$\left\{ \begin{array}{l} V^{(q;u)T} [\lambda^{(u)}_{ij}] \, V^{(u;u)} = (\text{diag}) \\ V^{(q;d)T} [\lambda^{(d)}_{ij}] \, V^{(d;d)} = (\text{diag}) \end{array} \right. \longrightarrow \text{eg. values.}$$

$$\boxed{V_{CKM} = V^{(q.u)\dagger} \, V^{(q.d)}}$$

$\Uparrow$

sensitive only to hierarchy among q's.

generation hierarchy contribution from both (q, u^c), (q, d^c)

- $\lambda^{*}_u / \lambda^{*}_t \ll \lambda_d / \lambda_b$. stronger hierarchy in u^c than d^c
- small angle mixing V_{CKM} strong hierarchy in q.

$$\lambda^{(e)}_{ij} L_i E^c_j H_d + \frac{\kappa_{ij}}{M} L_i L_j H_u H_u .$$

~~Large m.~~ V_{PMNS} (lepton sector) $= V^{(l;e)\dagger} V^{(l;\nu)}$

sensitive to L alone. not to E^c

$\Downarrow$ large mixing.

✓ no (or weak) hierarchy among L

✓ hierarchy in E^c.

$(Q, U^c, E^c) = \wedge^2 5 = 10$ hierarchical	v.s.	$(d^c, L) = \bar{5}$ no (or weak) hierarchy

$$W = \lambda^{(u)}_{ij}\, 10^{ab}_i\, 10^{cd}_j\, H(5)^e\, \varepsilon_{abcde} + \lambda^{(d,e)}_{ij}\, (\bar{5}_i)_a\, (10_j)^{ab}\, \bar{H}(\bar{5})_b$$

$$+ \frac{K_{ij}}{M} \left[\bar{5}_i H(5)\right]\left[\bar{5}_j H(5)\right].$$

- RH ν mass scale... below 10^{15} GeV how?
- origin of hierarchical 10's vs anarchy of $\bar{5}$
- How is 10-10-H(5) Yukawa generated?

$\left\{\begin{array}{l} SO(10) \text{ GUT.} \qquad \text{all combined into } (16 = \text{spinor}) \\ \text{Pati-Salam } (SU(4)_C \times SU(2)_L \times SU(2)_R) \end{array}\right.$

$\hookrightarrow (Q, L) : (4 \otimes 2 \otimes 1)$

flipped $SU(5)$

$10 = (Q, D^c, N^c)$, $\quad \bar{5} = (U^c, L)$, $\quad E^c$

$\left\{\begin{array}{l} \text{e.g. value hierarchy pattern. } \left(\begin{array}{l}3 \text{ data} \\ u.d.e\end{array}\right) \\ \text{mixing angles } (2 \text{ data } q.\ell) \end{array}\right.$

5 pattern cannot be explained by

3 free choice of hierarchy for

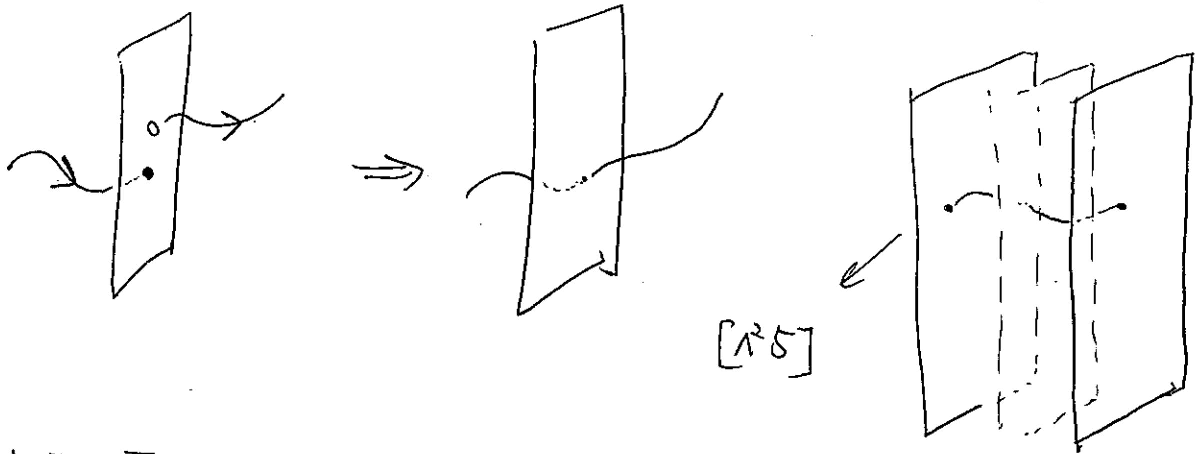
$10, \bar{5}, E^c$.

Day 3

Het and M / IIA

How to generate $\Delta W = \lambda^{(u)}_{ij}\, 10^{ab}_i\, 10^{cd}_j\, H(5)^e\, \varepsilon_{abcde}$?

$+\ \lambda^{(d,e)}_{ij} (\bar{5}_i)_a (10_j)^{ab} (\bar{H}(\bar{5}))_b$



[$\wedge^2 5$]

Het $E_8 \times E_8$

consider vector bdle bg. in $SU(5)_{str}$ on $X = CY_3$ V (rk=5)

[$E_8 \supset SU(5)_{str} \times SU(5)_{GUT}$]

E_8-adj. $\Rightarrow$ $(\text{adj.}, 1) + (1, \text{adj.}) + [(5, \wedge^2\bar{5}) + (\wedge^2 5, \bar{5})]$ + h.c.

RH ν $SU(5)_{GUT}$-vector 10-matter $\bar{5}$-matter, $\bar{H}(\bar{5})$ $H(5)$

$$\left[\ W \propto \int_{CY_3} \mathrm{Tr}_{E_8}\left(A dA + \tfrac{2}{3} AAA\right) \wedge \Omega\ \right]$$

$H^1(X; V)$
$H^1(X; \wedge^2 V)$

$\Uparrow$

$A_M \Rightarrow (A_\mu, A_\alpha, A_{\bar{\alpha}})$ dz^α + $d\bar{z}^{\bar{\alpha}}$

Gaugino the same eq.

$\Rightarrow$ Yukawa $\int [(0,1)\text{-form}]^{\times 3} \wedge \Omega^{(3,0)}$

$$\int d^{10}x \left[\bar{\Psi} D_M \Gamma^M \Psi\right] \Rightarrow \left(\bar{\Psi}\, [A_M \Gamma^M, \Psi]\right)$$

Lie alg. of E_8.

$SO(6)$ spinor $4 + \bar{4}$

$SU(3)$ $(3+1)$ $\boxed{(\bar{3}+1)}$

$$\boxed{(T^*X) \otimes (\det TX)^{1/2} + (\det TX)^{-1/2}}$$

to digress

$E_8 \supset SU(3) \times E_6$.

$\Rightarrow (adj.,1) + (1,adj.) + (3,27) + h.c.$

$H^1(X;T^*X) = H^{1,1}(X)$

$H^1(X;TX) \simeq H^{2,1}(X)$

if spin conn embeded.

→ Yukawa.

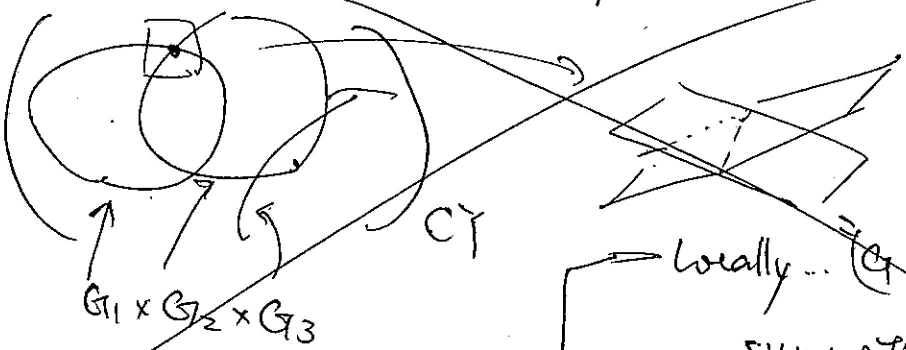
$\Rightarrow \underset{\text{divisor.}}{H^{1,1}(X)} \times H^{1,1}(X;\mathbb{R}) \times H^{1,1}(X;\mathbb{R}) \Rightarrow$ intersection #

• Question 1. Is E_8 nesecessary?

duality w/ M & F. (~~type II~~)

$\Longrightarrow$ more flexible. choice of gauge group. (brane-config)

• Observation 2 absence of overall GUT



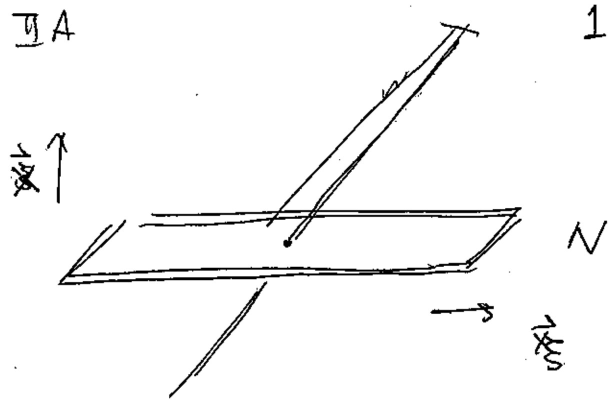
not here

Locally... "$(G_1 \times G_2)$" $\longrightarrow G_1 \times G_2$.

symmetry breaking picture.

but globally... ??

IIA / CY_3 w/ D6. $\overline{D6}$. & M / G_2-hol mfd.



$\boxed{\text{D6 on s.Lag 3-cycle of } CY_3 \Rightarrow \mathcal{N}=1 \text{ SUSY}}$

two 2-cycles
$\Rightarrow$ # chiral − # anti-chiral
= intersection #

$\overset{N}{D6} + \overset{1}{\overline{D6}}$. in $\mathbb{C}^3 \times \mathbb{R}^{3,1}$

$\uparrow \quad \uparrow \qquad 3 + 3+1$.

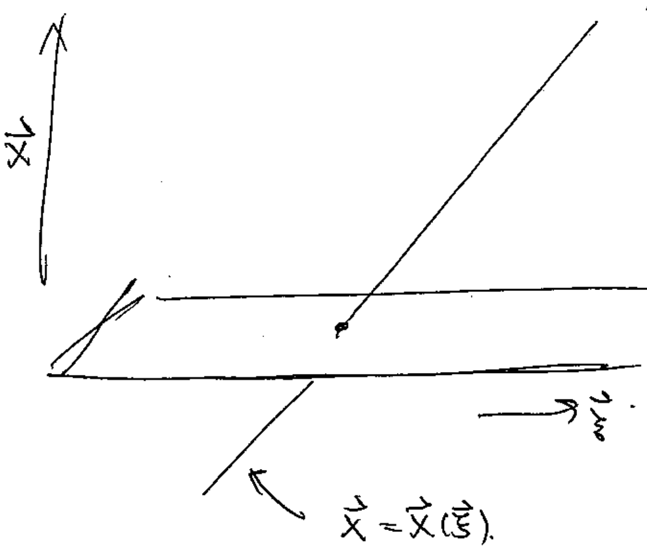
$\vec{X} = \vec{0} \qquad \vec{X} = \vec{X}(\vec{\xi})$.

linear. $\vec{X}(\vec{\xi})$

$\Downarrow$

$\boxed{\text{1 chiral multiplet (or anti chiral)}}$ dep. on $\mathrm{sgn}\left|\frac{\partial \vec{X}}{\partial \vec{\xi}}\right|$.

Berkooz Douglas Leigh.



IIA $\Rightarrow$ M

$$ds^2 = U^{-1}(\vec{X}(\vec{\xi}))(d\tau - \omega_i dX^i)^2 + U(\vec{X}(\vec{\xi}))\, dX^i dX^i$$

$$U(\vec{X}) = 1 + \frac{1}{|\vec{X} - \vec{X}(\vec{\xi})|} + \frac{N}{|\vec{X}|}$$

$(\vec{X}(\vec{\xi}) - \vec{0})$ change as $\vec{\xi}$.

(chiral) should be 1

$\boxed{\text{explain het-M duality}}$

G₂-holonomy mfd

locally… ALE fibration over real 3-cycle. Q.

$$SO(3) \times [SO(4) = SU(2)_L \times SU(2)_R]$$

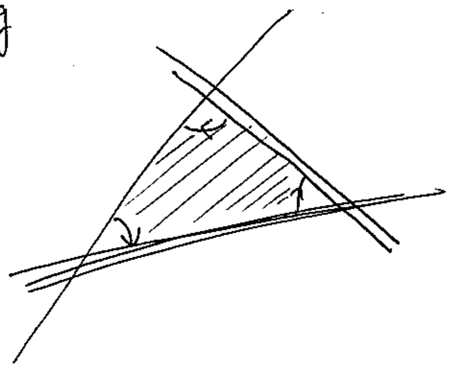
of SU(2) spin conn in SO(3) & $SU(2)_L$

32-SUSY $\Rightarrow$ 4-SUSY
charge.
8 charge?
SO(7) spinor

$$2 \otimes (2, 1) \oplus 2 \otimes (1, 2) \Rightarrow (\underline{1} \oplus \underline{3}) \oplus \underline{2} \times 2$$

Yukawa coupling

eg

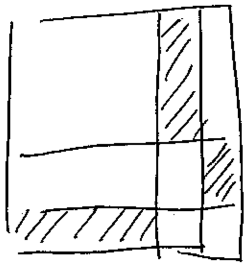


$^3D6 + {}^2D6 + {}^1D6$.

$\lambda \propto e^{-(\text{Area}/\alpha')}$ → not bad ~~for~~ in generating hierarchical Yukawa of SUSY SM

~~world sheet disc~~ stretched disc amplitude.

Yukawa generated because.



$U(6) \to U(3)\times U(2)\times U(1)$ symmetry breaking.

$\mathrm{Tr}(A[A,A]) \neq 0$.

M-theory ... $\mathbb{Z}(\text{2-cycles}) = 0$ topologically $\Leftrightarrow$ Lie alg. correspondig to ADE ~~type of ALE~~

hope to have $\boxed{5\times D6}$ + others for unification. | the same cycle

- A_n or D_n : no contraction w/ ε^{abcde}.
- $E_6 \supset U(2)\times SU(5)_{GUT}$.

$$78 = (3,1) + (1,1) + (1, 24{=}\mathrm{adj}) + (2, \cancel{10}\, \Lambda^2 5) + (\Lambda^2 2, \Lambda^4 5) + (\bar{2}, \Lambda^2\bar{5}) + (\Lambda^2\bar{2}, \underset{\underset{5}{\parallel}}{\Lambda^4\bar{5}})$$

from E_6 algebra.

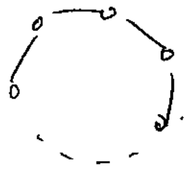
$$\mathrm{tr}\Big((\Lambda^2\bar{2}, \Lambda^4\bar{5})\ [(2,\Lambda^2 5), (2, \Lambda^2 5)]\Big) \neq 0. \Longrightarrow \text{up-type Yukawa.}$$

Douglas Moore hep-th/9603169 5.

ALE space: quiver gauge theory construction.

Kronheimer J. Diff. Geom. ~~299~~ 29 665 ('89) ($N=2$ SUSY)

extended Dynkin diagram



gauge group.

matter

$\Rightarrow$ impose D-term & F-term

$\Rightarrow$ moduli space.

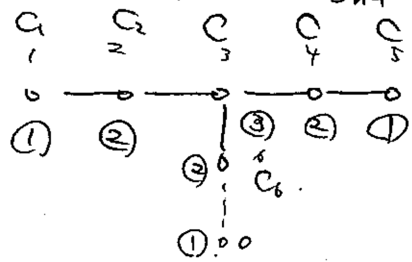
for each node $\vec{\xi}_i$

FI parameter.

(satisfying $n_i \vec{\xi}_i = \vec{0}$)

A_{N-1} : $\vec{\xi}_1 = \vec{X}_1 - \vec{X}_2$, ... $\vec{\xi}_{n-1} = \vec{X}_{n-1} - \vec{X}_n$, $(\vec{\xi}_n = \vec{\xi}_0) = \vec{X}_n - \vec{X}_1$

E_6 :



$\in n_i$ $n_i \vec{\xi}_i = \vec{0}$.

set $\vec{\xi}_{3,4,5,6} = \vec{0}$ everywhere on Q. (a local patch).

where

C_2 shrinks to 0

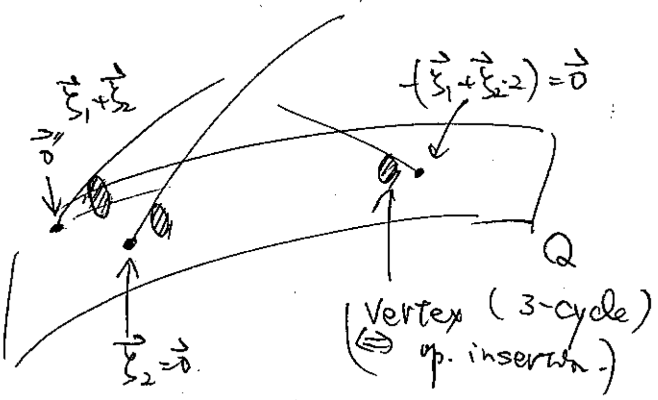
$\vec{\xi}_2 = 0 \Rightarrow$ 10 repr.

$(\vec{\xi}_1 + \vec{\xi}_2) = 0 \Rightarrow$ 10 repr.

$\vec{\xi}_0 = -(\vec{\xi}_1 + 2\vec{\xi}_2) = \vec{0} \Rightarrow$ 5-repr.

C_0 shrinks to 0-size.

$C_0 + \sum_{i=1}^{6} n_i C_i = 0$ $(C_1 + C_2)$ shrinks to 0-size.



$\vec{\xi}_1$, $\vec{\xi}_2$ varying over a local patch of Q.

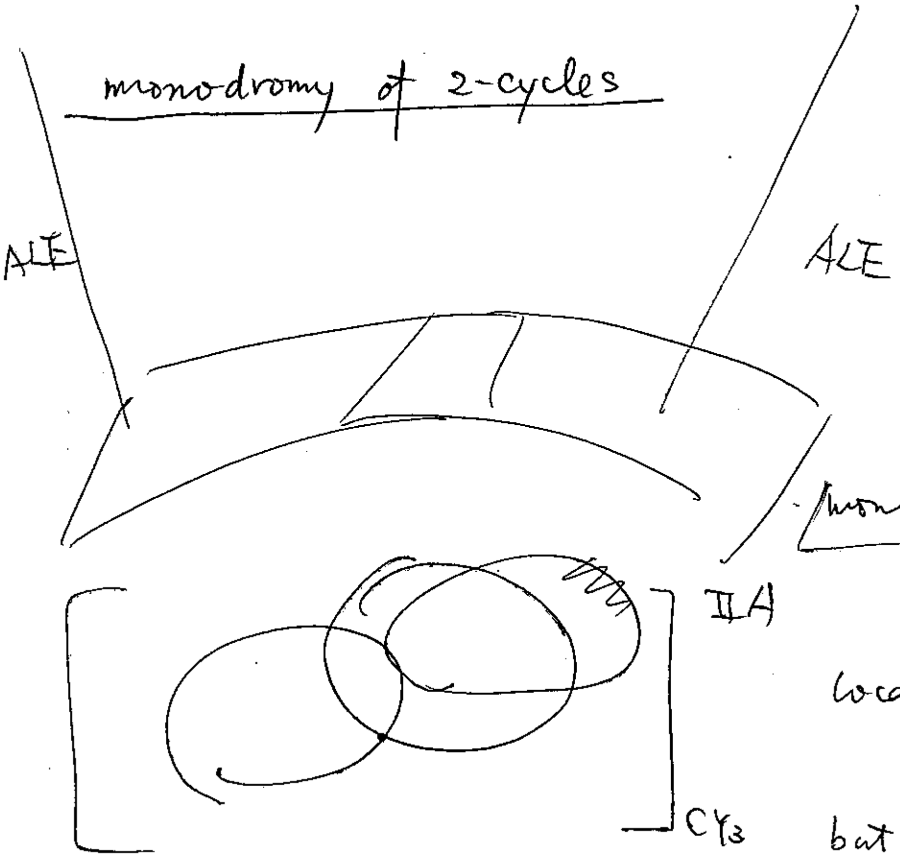
$\lambda \sim e^{-(\mathrm{vol}/\ell_s^3)}$

problem

$$\begin{bmatrix} & A & c \\ A & & b \\ c & b & \end{bmatrix} \cdots \quad ?$$

[Tatar–Watari hep-th/0602238]

monodromy of 2-cycles



glued together

- 2-cycles : gauge-inv

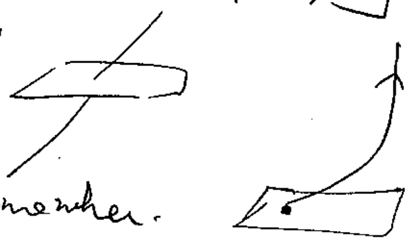
$\boxed{\text{glued mod Weyl reflection.}}$

$\Longleftarrow$

monodromy → diagonal : generated. some cycles go to infinity.

but still small

locally



but somewhere.

Summary

Het : E_8 :

up-type: $H^1(X;V)\times H^1(X;V)\times H^1(X;\Lambda^2 \bar{V})$

$\int \Omega \wedge (AAA)$ hard to calculate.

- toroidal orbifold ... ~~small~~
- how to generalize?

M/G_2. w/ E_6 (locally)

☆ cycles glued together up to Weyl reflc.
no global choice of algebra.

w/ E_6 ... generated

but not in an expected str.

☆ alternative :
D-brane instanton effect.

$\boxed{\text{Day 4}}$ Yukawa coupling in F-theory / IIB

IIB w. D7 . O7 . + bdle on them

$\Longrightarrow$ A-type or D-type algebra.

[use F-theory (E-type algebra)

D-brane instanton]

$\boxed{\text{M-theory}/T^2 \text{ — IIB}/S^1 \text{ duality}}$

$$\left[M/S^1 = \text{IIA on 10D} \right]/S^1 \underset{\text{T-dual}}{=} \text{IIB}/S^1$$

$vol(T^2) \to 0 \quad \Longleftrightarrow \quad$ radius $S^1 \nearrow \infty$.

cpx str. $T^2 \quad \Longleftrightarrow \quad \tau = \left(C^{(0)} + i e^{-\phi}\right)$

M2-branes on 1-cycle $\Longleftrightarrow$ $\left\{ \begin{array}{l} F1 \\ D1 \end{array} \right.$ string

$C^{(3)}$ on 1-cycle $\Longleftrightarrow$ $B^{(2)}$, $C^{(2)}$

$SL(2;\mathbb{Z})$ modular transformation on T^2 $\Longleftrightarrow$ $SL(2;\mathbb{Z})$ transformation of IIB.

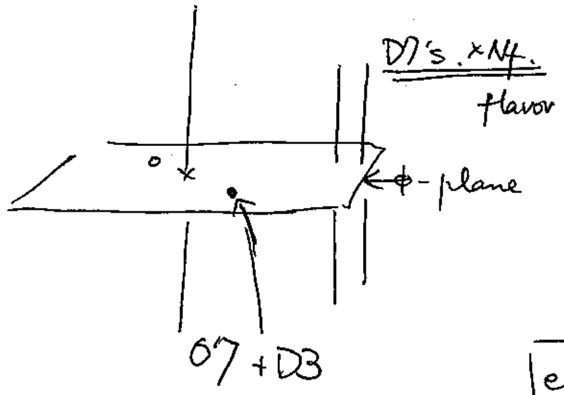
$$\boxed{\begin{array}{ll} \text{M-theory}/\text{elliptic } CY_4 & \text{— "IIB}/B_3 \times S^1\text{"} \\ \pi : X_4 \longrightarrow B_3 & \equiv \text{F-theory} \end{array}}$$

"IIB" but [T^2 (fiber) glued together after $SL(2;\mathbb{Z})$ monodromy transformation]

F1 & D1 string $\quad B^{(2)}$ & $C^{(2)}$ mixed up. on B_3.

elliptic fibered geometry (local picture I)

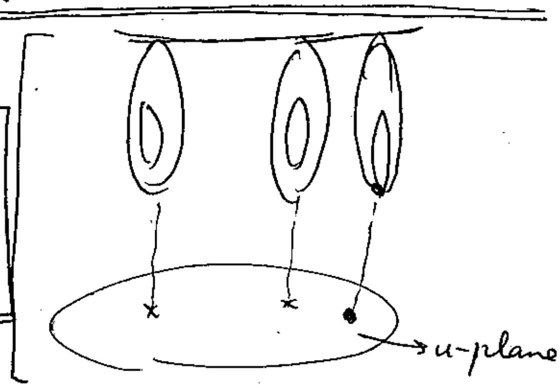
Seiberg-Witten theory $\Longleftrightarrow$ [$N=2$ $D=4$ SUSY $SU(2)$ gauge theory w N_f flavor]



$u = \mathrm{Tr}[\phi^2]$

$$y^2 = x^3 + A(u)\,x^2 + B(u)\,x + C(u).$$

elliptic fibered geometry



~~but~~ electron-massless singularity = D7-brane locus in u-plane. (orientifold picture)

Now ... throw away probe D3 and focus on "flavor brane".

free D3 : either $N=2$ or $N=4$ SUSY — not for pheno

(fract. D3 $\approx$ 7-brane + 5-brane + 3-brane)

elliptic fibered geometry (local picture II)

relation between M-theory & F-theory

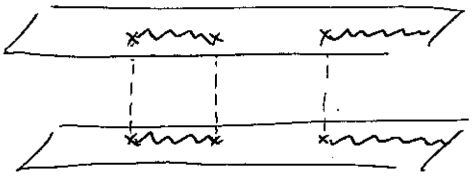
elliptic fibration. ; parametrized by a coordinate z

e.g. $y^2 = x^3 - x^2 + \varepsilon^2 (z^2 + bz + c)$

for given value of z.

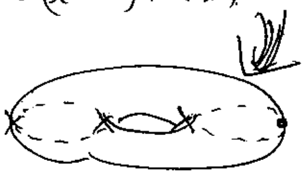
RHS : cubic in x

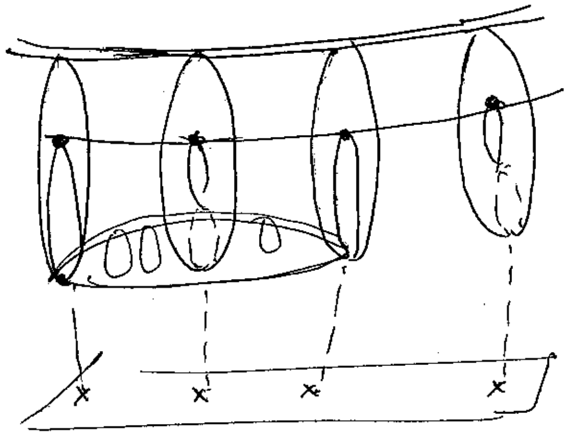
$x \sim (\infty,\ 1,\ \varepsilon\sqrt{\cdots},\ -\varepsilon\sqrt{\cdots})$



x-plane.

$y^2 = (x-e)\cdot(\ast)$





z

Shrinking 1-cycle

$\boxed{D6 \longrightarrow D7}$
(T-dual.)

M2-brane on 2-cycle
$\hookrightarrow$ openstring.

but now...

$H^1(T^2;\mathbb{Z})$. $\langle \alpha, \beta \rangle$

$(p\alpha + q\beta)$ cycle shrinks : $(p\text{-}q)$-7brane
$(p\text{-}q)$-string

7-brane Locus : characterized by $\Delta = 0$.

$y^2 = ax^2 + bx + c \Rightarrow \Delta = (e_+ - e_-)^2 \propto (b^2 - 4ac)$

cubic in $x \Rightarrow \Delta = [(e_1 - e_2)(e_2 - e_3)(e_3 - e_1)]^2$: fn of (coefficients of x.)

$\Delta = 0 \Rightarrow$ 2 pts. in z-plane.

globally ...

$\pi: X_4 \longrightarrow B_3$ elliptic fibration

cpx 3-fold.

$y^2 = x^3 + \quad x^2 + \quad \cdots$

"fn" of 3 coordinates of B_3 $\Rightarrow$ $\boxed{(\Delta = 0) \text{ in } B_3}$

$\|$

7-brane Locus.

$[\Delta = 0 :$ cpx surface.$] \times \mathbb{R}^{3,1}$

How to get Non-Abelian gauge group ...

(back to Local picture)

$N \times D7$ at the same place ...

$$y^2 = x^3 - x^2 + (z^N + z^{N-1} + \cdots + 1) \longrightarrow z^N. \Rightarrow (y^2 = -x^2 + z^N)$$

A_{N-1} singularity.

analogue

D_n-type singularity.

E_n-type singularity

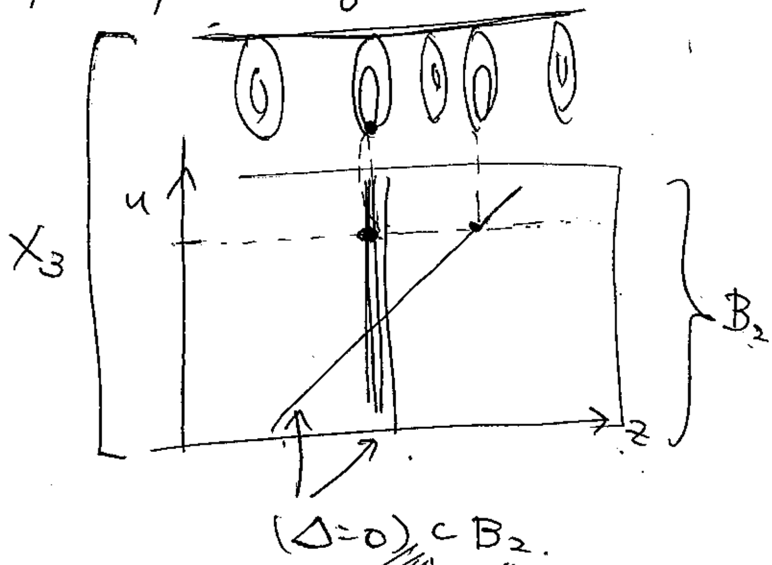
Def. eq. of elliptic fibered CY may reduce locally to

(brane direction)

$|$ A_n-type. D_n-type and E_n-type gauge theory

$\boxed{\text{deformation of singularity} \Longleftrightarrow \text{symmetry breaking}}$

$$y^2 = x^3 - x^2 + z^N(z+u)$$



<u>fixed u slice.</u>

A_N at $u=0$.

deformed (~~by u~~ dep-on u)

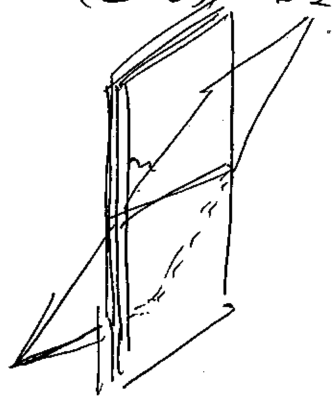
$\downarrow$

A_{N-1} at $u \neq 0$.

$\Updownarrow$

intersecting $D7^{\times N} + D7^{\times 1}$

$\Longleftrightarrow$ $\boxed{\text{localized matter}}$



$\boxed{\text{For } SU(5)_{GUT} \text{ in 4D}}$

elliptic fibration $\pi : X_4 \longrightarrow B_3$.

$(\Delta = 0) \subset B_3$ has an irr. component

local def. eq. $(z.u.v)$ for B_3

$$y^2 = x^3 - A_1 xy + A_2 x^2 - A_3 y + A_4 x + A_6 .$$

$$\begin{cases} A_1 = a_5(u.v) + \mathcal{O}(z) \\ A_2 = \cancel{a_4} z \cdot a_4(u.v) + \mathcal{O}(z^2) \\ A_3 = z^2 a_3(u.v) + \mathcal{O}(z^3) \\ A_4 = z^3 a_2(u.v) + \mathcal{O}(z^4) \\ A_6 = z^5 a_0(u.v) + \mathcal{O}(z^6) \end{cases}$$

$$\Rightarrow \Delta = z^5 \left[a_5^4 \cdot P_{(5)} + \mathcal{O}(z) \right]$$

$$P_{(5)} = (a_0 a_5^2 - a_2 a_5 a_3 + a_4 a_3^2)_{(u.v)}$$

$$(X_4 , G^{(4)})$$
$$\uparrow$$
$$[dC^{(3)} \text{ in M-theory } CY_4.]$$

chiral matter in F-theory

☆ compactification to 6D (hypermultiplets)

$(z, u) \subset$ Base B_2.
z ↓ normal coordinate

$a_5(u) = 0$: $D_5 \to A_4$ deformation [$SO(10) \to SU(5)$]

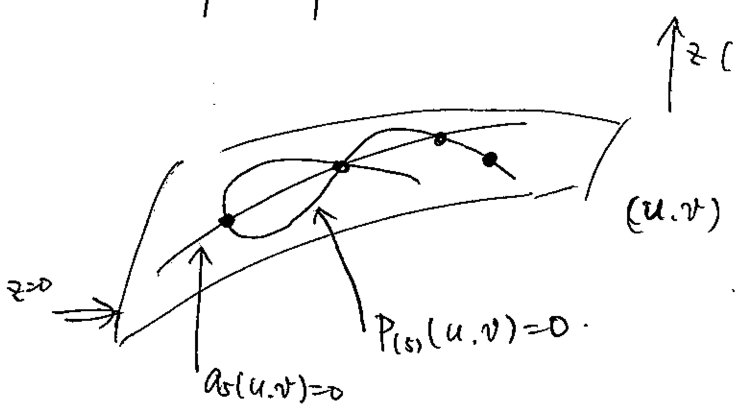
$P_{(5)}(u) = 0$ $A_5 \to A_4$ deformation [$SU(6) \to SU(5)$]

using Het + duality.

$\begin{cases} 10 (+\overline{10}) \text{ repr. hyper at } a_5 = 0 \text{ pt} \\ 5 (+\bar{5}) \text{ repr. hyper at } P_{(5)} = 0 \text{ pt.} \end{cases}$ ⟸ (understood from IIB D7+O7)

non-trivial in general. e.g. $E_8 \to E_7$.
each pt: half hyper of E_7 – 56 repr.

☆ compactification to 4D. (chiral multiplets)



4

<u>chiral matter</u> & <u>Yukawa</u> in F-theory ('08 ~) (flux cpt moduli stab. smooth CY.)

☆ <u>compactification to 6D</u>
<u>1-parameter deformation</u> (hypermultiplet)

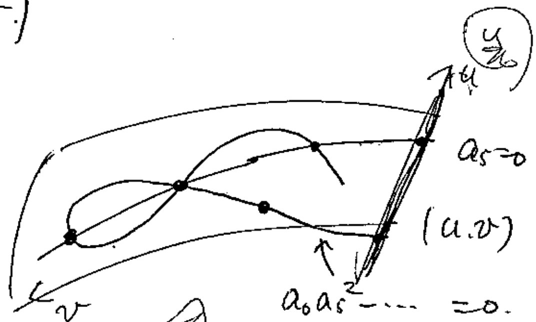
$a_5(u) = 0.$ ($D_5 \to A_4$ deformation)

$a_0 a_5^2 + a_2 a_5 a_3 + a_4 a_3^2 = 0$ ($A_5 \to A_4$ def).

10+$\overline{10}$ ~~repr.~~ hyper   5+$\bar{5}$ hyper -repr

(~~not surprising~~) ⟸ IIB.

determined by <u>using Het-F duality</u> [th/9605200]



non-trivial. $E_8 \to E_7$.
half hyper (E_7-56 rep.)

☆ compactification to 4D.

rk 2 enhancement possible.

[3 types of points : what happens?
along curves : hypermultiplets.

$\boxed{\text{mess.}}$ <u>Complicated behavior</u>. of descriminant locus.

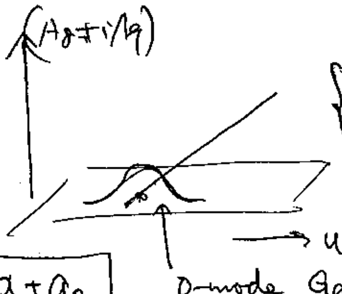
<u>use</u> ~~Het~~ /(elliptic fibration over B_2) — F/(elliptic K3) fibered over B_2) <u>duality</u>

[1 + 2 = 3 CY_3]   [2 + 2 = 4 CY_4]

[Wilson line ($A_8 + iA_9$) in T^2 direction varying over B_2.] ⇒ [cpx str. of elliptic K3 varying over B_2]

$E_8 > SU(5)_{str}$ ⟸ Wilson line

T^2-fiber ⟹ dual torus 5 pts in



$\left\{ \begin{array}{l} (V, 10) \\ (\wedge^2 V, \bar{5}) \end{array} \right.$

[zero locus of] $\boxed{y^2 = x^3 + a_5 xy + a_4 x^2 + a_3 y + a_2 x + a_0}$

$E_8 \supset SU(5)_{\rm str.}$ ← Wilson line.

T^2-fiber ⇒ dual torus ∋ 5 pts ... for ~~rk~~ rk 5 bdle V

$$y^2 = x^3 + a_5 xy + a_4 x^2 + a_3 y + a_2 x + a_0$$

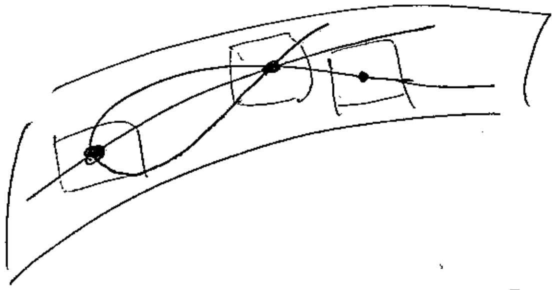
0-locus of this elliptic fn..

[$10\,\cancel{s} = H^1(X; V)$

$\bar{5} = H^1(X; \wedge^2 V)$]

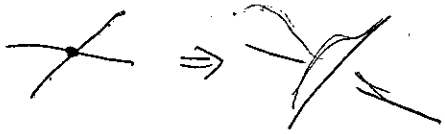
↑ use the same data.

result



10's : hol. ~~smooth~~ 1-component wave fn on $a_5 = 0$ curve.

$\bar{5}$'s : ~~smooth~~ hol. 1-component wave fn. on $a_0 a_5^2 - a_2 a_5 a_3 + a_4 a_3^2 = 0$ curve.



w/. D_6-pts resolved.

no chiral matter components localized at codim 3 singularities.

[mod relation between $G^{(4)}$ & line bdles on the curves.]

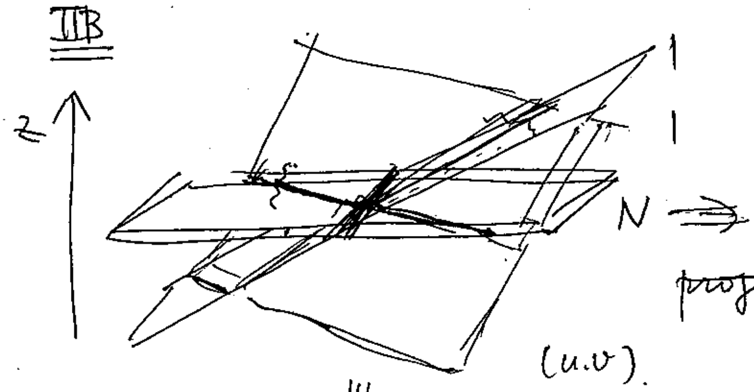
have chirality formula.

$$\begin{cases} \#10 - \#\overline{10} = \int_{4\text{-cycle}} G^{(4)} \\ \#\bar{5} - \#5 = \int_{4\text{-cycle}} G^{(4)}. \end{cases}$$

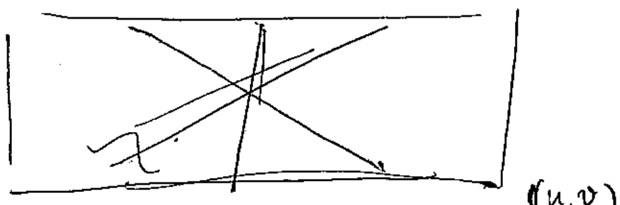
[IIB $\qquad \chi = \int_{\rm curve} \left(\frac{F}{2\pi}\right)$]

How to calculate Yukawa?

IIB



⇓



- open string vertex op.
  using hol. wave fn
  on intersection curve.
  ⇒ val. of hol. wave fn
  at the codim-3 pts.

- Alternative.

SU(N+2) w ⟨Higgs field⟩

$\begin{cases} A_{m=0\sim7}(u.v.x) \\ \Phi \ (\text{cpx}) \ (u.v.x) \end{cases}$

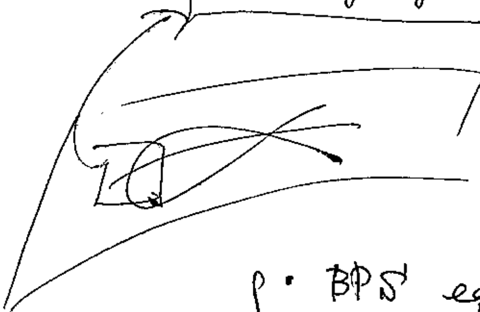
⇐ w. $\langle \Phi \rangle \neq 0$. (u.v)-dep.

matter: [Gaussian in transv.
hol. along. ⇒ overlap integral over (u.v) plane
in SU(N+2) YM theory

⇓ generalize.

E_6 gauge theory w $\langle \Phi, A_m \rangle \neq 0$ in U(2)

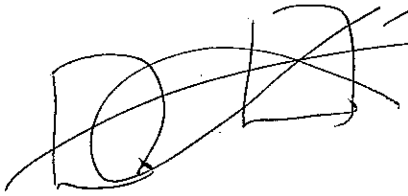
to get Yukawa



D6 gauge theory U(1)×U(1) bg. $\langle \Phi, A_m \rangle \neq 0$.

- BPS eq. for the bg.
- gauge inv. from Φ. ⟺ coeff. of CY4 def. eq.
  (as in Seiberg-Witten. mass parameter)
  not as intuitive. (naive lin. assumption goes wrong)

each sing



$\lambda_{ij} \sim \begin{bmatrix} f_1 & \\ f_2 & \end{bmatrix}$ $f_i g_j$

(f

$\lambda^{(d)}_{ij\cdot A} = f_i(A)\, g_j(B)$ $\boxed{\text{rk } 1}$

$\lambda^{(d)}_{ij} = \sum_A \lambda^{(d)}_{ij\cdot A}$

$\lambda^{(u)}_{ij\cdot A} \simeq f_i(p)\, f_j(p)$ $\Rightarrow \boxed{\text{not rk } 1}$

$\lambda^{(u)}_{ij} = \sum_p \lambda^{(u)}_{ij\cdot p}$

- splitting
- torus (g = 1.)