§1. SUSY GUT.

§1.1. Supersymmetric Standard Model.

$D=4$ $N=1$ SUSY.

☆ Gauge group : $SU(3)_C \times SU(2)_L \times U(1)_Y$ vector multiplet.

☆ Matter Representation.

$$
\left.\begin{array}{ll}
Q & (3, 2)^{1/6} \\
\bar{U} & (\bar{3}, 1)^{-2/3} \\
\bar{D} & (\bar{3}, 1)^{+1/3} \\
L & (1, 2)^{-1/2} \\
\bar{E} & (1, 1)^{+1}
\end{array}\right\} \text{ multiplicity } = 3.
$$

path integral

$\mathcal{D}\tilde{q}$ over $C^{(3)}(\mathbb{R}^{3,1}; (3\otimes 2)\otimes \mathcal{L}_Y^{1/6} \otimes \mathbb{C}^3))$

$\mathcal{D}q$ over $((\quad = \quad)\otimes \mathbb{C}^3)\otimes(\text{spinor})_L$

< basis : chosen arbitrarily from $\mathbb{C}^3$. >

$$
\left.\begin{array}{ll}
H_u & (1, 2)^{+1/2} \\
H_d & (1, 2)^{-1/2}.
\end{array}\right\} \text{ multiplicity } = 1.
$$

(doubled from the Standard Model)

☆ Interactions.

gauge interaction.
$$
\begin{cases}
D_\mu = (\partial_\mu - i g_r \rho^r(t^a) A_\mu^{r,a}) \\
V_{\text{Higgs}} = \dfrac{g_L^2 + g_Y^2}{8}\left(|\phi_u|^2 - |\phi_d|^2\right)^2
\end{cases}
$$

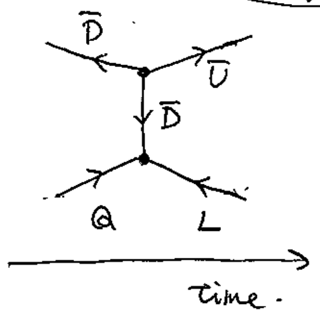
Yukawa interaction

$$
W = \lambda_{ij}^{(u)} (Q_j \bar{U}_i H_u) + \lambda_{kj}^{(d)} (Q_j \bar{D}_k H_d) + \lambda_{kj}^{(e)} (L_j \bar{E}_k H_d)
$$

☆ Remark 1 : Dimension-4 Proton Decay Problem

if there is an interaction

$$\Delta W_{dim 4} = \underbrace{\lambda_{ijk} \bar{D}_i \bar{U}_j \bar{D}_k}_{\Delta B = 1} + \underbrace{\lambda'_{ijk} \bar{D}_i Q_j L_k + \lambda''_{ijk} \bar{E}_j L_i L_k}_{\Delta L = 1}.$$



(supergraph)

$$\Gamma \sim \left(\frac{\lambda\lambda'}{m_{\tilde{q}}^2}\right)^2 m_p^5$$

$(d + u \to \bar{u} + e^+)$

$\Delta B = -1 ; \quad \Delta L = -1$

but acceptable only if

$$\Gamma \lesssim \frac{m_p^5}{(10^{16}\,\mathrm{GeV})^4}$$

i.e.

$$\boxed{(\lambda\lambda') \lesssim \left(\frac{m_{\tilde{q}}}{10^{16}\,\mathrm{GeV}}\right)^2}$$

solutions

- $\mathbb{Z}_2$ symmetry. $(Q, \bar{U}, \bar{D}, L, \bar{E})^-$ $(H_u, H_d)^+$.
  only $\mathbb{Z}_2$ even terms allowed.
- spontaneous R-parity violation at high energy.
  U(1) symmetry with large FI parameter ξ

  $$\Longrightarrow \frac{g^2}{2}\left(|\phi_+|^2 - |\phi_-|^2 - \xi\right)^2 \quad \text{from D-term.}$$

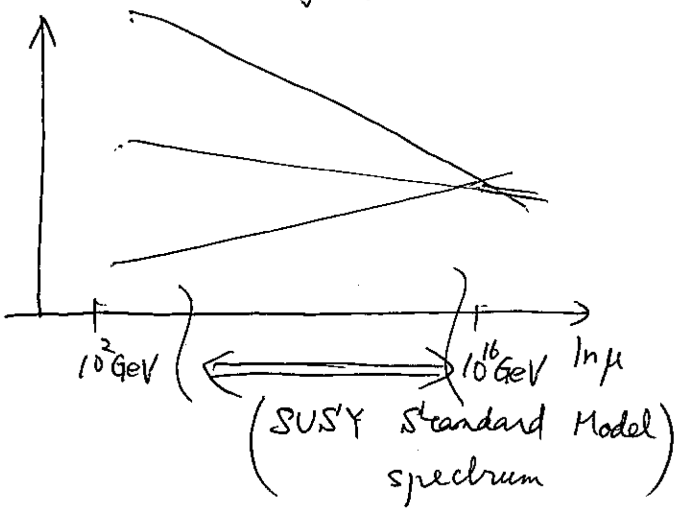
  if. $\langle\phi_+\rangle \neq 0$. $\langle\phi_-\rangle = 0$.

  U(1) spontaneous breaking.

  but $\Delta W_{dim\text{-}4}$ not allowed if $(\bar{D}\bar{U}\bar{D})$ etc. are positively charged.
- continuous U(1) symmetry with
  $\begin{cases} \text{small explicit breaking} \\ \text{spontaneous breaking.} \end{cases}$

§1.2 Unification : various ideas.

RG running of $(1/\alpha_i)$



- Pati – Salam
- Georgi – Glashow $SU(5)$
- flipped $SU(5)$
- $SO(10)$

cf. Weinberg – Salam.

$$\begin{matrix}(\nu_L^0) \\ (e_L^-)\end{matrix} \Rightarrow \begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$$

irr. under $SU(3)_c \times U(1)_{QED}$ $\qquad U(1)_{QED} \subset U(1)_Y \times SU(2)_L$

- Pati – Salam.

$$SU(4)_c \times SU(2)_L \times SU(2)_R$$

$$(\ 4\ ,\ 2\ ,\ 1\ ) = (Q+L)$$

$$+\ (\ \bar{4}\ ,\ 1\ ,\ 2\ ). = \left(\begin{matrix}\bar{U} \\ \bar{D}\end{matrix},\ \begin{matrix}\bar{N} \\ \bar{E}\end{matrix}\right)$$

$$SU(4)_c \supset SU(3)_c \times U(1)_{B-L}.$$

$U(1)_Y$: lin. comb. of $U(1)_{B-L}$ & Cartan of $SU(2)_R$

- Georgi – Glashow $SU(5)$.

gauge group : $SU(5)$.

$$\begin{pmatrix} -1/3 & & & & \\ & -1/3 & & & \\ & & -1/3 & & \\ & & & 1/2 & \\ & & & & 1/2 \end{pmatrix}$$

($U(1)_Y$ gen in $su(5)$.)

matter

$$3\times\begin{cases} \left(10 = \square = \wedge^2 5\right) = \left(\begin{array}{c|c} \bar{U}_{ab} & Q_{ai} \\ \hline -Q_{ai} & \begin{pmatrix} 0 & \bar{E} \\ -\bar{E} & 0 \end{pmatrix} \end{array}\right) \\ \left(\bar{5} = \square = 4\right) = \begin{pmatrix} \bar{D}_{\bar{a}} \\ \hline L_{\bar{i}} \end{pmatrix} \end{cases}$$

Higgs.

$$\begin{cases} H(5) = \begin{pmatrix} H_c \\ H_u \end{pmatrix} \\ \bar{H}(\bar{5}) = \begin{pmatrix} \bar{H}_{\bar{c}} \\ H_d \end{pmatrix} \end{cases}$$

☆ generator normalization.

- $SO(10)$

digression: $U(n)$ and $SO(2n)$.

orthogonal group. $g^T_{2n\times 2n} \cdot g_{2n\times 2n} = \mathbb{1}_{2n\times 2n}$.

consider $\mathbb{J}_{2n\times 2n} = \left(\begin{array}{c|c} & \mathbb{1}_{n\times n} \\ \hline \mathbb{1}_{n\times n} & \end{array}\right)$.

then. $\{g' \in 2n\times 2n \text{ matrix} \mid g'^T \cdot \mathbb{J} \cdot g' = \mathbb{J}\}$ forms a group.

$\because$ $\left[\; \mathbb{P} = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathbb{1} & i \\ \mathbb{1} & -i \end{pmatrix} \qquad \mathbb{P}^T \mathbb{J} \mathbb{P} = \mathbb{1}_{2n\times 2n}. \right.$

$\Rightarrow \quad g \longmapsto g' = \mathbb{P} g \mathbb{P}^{-1}$

$$\left.\begin{array}{l} SU(5)_{GUT} \subset SO(10) \\ SU(8) \subset SO(16) \subset E_8. \\ SU(3) \subset U(3) \subset SO(6). \end{array}\right\}$$

$$g' = \left(\begin{array}{c|c} h & \\ \hline & h^{T-1} \end{array}\right)$$

$h : U(n)$.

$\Rightarrow g'^T \mathbb{J} g' = \mathbb{J}$

gauge group : $SO(10)$

matter : spinor repr $\times 3$

EW higgs : vector repr $\times 1$.

$$\mathrm{Res}^{SO(10)}_{SU(5)}(\text{spinor}) = 1 \oplus \wedge^2 5 \oplus (\wedge^4 5 = \bar{5})$$

$$\mathrm{Res}^{SO(10)}_{SU(5)}(\text{vector}) = 5 \oplus \bar{5}$$

GUT breaking Higgs : $SO(10)$ – spinor + $\overline{\text{spinor}}$
(chiral multiplet)

$$(\underline{\langle 1 \rangle} \oplus \wedge^2 5 \oplus \wedge^4 \bar{5}) \oplus (\underline{\underline{\langle \bar{1} \rangle}} \oplus \wedge^2 \bar{5} \oplus \wedge^4 \bar{5})$$

$$SO(10) \xrightarrow[\text{Higgs vev}]{} SU(5).$$

- flipped $SU(5)$

gauge group : $SU(5)' \times U(1)'_X$

matter $\left[(\bar{E} = 1)^{-5} \oplus (\bar{D}, Q, \bar{N})^{-1} \oplus (\bar{U}, L)^{+3} \right] \times 3$

$\bar{D} \leftrightarrow \bar{U}$
$\bar{E} \longleftrightarrow \bar{N}$ flipped from Georgi – Glashow $SU(5)$

$U(5) \subset SO(10)$ different from (:)

$$t_Y = \frac{-1}{5}(t_{X'} + t_{Y'})$$

$$\frac{1}{g_Y^2} = \frac{1}{25}\left(\frac{1}{g_{X'}^2} + \frac{1}{g_{Y'}^2}\right) \overset{?}{=\!=} \frac{5}{3} \times \left(\frac{1}{g_c^2} \text{ or } \frac{1}{g_L^2}\right)$$

solution : $SU(5)' \times U(1)'_X \subset SO(10)$

$$\Rightarrow \quad \frac{1}{g_{5'}^2} = \frac{1}{g_{10}^2} \quad \text{and} \quad \frac{1}{g_{X'}^2} = \frac{40}{g_{10}^2}$$

$$\frac{1}{g_{Y'}^2} = \frac{5/3}{g_{5'}^2}$$

$$\Rightarrow \frac{1}{g_Y^2} = \frac{1}{25}\left(\frac{40}{g_{10}^2} + \frac{5/3}{g_{10}^2}\right) \Rightarrow \frac{5/3}{g_{10}^2} = 5/3\left(\frac{1}{g_c^2} \text{ or } \frac{1}{g_L^2}\right)$$

matter $\quad {}^{EW}$Higgs $\quad H(5') + H(\bar{5}')$

$$\begin{array}{ccc} H(5') & + & H(\bar{5}') \\ \cup & & \cup \\ H_d & & H_u \end{array}$$

$$10' \cdot 10' \cdot H(5') + \bar{5}' \cdot 10' \cdot H(\bar{5}') + 1' \cdot \bar{5}' \cdot H(5')$$

d-type $\qquad$ u-type. $\qquad$ e-type

GUT breaking Higgs $\quad H(10) + H(\overline{10})$.

$$\left(\begin{array}{c|c} & \\ \hline & \begin{smallmatrix} & v \\ -v & \end{smallmatrix} \end{array}\right) \Rightarrow \begin{array}{c} SU(5)' \times U(1)'_X \\ \cup \\ \underline{\underline{SU(3)_c \times SU(2)_L}} \times U(1)'_Y \end{array}$$

$$U(1)'_Y \times U(1)'_X \Downarrow \underline{\underline{U(1)_Y}}$$

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conventional normalization of Lie alg. generators.

$$2\,\mathrm{tr}\left[\left(\frac{\tau^a}{2}\right)\left(\frac{\tau^b}{2}\right)\right] = \delta^{ab} \qquad \text{for Pauli matrix.}$$

$SU(N)$:

$$2\,\mathrm{tr}_{\Box}(t^a t^b) = \delta^{ab}.$$

$$2\,\mathrm{tr}_{\rho}\left(\rho(t^a)\,\rho(t^b)\right) = (2T_\rho)\,\delta^{ab}$$

In $SU(5)$

$$2\,\mathrm{tr}_{\Box}(t_Y t_Y) = \left(\frac{5}{3}\right) \quad \Rightarrow \quad t_1 = \sqrt{\frac{3}{5}}\, t_Y$$

$$\frac{1}{\alpha_1} = \frac{3/5}{\alpha_Y}$$