§1.3 Flavor Structure (in Supersymmetric interactions)

$$\Delta K = \bar{U}^\dagger \bar{U} + \bar{D}^\dagger \bar{D} + Q^\dagger Q + \bar{E}^\dagger \bar{E} + L^\dagger L + \cdots$$

$$\Delta W = \lambda^{(u)}_{ij}\, \bar{U}_i Q_j H_u + \lambda^{(d)}_{jk}\, Q_j \bar{D}_k H_d + \lambda^{(e)}_{jk}\, \bar{E}_j L_k H_d + \frac{\kappa_{kk'}}{M}(H_u L_k\, H_u L_{k'})$$

hierarchy. $\left[\begin{array}{l} V_{CKM} \\ \text{mixing} \\ \text{angles.}\end{array}\right]$ eigenvalues. $\left[\begin{array}{l} V_{PMNS} \\ \text{mixing} \\ \text{angles}\end{array}\right]$ $\left(\begin{array}{l}\text{hierarchy} \\ \text{not measured} \\ \text{yet}\end{array}\right)$.

☆ "eigen values" and mixing angles.

$$\begin{cases} (P^{(\bar{U})})_{i\hat{i}} \cdot \lambda^{(u)}_{ij} \cdot P^{(Q-u)}_{j\hat{j}} = \delta_{\hat{i}\hat{j}}\, y^{(u)}_{\hat{i}} \\ (P^{(\bar{D})})_{k\hat{k}} \lambda^{(d)}_{kj} \cdot P^{(Q-d)}_{j\hat{j}} = \delta_{\hat{k}\hat{j}}\, y^{(d)}_{\hat{k}} \end{cases}$$ "diagonalize"

$$\underline{V_{CKM} = \cancel{(P^{(Q-d)})^\dagger (P^{(Q-u)})}\ (P^{(Q-u)})^\dagger (P^{(Q-d)})}$$

$$\begin{cases} Q_j \supset (u_L)_j = (P^{(Q-u)})_{j\hat{j}} (\hat{u}_L)_{\hat{j}} \\ Q_j \supset (d_L)_j = (P^{(Q-d)})_{j\hat{j}} (\hat{d}_L)_{\hat{j}} \end{cases}$$ mass-eigen basis.

$$\underline{V_{PMNS} = (P^{(L-e)})^\dagger (P^{(L-\nu)})}$$ (similarly.)

(LFM)

☆ QCD renormalization. and $\tan\beta \equiv \left(\langle H_u\rangle / \langle H_d\rangle\right)$.

$\Rightarrow$ focus on $\begin{cases}\text{relative hierarchy among } \{y^{(u)}_j\text{'s}\},\ \{y^{(d)}_j\text{'s}\},\ \{y^{(e)}_j\text{'s}\} \\ \text{mixing angles. in } (\hat{u}_L - \hat{d}_L) \text{ and } (\hat{e}_L - \hat{\nu}_L)\end{cases}$

neutrino mass eigenvalues : not measured yet.

Yukawa eigenvalues.

- up sector: $\hat{u}$ a few MeV $\hat{c}$ 1.27 GeV $\hat{t}$ 172 GeV
- down sector: $\hat{d}$ $\left(\begin{smallmatrix}\text{a few} \\ \text{-several}\end{smallmatrix}\right)$ MeV $\hat{s}$ 0.1 GeV $\hat{b}$ 4.2 GeV
- charged lepton sector: $\hat{e}$ 0.51 MeV $\hat{\mu}$ 0.105 ~~G~~eV $\hat{\tau}$ 1.8 GeV.

$y^{(d)}/y^{(e)} \sim 3$ on average. (QCD renormalization)

$y^{(u)}_1/y^{(u)}_3 \sim 10^{-5}$ v.s. $y^{(d)}_1/y^{(d)}_3 \sim 10^{-3}$ or $y^{(e)}_1/y^{(e)}_3 \sim 10^{-3.5}$

strong hierarchy. mild hierarchy

mixing angles.

$$V_{CKM} \sim \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 1.0 & 0.04 \\ 0.008 & 0.04 & 0.74< \end{pmatrix}$$ complex phase ignored.

$$\begin{cases} \sin^2(2\theta_{23}) > 0.92 \\ \sin^2(2\theta_{12}) \sim 0.86 \pm 0.02 \\ \sin^2(2\theta_{13}) \sim 0.09 \pm 0.01 \end{cases}$$ lepton sector.

unexpectedly large mixing angles.

$\boxed{\text{Observation}}$

$\left\{\begin{matrix}\text{CKM} \\ \text{PMNS/LFM}\end{matrix}\right\}$ matrix reflects the structure among $\left\{\begin{matrix}(q_L) \\ (\ell_L)\end{matrix}\right\}$.

$\bar{u}, \bar{d}, \bar{e}$: irrelevant.

$\Delta m^2_{atm} \simeq (\cancel{2\sim3}\ 2.4) \times 10^{-3} (eV)^2$; $\Delta m^2_{solar} \simeq \cancel{8.0}\ 7.5 \times 10^{-5} (eV)^2$

an idea: Froggatt Nielsen model.

up-type. $H_u \cdot (\bar{U}_1, \bar{U}_2, \bar{U}_3) \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}$

$\times\, \mathcal{O}(1)$ coefficients.

down-type $H_d\, (\bar{D}_1, \bar{D}_2, \bar{D}_3) \begin{pmatrix} \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}$

assign hierarchy to (Q_1, Q_2, Q_3), $(\bar{U}_1, \bar{U}_2, \bar{U}_3)$. by $3:2:0$

$(\bar{D}_1, \bar{D}_2, \bar{D}_3)$ by $0:0:0$

$\Rightarrow$ "eigenvalues" $y^{(u)}: (\lambda^{3+3}, \lambda^{2+2}, \lambda^{0+0})$ strong hierarchy

$y^{(d)}: (\lambda^{3+0}, \lambda^{2+0}, \lambda^{0+0})$. mild hierarchy.

diagonalization matrices $P^{(Q\text{-}u)} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$

$P^{(Q\text{-}d)}$, $P^{(\bar{U})}$ also like this.

$P^{(\bar{D})} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

$\rightarrow V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$.

$\angle$ determined by the hierarchy assigned to (Q_1, Q_2, Q_3)

A solution

10_i's $= (\bar{U}_i, Q_i, \bar{E}_i)$: hierarchical $\Rightarrow$ up-type hierarchical $\longrightarrow$ small angle

$\bar{5}_i$'s $= (\bar{D}_i, L_i)$: not so much $\Rightarrow$ d/e-type mild. $\longrightarrow$ large angle. ?

consistent with Georgi-Glashow SU(5)

disfavor (PS. SO(10), flipped.)

2 assumptions for 5 patterns. (data).

Q1: where do this come from? What is the meaning of the basis 10_i's.

Q2: In Georgi-Glashow SU(5), how do we account for s/μ ?

§1.4 GUT Breaking (doublet–triplet splitting problem)

Supersymmetric Standard Models.

$$\begin{cases} H_u & \text{but not } (H_c, H_u) = H(5) \\ H_d & \text{but not } (\bar{H}_c, H_d) = \bar{H}(\bar{5}) \end{cases}$$ in the spectrum below the unif. scale.

but ... H_c, $\bar{H}_c$ will appear at the unif. scale...

How can they be massive?

Minimal SU(5) model.

$$\Delta W = \bar{H}(\bar{5}) \cdot \mu_0 H(5) + \kappa \bar{H}(\bar{5}) \cdot \Sigma \cdot H(5).$$

effective mass :
$$\begin{pmatrix} \mu_0+2\kappa v & & & & \\ & \mu_0+2\kappa v & & & \\ & & \mu_0+2\kappa v & & \\ & & & \mu_0-3\kappa v & \\ & & & & \mu_0-3\kappa v \end{pmatrix}$$

if $|\mu_0 - 3\kappa v| \ll |\mu_0|$ or $|\kappa v|$ ... H_u & H_d : light.

fine tuning? any explanation?

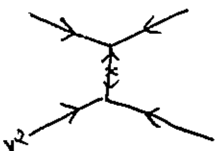
dimension-5 proton decay problem.

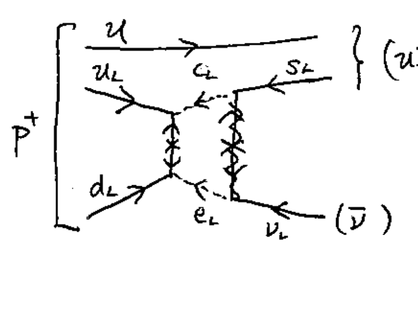
$$\begin{bmatrix} \lambda^{(u)} \; 10^{ab} \cdot 10^{cd} \; H^e(5) \; \varepsilon_{abcde} \\ \lambda^{(d/e)} \; \bar{5}_a \; 10^{ab} \; \bar{H}_b(\bar{5}) \end{bmatrix}$$

$\Rightarrow \begin{cases} (\bar{U}_a \bar{E} H_c^a) & \lambda^{(u)} \\ (Q^a Q^b H_c^c)\,\varepsilon_{abc} & \lambda^{(u)} \end{cases}$

$\Rightarrow \begin{cases} (L\, Q^a \bar{H}_{ca}) & \lambda^{(d/e)} \\ (\bar{D}_a \bar{U}_c \bar{H}_{c,b})\,\varepsilon^{abc} & \lambda^{(d/e)} \end{cases}$

supergraph





$\} \; (u\bar{s}) = K^+$

$(p^+ \to K^+ + \bar{\nu})$

$$\frac{\lambda^2}{M_{H_c}} (QQQL)$$
$$+ \frac{\lambda^2}{M_{H_c}} (\bar{U}\bar{E} \cdot \bar{U}\bar{D})$$

a big problem

$$\boxed{\text{if } \frac{\lambda^2}{M_{H_c}} \sim \frac{1}{\text{GUT scale}}}$$

$\underset{\text{depends on}}{\Rightarrow} \begin{Bmatrix} H_c\text{–matter–matter coupling} \\ \text{mass matrix among Higgs.} \end{Bmatrix}$

An idea

$$-\begin{pmatrix} H_c'' \\ H_u'' \end{pmatrix} - \begin{pmatrix} \bar{H}_c'' \\ H_d'' \end{pmatrix} - \begin{pmatrix} H_c \\ H_u \end{pmatrix} \quad \begin{pmatrix} \bar{H}_c \\ H_d \end{pmatrix} - \begin{pmatrix} H_c' \\ H_u' \end{pmatrix} - \begin{pmatrix} \bar{H}_c''' \\ H_d''' \end{pmatrix} -$$

or

$$\begin{array}{cccccc} & H_c''' & H_c' & \bar{H}_c' & & \bar{H}_c''' \\ H_u''' & H_u'' & H_u & H_d & H_d'' & H_d''' \end{array}$$

with infinitely many Higgs fields

e.g. Kaluza Klein states from Higher Dimensions ($\mathcal{M}$.)
internal mfd.

Two distinct options.

hyper charge flat connection $\mathcal{L}_Y$ on $\mathcal{M}$. $(\pi_1(\mathcal{M}) \neq \{1\})$

$\Leftrightarrow \mathcal{M} = \tilde{\mathcal{M}}/\Gamma$ (Γ : discrete group acting freely on $\tilde{\mathcal{M}}$ simply conn.)

$$P\left[\exp\left(i \oint_\gamma A\right)\right] \neq \mathbb{1}.$$

Wilson line. (SU(5) breaking)

for $\gamma \in \pi_1(\mathcal{M})$

$\Longrightarrow$ vector like $H(5) - \bar{H}(\bar{5})$. different spectrum $\left\{\begin{matrix} \text{triplet} \\ \text{doublet} \end{matrix}\right\}$

(dim. 5 p decay : possibly an issue.)

hyperchange flux $\mathcal{L}_Y$ on $\mathcal{M}$ $\quad (c_1(\mathcal{L}_Y) \neq 0)$.

different net chirality in $\left\{\begin{matrix} \text{triplet} \\ \text{doublet} \end{matrix}\right\}$ in the index formula.

$H_u - H_d$ not vector like.

(note: $\left\{\begin{matrix} \text{discrete Wilson line} \\ \text{continuous Wilson line} \end{matrix}\right\}$ in toroidal orbifold have nothing to do with the flat connection above.