§2 String Compactification and Duality

§2.1 Five Superstring Theories. ($D=10$)

☆ list of massless particles.

	bosonic massless.
Het $E_8\times E_8$.	g_{mn}, B_{mn} ϕ. ; $E_8\times E_8$ A_m
〃 $SO(32)$	〃 〃 〃 ; $SO(32)$ A_m.
Type I	g_{mn} $C^{(2)}_{mn}$ ϕ ; $SO(32)$ A_m
Type IIA	g_{mn} B_{mn} ϕ ; $C^{(3)}_{kmn}$. $C^{(1)}_m$
Type IIB.	g_{mn} B_{mn} ϕ ; $C^{(4)}_{klmn}$ $C^{(2)}_{mn}$ $C^{(0)}$

(Het $E_8\times E_8$, 〃 $SO(32)$, Type I: 16-SUSY charges; Type IIA, Type IIB: 32-SUSY charges.)

---- and infinitely many massive particles.

$$\tilde{F}^{(5)} \equiv dC^{(4)} - \frac{1}{2} C^{(2)}\wedge H^{(3)} + \frac{1}{2} B^{(2)}\wedge F^{(3)}$$

$$\boxed{\tilde{F}^{(5)} = *\tilde{F}^{(5)}}$$

§2.2 Compactification.

★ non-linear σ-model.

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma \sqrt{h}\, h^{ab} G_{MN}(X)(\partial_a X^M)(\partial_b X^N)$$

Is this a CFT?

expand $G_{MN}(X) = \delta_{MN} + \cdots \left(\frac{X}{R}\right)$.

redefine fields. $X^M \Rightarrow \sqrt{\alpha'}\phi^M$ for canonical kin. terms.

$\Rightarrow$ interaction $(\partial\phi^M)(\partial\phi^N)\,\phi \times \left(\frac{\sqrt{\alpha'}}{R}\right)$

perturbation in $\left(\alpha'/R^2\right)$

$\beta = 0$. at 1-loop : Ricci flat. $R^m{}_{kml} \equiv R_{kl} \Leftrightarrow 0$

For a Kähler manifold. $\mathcal{M}$. (complex coordinates $\{z^\alpha\}$)

$$\begin{pmatrix} R^\alpha{}_{\beta\gamma\bar\delta}\, dz^\gamma \wedge d\bar z^{\bar\delta} & \\ & \oplus \quad R^{\bar\alpha}{}_{\bar\beta\gamma\bar\delta}\, dz^\gamma \wedge d\bar z^{\bar\delta} \end{pmatrix} \Leftarrow \text{curvature form.}$$

$$R_{\bar\alpha\beta\gamma\bar\delta} = -K_{\bar\alpha\beta\gamma\bar\delta} + K^{\bar\rho\sigma} K_{\sigma\bar\alpha\bar\delta} K_{\bar\rho\beta\gamma}$$

$\Longrightarrow$ symmetric under $\beta \leftrightarrow \gamma$.

$$\boxed{\begin{array}{ccc} R^\alpha{}_{\beta\alpha\bar\delta} = 0 & \Longleftrightarrow & R^\alpha{}_{\alpha\beta\bar\delta}\, dz^\beta \wedge d\bar z^{\bar\delta} = 0 \\ (\text{Ricci flat}) & & c_1(T\mathcal{M}) = 0. \end{array}}$$

Kähler mfd w/ vanishing $c_1(T\mathcal{M})$
$\Rightarrow$ Calabi–Yau manifold.
(sometimes $b_1 = 0$)

✓ classes of string vacua.

✓ $\beta_{1\text{-loop}} = 0$ enough for CFT? clearly not.
How is Kähler $\mathcal{M}$ special.? (or not special at all?)

✓ How many string vacua are still left out of this category?

§2.3 String – String Duality

* IIA/S^1 — IIB/S^1 T-duality

free scalar on world sheet

mode decomposition

$$X^\mu = x^\mu - i\frac{\alpha'}{2}p^\mu \ln(z\bar{z}) + i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\frac{1}{m}\left(\frac{\alpha_m^\mu}{z^m}+\frac{\tilde{\alpha}_m^\mu}{\bar{z}^m}\right)$$

$$z = e^{-iw} = e^{-i(\sigma_1+i\sigma_2)} \longleftrightarrow e^{-i(\sigma-\tau)}$$

$$\bar{z} = e^{i\bar{w}} = e^{i(\sigma_1-i\sigma_2)} \longleftrightarrow e^{i(\sigma+\tau)}$$

$$\Rightarrow -\frac{i}{2}\ln(|z|^2) \longleftrightarrow \tau$$

S^1 compactification.

- winding mode in $\mathcal{D}X^\mu(\sigma,\tau)$
  $$X^\mu(\sigma+2\pi) = X^\mu(\sigma) + 2\pi R\, w^\mu \qquad w\in\mathbb{Z}.$$
- eigenvalue of canonical conjugate of $x^\mu \Rightarrow \frac{n^\mu}{R}$ $\quad n\in\mathbb{Z}$.

$$X^\mu = x^\mu - i\frac{\alpha'}{2}p_L^\mu \ln(z) + \cdots$$
$$\qquad - i\frac{\alpha'}{2}p_R^\mu \ln(\bar{z})$$

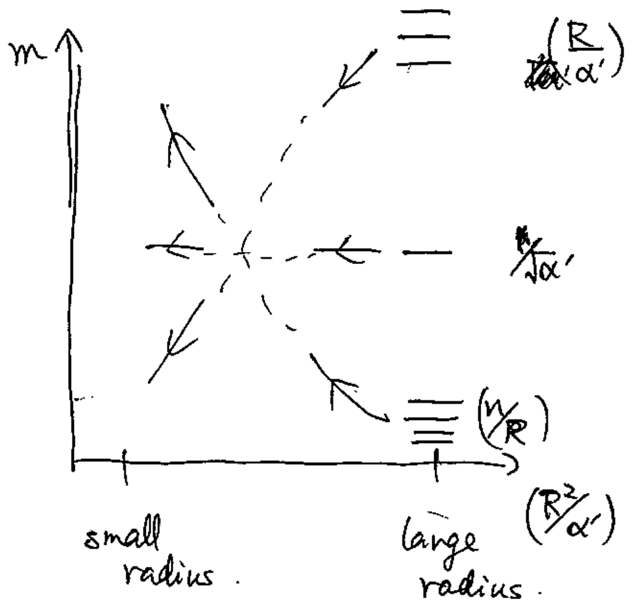
$$\begin{cases} p_L^\mu = \left(\frac{n^\mu}{R} + \frac{R w^\mu}{\alpha'}\right) \\ p_R^\mu = \left(\frac{n^\mu}{R} - \frac{R w^\mu}{\alpha'}\right)\end{cases}$$

$$\boxed{\frac{\alpha'}{2}(p_L^2 - p_R^2) = 2(\tilde{N}-N) \in 2\mathbb{Z}}$$

$$\begin{cases} \frac{\alpha'}{4}m^2 = \frac{\alpha'}{4}p_L^2 + N + \text{const} \leftarrow L_0 = 0 \\ \frac{\alpha'}{4}m^2 = \frac{\alpha'}{4}p_R^2 + \tilde{N} + \text{const} \leftarrow \tilde{L}_0 = 0\end{cases}$$

spectrum : <u>labeled by $n, w \in \mathbb{Z}$.</u>

<u>depends on (α'/R^2)</u> $\qquad \Big(g_s,\ \underbrace{(\alpha'/R^2)}_{\text{moduli}}\Big)$



T^k compactification.

$$\Rightarrow \begin{cases} k \ \text{KK index } n^{\mu_1} \cdots n^{\mu_k}. \\ k \ \text{winding index } w^{\mu_1} \cdots w^{\mu_k}. \end{cases}$$

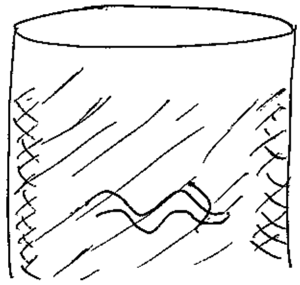
$$\text{vacum} \Longleftrightarrow \begin{pmatrix} \begin{pmatrix} p_L^{\mu} \end{pmatrix} & \cdots & \begin{pmatrix} p_L^{\mu} \end{pmatrix} \\ \begin{pmatrix} p_R^{\mu} \end{pmatrix} & & \begin{pmatrix} p_R^{\mu} \end{pmatrix} \end{pmatrix}_{2k\times 2k \text{ matrix}} \equiv \Lambda$$

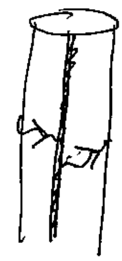
$$\left(\frac{\alpha'}{2}\right) \Lambda^T \begin{pmatrix} \mathbb{1} & \\ & -\mathbb{1} \end{pmatrix} \Lambda = \begin{pmatrix} & \\ & \end{pmatrix}$$

$\Uparrow$ (integral. diagonal $\Rightarrow$ even)

$$\boxed{\text{moduli } ([\Lambda]) = O(k)\times O(k) \backslash O(k,k;\mathbb{R}) / (\text{discrete})}$$

small radius limit





winding mode.

IIB

IIA w/ D8-brane.

$$m = \frac{|n|}{R_{IIB}} \Longleftrightarrow m = \frac{2\pi R_{IIA}}{2\pi\alpha'}|w|$$

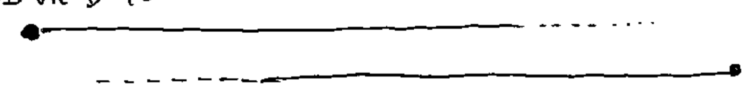
KK mode

dual if $\boxed{\frac{\sqrt{\alpha'}}{R_{IIB}} = \frac{R_{IIA}}{\sqrt{\alpha'}}}$ matching spectrum.

$$\boxed{\frac{1}{g_{s,B}} = \frac{\left(R_A/\sqrt{\alpha'}\right)}{g_{s,A}} \Longleftrightarrow \frac{\left(R_B/\sqrt{\alpha'}\right)}{g_{s,B}} = \frac{1}{g_{s,A}}}$$

$\boxed{\mathrm{IIB}/S^1 = \mathrm{IIA}/S^1}$ single class of theories.

IIB on D=10



$$O(1)\times O(1) \backslash O(1.1;\mathbb{R}) / \text{discrete}.$$

T-duality. transformation.

$$\left(\begin{array}{l} \text{small radius} \longleftrightarrow \text{large radius}. \\ \text{strong coupling} \longleftrightarrow \text{not necessarily strong coupling}. \end{array}\right)$$

☆ Het $E_8\times E_8$ — Het $SO(32)$ duality (S^1 compactification).

Heterotic string $\begin{cases} \text{left mover} & \mathcal{D}X_L^\mu \quad \mathcal{D}H^m_{m=1\sim16}. \\ \text{right mover} & \mathcal{D}X_R^\mu\ \mathcal{D}\Psi^\mu \\ & \mu=0\sim9 \end{cases}$

S^1 compactification.

g_s., (R, 16 Wilson lines.) $\longrightarrow$ 17 parameters. (in sugra picture).

__states__

$$e^{i(k_L^\mu\cdot X_{L\mu}+k_R^\mu\cdot X_{R\mu})}\ e^{ik_L^m\cdot H_L^m\sqrt{\frac{\alpha'}{2}}}$$

$$\Rightarrow \left|\begin{array}{lll} \frac{\alpha'}{4}m^2 = \frac{\alpha'}{4}(k_L^9)^2 + \frac{\alpha'}{4}(k_L^m)^2 + N & & (L_0=0) \\ \quad\ = \frac{\alpha'}{4}(k_R^9)^2. & + \tilde{N} & (\tilde{L}_0=0) \end{array}\right.$$

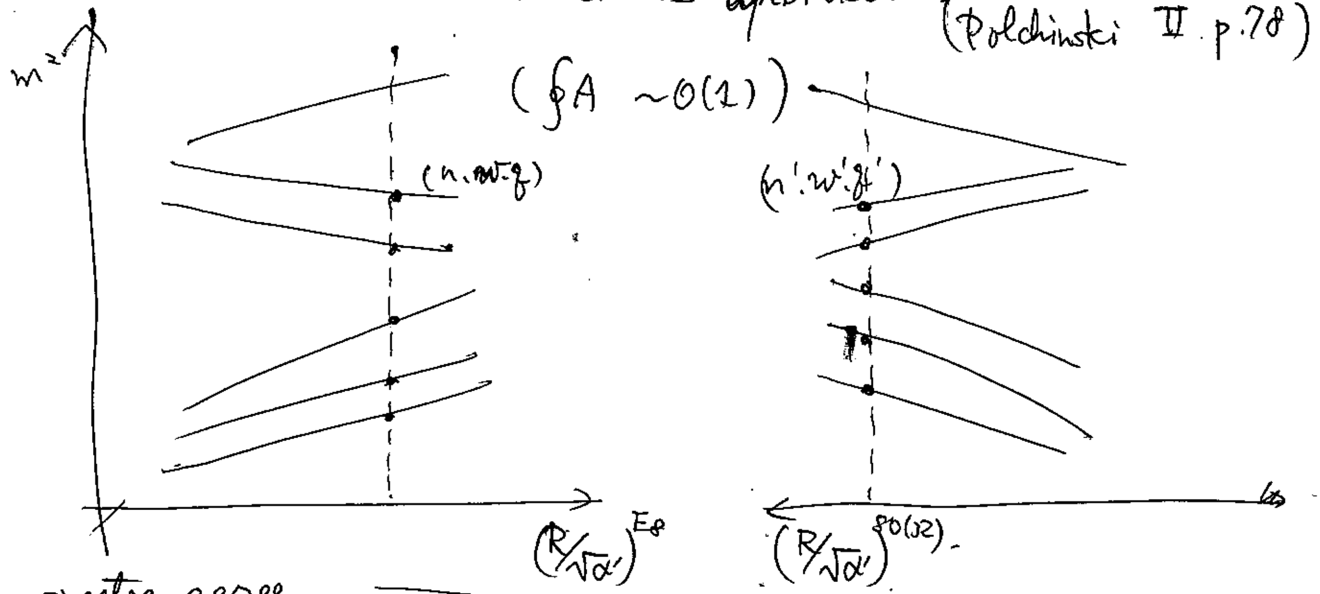
$$\left\{\begin{array}{l} k_L^9 = \frac{n}{R} + \frac{R}{\alpha'}w - g^m A_9^m - \frac{R}{2}w^9 A_9^m A^m{}_9 \\ k_L^m = \sqrt{\frac{2}{\alpha'}}\left(g^m + R A_9^m w^9\right) \\ k_R^9 = \frac{n}{R} - \frac{R}{\alpha'}w - g^m A_9^m - \frac{R}{2}w^9 A_9^m A_9^m \end{array}\right. \qquad \left(\vec{g}\in \begin{array}{l}\text{root lattice of} \\ SO(32) \text{ or } \cancel{E_8\times E_8}\end{array}\right)$$

once again

$$\boxed{\left(\begin{array}{c}\text{moduli}\\ \text{space}\end{array}\right) = O(1)\times O(17) \backslash O(1.17;\mathbb{R}) / (\text{discrete})} \Rightarrow \left(\begin{array}{c} R \,\&\, A_m^I \\ \text{mixed} \\ \text{moduli} \\ \text{space}. \end{array}\right)$$

turn on only $(g_s, R)_{E_8\times E_8}$ and $(g_s; R)_{SO(32)}$ and partially $\langle A_q^m \rangle$

so $SO(16)$ remains unbroken (Polchinski II p.78)



spectra agree

$$\boxed{\text{if} \quad \left(R^{E_8}\sqrt{\frac{2}{\alpha'}}\right) * \left(R^{SO(32)}\sqrt{\frac{2}{\alpha'}}\right) = 1}$$

$$\boxed{\frac{\left(R^{E_8}/\sqrt{\alpha'}\right)}{\left(g_s^{E_8}\right)^2} = \frac{\left(R^{SO(32)}/\sqrt{\alpha'}\right)}{\left(g_s^{SO(32)}\right)^2}}$$