Het$/T^4$ — IIA$/K3$ duality (16-SUSY) K3 : CY_2

Narain compactification.

$$\boxed{\left(O(4)\times O(20) \backslash O(4.20) \Big/ \text{discrete} \right)} \text{ for Het.}$$

On the other hand. in IIA

moduli space of metric on K3

- $\left(SO(3)\times SO(19) \backslash SO(3,19) \Big/ \text{discrete} \right)$.
- $\left\langle \begin{array}{l} h^0(K3) = h^4(K3) = 1 \\ h^2(K3) = 22 \end{array} \right\rangle$.
- B-field on the 22 2-cycles.
- 1 combination from ϕ_{10D} & vol(K3).

$$\boxed{\left(SO(4)\times SO(20) \backslash SO(4.20) \Big/ \text{discr.} \right)} \text{ for IIA}$$

$R^m{}_{nkl}\, dy^k \wedge dy^l$

SU(2) - valued

not SO(4) valued.

$\Downarrow$

$\langle dy^k \omega_k^{\alpha\beta} \Gamma^{[\alpha\beta]} \rangle$.

half non-trivial.

CY_3 : SU(3) - valued.

not SO(6) - valued.

$SO(6)$-spinor $= SU(4) - \underline{\underline{4}}$

$\cup$

$SU(3)$.

$\left(\boxed{\langle\omega\rangle} \right)$ 3/4 non-trivial

1/4 unbroken.

~~dimensional reduction~~. (massless spectrum in 6D eff. theory)

Het side.

$g_{\mu\nu}$ $g_{\mu i}$

$B_{\mu\nu}$ $B_{\mu i}$

ϕ.

4×4 scalars (B_{ij}, g_{ij}).

$16 \times A_\mu$.

16×4 scalar (Wilson line)

$\downarrow$

$\boxed{24 \text{ vector}}$ $= 80$ scalars.

IIA side.

$g_{\mu\nu}$

$B_{\mu\nu}$

ϕ'

$C^{(3)}$ on 22 2-cycles

$\Downarrow$

$22\times$ vector

$SO(4)\times SO(20) \backslash SO(4.20)$ scalars

$C^{(1)}_\mu$.

dual of $C^{(3)}_{\kappa\mu\nu}$

$\hookrightarrow$ 1-form

$\boxed{24 \text{ vectors.}}$

[the same massless spectrum in 6D.]

unbroken (enhanced) symmetry in Het.

there must also be in IIA ?

(in high codim. subspace of $\frac{O(4,20)}{O(4)\times O(20)}$ moduli space)

$\Updownarrow$

(for a special choice of moduli of K3 (ω, B))

Intersection form of 2-cycles. in K3.

$$\begin{pmatrix} \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \oplus \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \oplus \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \oplus (-C_{E_8}) \oplus (-C_{E_8}) \end{pmatrix} \begin{array}{l} 6 \\ 8 \\ 8 \end{array}$$

diagonal: -2.

total 22×22.

$C^{(3)}$ on these 2-cycles $\Rightarrow$ $U(1)$ vector in D=6 eff. theory.

D2-branes wrapped on these 2-cycles $\Rightarrow$ charged under these $U(1)$'s.

$\Rightarrow$ W-boson's of non-Abelian gauge theory.

from SUSY 16-SUSY ~~vector multiplet~~ super YM.

non-Abelian enhanced

when those cycles collapsed to a point (0-size)

- A-D-E singularity (isolated singul. pts in cpx surface)
- ALE space. of A-D-E type.
- multi centered Taub-NUT space of A-D-E type.

A_{N-1} singularity : $y^2 = x^2 + z^N$

D_N : $y^2 = -x^2 z + z^{N-1}$

E_6 : $y^2 = x^3 + z^4$

E_7 : $y^2 = x^3 + x z^3$

E_8 : $y^2 = x^3 + z^5$

§ 2.4 M-theory and Type ~~IIB~~ IIA String Theory

M-theory : something whose L.E. effective theory is D=11 supergravity.

bosonic massless modes (d.o.f.) $\left(g_{mn} \;,\; C^{(3)}_{lmn} \right)$

soliton $\begin{cases} \text{M2-brane} \\ \text{M5-brane} \end{cases}$

• 11D SUGRA / S^1

$$\Longrightarrow \begin{bmatrix} g_{\mu\nu} & g_{\mu,11} \to C^{(1)}_\mu & g_{11\,11} \to \phi \\ & C^{(3)}_{\lambda\mu\nu} & C^{(3)}_{11\,\mu\nu} \to B_{\mu\nu} \end{bmatrix}$$

massless d.o.f. of Type IIA.

• $\begin{cases} \text{M2-branes wrapped on } S^1 \longrightarrow \text{F1} \\ \quad\text{not} \quad " \quad " \quad " \longrightarrow \text{D2} \\ \text{M5-branes wrapped on } S^1 \longrightarrow \text{D4} \\ \quad\text{not} \quad " \quad " \quad " \longrightarrow \text{NS5} \end{cases}$

11D sugra.

$$\boxed{\begin{array}{ll} S = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g}\,R + \cdots & \frac{1}{2\kappa_{11}^2} \equiv \frac{(2\pi)}{\ell_{11}^9} \\ T_{M2} = \frac{2\pi}{\ell_{11}^3} & T_{M5} = \frac{2\pi}{\ell_{11}^6} \end{array}}$$

~~IIB~~ IIA

Dp-brane

$$T_p = \frac{1}{(2\pi)^p(\alpha')^{\frac{p+1}{2}} g}$$

S^1 compactification . radius R_{11}

S^1-KK state mass $= \frac{1}{R_{11}}$ charged under $g_{\mu 11} = C^{(1)}_\mu$ D0

$\hookrightarrow \frac{1}{\sqrt{\alpha'}\, g_{s.\mathrm{IIA}}} = T_0 .$

M2 on S^1

$$\frac{2\pi}{(\ell_{11})^3} \times (2\pi R_{11}) \longrightarrow \frac{1}{2\pi\alpha'}$$

dictionary between $(\ell_{11}, R_{11}) \longleftrightarrow (\alpha', g_{s.\mathrm{IIA}})$

then it follows that ....

M2 ~~on~~ w/o wrapping.

$$\frac{(2\pi)}{(l_{11})^3} \longrightarrow \frac{1}{2\pi\alpha'} \times \frac{1}{2\pi\sqrt{\alpha'}\, g_{s.IIA}} = \frac{1}{(2\pi)^2(\alpha')^{3/2} g_{s.IIA}}$$

$$= T_{D2}$$

M5 wrapping

$$\cancel{\frac{2\pi}{(l_{11})^6}} \cancel{\longrightarrow} \cancel{\frac{1}{(2\pi)}} \times$$

$$\frac{2\pi}{(l_{11})^6} \times (2\pi R_{11}) \longrightarrow \frac{T_{D2} \times T_{F1}}{(2\pi)} = \frac{1}{(2\pi)^4(\alpha')^{5/2} g_{s.IIA}} = T_{D4}$$

M5 w/o wrapping

$$\frac{2\pi}{(l_{11})^6} \longrightarrow \frac{(T_{D2})^2}{2\pi} = \frac{1}{(2\pi)^5(\alpha')^3 g_{s.IIA}^2}$$

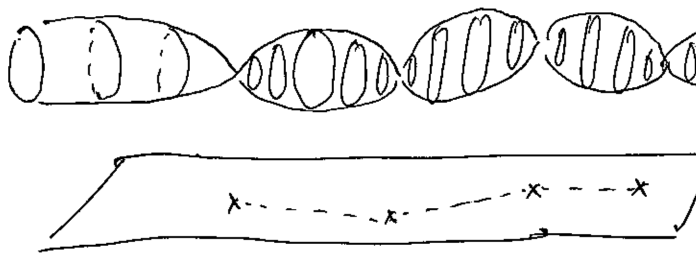
★ multi centered Taub-NUT space for M-theory — D6-brane in IIA

$(\vec{x} \in \mathbb{R}^3,\ \tau \in [0, 4\pi])$ coordinates.
$\quad\quad\quad\quad x_M$

$$ds^2 = U(\vec{x})^{-1}(d\tau + A_i(\vec{x})dx^i)^2 + U(\vec{x})\, dx^i dx^i$$

$$\begin{cases} U(\vec{x}) = 1 + \sum_{I=1}^{N} \dfrac{m}{|\vec{x} - \vec{x}_I|} \\ \vec{\nabla} \times \vec{A} = -\vec{\nabla} U(\vec{x}) \end{cases}$$

smooth if $\vec{x}_I$'s $(I = 1 \sim N)$ are distinct from one another



$\vec{x} \in \mathbb{R}^3$

- large $|\vec{x}|$:
  $U(\vec{x}) \simeq 1.$
  const. radius.
  $M \longrightarrow$ IIA.
- $\vec{x} \sim \vec{x}_I$:
  $U \longrightarrow +\infty$
  $U^{-1} \longrightarrow 0$
  S^1 radius $\Rightarrow 0$.

S^1-fibration over intervals between $\{\vec{x}_I\}$'s

N-center $\Rightarrow$ $(N-1)$-topological 2-cycles.

M-theory $(g_{mn},\ C^{(3)}_{lmn})$ on A_{N-1}-type $(N$-center$)$ Taub-NUT space.

M2-brane on the 2-cycles.

$\Rightarrow$ openstring (F1 string)

end points (center $\vec{x}_I \in \mathbb{R}^3$). D6-branes.

$SU(N)$ gauge theory (A_{N-1})

★ If we take $U(\vec{x}) = \sum_{i=1}^{N} \dfrac{m}{|\vec{x} - \vec{x}_I|}$ instead.

That's $\mathbb{C}^2/\mathbb{Z}_N$ in the limit of $\vec{x}_I \to \vec{0}$.

intersection form

$$\begin{pmatrix} -2 & 1 & & & 0 \\ 1 & -2 & 1 & & \\ & 1 & \ddots & \ddots & \\ & & \ddots & \ddots & 1 \\ 0 & & & 1 & -2 \end{pmatrix} \Rightarrow (-1) \times \text{ Cartan matrix of } A_{N-1}.$$

Atiyah-Hitchin metric $\Rightarrow$ D6-brane + O6-plane.

~~Het~~
Het$/T^3$ — M$/K3$ duality by "rolling up"

in 7D eff. theory $\quad 10-3=7D$ vs $11-4=7D$.
16 SUSY vs $32 \times \frac{1}{2} = 16$ SUSY.

metric 7D. $\quad g_{mn}$

2-form 7D. $\quad B^{Het}$. or C^{M}_{7D}

vector 7D. $\quad \begin{pmatrix} 16 \text{ Cartan} \\ + \\ \text{KK vector} \\ 3^{(T^3)} \times 2^{(g_{xm} \,\&\, B_{xm})} \end{pmatrix}$ or $\begin{pmatrix} C^{(3)}_{m\,ab} \hookrightarrow \text{2-cycles in K3} \\ h^2(K3) = 22 \end{pmatrix}$

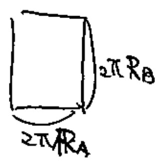
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \| \quad 22$

scalar 7D. $\quad \begin{pmatrix} g_s^{Het} \\ + \\ 3 \times 19 \text{ Narain moduli} \\ \approx 3 \times 16 \text{ Wilson line} \\ + 3 \times 3 \quad g_{ab}, B_{ab} \end{pmatrix}$ or $\begin{pmatrix} \text{metric on K3} \\ SO(3,19)/SO(3) \times SO(19) \\ + \text{ over all volume.} \end{pmatrix}$

unbroken $U(1)^{\times 22}$. @ generic pt in the moduli space.

enhanced sym. @ special pts

$\begin{cases} \text{Het: vanishing Wilson lines} \\ \text{M : collapsed 2-cycles.} \end{cases}$

☆ M-theory$/T^2$ = IIB$/S^1$: verify that T^2  for M $\Rightarrow \frac{1}{g_s} = \frac{R_B}{R_A}$ in IIB.